Notes for students attending the summer school in Barcelona, 2026

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Dear students,

here are some notes that I prepared following the discussions we had after my classes.

Somebody asked me how sure we are that most cosmic rays are Galactic. I suggest to read this paper by Andy Strong where, among other things, he discusses exactly this question.

Strong, A. W., Truths universally acknowledged? Reflections on some common notions in cosmic rays, Nuclear and Particle Physics Proceedings, Volume 297-299, p. 165-170 (2018)

The book I mentioned about self-similar phenomena in hydrodynamics is:

Zel'dovitch and Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena (especially chapter XII).

A gold mine for self similar problems is the famous review:

J. P. Ostriker and C. F. McKee, Astrophysical blastwaves, Rev Mod Phys, 60, 1 (1988)

1 Exercises on dimensional analysis, and with no or very little calculations

To warm up, derive the expansion law for a supernova remnant shock. A supernova remnant shock goes through four phases. The first two phases are adiabatic, and are the free expansion phase, when the mass of the ejecta dominates over the mass of the swept up gas, and the Sedov-Taylor phase, when the swept-up mass dominates (we saw these two cases during the course).

The following two phases are not adiabatic, and they are the snowplough phase and the momentum conserving phase. In the first of these phases the shell of compressed interstellar material[1] cools due to radiative losses, but the interior of the supernova remnant remains adiabatic. In the last phase, both the dense shell and the interior cool due to radiative losses.

The scaling $R_s \propto t^\alpha$ for the four phases gives $\alpha = 1$, $2/5$, $2/7$, and $1/4$, respectively.

I will not provide the solutions of the exercises below, but feel free to send me yours.

1.1 The expansion of interstellar bubbles

A supernova explosion releases in an impulsive event an amount of mechanical energy E. This results in the appearance of a spherical shock expanding in the ambient interstellar medium. If the supernova explodes in an homogeneous and cold medium of mass density ϱ, and if radiative losses can be neglected (i.e. we are in the Sedov/adiabatic phase), the radius of the shock R_s scales with time t as:

$$R_s \sim \left(\frac{E}{\varrho}\right)^{1/5} t^{2/5} \quad . \tag{1}$$

1. Using dimensional analysis, derive the scaling of the shock radius with time assuming that, instead of in an impulsive event, the mechanical energy is released continuously at a rate L (L is a *luminosity* and has the dimensions of an energy per unit time). This is what happens, for example, in stellar winds from massive stars. Analogously to supernova remnants, the wind induces the formation of a spherical shock expanding in the interstellar medium. The mass swept up by the shock is concentrated in a thin shell, and a low density region (an *interstellar bubble*, in astronomical jargon) forms inside the shell.

   To derive the scaling between shock radius and time, assume that:

   - the system is adiabatic;
   - the ambient interstellar medium is homogeneous and is characterised by a mass density ϱ;
   - the shock is strong, i.e., the pressure of the interstellar medium can be neglected;
   - the rate of energy injection L is constant in time, and;
   - the mass ejected by the wind is negligible with respect to the mass of the interstellar medium swept up by the shock.

2. Consider now a system where the shock is radiative but the fluid injected by the wind is adiabatic (interstellar bubbles spend most of their life in

[1] Can you provide a rough estimate of the thickness ΔR of the shell of compressed interstellar gas in terms of the ratio $\Delta R/R_s$, where R_s is the supernova shock radius?

this phase). Show that in this case the slope η of the scaling $R_s = A \times t^\eta$ is identical to that obtained in point 1 above, while the normalization factor A is affected. [Hint: repeat what has been done in point 1 above by substituting L with an effective luminosity $L_{eff} = L - L_{rad}$, where L_{rad} is the energy radiated away from the shock per unit time. For a radiative shock L_{rad} is equal to the flux of mechanical energy flowing across the shock surface.]

3. How does the scaling change if both the shock and the injected fluid are radiative? To answer the question, consider a stellar wind injecting an amount of mass per unit time $\dot{M}$ at a velocity v_w. Both $\dot{M}$ and v_w are constant in time. [Hint: the rate of injection of mechanical energy can be expressed as $L = (1/2)\dot{M}v_w^2$. The total momentum μ_{tot} of the system is not conserved, being the integral in time of the rate of momentum injection into the system: $\mu_{tot} = \int \mathrm{d}t\dot{M}v_w = \dot{M}v_w t$.]

4. How does the scaling change if the system is adiabatic but the energy injection rate is not constant in time, and scales as $L \propto t^a$?

1.2 Detonation waves

Detonation shock waves are explosive phenomena characterised by an increase of the energy of the system with time. This can happen, for example, if the passage of the shock trough the ambient medium induces chemical reactions in the medium itself (due for example to the increase of temperature and density downstream of the shock). If the reactions are esothermic, they will release energy, increasing the total energy of the system.

This phenomenon can be described in a simplified way as follows. Suppose to have an initial point-like explosion releasing an energy E_0 at a specific location in space. An expanding, spherical, and strong shock wave will form, characterized by a radius R_s and a velocity u_s. Assume that the ambient medium is uniform and of density ϱ, and that each gram of the medium swept by the shock releases an energy ϵ (ϵ has then the units of an energy per unit mass). Assume that this energy is released instantaneously at the passage of the shock. In other words, each elementary volume of the ambient medium will release an energy $\epsilon\varrho$ immediately after the passage of the shock. The system is in the adiabatic phase, which means that no energy is radiated away in form of photons.

If the total energy of the system E is much larger than the initial energy E_0, a scale free solution of the problem can be found, in the form $R_s = At^\eta$.

1. Find η and an approximate value for A using dimensional analysis.

2. Find η and an approximate value for A using the alternative method described in the following: find how the energy of the system E scales with the shock radius R_s, and then find η and A by using the well known expression $R_s \sim (E/\varrho)^{1/5}t^{2/5}$.

3. Can you compute η if the medium is not homogeneous, but characterized by a density that decreases with the distance R from the initial explosion as $\varrho \propto R^{-\alpha}$?

1.3 The Trinity test: the first atomic explosion

The first atomic bomb exploded on July 16th in the desert of New Mexico as part of the Manhattan project. The energy released in the explosion was of course a military secret. In 1947 some pictures of that atomic explosion were published by the *Life* magazine (see Fig. 2), and the British physicist Geoffrey Taylor used them to estimate the explosion energy. Taylor published his results in 1950, and the US army was annoyed by that.

1. Estimate the explosion energy by means of the information contained in Figure 2, which shows a picture of the shock wave that resulted from the explosion. Assume the shock wave to be spherical, and estimate its radius from the 100 m scale provided in the picture, which was taken 0.025 seconds after the explosion. Assume the expanding shock to be in the *adiabatic* phase. The only additional information needed to solve the problem is the density of the air, which is roughly equal to 1.2×10^{-3} g/cm^3. [Hint: atomic bombs and supernova remnants behave exactly in the same way. Energies of bombs are measured in Kilotons. 1 Kiloton = 4×10^{19} erg.]

2. Can you prove, a posteriori, that the explosion was not, at the time the picture was taken, in the free expansion phase? [Hint: the weight of the bomb was 214 tons.]

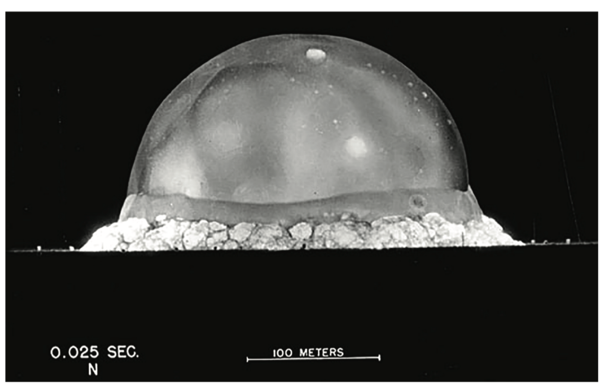

Figure 1: Picture of the first atomic explosion ever: the Trinity test.

1.4 Radiation-pressure dominated bubbles

Consider a star cluster (a group of stars concentrated in a very small region of space) emitting energy in form of photons at a constant rate L. L is a luminosity and has the units of an energy per unit time. We assume the star cluster to be point like, located at $R = 0$, and surrounded by a gas of uniform mass density ϱ. Under certain circumstances, the radiation pressure exerted by the photons onto the surrounding gas can create a cavity around the cluster. This is because the matter is pushed away from the star cluster and accumulates in a very thin spherical shell of radius $R = R_s$. The shell expands supersonically and, as in the case of supernova remnants, is bounded by a spherical shock wave. The expansion velocity of the shock is u_s.

If radiation pressure is the dominant force acting on the shell, a scale free solution of the problem can be found: $R_s \sim At^\alpha$ and $u_s = \alpha R_s/t$. Here, t is the time since the star cluster began to emit photons. We will assume that the shell radiates photons (i.e. the energy of the system is not conserved), that the density in the cavity is so low that photons are not absorbed there, and that the shell is optically thick (i.e. all the photons emitted by the stars are absorbed in the shell). Remember that if a photon flux F (energy per unit surface per unit time) is absorbed by a gas, the momentum transferred to the gas per unit surface and unit time is F/c, where c is the speed of light. Finally, assume that the expanding spherical shock is strong (i.e. you can neglect the pressure of the gas surrounding the star cluster).

1. Find the rate $\dot{\mu}$ at which momentum is transferred to the entire shell, and show that such quantity does not depend on time.

2. Estimate A and α.

3. Call E_k the kinetic energy of the shell, and E_{rad} the total energy radiated by the stars since $t = 0$. Show that, while E_k increases with time, the ratio E_k/E_{rad} decreases and tends to zero for late times.

4. Estimate A and α considering a situation where the density of the plasma around the star cluster is not uniform, but scales as $\varrho = \varrho_0 R^{-2}$, where R is the distance from the cluster.

1.5 Two shocks

Consider a plane, infinite, and non-radiative shock moving from left to right along the z axis with a constant velocity v_1. The current position of the shock is z_1, and the gas upstream of the shock ($z > z_1$, zone A in Fig. 2) is uniform and at rest. Consider now a second plane, infinite, and non-radiative shock located at $z_2 < z_1$, which is also moving from left to right with a constant velocity v_2. Will the second shock eventually catch the first one?

[Hint: there is a very simple argument that involves no calculations that can be used to answer this question. In alternative, you can do the following:

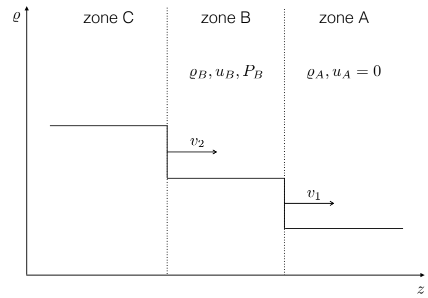

Figure 2: Setup of the problem for Exercise 2.

1. For simplicity, consider that the shock moving at a velocity v_1 is strong (its Mach number is $\mathcal{M}_1 \gg 1$).

2. Remember that the jump conditions for a strong shock are:

$$\frac{\varrho_d}{\varrho_u} = \frac{u_u}{u_d} = 4 \tag{2}$$

$$P_d = \frac{3}{4}\varrho_u u_u^2 \tag{3}$$

   where ϱ_i and u_i are the gas density and velocity of the fluid upstream ($i = u$) and downstream ($i = d$) of the shock, respectively, and P_d is the gas pressure downstream of the shock. Such conditions are valid *in the rest frame of the shock* (where the shock is at rest). A shock must have a Mach number $\mathcal{M} = u_u/c_{s,u} > 1$, where $c_{s,u}$ is the sound speed in the upstream fluid.

3. Apply the jump conditions to the first shock, and compute the velocity that the second shock should have in order to be a shock (Mach number larger than 1).]

1.6 The implosion of a bubble

In the class, we studied a number of explosive phenomena. In this exercise we will study the opposite phenomenon: an implosion.

Consider an infinite and non-radiative plasma of uniform density ϱ_0 and pressure P_0. The plasma contains a spherical cavity (a bubble) of initial radius R_0. At some initial time the plasma outside of the bubble is at rest. The pressure inside the bubble is equal to 0 (the cavity is empty!), while outside of the bubble the pressure is finite (equal to P_0). Summarising: the initial density and pressure of the plasma are equal to ϱ_0 and P_0 for any $r > R_0$. On the other hand, for $r < R_0$ both density and pressure are equal to 0. Here, we have chosen the origin of the spherical coordinate system to be the centre of the bubble Due to this difference of pressure, the bubble will collapse.

The goal of this exercise is to study the spherical collapse, and determine the radius of the bubble as a function of time, $R(t)$, as well as its velocity $\dot{R}(t) = \mathrm{d}R/\mathrm{d}t$. We will chose the time coordinate so that time $t = 0$ corresponds to the end of the implosion, when $R = 0$ and the bubble disappears. Finally, we assume that the plasma in *incompressible*, i.e. the density is equal to ϱ_0 at any time and any position outside of the bubble ($r > R$). Following the steps below, you will show that as time approaches $t = 0$ the solution becomes scale free: $R = A(-t)^\alpha$. We used $-t$ instead of t in the expression because the collapse happens for times $t < 0$.

1. Use dimensional analysis to show that the problem possesses a characteristic time scale τ_0.

2. How much energy E_{tot} had to be used to create the bubble? [Hint: to create the bubble a certain volume of space had to be emptied. Therefore, the energy was used to perform a "$P\mathrm{d}V$" work.]

3. Notice that E_{tot} is a conserved quantity. It represents all the energy that has been added to the system in order to create the bubble. Therefore it will also represent the total energy of the system during the collapse. At this stage, are you allowed to find A and α using dimensional analysis? After replying, follow the next steps to find a formal derivation of these two parameters.

4. At the initial time $t = t_0$ the kinetic energy E_k of the system is 0 (the plasma is at rest) and the total energy is purely in the form of a potential energy, $E_{tot} = E_p$ (you computed such potential energy in point 2 above). During the collapse the potential energy will be gradually converted into kinetic energy as the plasma collapses. At $t = 0$ the energy will be entirely kinetic, $E_{tot} = E_k$. Can you compute the expression of the kinetic energy E_k as a function of time? [Hint: instead of finding $E_k(t)$ it is more convenient to find $E_k(R(t), \dot{R}(t))$. This can be done following these steps:]

   (a) Use the continuity equation to find an expression for the plasma velocity u for $r > R$. The expression for u will contain also R and $\dot{R}$. The continuity equation in spherical coordinates is:

   $$\frac{\partial \varrho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \varrho u\right) = 0 \tag{4}$$

   Remember that the plasma density $\varrho = \varrho_0$ is a constant (both in time and space, outside of the bubble). This simplifies a lot the equation.

   (b) Once you have the expression of u find the expression of the kinetic energy of the system [Hint: this involves an integration on the entire space outside of the bubble].

5. Now that you have computed E_{tot} in point 1 above, and $E_{kin}(t)$ you need to find an expression for the potential energy $E_p(t)$. Once you have found it, use the equality $E_{tot} = E_k(t) + E_p(t)$ to obtain the expression:

$$\dot{R}^2 = \frac{2P_0}{3\varrho_0}\left[\left(\frac{R_0}{R}\right)^3 - 1\right] \tag{5}$$

6. Show that $R = A(-t)^\alpha$ is *not* a solution of the equation above. However, is there a limit you can consider where this is the case? In such a limit, find A and α. Are these numbers familiar to you?

1.7 Supernova explosion in a cloud

Consider a supernova exploding in the centre of a spherical cloud of gas of radius R and mass density ϱ. The total mass of the cloud is then $M = (4\pi/3)\varrho R^3$. Outside of the cloud the gas density is extremely, small: $\varrho \sim 0$. The supernova explosion releases an amount of mechanical energy E, and the mass of the stellar ejecta is $M_{ej} \ll M$. As a result of the explosion, the interstellar gas will be pushed away and it will accumulate in an expanding and dense spherical shell of radius R_s. The shell moves at a velocity u_s. The goal of this exercise is to investigate the evolution of the shell when its radius exceeds that of the cloud. We will consider the system to be adiabatic (no energy is radiated away), and the shock to be strong at any time (the pressure of the ambient gas can be neglected).

1. Estimate the evolution of the shell velocity as a function of its radius, $u_s(R_s)$ for $R_s > R$. [Hint: use energy conservation, and remember that, for an adiabatic gas, pressure P and volume V are connected through $PV^{5/3} = const.$]

2. Under which conditions the problem allows a scale free solution of the form $R_s = At^\alpha$? Which are the values of A and α?

3. If v_0 is the initial velocity of the ejecta just after the supernova explosion, and v_∞ the velocity of the shell for $t \to \infty$, how large is the ratio v_∞/v_0?.