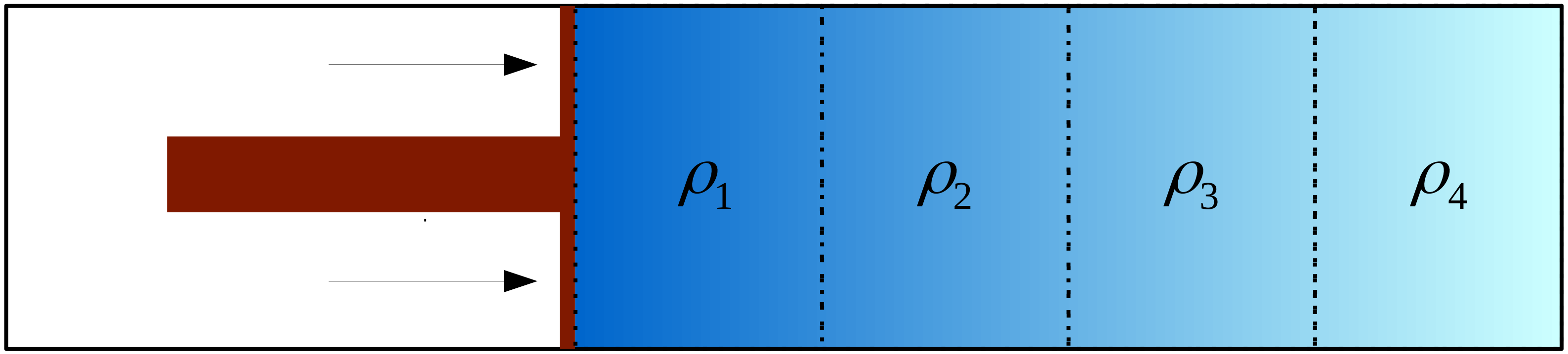
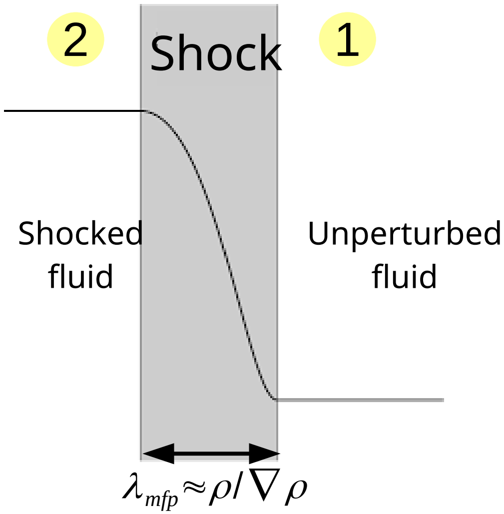
Acceleration Processes





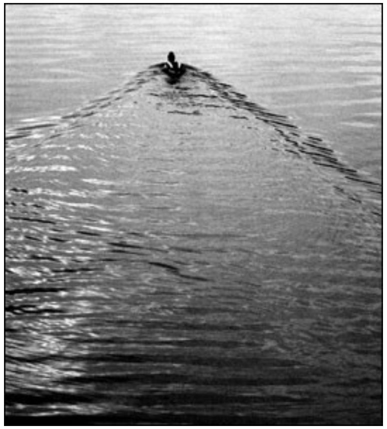
Mass conservation across the fluid (1→ upstream flow; 2→ downstream flow):

$$\rho_1 u_1 = \rho_2 u_2$$

Compression ratio R:

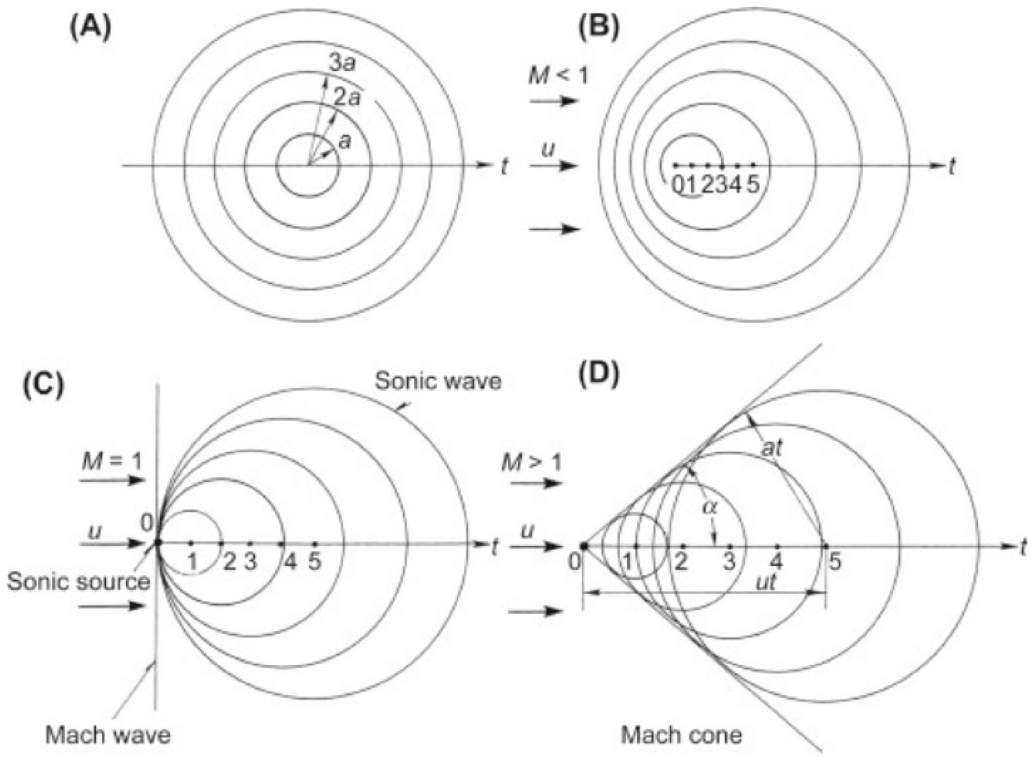
$$R \equiv \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2}$$

Therefore: $u_2 = \dfrac{u_1}{R}$



Or if we use V_shock for the upstream speed u1: $u_2 = \dfrac{V_{shock}}{R}$

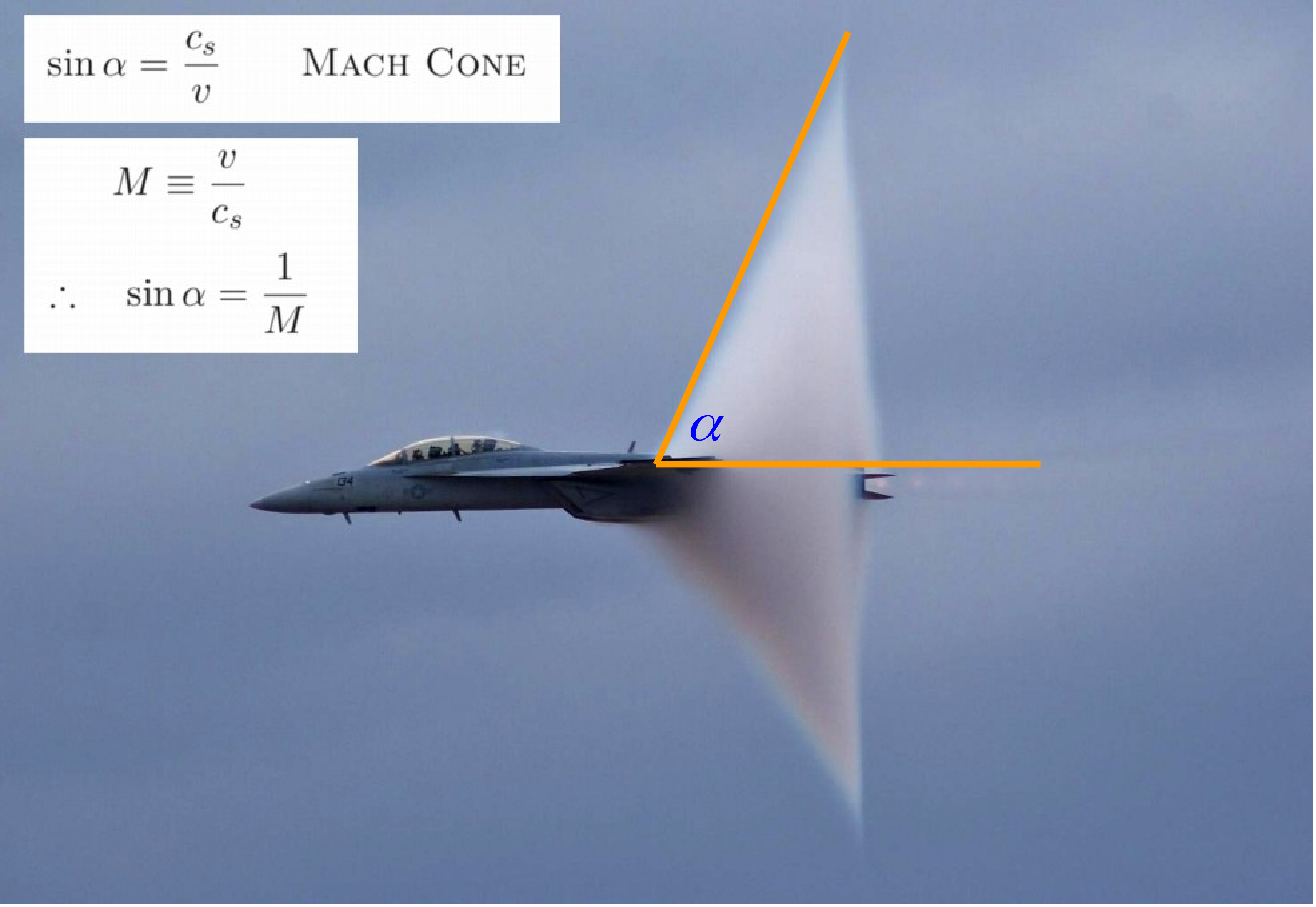
Mach Number and Mach Cone

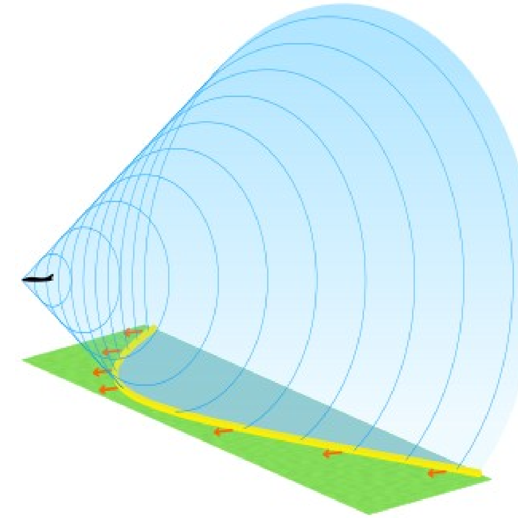


(A) & (B)
subsonic flow

(C) & (D)
supersonic flow

From Science Direct





Adapted from
lecture of
Vasily Belokurov

https://people.ast.cam.ac.uk/~vasily/Lectures/AFD_2021_Reynolds/AFDLecture09_10Feb2021.pdf

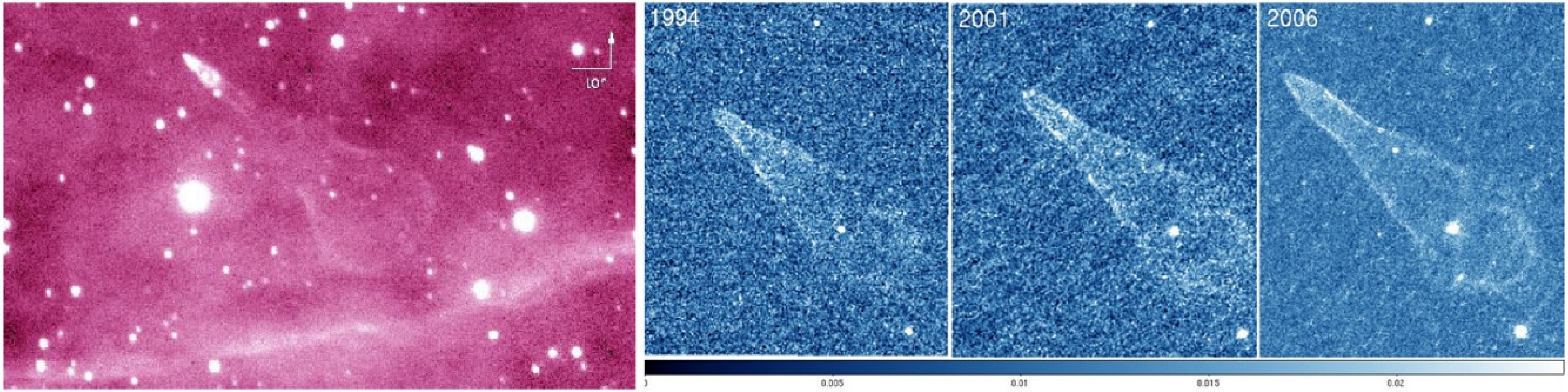
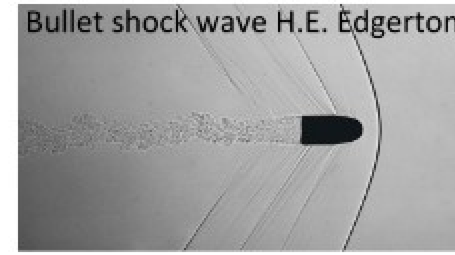


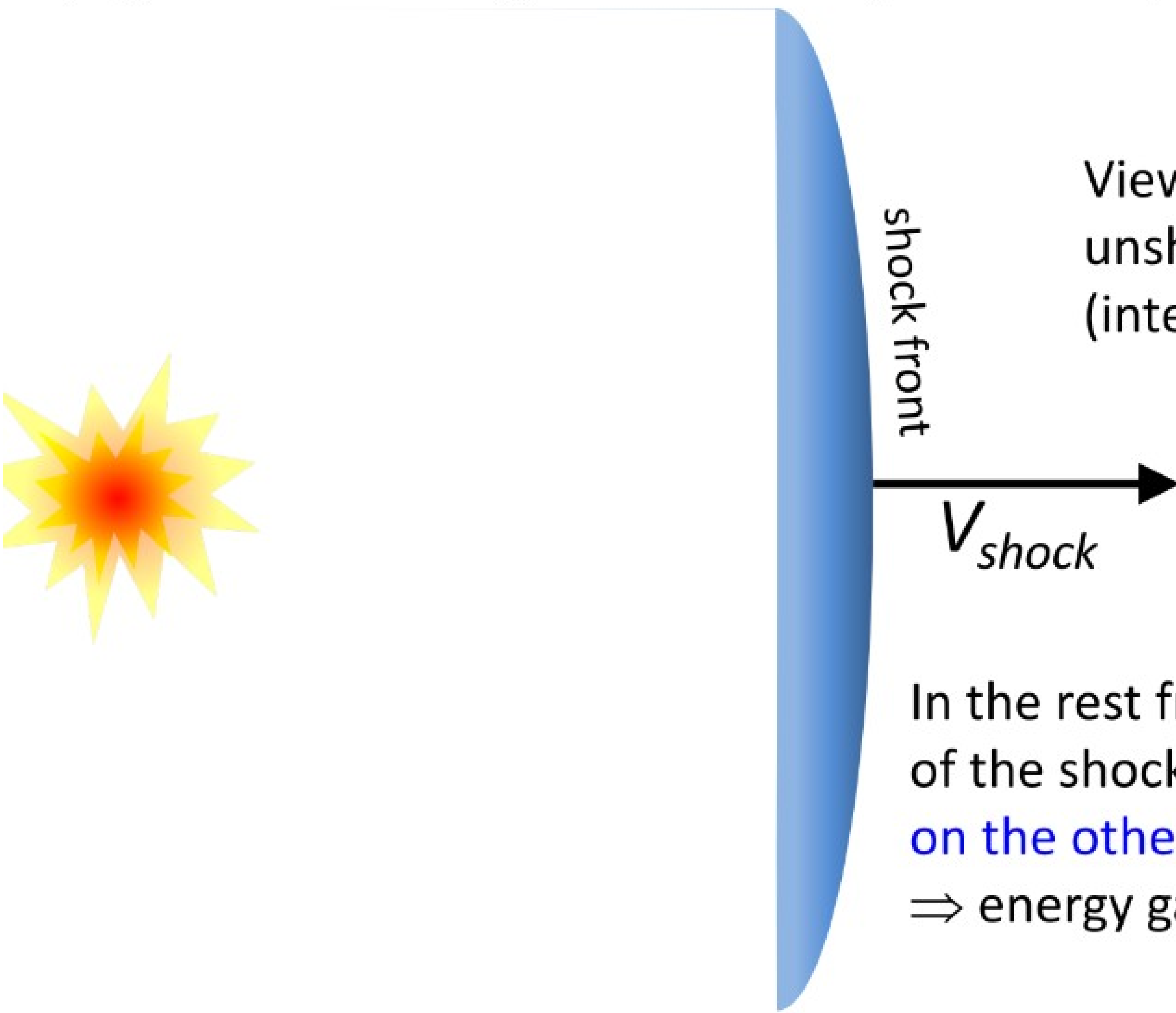
Figure 1. Left-hand panel: The Guitar Nebula in H α, imaged with the 5-meter Hale Telescope at the Palomar Observatory, 1995 (Cordes et al. 1993). Right-hand panel: The head of the Guitar Nebula in H α, imaged with the Hubble Space Telescope in 1994, 2001, and 2006 (Gautam et al. 2013). The change in shape traces out the changing density of the ISM.

Toropina, Romanova & Lovelace 2019, MNRAS 484, 1475

First order Fermi acceleration
(Bell 1978, Blandford & Ostriker 1980)

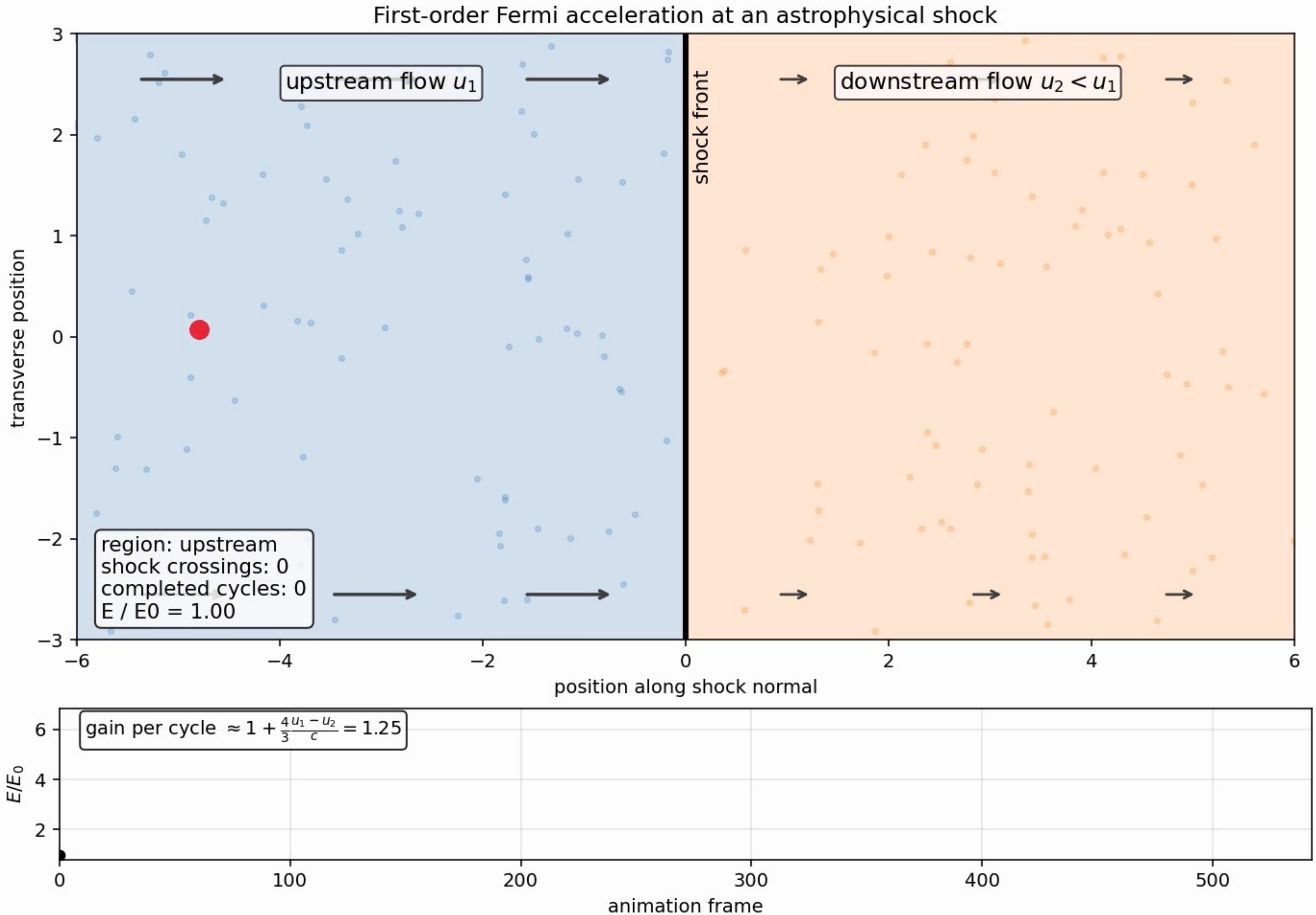
Supersonic shocks arise in various astrophysical scenarios (e.g. shock travelling ahead of the ejecta of a supernova)





Viewed in the frame of the unshocked medium (interstellar medium)

In the rest frame of the fluid in either side of the shock, a particle sees the medium on the other side of the shock approaching
$\Rightarrow$ energy gain in *every* shock crossing

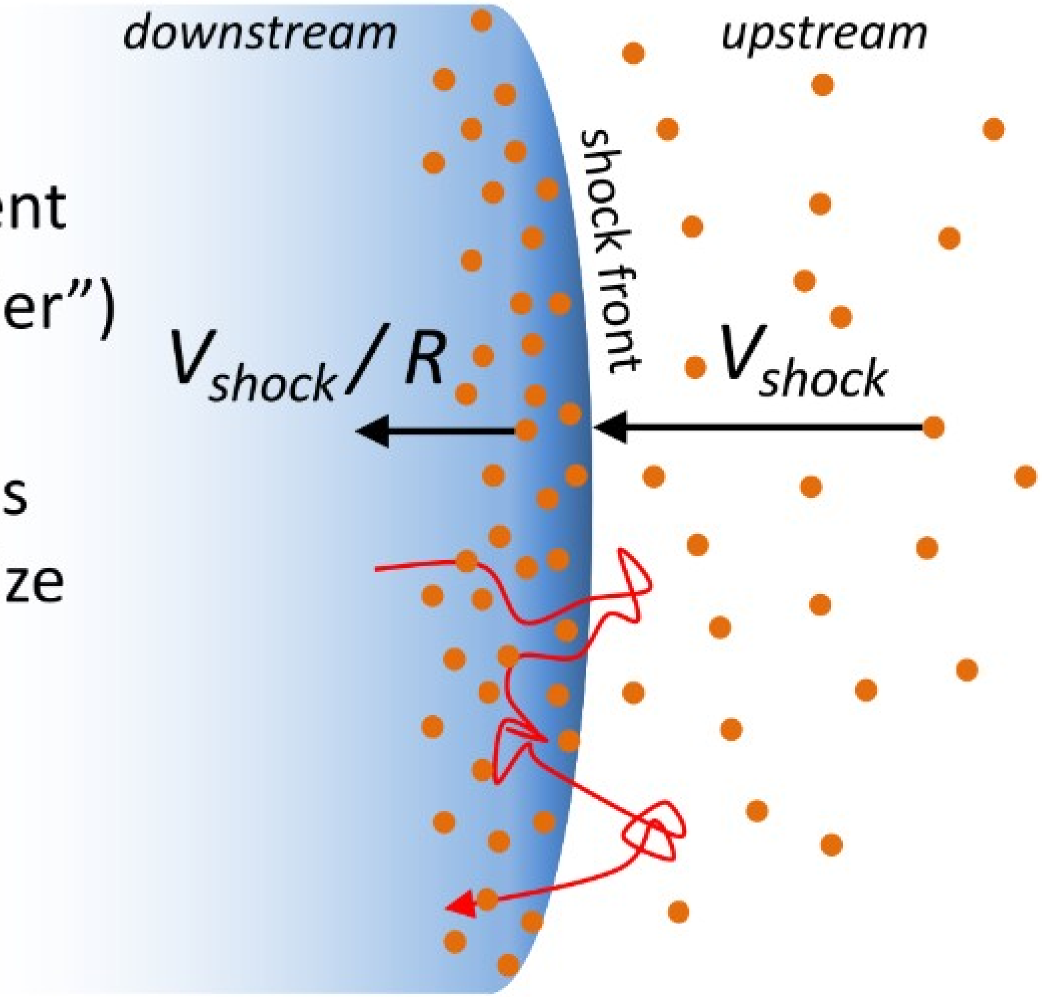


First-order Fermi acceleration at an astrophysical shock
upstream flow u_1
shock front
downstream flow $u_2 < u_1$
region: upstream
shock crossings: 0
completed cycles: 0
E / E0 = 1.00
transverse position
position along shock normal
gain per cycle $\approx 1 + \frac{4}{3}\frac{u_1 - u_2}{c} = 1.25$
E/E_0
animation frame

First order Fermi acceleration

R : shock compression ratio

Particle energy gain is more efficient
$\langle \Delta E/E \rangle \propto (V/c)$ (hence "first order")



Particles keep being accelerated as long as their gyroradius is $\lesssim$ the size of the acceleration region

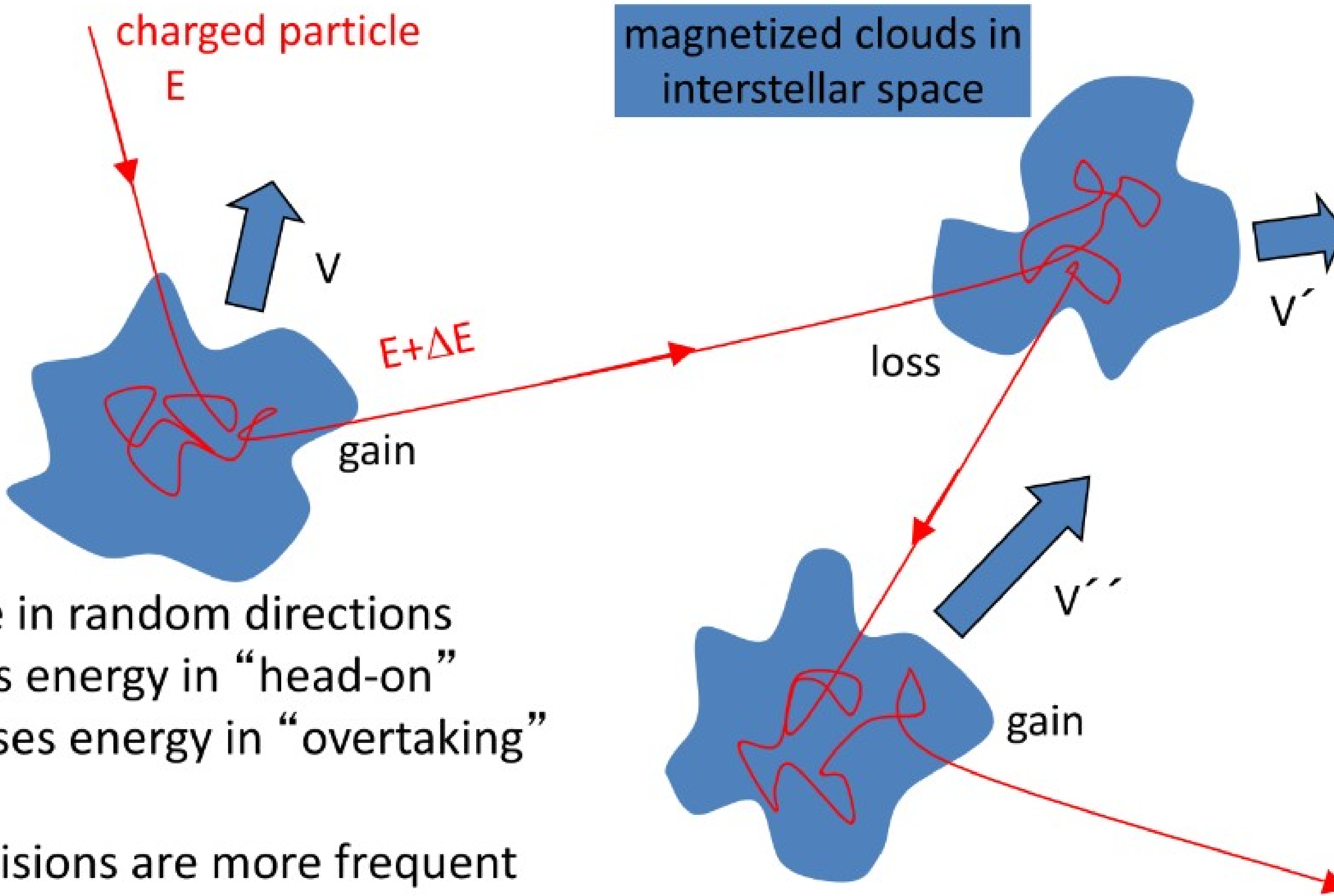
Produces power-law spectrum:

$$dN/dE \propto E^{-\frac{R+2}{R-1}}$$

Viewed in the <u>shock rest frame</u>
(E-*loss* when reflected in downstream medium, but larger E-gain when reflected in upstream)

Strong shock *(R=4)* $\Rightarrow$ $dN/dE \propto E^{-2}$

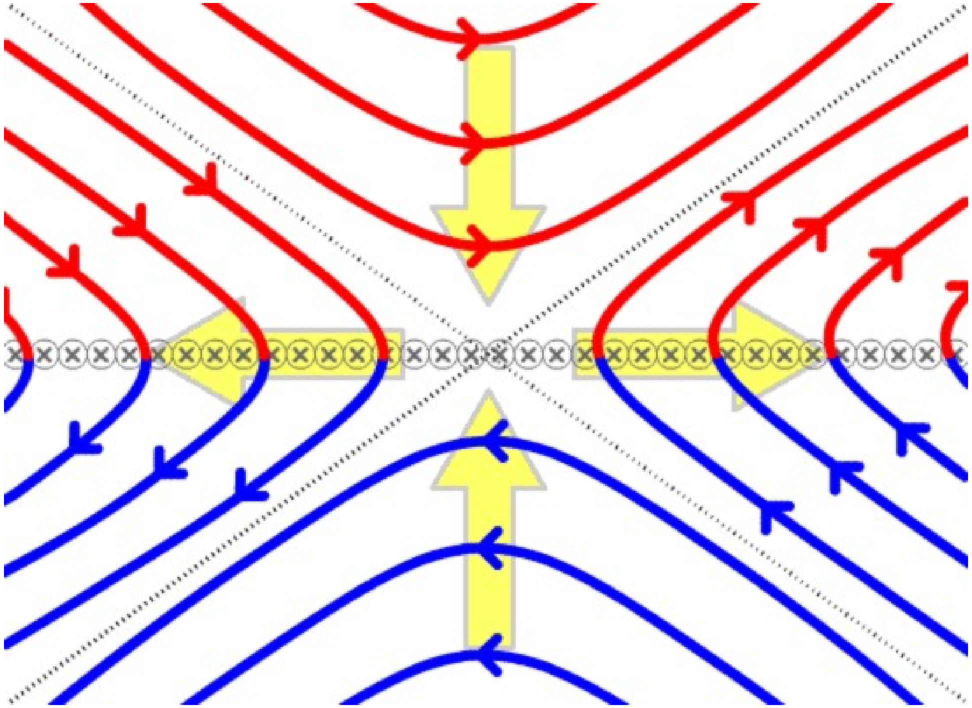
Second order Fermi acceleration



- Clouds move in random directions
- particle gains energy in "head-on" collisions, loses energy in "overtaking" collisions
- Head-on collisions are more frequent

- Therefore in average there is a net energy gain $\langle \Delta E/E \rangle \propto (V/c)^2$
- Inefficient: $(V/c) \sim 10^{-4} \Rightarrow \langle \Delta E/E \rangle \sim 10^{-8}$ Insufficient to overcome ionization losses

Magnetic reconnection



Ideal MHD:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

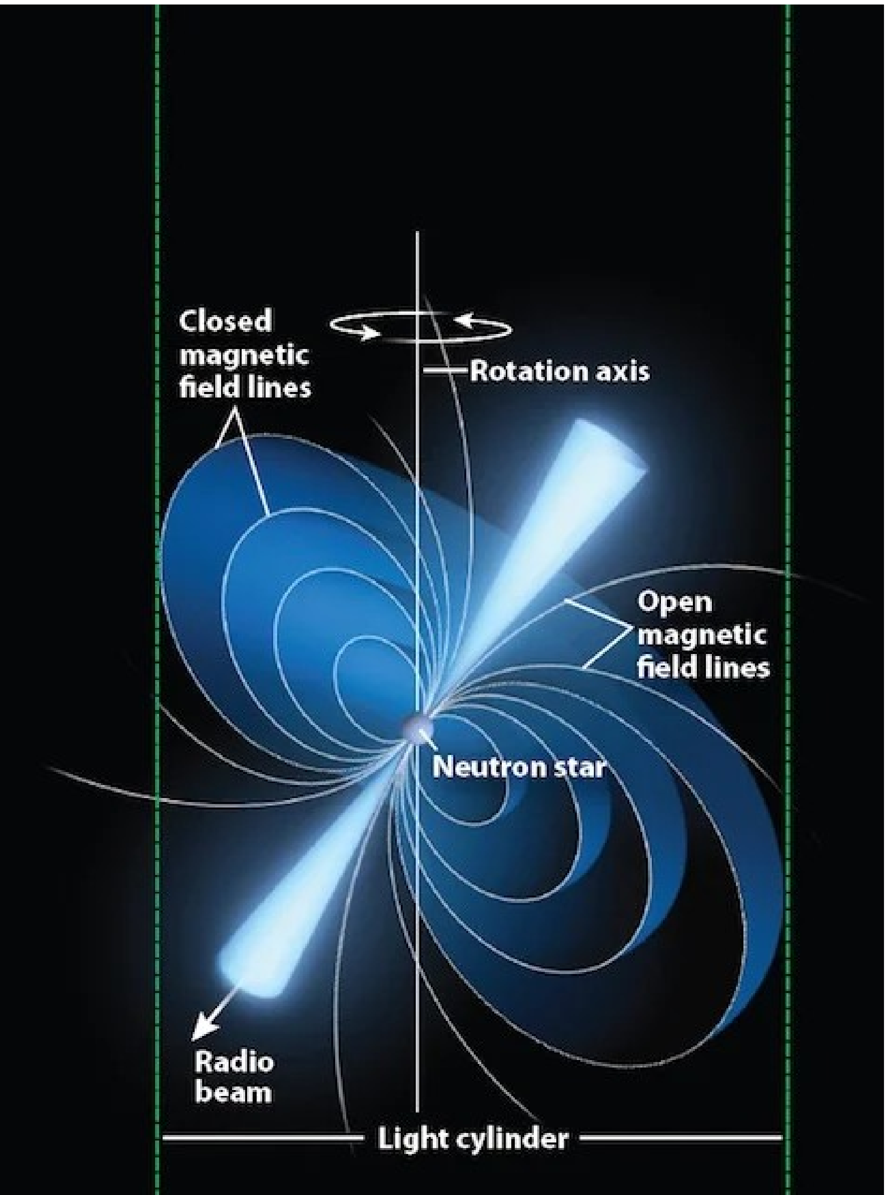
Non-ideal MHD:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left[\eta \boldsymbol{J} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{n_e} - \frac{\nabla p_e}{n_e} \right]$$

Ideal MHD means the plasma is a perfect conductor. B frozen in plasma.
So if magnetic field lines are separate, they stay separate.
In realistic MHD you have resistive diffusion, Hall term, electron-pressure term, etc. → reconnection. Outflow leaves with ~Alfvenic speed: $v_A = \frac{B}{\sqrt{4\pi\rho}}$

Inflow → current sheet ←Inflow  ↑ Outflow ↓

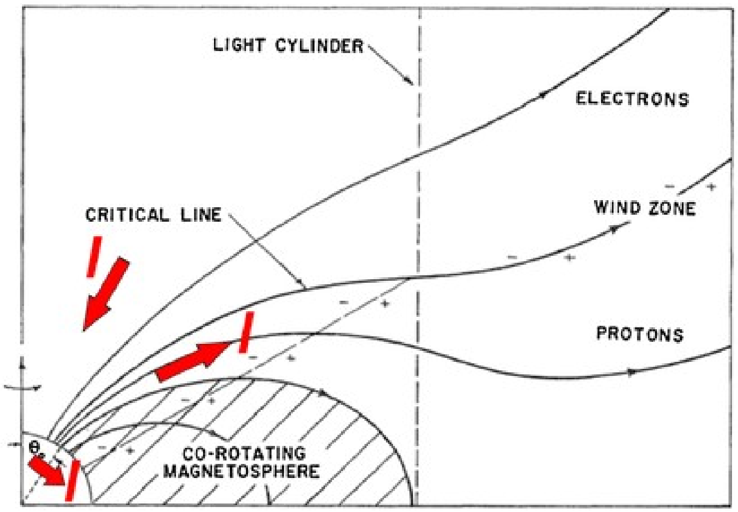
Direct electric-field acceleration in magnetospheres



Acceleration of charges occurs also in pulsar magnetospheres.

The charges are pushed along the open Magnetic field lines

Goldreich-Julian Density



Let's assume that the magnetic field is a dipole with the magnetic axis aligned with the rotation axis. The magnetic field at a distance r from the neutron star is:

$$\mathbf{B} \approx B_0 \left(\frac{R}{r}\right)^3 \hat{z}$$

The plasma within the magnetosphere is assumed to co-rotate with the neutron star. The co-rotation velocity at a point is:

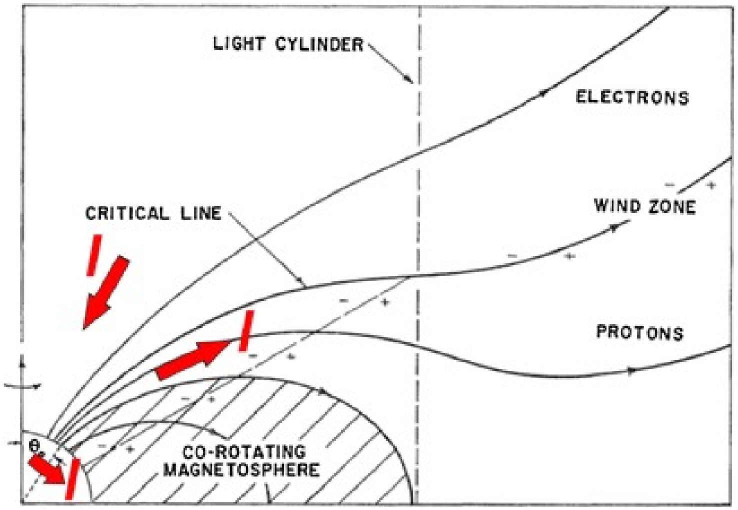
$$v = \Omega \times r$$

According to Faraday's law, a changing magnetic flux induces an electric field.

In the co-rotating frame, this electric field can be expressed as:

$$E = -\frac{\Omega \times (r \times B)}{c}$$

Goldreich-Julian Density



This becomes (for a simple case where Omega and r are parallel):

$$E = -\frac{\Omega r B \sin\theta}{c}\hat{\Phi}$$

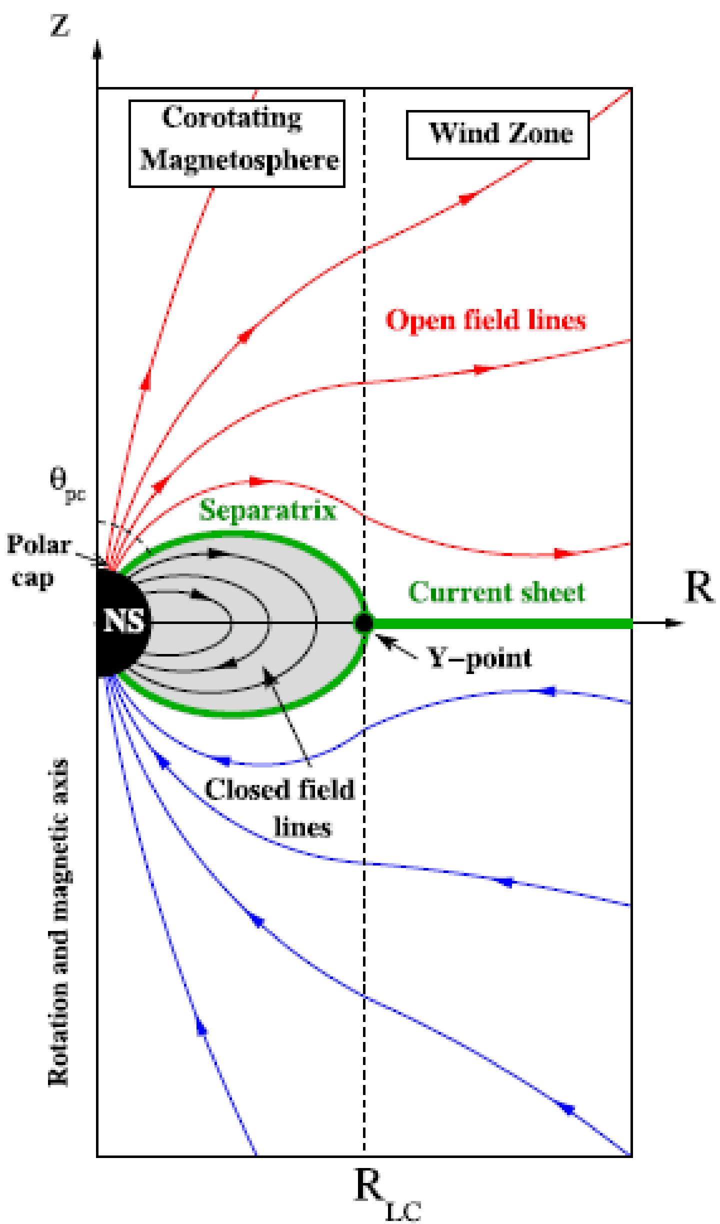
where phi is the azimuthal direction.

Now, in a force-free magnetosphere (i.e. no parallel electric & magnetic fields):

$$E \cdot B = 0$$

To eliminate the parallel electric field, we need the right charge density to screen out the component of the electric field that would accelerate particles along magnetic field lines. We can then use Gauss law to find which charge density we need to do that.

Goldreich-Julian Density

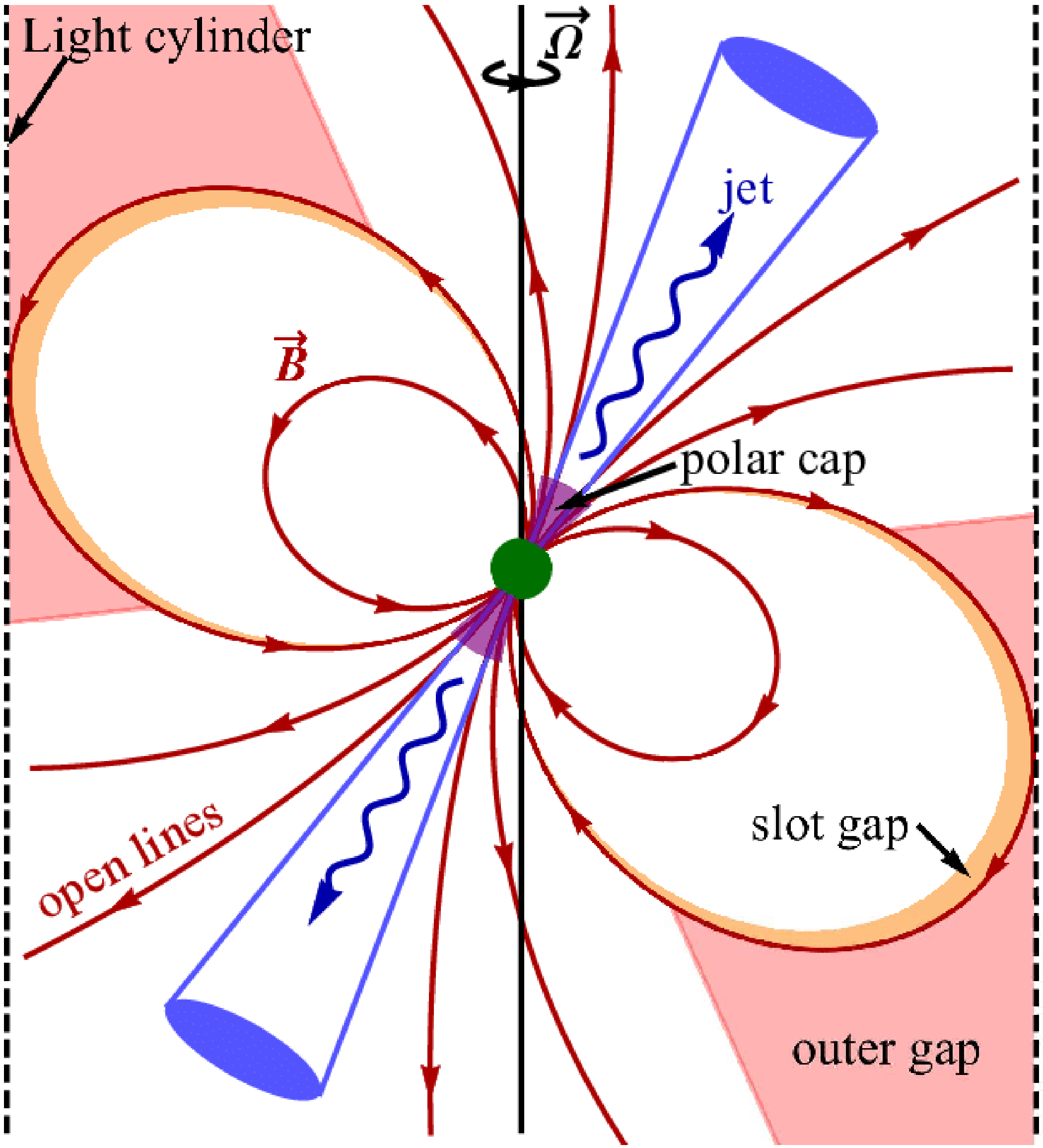


From Gauss' law we find the so-called Goldreich-Julian density:

$$\rho_{GJ} = -\frac{\Omega \cdot B}{2\pi c}$$

(To find it, remember the triple cross product A x (B x C) = B(AC) – C(AB), and apply it to the Electric field formula, then take the divergence).

The force-free magnetosphere means no parallel electric field. Gauss's law provides the correct charge density to ensure this by neutralizing the electric field component along the magnetic field.
Without satisfying Gauss's law, the magnetosphere would develop accelerating fields, breaking co-rotation and leading to **particle outflows and gaps**.



Gaps

The pulsar magnetosphere cannot perfectly maintain the force-free condition everywhere because some regions are “charge-starved”, especially near the magnetic poles and light cylinder.
The force-free condition is locally violated in specific regions called **gaps**, where the required charge density is not present.

A. **Inner Gap** (Polar Cap Region):
Located near the neutron star surface, around the magnetic poles.
The neutron star surface cannot supply enough charges to meet the Goldreich-Julian density. As a result the parallel electric field is not completely canceled. Particles are accelerated along magnetic field lines, producing curvature radiation and pair production.

B. **Outer Gap** (Light Cylinder Region):
Located further out near the light cylinder, where magnetic field lines are open. Plasma created in the inner gap or polar region cannot fill the entire magnetosphere. The plasma density decreases as field lines stretch, causing a deficit of charges. The parallel electric field reappears, accelerating particles to high energies and emitting gamma rays.