

Pulsars

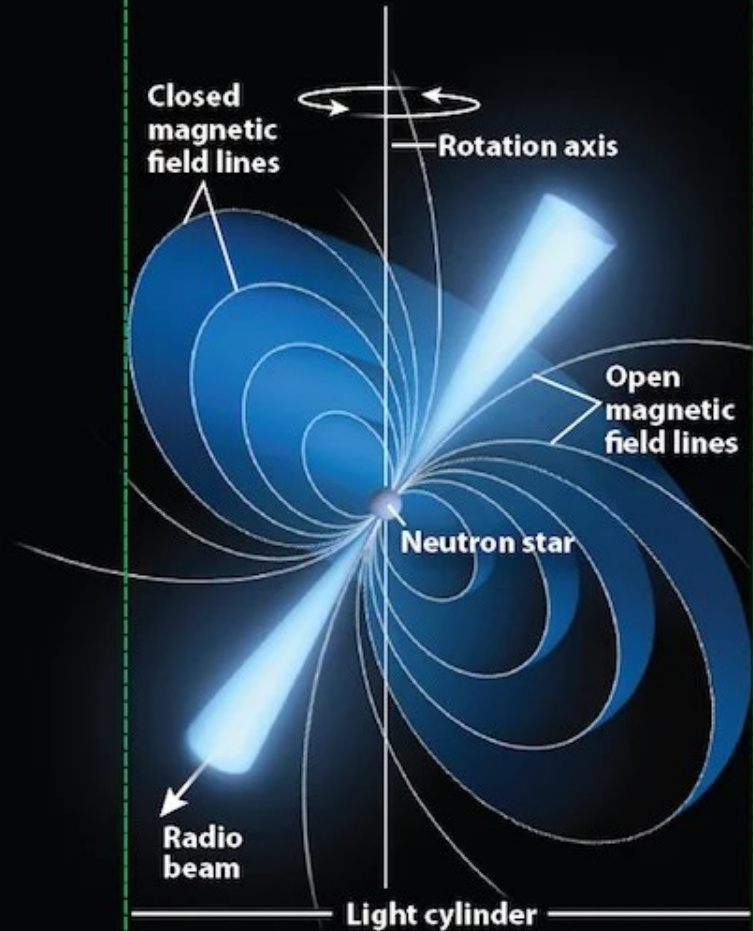
The magnetic field lines are (approximately!) dipolar and open at the light cylinder.

A pure magnetic dipole field at the magnetic pole of the star B is related to the magnetic moment $\mathbf{m}$ by:

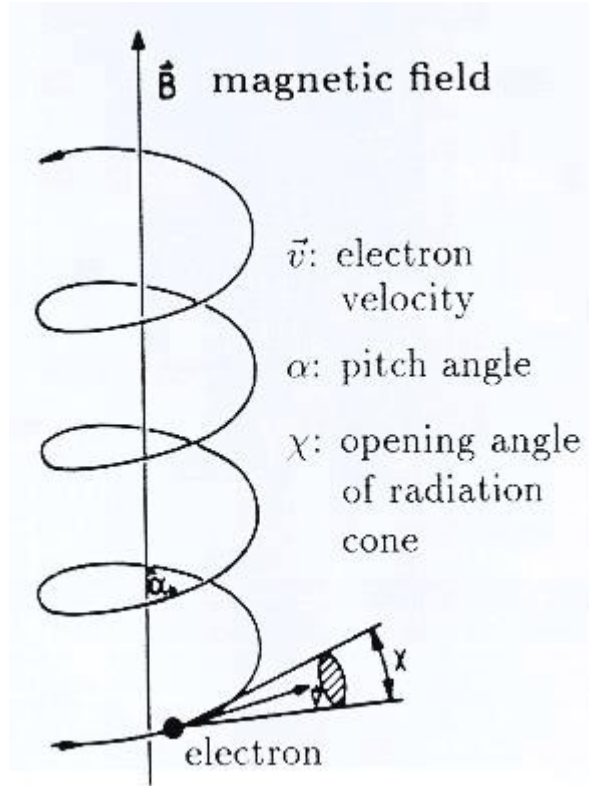
$$\mathbf{m} = \frac{B R^3}{2} \hat{m} e^{-i\Omega t} \quad |\mathbf{m}| = \frac{B R^3}{2}$$

At infinity, such configuration is seen to emit radiation because of the time-varying dipole moment:

$$P_{rad} = \dot{E} = -\frac{2}{3c^3} |\ddot{\mathbf{m}}|^2 = -\frac{B^2 R^6 \Omega^4 \sin^2 \alpha}{6c^3}$$



Curvature Radiation

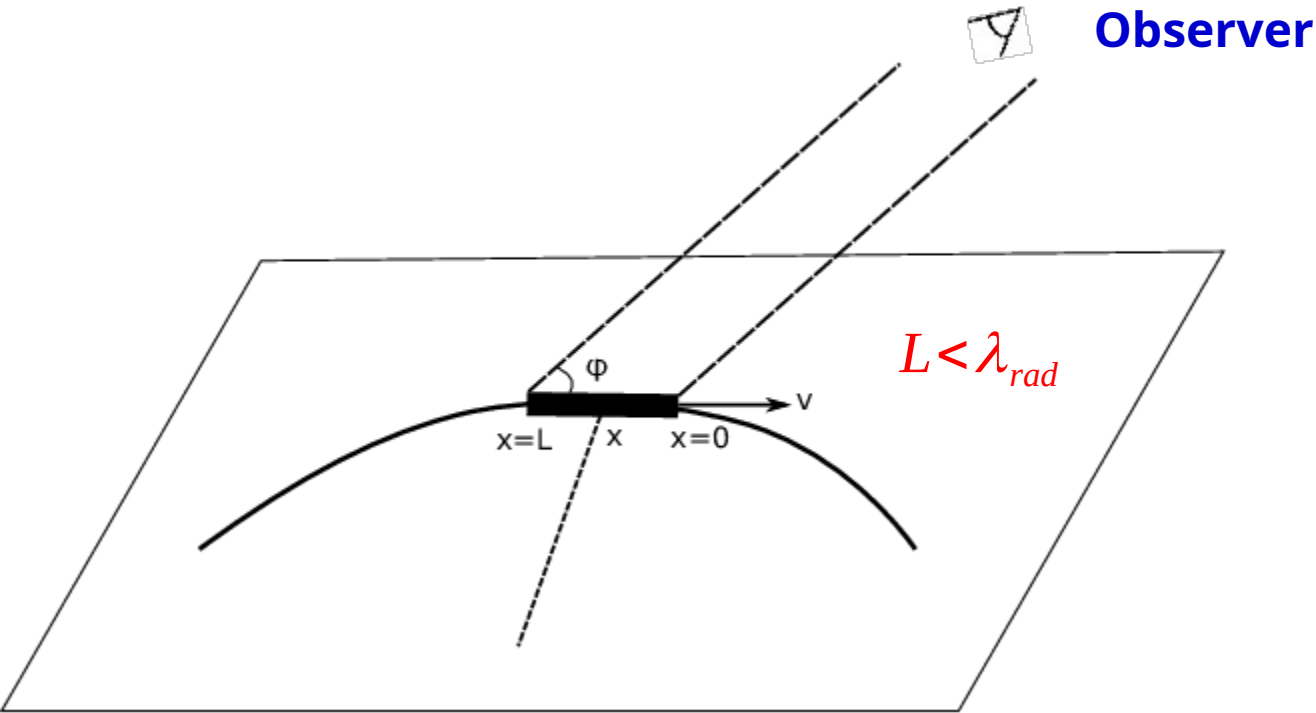


For synchrotron radiation the pitch angle α defines the direction of emission when the electron moves around a magnetic field line.

The parallel velocity is undisturbed and follows the straight parallel magnetic field component. The perpendicular component of the velocity changes because of the Lorentz force and the relativistic circular motion produces a beaming with half-width equal to $1/\text{Lorentz factor}$.

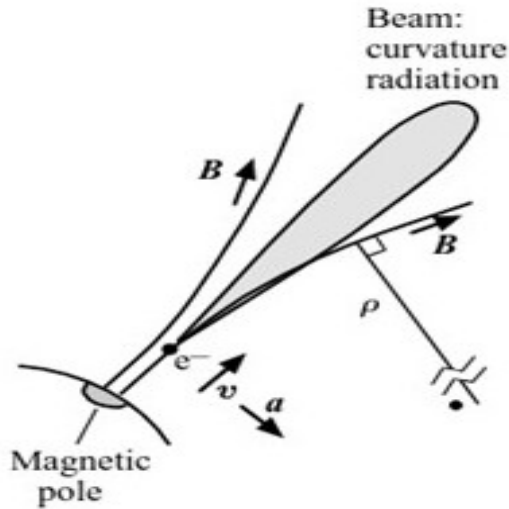
But what happens if $B_{\parallel}$ is also curved?

Curvature Radiation



If the electrons are very energetic and bunched together, in the sense that they are very close together (closer than the typical wavelength of the radiation) then each bunch (in the figure is the black bar) radiates as a single *super-electron* and the power emitted can dominate.

Curvature Radiation



Because the electron bunches emit radiation while they are confined to a length $L < \lambda_{rad}$ their radiation might be **coherent**.

Note that synchrotron, Compton, bremsstrahlung and blackbody emission are all incoherent emission processes.

Because the length of the bunch is fixed, while the emitted power covers a broad range, coherent curvature radiation is observed typically in radio, while the optical/X-ray/gamma-ray photons will be incoherent.

Take Rayleigh-Jeans approximation and check if the temperature exceeds that of a blackbody. Blackbody is the most efficient radiator (maximum entropy). No other incoherent emission process can be more efficient.

$$I(\nu) = \frac{2kT\nu^2}{c^2}$$

Characteristic Frequency of Emission

The radius of curvature is:

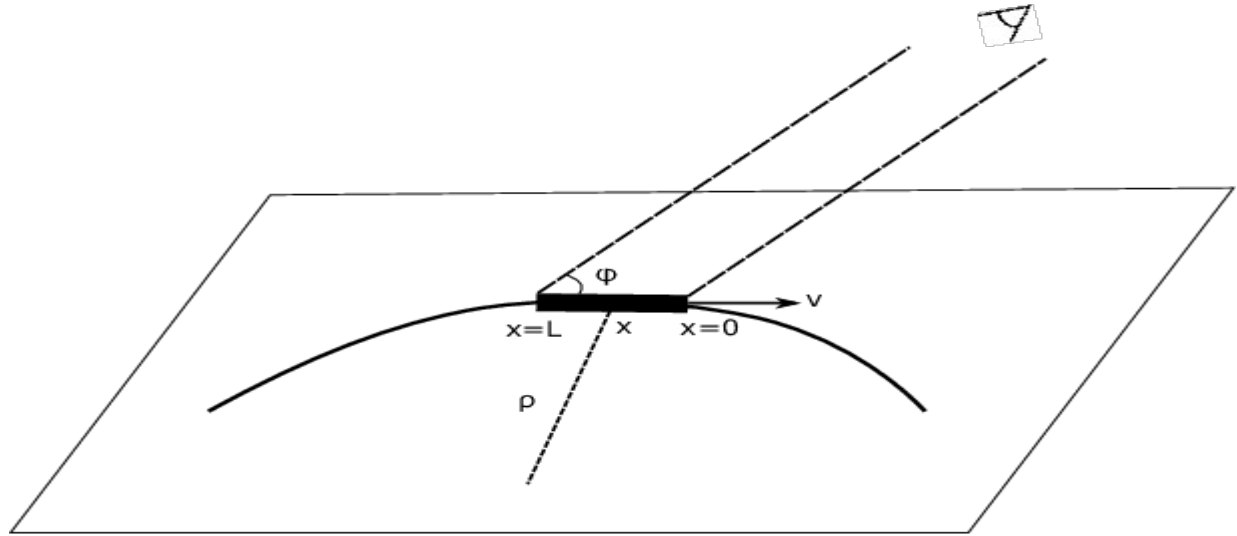
$$\rho \simeq c u^2 / a$$

This generates a radiation
at an effective frequency of
emission:

$$\nu_{cur} = \nu_L \gamma^2$$

where now the Larmor frequency is:

$$\nu_L = \frac{c}{2\pi\rho}$$



Curvature Radiation Power

Remember that emitted power of an accelerating charge is:

$$P \propto q^2 a^2$$

If you have N charges emitting *incoherently* (e.g., synchrotron):

$$N \cdot P \propto N q^2 a^2$$

If you have N charges emitting *coherently* (e.g., curvature radiation):

$$N \cdot P \propto N^2 q^2 a^2$$

where (Nq) is a sort of “single charge” for our electron bunch.