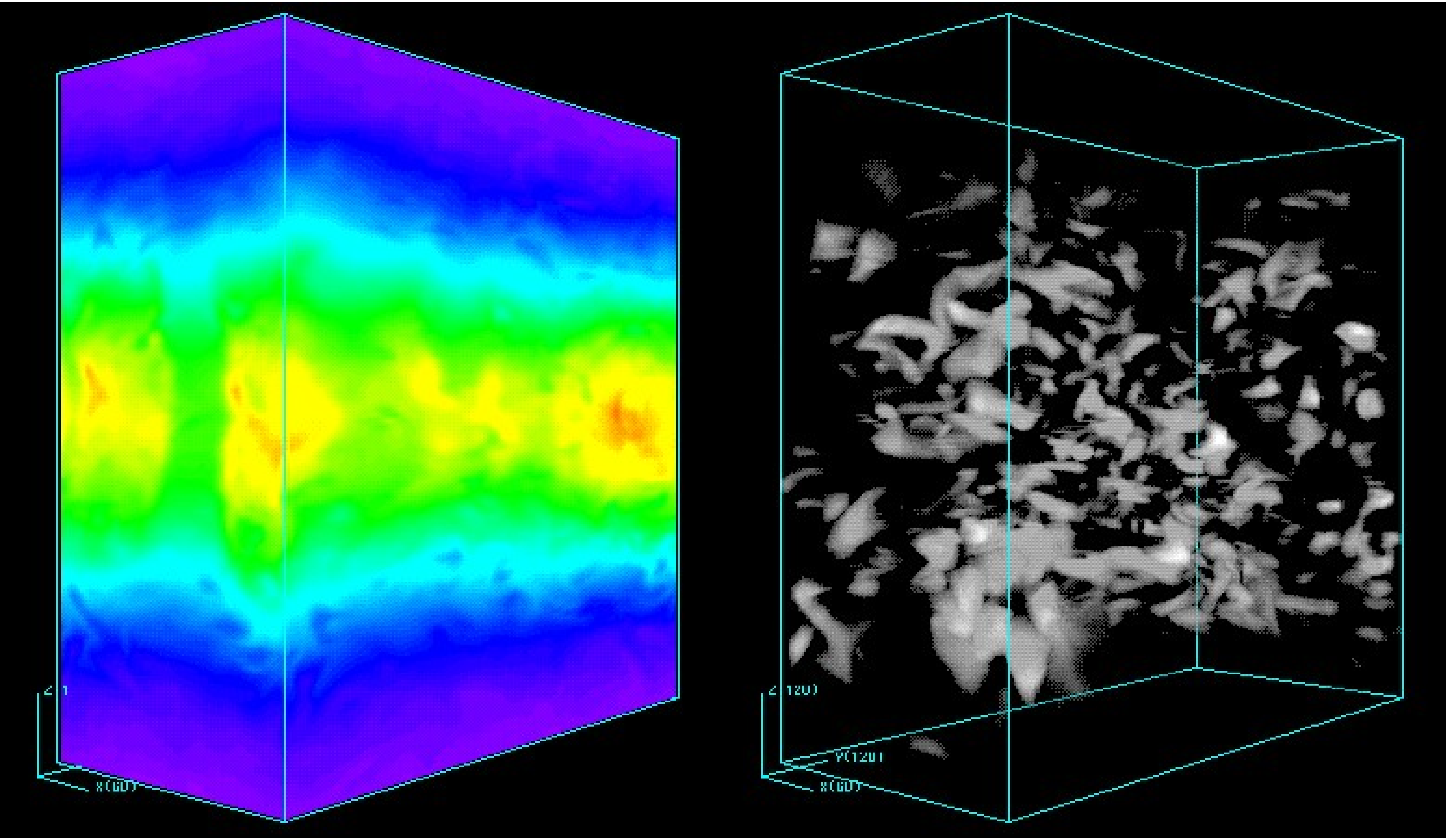
Radiation processes and basic processes

A. Patruno

Outline

- I. Radiative Transport Equation
- II. Accelerating Charge
- III. Radiative Processes:
  - Thermal Emission
  - Non-Thermal Emission
- IV. Accelerating Processes

I. Radiative Transport (Equation)



What is meant by *Radiative Transport?*

In principle, radiation is described by Maxwell’s equations:

$\nabla \cdot \mathbf{D} = \rho$	(1)	Gauss’ Law
$\nabla \cdot \mathbf{B} = 0$	(2)	Gauss’ Law for magnetism
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	(3)	Faraday’s Law
$\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	(4)	Ampère-Maxwell Law

Together with particle dynamics, this is the full microscopic problem.

However, this is usually the wrong level of description for high-energy astrophysics:

- **astronomical radiation is usually incoherent**, so the phase information averages out. What survives observationally is not the field amplitude E(t), but an intensity, spectrum, light curve and polarization.
- **Microscopic sources are inaccessible**: to solve the full field problem, one would need the instantaneous positions, velocities and accelerations of all radiating charges, plus the plasma response. That is not what we know observationally.
- **Light emission if often described in quantum/statistical way**: emission processes are often more naturally described as particle interaction rates and cross sections, not as a classical coherent electromagnetic wave problem.

What is meant by *Radiative Transport?*

Radiative transfer describes how radiation propagates through matter.

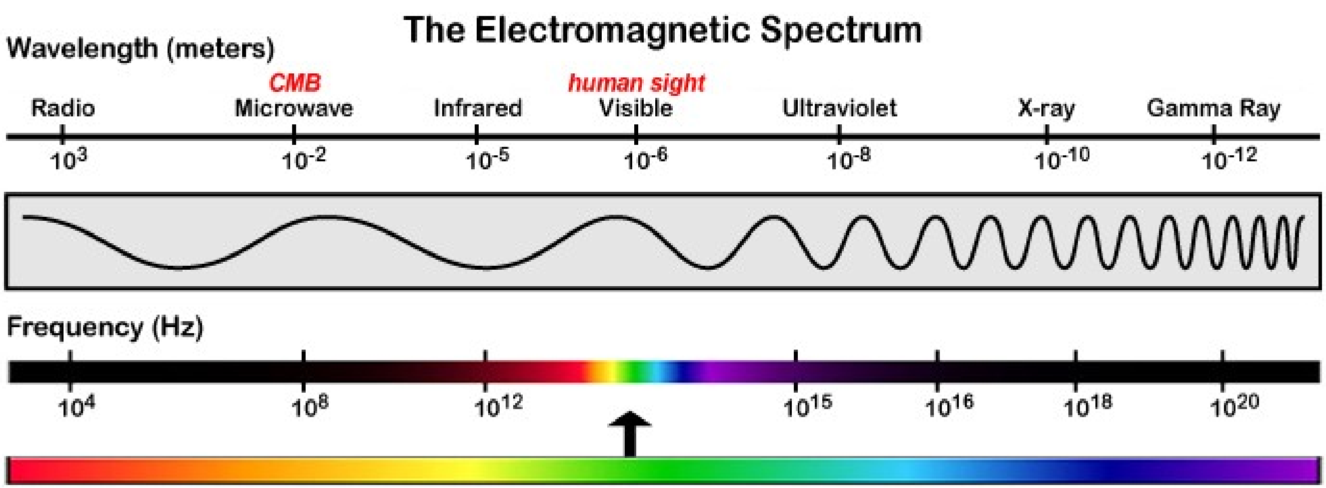
It answers questions like: *how does the radiation field change when propagating through matter?*

What is the energy spectrum of a source? What is the cooling/heating rate of a source?

Before writing the transfer equation, we need a few precise definitions.

$$\lambda = \frac{c}{\nu} \qquad E = h\nu = \hbar\omega$$

$$1\,eV \rightarrow 11{,}000\,K\,(E = k_B T)$$



Radio > 1 mm

IR $700\text{nm} - 1\text{mm}$

Optical $400 - 700\text{nm}$

UV $100 - 400\text{nm}$

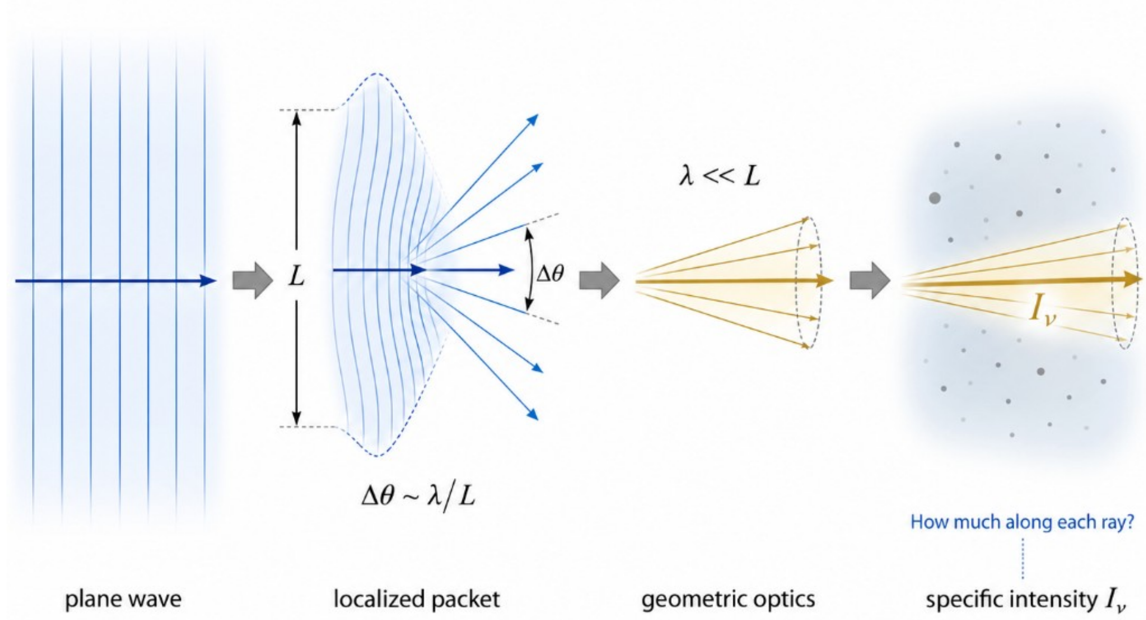
X-Rays $0.1 - 100$ keV

Gamma > 1 MeV

What is meant by *Radiative Transport?*

A light ray is not a fundamental object.
It is an approximation.

A perfectly plane wave has a well-defined propagation direction, but is **completely delocalized in space**. A photon localized within a finite transverse size L has an unavoidable spread in transverse momentum. Therefore, its propagation direction is not perfectly sharp. The angular uncertainty is approximately $\Delta\theta \sim \lambda$ L.

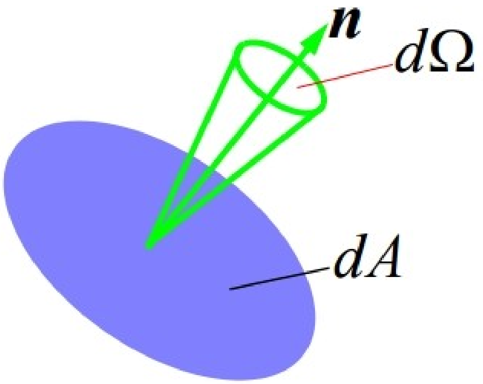


Therefore, the ray approximation is valid only when $\lambda \ll L$ where L is the macroscopic scale over which the radiation field, the source geometry, or the medium changes. This is the *geometric-optics limit*. **A ray represents the local direction of photon propagation.** A narrow bundle of rays represents radiation propagating within a small solid angle around a given direction. Radiative transfer then asks: *How much radiation is carried along each ray?*

The central quantity is not the electromagnetic field amplitude, but the **specific intensity I(ν)**

Specific Intensity (Brightness)

The flux is a measure of the energy carried by **all rays** passing through a given area. What if we want to follow each ray instead?
Note: in transfer theory (which is the one we're using here) a single ray carries no energy, so we need to consider an infinitesimal amount of energy carried by a set of rays which differ infinitesimally from the given ray.



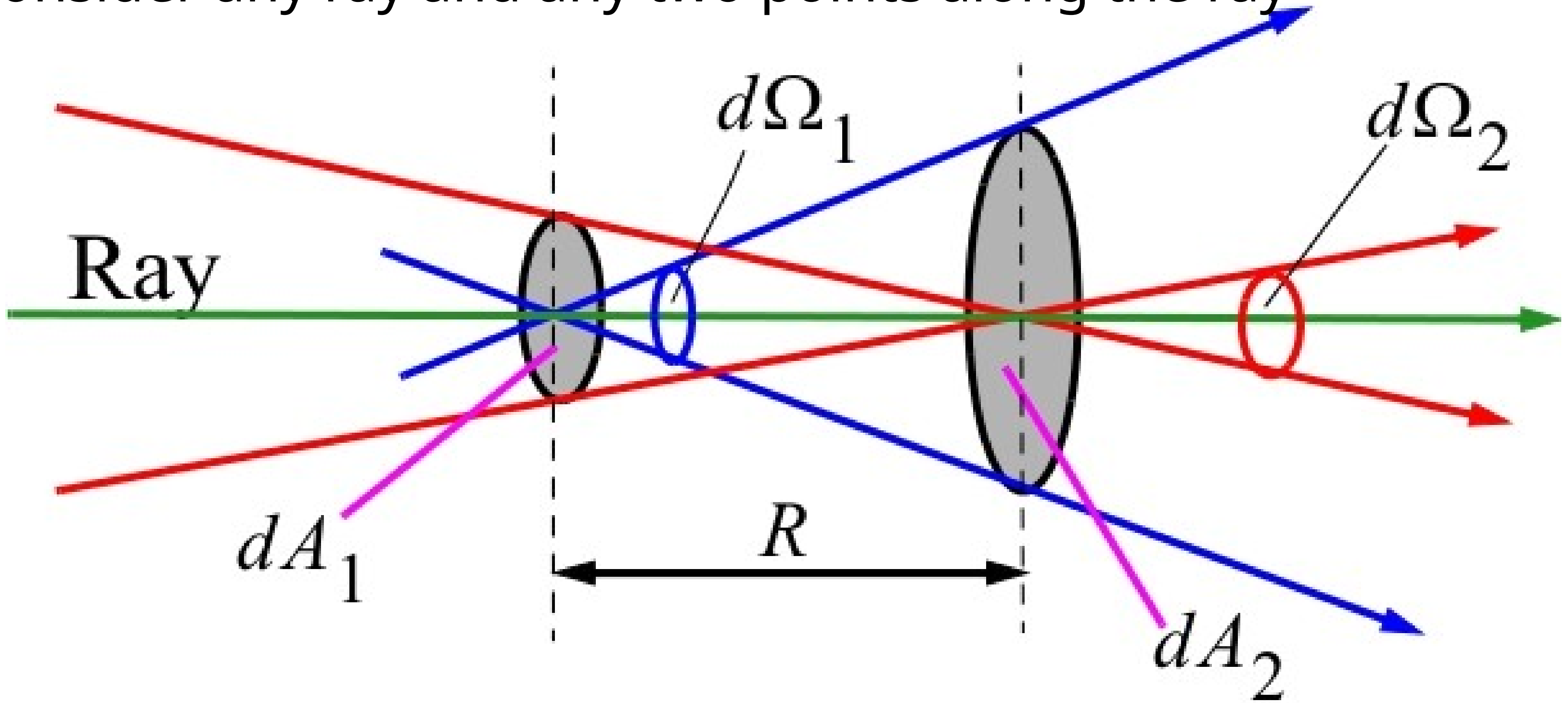
Specific Intensity: energy passing through an area dA normal to the direction of the given ray and consider all rays passing through dA whose direction is within an infinitesimal solid angle of the given ray:

$$dE = I_\nu \, dA \, d\Omega \, dt \, d\nu$$

$$I_\nu = \frac{dE}{dA \, d\Omega \, dt \, d\nu}$$

Constancy of Brightness in Free Space

Consider any ray and any two points along the ray



Consider the energy flux through an elementary surface dA1 within solid angle $d\,\Omega_1$
Consider all of the photons which pass through dA1 in this direction which then pass through dA2 . We take the solid angle of these photons to be $d\,\Omega_2$

Constancy of Brightness in Free Space

We use now energy conservation:

$$dE = I_{\nu 1}\, dA_1\, d\Omega_1\, dt\, d\nu_1 = I_{\nu 2}\, dA_2\, d\Omega_2\, dt\, d\nu_2$$

Remember the definition of solid angle:

$$d\Omega_1 = dA_2 / R^2 \qquad d\Omega_2 = dA_1 / R^2$$

Since the frequency of the ray is the same ($d\nu_2 = d\nu_1$) we have:

$$I_{\nu 1} = I_{\nu 2} = constant$$

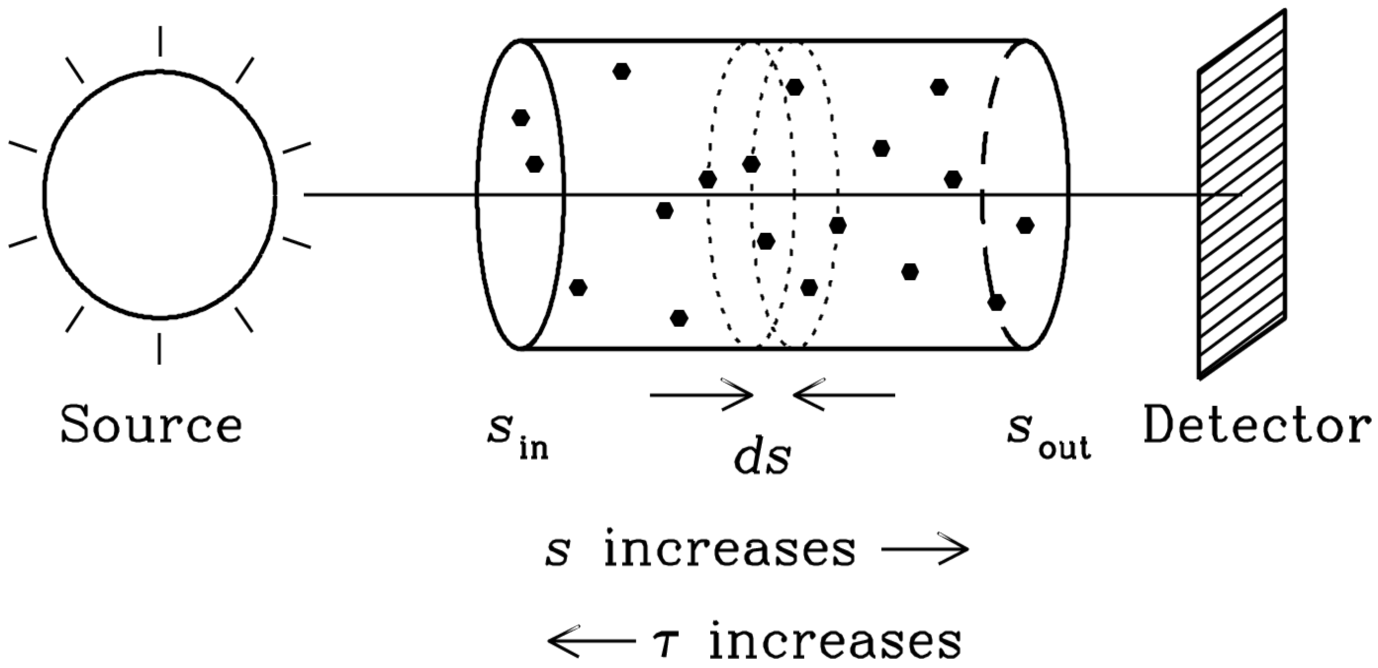
Brightness does NOT depend on distance

Why do we use radiative transfer?

The light we detect arrives at us in two steps:

- first, it is created by some radiative process (e.g., blackbody, synchrotron, etc etc...)
- then it propagates through space where it might be (partially) scattered and absorbe

Scattering, absorption and emission are thus three fundamental steps to generate the light we see.



Transfer theory tells us how the specific intensity of an object is affected by **absorption**, **scattering** and **emission**.

Emission (1)

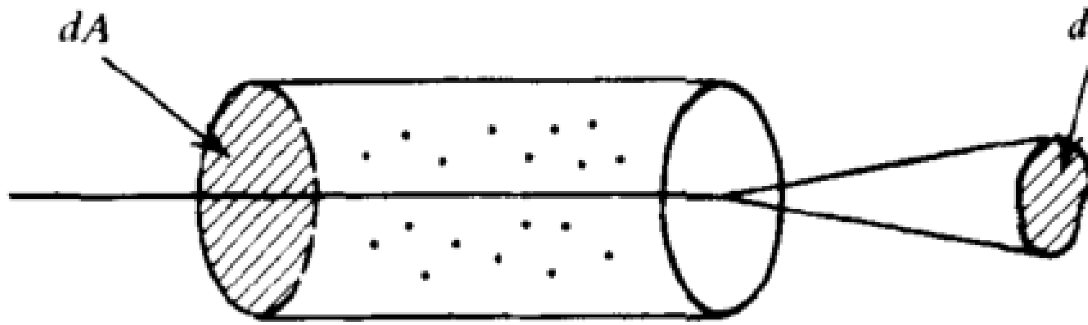
- The spontaneous emission coefficient j is defined as the energy emitted per unit time per unit solid angle per unit volume:
  $dE = j\, dV\, d\Omega\, dt$
- Or when the emission is monochromatic:
  $dE = j_v\, dV\, d\Omega\, dt\, dv$
- If the emission is isotropic, we can write:
  $j_v = 1/(4\pi)\, P_v$, with the power per unit volume per unit frequency.

Emission (2)

- In going a distance *ds*, a beam of cross section dA travels through a volume $dV = dA\, ds$
- The intensity added to the beam is then:
  $dI_\nu = j_\nu\, ds$
- Or compare the specific intensity and the emission coefficient:
  j_ν [erg cm^{-3} s^{-1} ster^{-1} Hz^{-1}] to
  I_ν [erg cm^{-2} s^{-1} ster^{-1} Hz^{-1}]

Absorption

- The absorption coefficient α [cm^{-1}] is defined by the following equation: $dI_\nu = -\alpha_\nu I_\nu ds$, representing the loss of intensity in a beam as it travels a distance *ds*.
- This α can be defined by: $\alpha_\nu = n\sigma_\nu$ or $\alpha_\nu = \rho\kappa_\nu$



(a)

Radiation transport

- The decrement of I_v when passing through a path of length *ds*:
  $dI_v = -\alpha_v I_v ds$
- Inside a source, a contribution to I_v can be made from emitters. The increment is:
  $dI_v = j_v ds$
- The basic equation of transport is:

$$\boxed{\frac{dI_v}{ds} = -\alpha_v I_v + j_v}$$

Simple solutions (1)

- Emission only:

$$\frac{dI_\nu}{ds} = j_\nu \rightarrow I_\nu = I_{\nu,0} + \int_0^S j_\nu ds$$

with S the total emission path.

- The increase in the specific intensity is equal to the emission coefficient integrated along the line of sight

Simple solutions (2)

- Absorption only:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \rightarrow \frac{dI_\nu}{I_\nu} = -\alpha_\nu ds$$

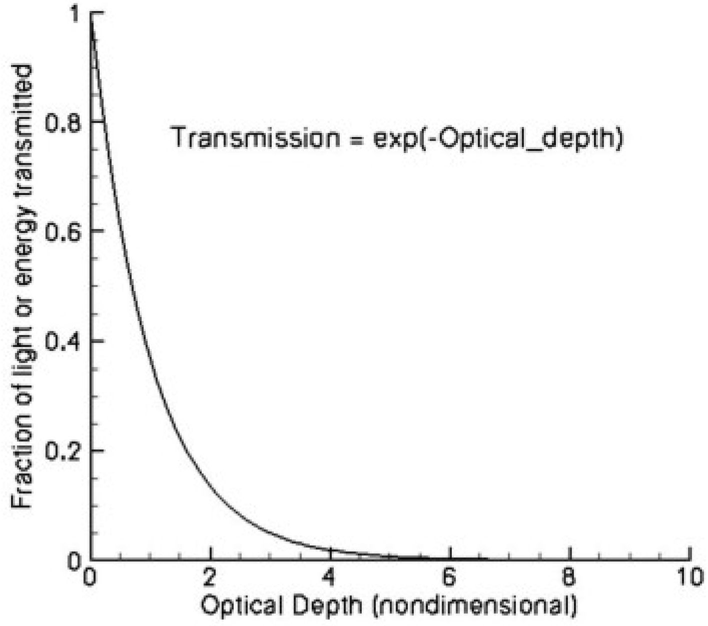
$$I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$$

- The brightness decreases along the ray by the exponential of the absorption coefficient integrated along the line of sight

Optical depth (1)

- We now introduce the quantity **optical depth:**

$$d\tau_\nu = \alpha_\nu ds = n\sigma_\nu ds$$

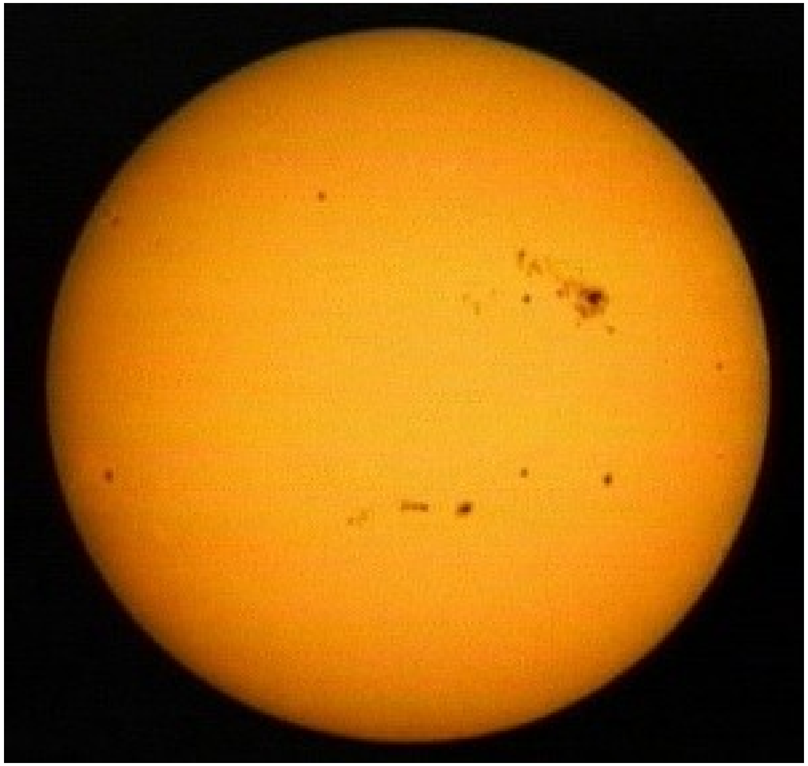


Optical depth (2)

- The intensity decreases as follows:
  $I_\nu = I_{\nu,0} \exp(-\tau_\nu)$

- What follows from this:
  - τ=1 → 1/e (37%)
  - τ>>1 → optically thick
  - τ<<1 → optically thin

The “surface” of the Sun



The “surface” of the Sun is usually defined as the location in the photosphere where the optical depth is equal to 2/3.
This is just a definition, and it is used because about 50% of photons we see are coming from this region.

Transport equation and source function

- The rewritten equation of transport becomes

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu} \rightarrow \frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

- We define S_ν the source function as follows

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

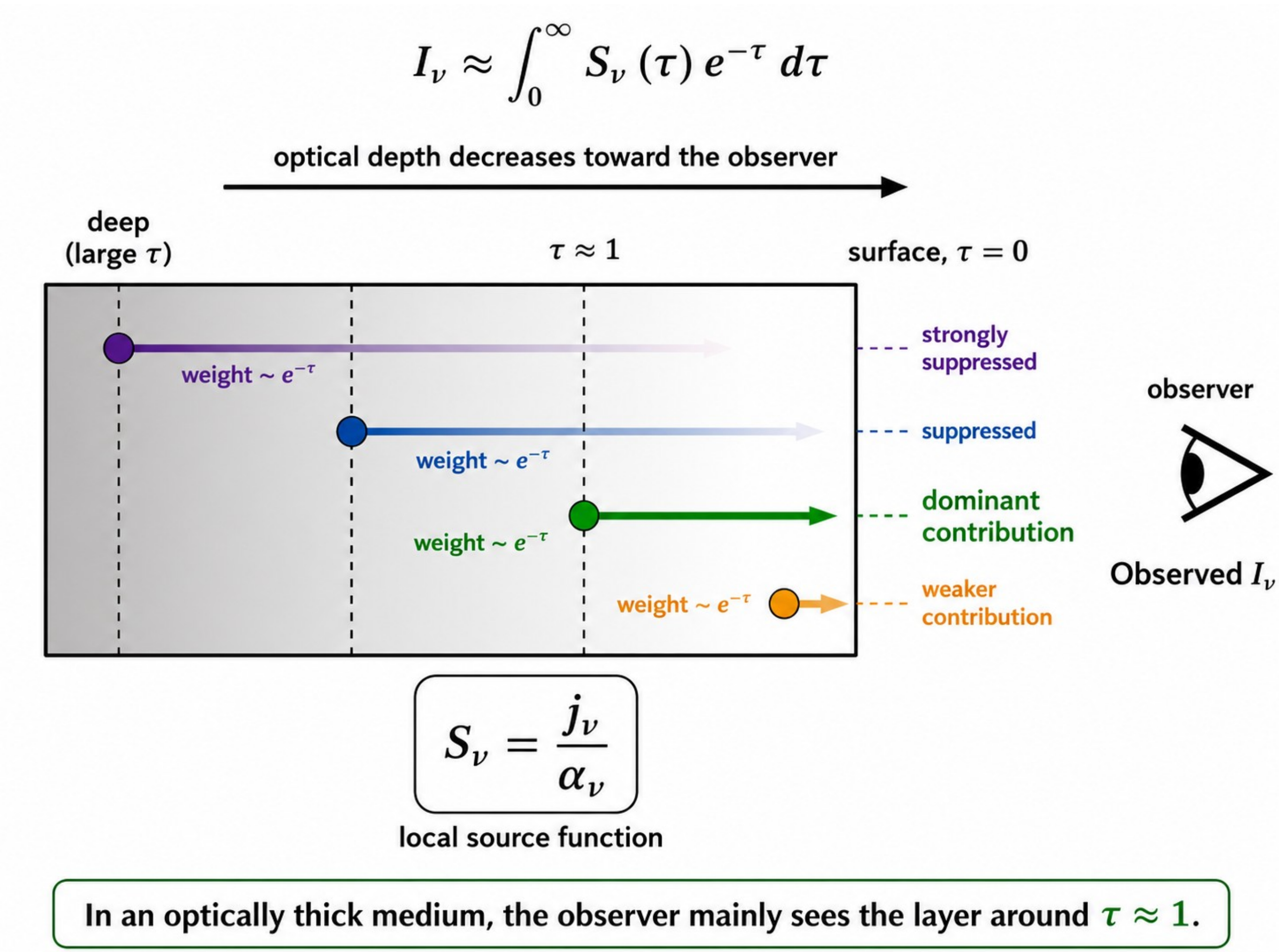
Equation of transfer

- This yields the format solution of the EOT:

$$I_\nu(\tau_\nu) = I_{\nu,0}\, e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$$

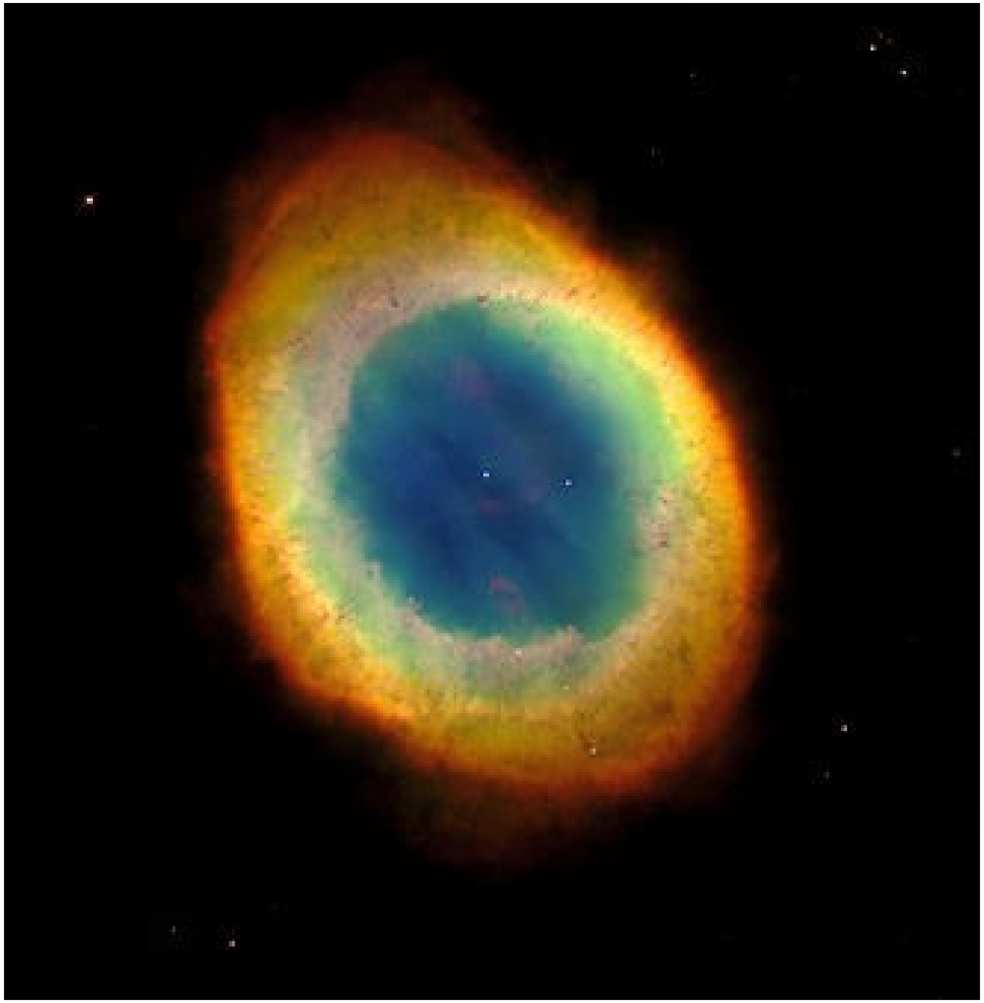
- When S_ν constant:
  $I_\nu(\tau_\nu) = I_{\nu,0}\exp(-\tau_\nu) + S_\nu(1-\exp(-\tau_\nu))$
- $\tau >> 1$: $I_\nu \rightarrow S_\nu$
- $\tau << 1$: $I_\nu \rightarrow I_{\nu,0} + S_\nu \tau_\nu$

(The source function can be thought of as the specific intensity that the local material tends to impose on the radiation field over an optical depth of order one.)



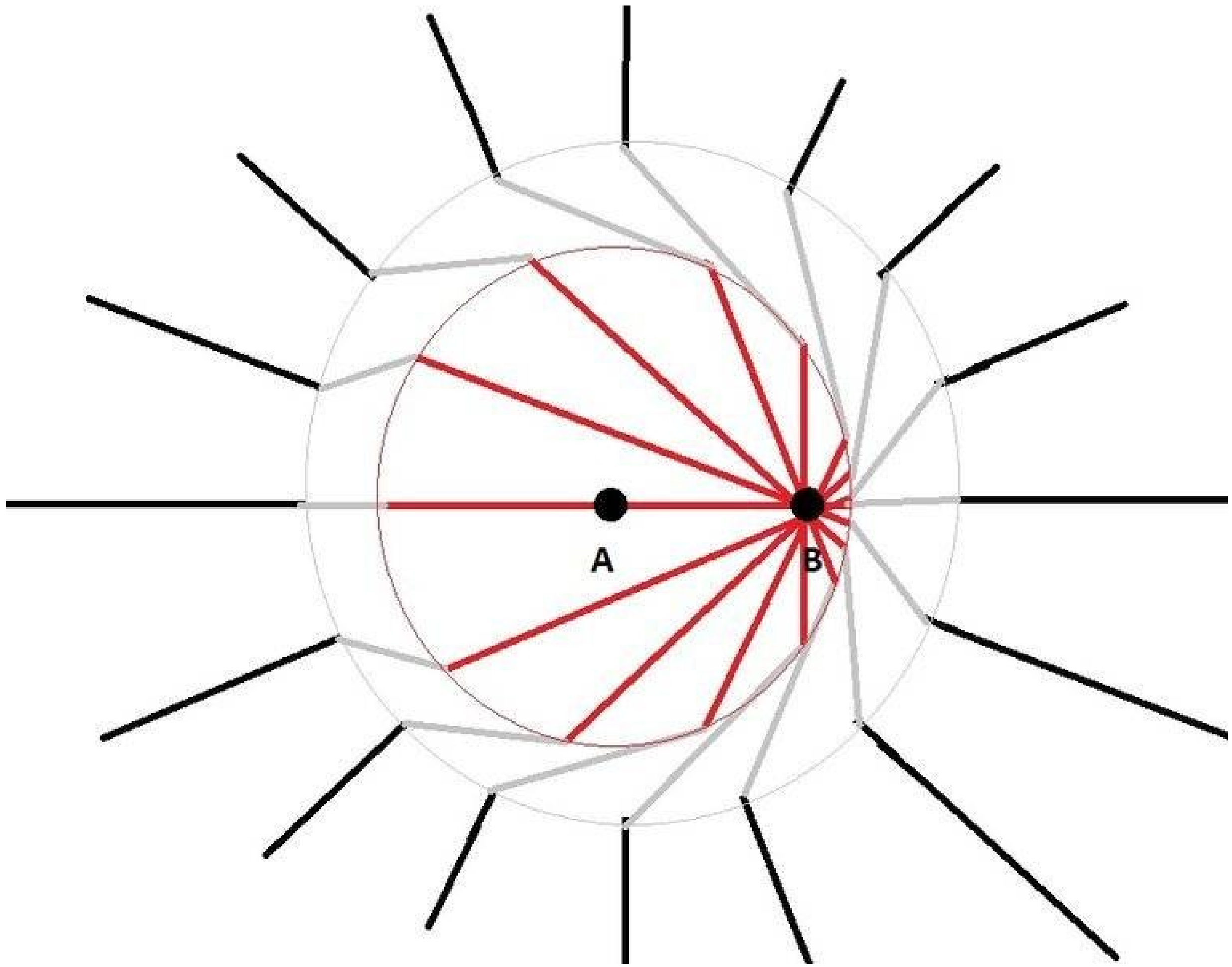
Light comes from a white dwarf illuminating the surrounding material

Helix Planetary Nebula
(remnant of a low mass star)
Distance: 220 pc
Size: about 0.8 pc across



Ring Planetary Nebula
(remnant of a low mass star)
Distance: 700 pc
Size: about 0.4 pc across

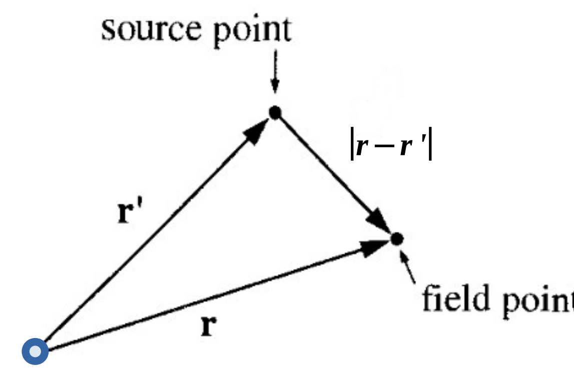
II. Accelerating Charges



Retarded Time

First of all let's introduce the symbols **r** and **r'** are.
In electrodynamics one frequently encounters problems involving two points, typically, a source point, **r'**, where an electric charge is located, and a field point, **r**, at which you are calculating the electric or magnetic field.

This is the origin of our Cartesian reference frame. We are measuring the field at the distance **r** from the origin of our frame. And we are measuring this field that is generated by the charge at the point **r'**.



The *retarded time* refers to the conditions at the point **r'** that existed at a time *earlier* than *t*
By just the time required for light to travel between **r** and **r'**.

In other words: the field at a certain point in space is not determined by where the charge is NOW (time *t*) but it depends on the state of the charge in the past. How far in the past? Just the time it takes to the fields to propagate from the charge to the point we're measuring.

Accelerations: Retarded Potentials

We know from Maxwell equations that the field **E**(r,t) and **B**(r,t) can be expressed in terms of two potentials, $\phi(r,t)$ and **A**(r, t).

$$\boldsymbol{B} = \nabla \wedge \boldsymbol{A} \qquad \boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}$$

We also know that the two potentials satisfy the equations:

$$\begin{cases} \nabla^2 \boldsymbol{A} - \dfrac{1}{c^2}\dfrac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu_0 \boldsymbol{J} \\ \nabla^2 \phi - \dfrac{1}{c^2}\dfrac{\partial^2 \phi}{\partial t^2} = -\dfrac{\rho_e}{\varepsilon_0} \, . \end{cases}$$

And we know that the solution for these two equations is:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{J}(\boldsymbol{r}', t - |\boldsymbol{r} - \boldsymbol{r}'|/c)}{|\boldsymbol{r} - \boldsymbol{r}'|}\, \mathrm{d}^3\boldsymbol{r}' \, ,$$

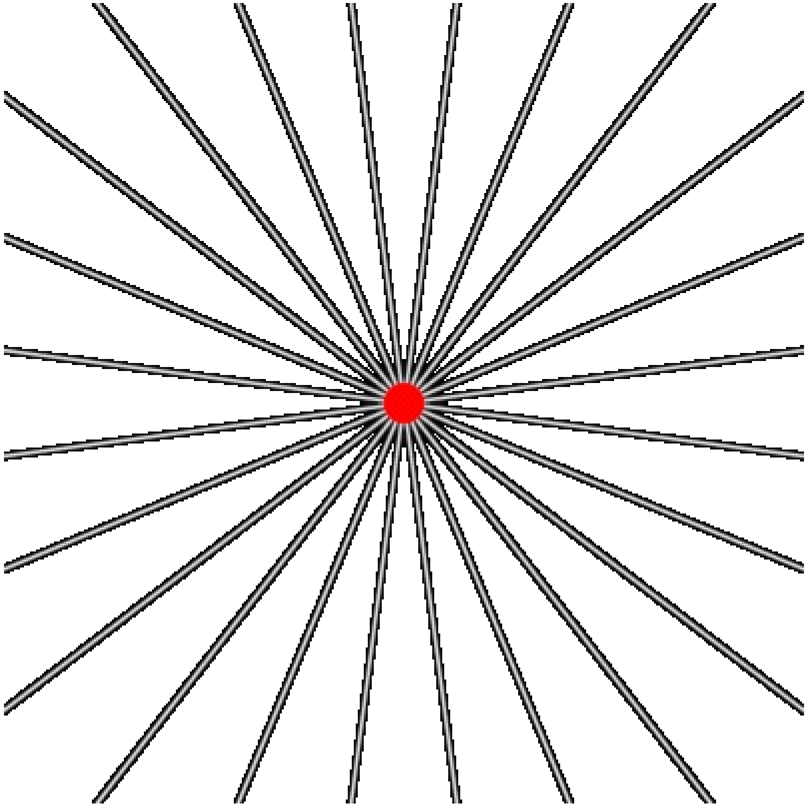
$$\phi(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_e(\boldsymbol{r}', t - |\boldsymbol{r} - \boldsymbol{r}'|/c)}{|\boldsymbol{r} - \boldsymbol{r}'|}\, \mathrm{d}^3\boldsymbol{r}' \, .$$

Here ***r*** is the point at which the fields are measured. The integration is over the electric current and charge distributions throughout space. The terms |***r-r'***|/*c* take account of the fact that the current and charge distributions should be evaluated at ***retarded times***.

Accelerations: Retarded Potentials

Electric and magnetic fields move at the speed of light, which is finite.

If you take a charge and move it, the field lines will change. The disturbance will take time to propagate. *It is this “retardation” that makes possible for a charge to radiate!*



See animation!

Velocity and Acceleration Fields

If one calculates the **E** and **B** fields from the retarded potentials one finds the following:

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R} \times \left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right],$$

$$\mathbf{B}(\mathbf{r},t) = \left[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)\right].$$

Here **u** is the velocity of the charge, **n** is a unit vector from the charge to the field point.

$$\boldsymbol{\beta} \equiv \frac{\mathbf{u}}{c}, \qquad \kappa \equiv 1 - \mathbf{n}\cdot\boldsymbol{\beta}$$

We have aso used the notation **R = |r-r'|** (so that **n** = **R**/R)

Velocity and Acceleration Fields

If one calculates the **E** and **B** fields from the retarded potentials one finds the following:

$$\mathbf{E}(\mathbf{r},t)=\boxed{q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]}+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right],$$

$$\mathbf{B}(\mathbf{r},t)=[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)].$$

This field falls off as 1/R^2, it is called the *velocity field* and it is a generalization of the Coulomb law for moving particles. For **u**<<c then it becomes precisely Coulomb's law. Note that there is no acceleration in this term, i.e., this field is generated by charges at rest or with constant velocity.

it vector from the charge to the field point.

so that **n** = **R**/R)

Velocity and Acceleration Fields

If one calculates the **E** and **B** fields from the retarded potentials one finds the following:

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right],$$

$$\mathbf{B}(\mathbf{r},t) = \left[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)\right].$$

This field falls off as 1/R^2, it is called the *velocity field* and it is a generalization of the Coulomb law for moving particles. For **u**<<c then it becomes precisely Coulomb's law. Note that there is no acceleration in this term, i.e., this field is generated by charges at rest or with constant velocity.

t vector from the charge to the field point.

o that **n** = **R**/R)

When **u**~c then the term *k* becomes very important and concentrates the fields in a narrow cone (*beaming effect*).

Velocity and Acceleration Fields

If one calculates the **E** and **B** fields from the retarded potentials one finds the following:

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \boxed{\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]},$$

$$\mathbf{B}(\mathbf{r},t) = [\mathbf{n}\times\mathbf{E}(\mathbf{r},t)].$$

Here **u** is the velocity of the charge, **n** i … nt.

$$\boldsymbol{\beta} \equiv \frac{\mathbf{u}}{c}, \qquad \kappa \equiv 1 - \mathbf{n}\cdot\boldsymbol{\beta}$$

We have aso used the notation **R = |r** …

This is the *acceleration field, i.e.*, it appears when the charges are accelerated.
Note that it falls off as 1/R, not as 1/R^2.
The *acceleration field* is also known as the *radiation field* and it is orthogonal to **n**.

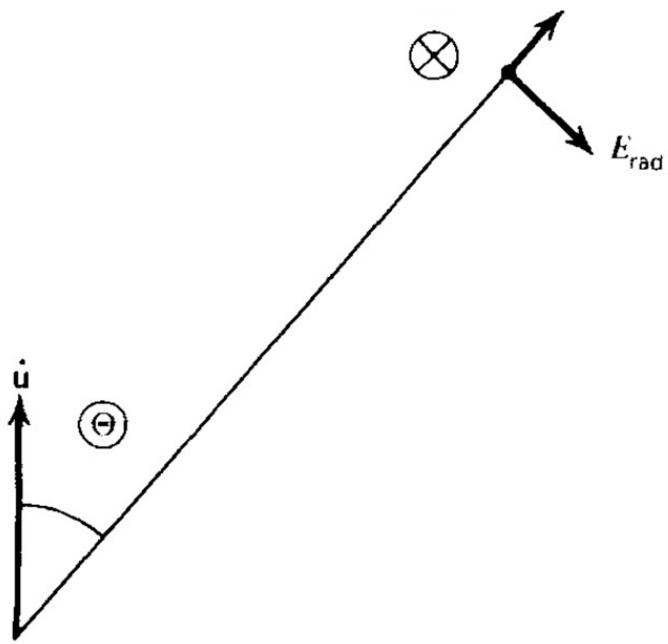
Larmor’s Formula

What can we say about the radiation field when the velocity is <<c? (non-relativistic case)

In this case beta<<1 and thus we can simplify the electric and magnetic field expressions and obtain:

$$\mathbf{E}_{\text{rad}} = \left[(q/Rc^2)\mathbf{n} \times (\mathbf{n} \times \dot{\mathbf{u}}) \right]$$

$$\mathbf{B}_{\text{rad}} = \left[\mathbf{n} \times \mathbf{E}_{\text{rad}} \right].$$



What is the Poynting vector **S**? (remember that the Poyinting vector defines the direction towards which the energy carried by the em fields is directed. Here **S** is parallel to **n;**
S has units of erg/s/cm^2, i.e., energy flux).
Since:

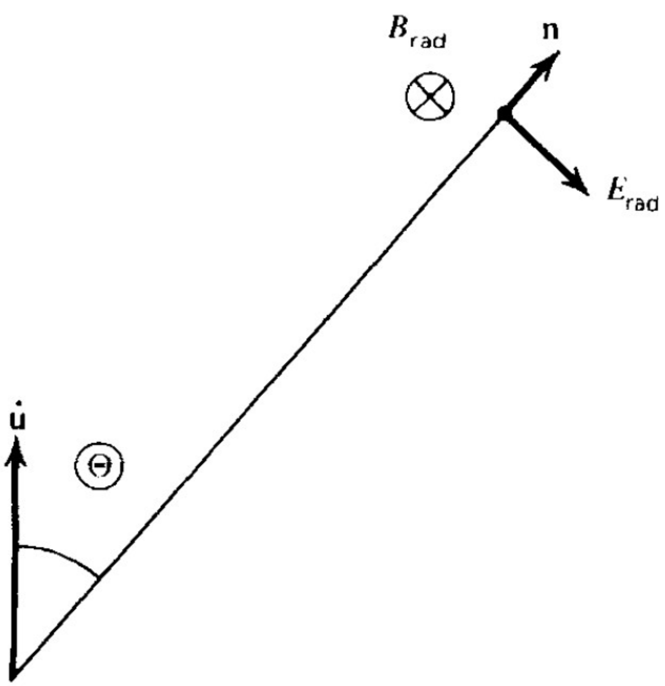
$$|\mathbf{E}_{\text{rad}}| = |\mathbf{B}_{\text{rad}}| = \frac{q\dot{u}}{Rc^2} \sin\Theta$$

the Poynting vector has magnitude:

$$S = \frac{c}{4\pi} E_{\text{rad}}^2 = \frac{c}{4\pi} \frac{q^2 \dot{u}^2}{R^2 c^4} \sin^2 \Theta.$$

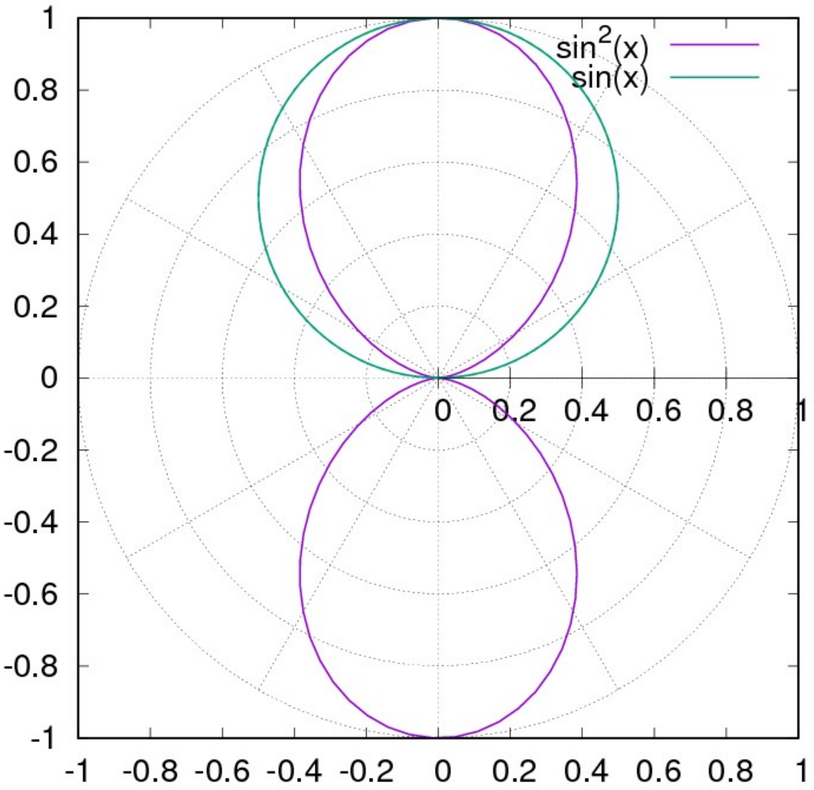
Larmor's Formula

$$S = \frac{c}{4\pi} E_{\mathrm{rad}}^2 = \frac{c}{4\pi} \frac{q^2 \dot{u}^2}{R^2 c^4} \sin^2 \Theta.$$



Note the angle theta!

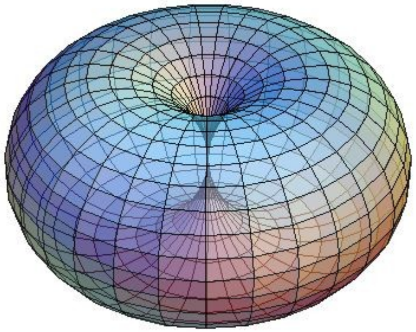
The energy of the em. field is not isotropic but there is a sin^2 !!



Larmor's Formula

Now let's calculate the power in a unit solid angle about **n**. To do this we multiply the Poynting vector (units: erg/s/cm^2) by an area dA (cm^2) to get a power (erg/s). How do we choose dA? We know that the solid angle dOmega = dA/R^2. Therefore:

$$\frac{dW}{dt\,d\Omega} = \frac{q^2\dot{u}^2}{4\pi c^3}\sin^2\Theta$$

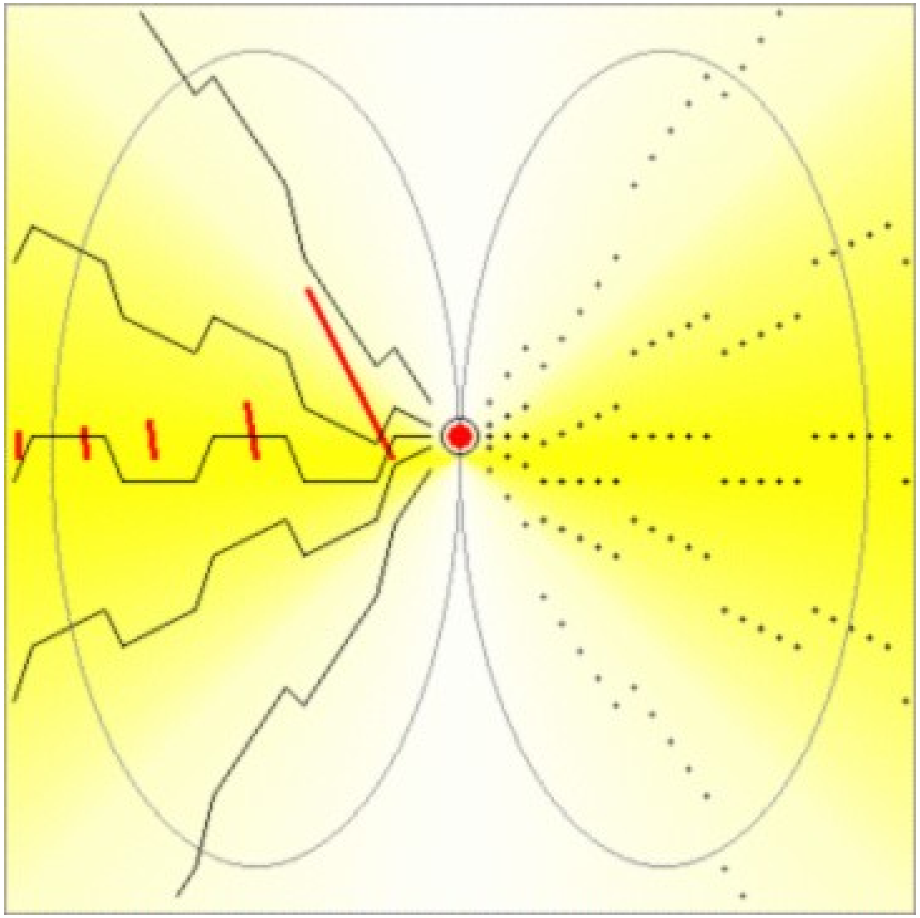


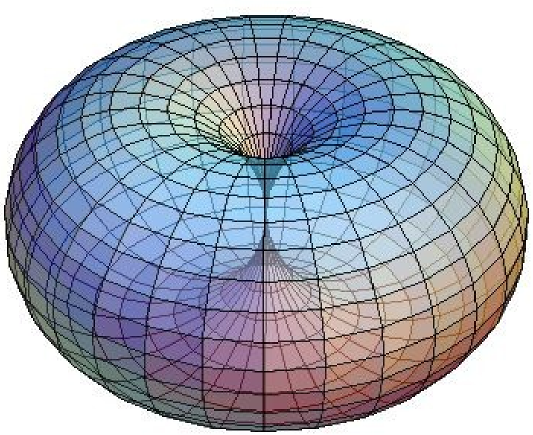
And now we integrate the above expression over the whole solid angle Omega=4*pi and we obtain the total power emitted by an accelerated charge in the non-relativistic approximation:

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

Larmor's Formula

IMPORTANT: The power emitted is proportional to <u>the square of the charge</u> and <u>the square of the acceleration</u>.





The animation represents a charged particle being switched up and down in a very strong electric field, such that the shape is traced out in time and aligns to an approximate square wave. The ovals’ reference lines are drawn to the left and right of the charge and correspond to a cross-section through the doughnut toroid, as illustrated in the previous diagram. Based on the criteria of the Larmor formula, when a charge is subject to acceleration, i.e. during the transition positions, it radiates power also subject to the angle θ with respect to the axis of charge motion. As such, the energy density is reflected by the depth of the yellow shading, symmetrical about the axis of motion. However, the intention of the left-right sides of the animation is to be somewhat illustrative of wave-particle duality in that the left reflects the electric field lines, while the right reflects the streams of photons being emitted by the charge. The field lines or photon streams are shown at different angles, e.g. 0, 30, and 60 degrees, from the maximum, which is always perpendicular to the axis. Finally, the oscillating red lines on the left reflect the total electric field E=E_rad + E_vel as a function of distance. So what you see is the effects of E_vel reducing by 1/R^2, while E_rad only reduces by 1/R and so quickly becomes the dominant field as the radius from the charge increases.