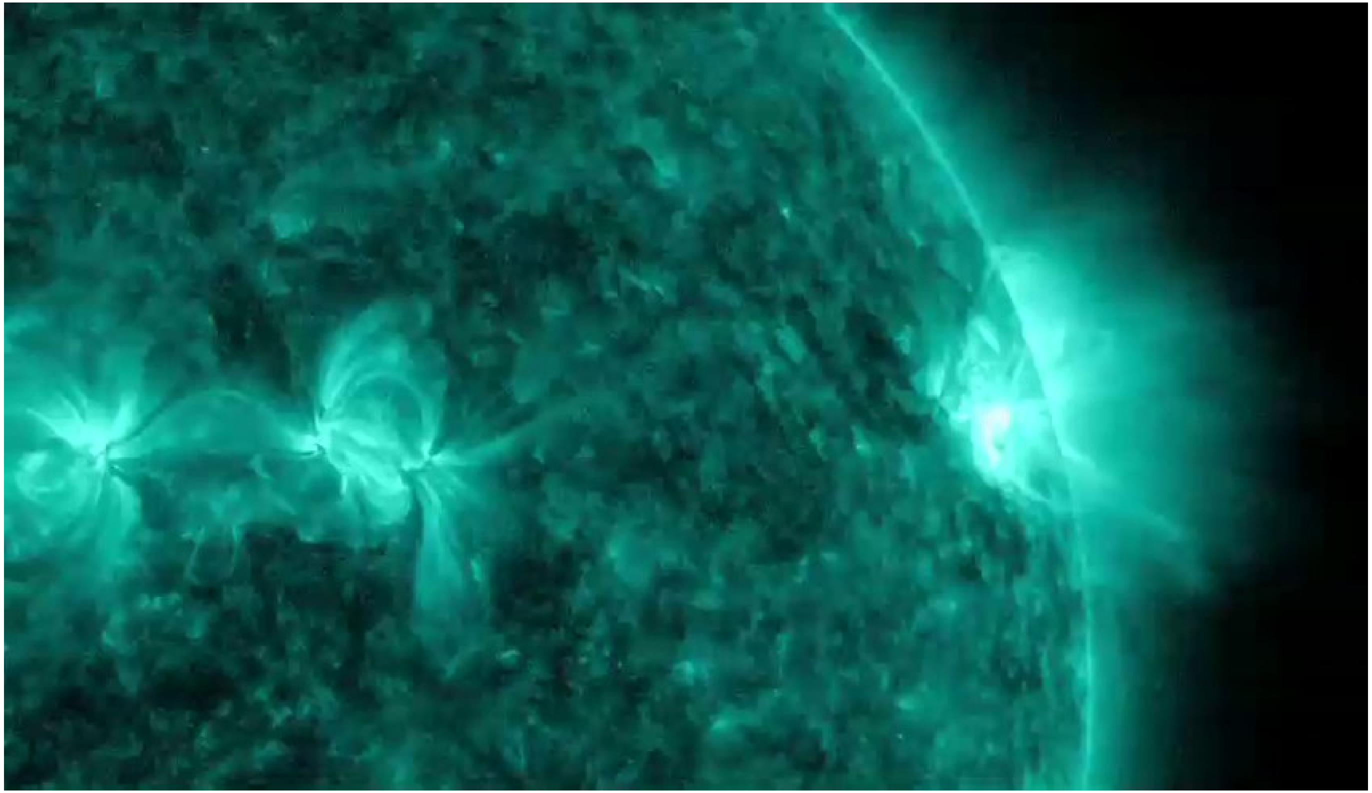
# Radiative Processes

Outline:

Leptonic radiative processes:
- Bremsstrahlung (free-free) radiation (thermal & non-thermal)
- Synchrotron Radiation (non-thermal)
- Curvature Radiation (non-thermal)
- Inverse Compton (thermal and non-thermal)
- Synchrotron Self-Compton
- Cherenkov Radiation

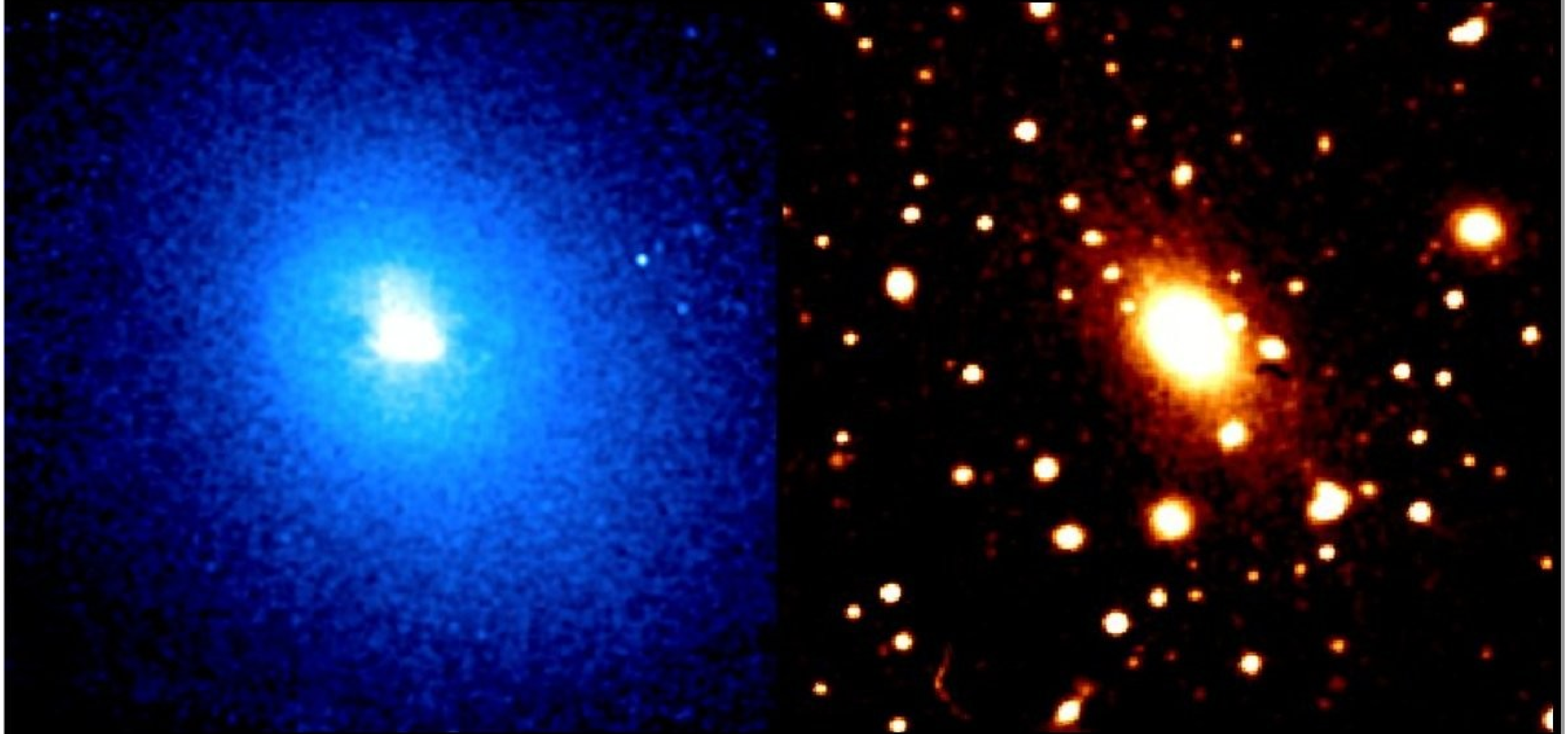
Hadronic high-energy emission channels:
- Proton-proton
- Proton-photon
- Proton-synchrotron

# Bremsstrahlung Radiation



# Abell 2199

Chandra (X-ray) DSS

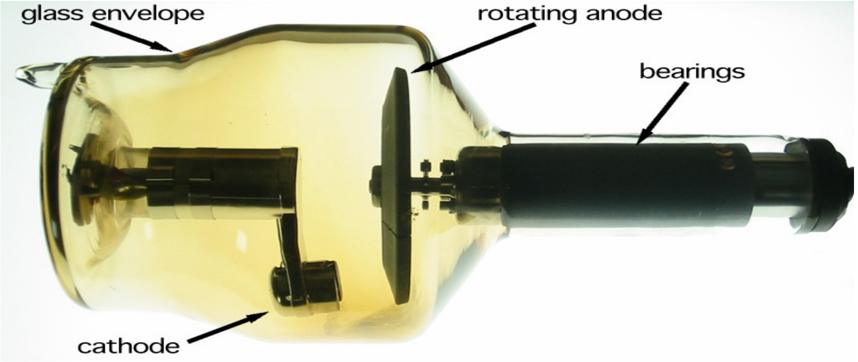


redshift, $z$ = 0.0309

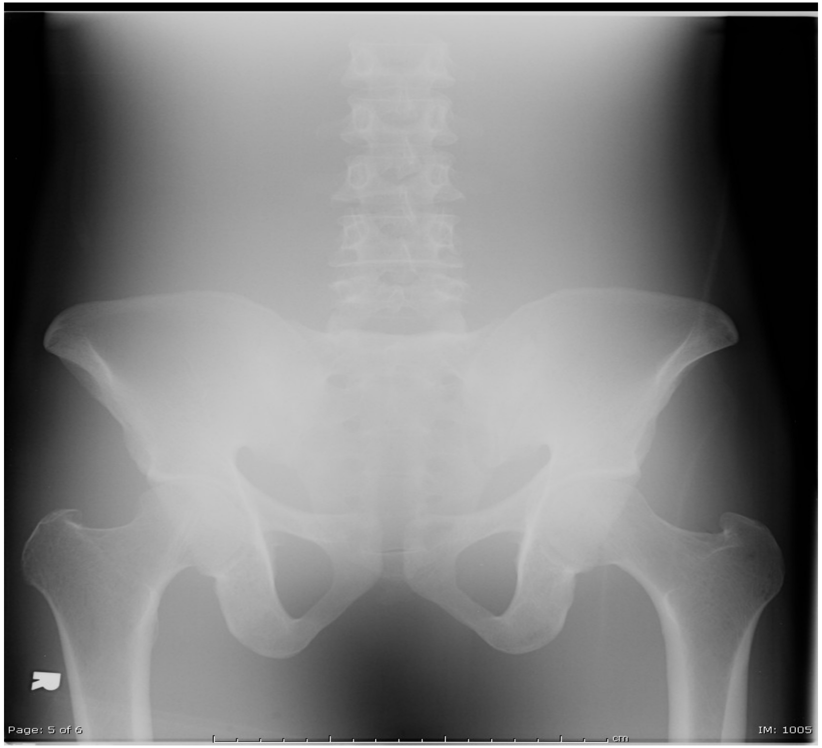
50 thousand light years

X-ray image of Perseus A taken with the Chandra telescope, showing a central bright source, two ``bubbles'' above and below this source, and the shadow of an infalling galaxy (top right). The remaining emission is from the central part of a cooling flow (the inward flow of hot gas into the potential well of the elliptical) which is losing energy (i.e. cooling) via thermal bremsstrahlung emission in the X-ray region.

# An Example in Everyday Life

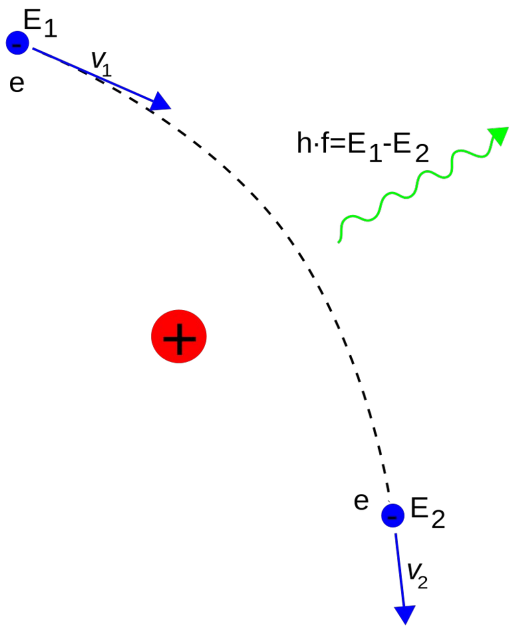



X-Rays used in medicine (radiographics)
are generated via the Bremsstrahlung process.



# In a nutshell:

Bremsstrahlung radiation is emitted when a charged particle is deflected by another charge.



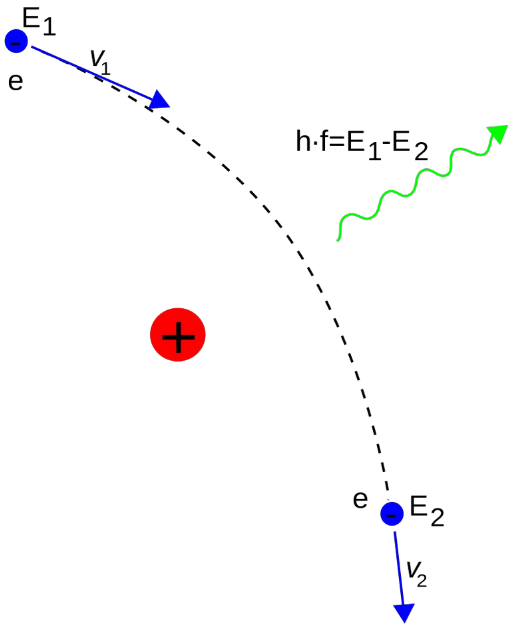

If plasma produces radiation in this way and the radiation can escape the environment without further interaction (i.e., the plasma is optically thin) then you will see Bremsstrahlung radiation.

This type of radiation is seen *very often* in astrophysical phenomena:

- Intracluster medium (X-rays)
- solar flares (X-rays)
- isolated neutron stars (X-rays)
- neutron star binaries (X-rays)
- black hole binaries (X-rays)
- supermassive black holes (X-rays)
- HII regions in the Milky Way (radio)
- Astrophysical Jets (radio)

# Bremsstrahlung Radiation

Bremsstrahlung seems a simple process, but in reality is complicated because the energy of the radiation emitted might be comparable to that of the electron producing it.




This means we need a quantum treatment. Particles can also move relativistically, so we might need a relativistic treatment.

**However, a classical treatment works well** in most cases and the quantum corrections can be introduced as corrections (Gaunt factor)

Finally, the relativistic treatment will be seen later as a special case.

# Ensemble of Particles

So far so good, but what about an ensemble of particles? After all if we want to calculate the properties of Bremsstrahlung radiation we need to consider a lot of particles...

There is a complication here, because the expressions for the radiation fields refer to conditions at retarded times, and these retarded times will differ for each particle and we have an *enormous* amount of particles...

Abell 1689 (z=0.18, i.e., about 2 billion light years away)

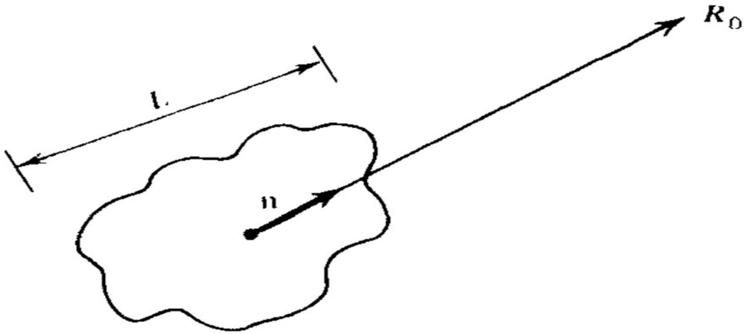
Solution: Let the typical size of the system be L, and let the typical time scale for changes within the system be T. If T is much longer than the time it takes light to travel a distance L, $T \gg L/c$, then the differences in retarded time across the source are negligible.

Does this happen for example in an intra-cluster plasma?

# Ensemble of Particles

Now our radiation field is:

$$\mathbf{E}_{\rm rad} = \sum_i \frac{q_i}{c^2} \frac{\mathbf{n}\times(\mathbf{n}\times\dot{\mathbf{u}}_i)}{R_i}$$




Of course we have *no idea* what are the single velocities of each particle, neither we know how many particles there are!

Solution: call L the size of our cluster.
Call R0 the distance from some point in the system to the field point (i.e., where we are since we are measuring this field).
But now you see that the difference between each Ri tends to zero as R0 → infinity (since we are very far away from the cluster!). So we can write:

$$\mathbf{E}_{\rm rad} = \frac{\mathbf{n}\times(\mathbf{n}\times\ddot{\mathbf{d}})}{c^2 R_0}$$

where $\mathbf{d} = \sum_i q_i \mathbf{r}_i$. is the dipole moment of the charges.

# Dipole Approximation

Following the same procedure as for the single particle case, we can find the total power emitted by an ensemble of particles (in the **non-relativistic** limit)
in the so-called *dipole approximation:*

$$\frac{dP}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3} \sin^2\Theta$$

$$P = \frac{2\ddot{\mathbf{d}}^2}{3c^3} .$$

Using Fourier transform we can easily find that:

$$\ddot{d}(t) = -\int_{-\infty}^{\infty} \omega^2 \hat{d}(\omega) e^{-i\omega t} d\omega,$$

$$\hat{E}(\omega) = -\frac{1}{c^2 R_0} \omega^2 \hat{d}(\omega) \sin\Theta.$$

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} |\hat{d}(\omega)|^2.$$

Now we have the key to understand Bremsstrahlung radiation…

# Dipole Approximation

Following the same proc
emitted by an ensemble
in the so-called *dipole ap*

**Remember how the Fourier Transform of a derivative works:**

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t} d\omega$$

$$f'(t) = \frac{d}{dt}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t} d\omega\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\omega\, F(\omega)\, e^{i\omega t} d\omega$$

Using Fourier transform we can easily find that:

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$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2.$$

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# Dipole Approximation

**This term instead comes from:**

$$\mathbf{E}_{\mathrm{rad}} = \frac{\mathbf{n}\times(\mathbf{n}\times\ddot{\mathbf{d}})}{c^2 R_0}$$

$$E(t) = \ddot{d}(t)\frac{\sin\Theta}{c^2 R_0}$$

d the total power

Using Fourier transform we can easily find that:

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# Dipole Approximation

Following the same procedure as for the single particle case, we can find the total power emitted by an ensemble of particles (in the **non-relativistic** limit) in the so-called *dipole approximation:*

$^2\Theta$

**This comes from Parseval's Theorem.**

$$\int_{-\infty}^{\infty} E^2(t)\,dt = 2\pi \int_{-\infty}^{\infty} |\hat{E}(\omega)|^2\, d\omega$$

**Total energy per unit area in a pulse:**

$$\frac{dW}{dA} = \frac{c}{4\pi}\int_{-\infty}^{\infty} E^2(t)\,dt.$$

$$\ddot{d}(t) = -\int_{-\infty}^{\infty} \omega^2 \hat{d}(\omega) e^{-i\omega t}\, d\omega,$$

$$\hat{E}(\omega) = -\frac{1}{c^2 R_0}\omega^2 \hat{d}(\omega)\sin\Theta.$$

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2.$$

Now we have the key to understand Bremsstrahlung radiation…

# Spectrum of an ensemble of particles with a single velocity v

Now the treatment on how to choose the boundaries of the integration becomes quite lengthy and complicated. We are interested only in a few features that will determine how the final spectrum will look like.

The final spectrum of plasma with electron having a <u>single velocity **v**</u> will look like the following:

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\; c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega).$$

$$\left( P_\nu = 4\pi\, j_\nu = \frac{dW}{d\nu\, dV\, dt} \right)$$

The Gaunt factor contains quantum corrections which we have not taken properly into account here, but can be approximated as:

$$g_{ff}(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\left( \frac{b_{\max}}{b_{\min}} \right)$$

# Thermal Bremsstrahlung

How do we go from the spectrum of an ensemble of ions and electrons (with the latter all having a single velocity **v)** to the spectrum of an ensemble of ions and electrons with a distribution of velocities?

First we need to know *which distribution* of velocities.

Let's take the most common case (almost always the case in astrophysics) which is that of a thermal plasma, i.e., electrons and ions with velocities distributed according to the Maxwell-Boltzmann distribution.

$$F(v)\,dv = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\,dv$$

We then need to integrate $\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\, c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega).$ over this distribution

# Spectrum:Thermal Bremsstrahlung

$$\frac{dW(T,v)}{dV\,dt\,dv} = \frac{\int_{u_{\min}}^{\infty} \frac{dW(u,v)}{dV\,dt\,dv} u^2 \exp\left(\frac{-mu^2}{2kT}\right) du}{\int_{0}^{\infty} u^2 \exp\left(\frac{-mu^2}{2kT}\right) du}$$

Why there is a minimum velocity in the integral? Shouldn’t we use zero as the minimum?

# Spectrum:Thermal Bremsstrahlung

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Why there is a minimum velocity in the integral? Shouldn't we use zero as the minimum?

The photons need to be created during the deceleration of the electron. So the initial kinetic energy of the electron *must be* larger than the photon energy.
This creates a cutoff in the spectrum and this is due to the *discreteness of photons*, i.e., they are discrete and not continuum entities.

$$u_{min}=(2hv/m)^{1/2}$$

# Spectrum:Thermal Bremsstrahlung

Performing the integration one gets:

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

$\varepsilon_\nu^{ff}$ → (energy/frequency/volume/time).

$j_\nu$ → (energy/frequency/volume/time/*solid angle*).

Also, the symbol $\varepsilon_\nu^{ff}$ is exactly the same as $P_\nu$ in $j_\nu = \frac{1}{4\pi} P_\nu$

# Spectrum:Thermal Bremsstrahlung

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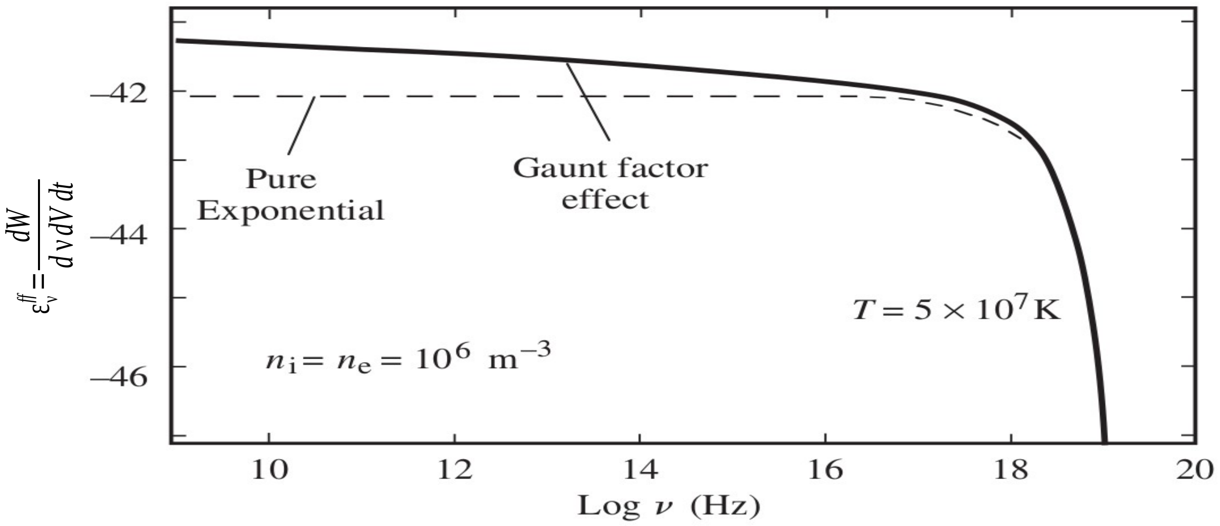
What do we see here?
The emission coefficient seem to depend on the temperature (be careful because T is also in the exponential), on the density of ions and electrons and on the ion charge.
The frequency dependency is only in the exponential. The average Gaunt factor can be considered very close to unity since
this is its order of magnitude.

**The spectrum will be basically flat, except when exp(-h*nu/kT) becomes dominant.**
**This happens when the thermal energy of electrons is basically insufficient to generate high energy photons.**



# Thermal Bremsstrahlung: Absorption

What happens at low frequencies?

Hint: What law we can ***always*** use when we have some emission from matter in thermal equilibrium?

# Thermal Bremsstrahlung: Absorption

If we have thermal emission then we can use Kirchhoff's law!

$$j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T)$$

$$\varepsilon_\nu^{ff} = \frac{dW}{d\nu \, dV \, dt} = 4\pi j_\nu^{ff} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

$$\alpha_\nu^{ff} = 3.7\times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT}) \bar{g}_{ff}$$

We see that when $h\nu \ll kT$ we are in the Rayleigh-Jeans regime:

$$\alpha_\nu^{ff} = 0.018 T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

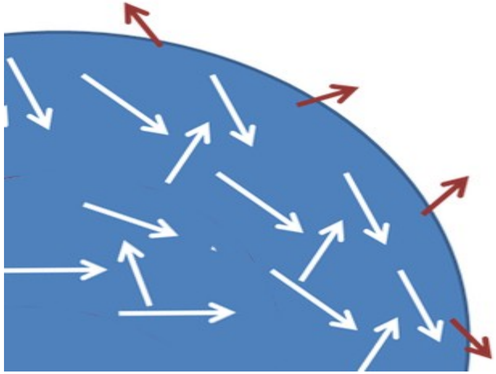
This is telling us that the spectrum of Bremsstrahlung is *self-absorbed* at low frequencies.
Why?

# Thermal Bremsstrahlung

Remember that the optical depth is defined as: $d\tau_\nu = \alpha_\nu ds$

Therefore since $\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}$. we have that $\tau_\nu \propto \nu^{-2}$ as well.

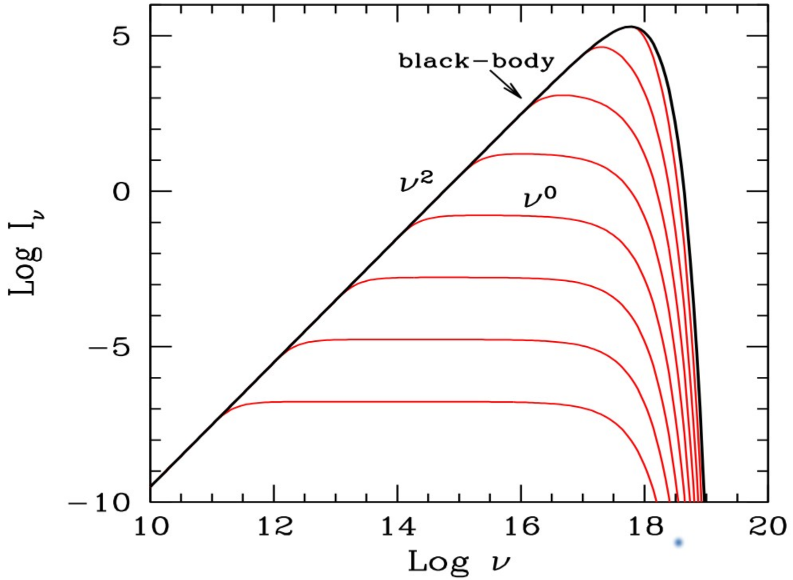
The smaller the frequencies, the larger the optical depth. This means that radiation is absorbed more and more before leaving the system. But this is precisely what a blackbody is! So at *low frequencies* we expect a blackbody like spectrum.



So what is the *specific brightness* of Bremsstrahlung radiation?

At low frequency we expect it to look like blackbody.
At intermediate frequencies it has to be flat
At high frequencies there must be an exponential cutoff

# Thermal Bremsstrahlung




Now we can understand the spectrum of Bremsstrahlung!

At large optical depths: $\dfrac{j_\nu}{\alpha_\nu} = S_\nu = B_\nu$

At small optical depths: $I_\nu(\tau_\nu) = j_\nu R = \dfrac{\varepsilon_\nu^{ff} R}{4\pi}$

$$\varepsilon_\nu^{ff} = \frac{dW}{d\nu\, dV\, dt} = 4\pi j_\nu^{ff} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

If the region of size R has large optical depth at any frequency then Bremsstrahlung becomes Blackbody spectrum (black solid line). Otherwise is will show the typical flat spectrum in the intermediate frequencies (optically thin), blackbody spectrum at low frequencies (optically thick part) at cutoff at high frequencies

# Cooling Time

Since we know how much does a thin plasma radiate we can calculate the energy losses and the so-called cooling time:

$$\frac{\text{energy density of the plasma}}{\text{rate of energy loss}} = \frac{\frac{3}{2}(n_e+n_i)kT}{\varepsilon^{ff}} = \frac{3nkT}{\varepsilon^{ff}}$$

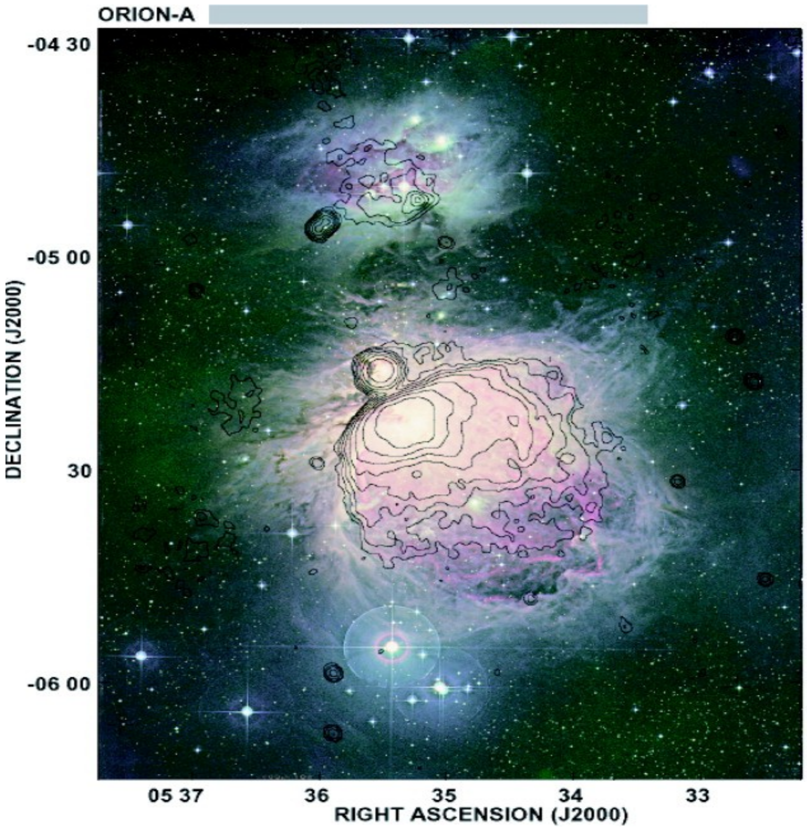
Here we have integrated the emission coefficient $\varepsilon_\nu^{ff}$ over all frequencies:

$$\varepsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times10^{-27}T^{1/2}n_e n_i Z^2 \bar{g}_B \longrightarrow L=\varepsilon^{ff}V$$

This is useful to calculate the cooling time:

$$\tau_{cool}=6\times10^3 T^{1/2} n_e^{-1} \bar{g}_{ff}\, yr$$

# Cooling Time: HII regions




The Orion nebula is an HII region.
Here you see the radio continuum overlaid to the optical image. The radio continuum is Bremsstrahlung emission.

What is the cooling time of the nebula?

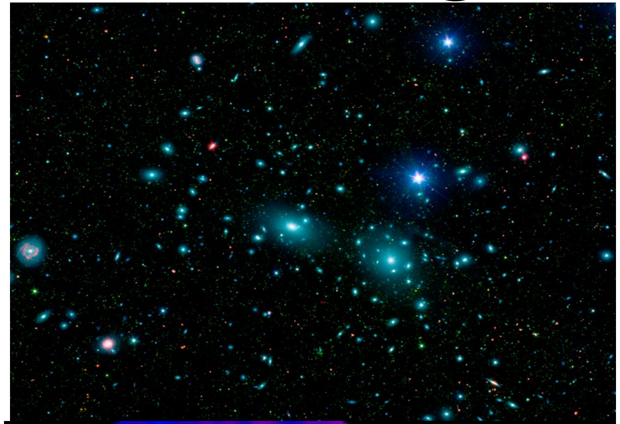
$$\tau_{cool} = 6 \times 10^3 T^{1/2} n_e^{-1} \bar{g}_{ff} \, yr$$

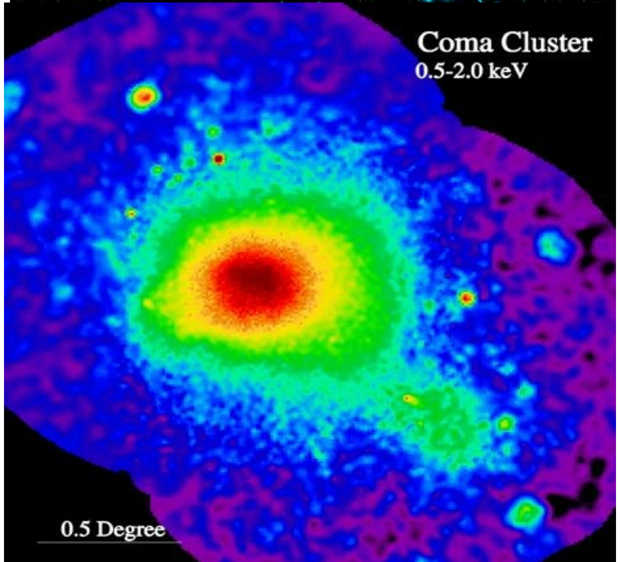
Here ne ~ 100-1000 cm^-3
T ~ 10,000 K

The cooling time is of the order of a few thousands years.

But the nebula has an age of 3 Myr.
So what does this mean?

# Cooling Time: Intra-cluster medium






Here the typical temperatures are 10^7 K (indeed we see most radiation in X-rays, whereas in the Orion nebula it was mostly at radio waves).
The typical densities are also very low:

ne ~ 0.001 cm^-3

$$\tau_{cool} = 6 \times 10^3 T^{1/2} n_e^{-1} \bar{g}_{ff} \, yr \approx 10 \, Gyr$$

Intracluster gas takes a very long time to cool down!

# Non-thermal Bremsstrahlung

After the thermal case, the generalization is simple: keep the same electron-ion collision physics, but replace the Maxwellian by a non-thermal power-law electron population. The result is radiation that is strongest in dense gas.

$$N_e(E_e) = K\,E_e^{-p}$$

The non-thermal bremsstrahlung emissivity can then be written as:

$$\epsilon_\nu^{ff,NT} = h\nu\, n_i\, c \int_{h\nu}^{\infty} N_e(E_e)\,\frac{d\sigma_{ff}}{d\nu}\,dE_e$$

In the rel. limit, the cross section scales as 1/frequency, so the final emissivity is also a power-law:

$$\boxed{\epsilon_\nu^{ff,NT} \propto \nu^{1-p}}$$