

Impossible Twin Stars: How can we see phase transitions?

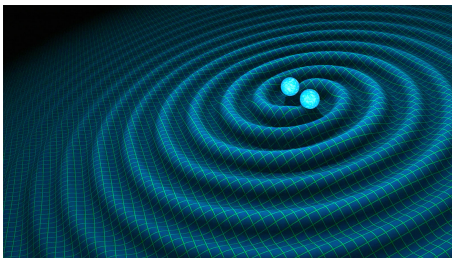
Jan-Erik Christian



Barcelona
May 20th 2026

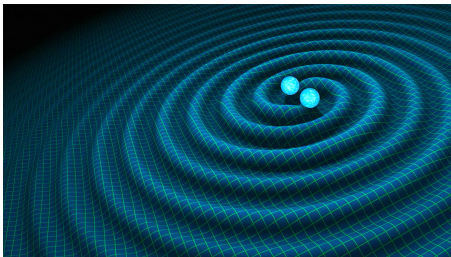
Compact Stars in the QCD phase diagram

Outline



Topics of this talk:

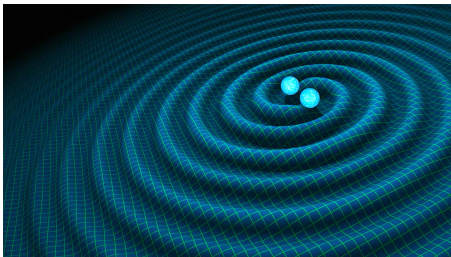
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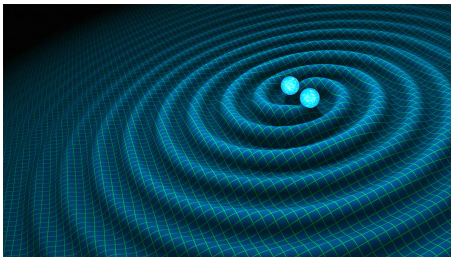
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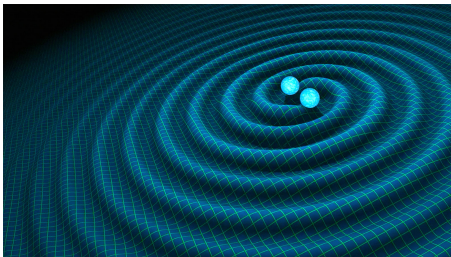
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- Are twin stars possible?

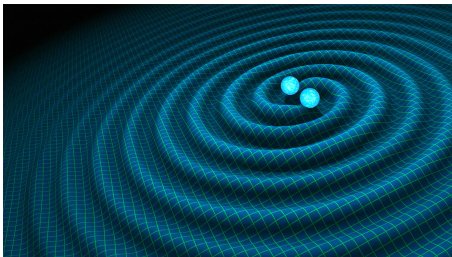
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Topics of this talk:

- What are twin stars and why do we care?
Strong, mass-radius indicators for phase transitions.
- Are twin stars possible?
Yes, but very only in very specific cases.

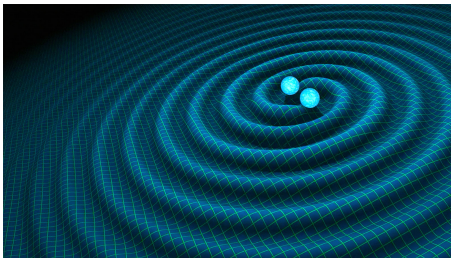
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Topics of this talk:

- What are twin stars and why do we care?
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Yes, but very only in very specific cases.
- Are twin stars possible from a proto-neutron stars perspective?

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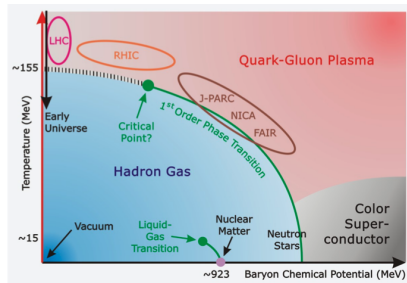
Topics of this talk:

- What are twin stars and why do we care?
Strong, mass-radius indicators for phase transitions.
- Are twin stars possible?
Yes, but very only in very specific cases.
- Are twin stars possible from a proto-neutron stars perspective?
Still yes, but even more limited.

Motivation

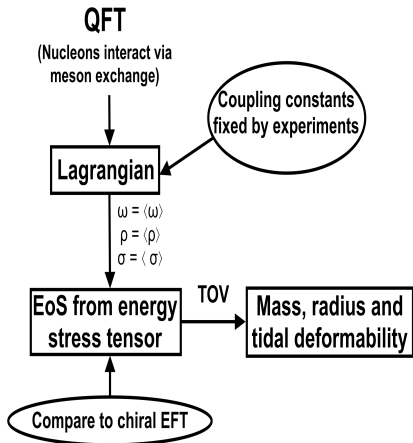
We know:

- Low density from terrestrial experiments and theory.
 - Astrophysical constraints work at high density.
 - A phase transition to QM **will** take place at some point.
- Where is the phase transition and how can we tell from mass, radius and tidal deformability constraints?



[QCD phase diagram sketch, GSI]

Relativistic Mean Field Approach



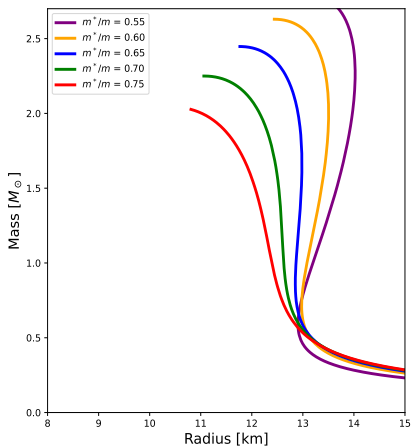
n_0	0.16 fm^{-3}	g_σ, g_ω, b, c
E/A	-16.3 MeV	
K	240 MeV	
m^*/m	$0.55 - 0.75$	
J	$30 - 32 \text{ MeV}$	g_ρ, Λ_ω
L	$40 - 60 \text{ MeV}$	

Values fixed at n_0 . Note:
 $m^* = m_{\text{nucleon}} - g_\sigma \sigma$

We can vary $J, L, m^*/m$.

- Setup following: [Hornick et al. 2018, Phys. Rev. C].

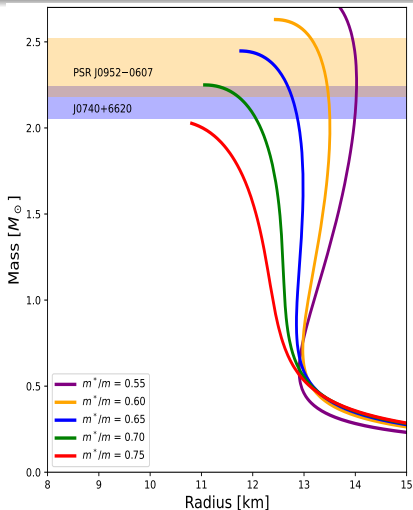
Mass-Radius Constraints



[Christian 2023]

Mass-Radius Constraints

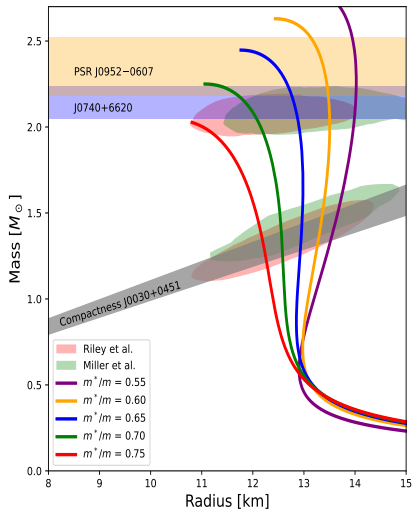
- Neutron stars with $2 M_{\odot}$ are known.



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Mass-Radius Constraints

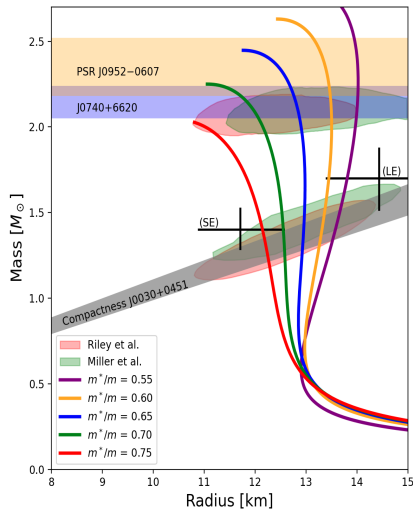
- Neutron stars with $2 M_{\odot}$ are known.
- NICER measured radii between 11 – 16 km.



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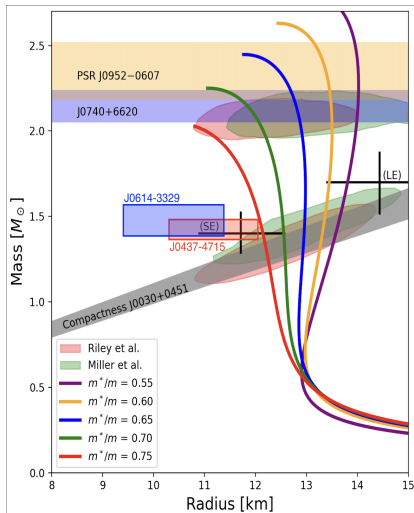
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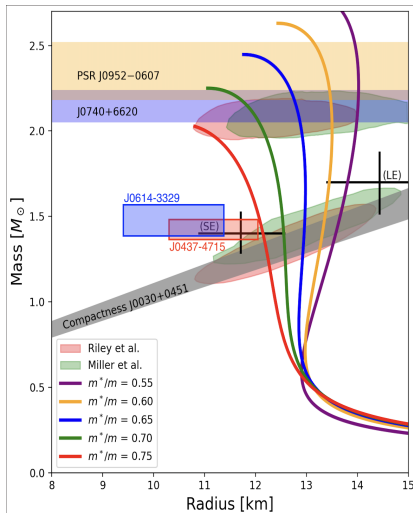
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- New NICER data similar to SE.



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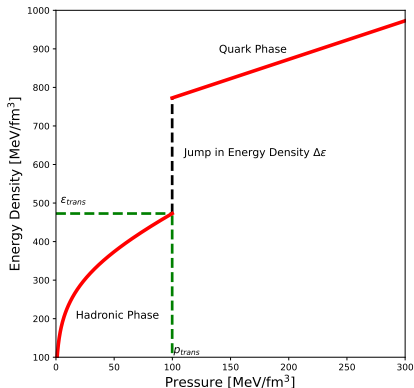
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- NICER measured radii between 11 – 16 km.
- Potential candidates after NICER reanalysis (Vinciguerra et al. 2023).
- New NICER data similar to SE.
- Tidal deformability rules out $m^*/m > 0.65$.



[Christian 2023]

Constant Speed of Sound Quark Matter

- First order phase transition at critical pressure p_{trans} .
- Parameterization is well known. [Alford et. al. 2013, Phys. Rev. D]
- We use $c_{QM} = 1$.

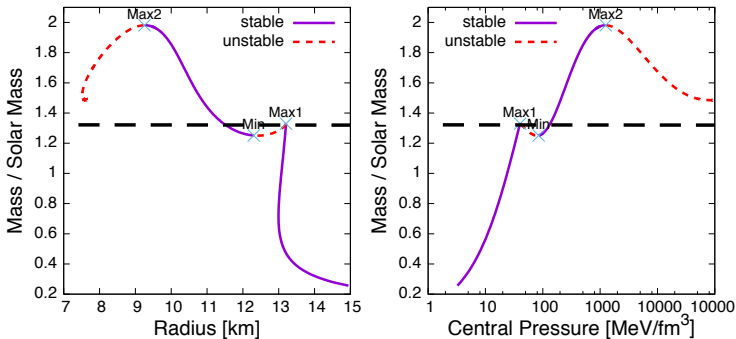


[Christian 2023]

$$\epsilon(p) = \begin{cases} \epsilon_{HM}(p) \\ \epsilon_{HM}(p_{trans}) + \Delta\epsilon + c_{QM}^{-2}(p - p_{trans}) \end{cases}$$

Twin Star Solutions

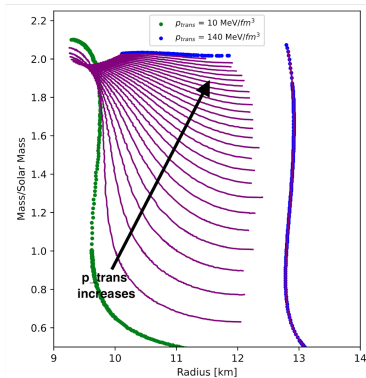
- Phase transition can lead to twin star solutions, where two stars have the same mass, but different radii.



[Christian, Zacchi and Schaffner-Bielich (2018), Eur. Phys. J. A]

Parameter Effects on MR Relation; Hybrid vs Twin

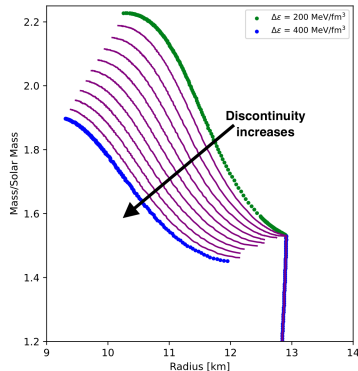
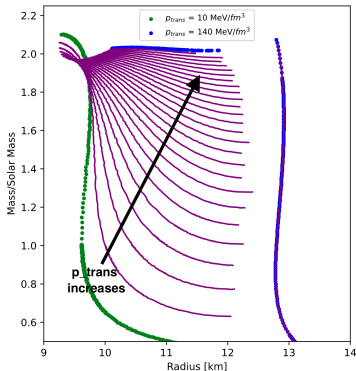
- ρ_{trans} determines the first branch's maximum and the shape of the second branch.



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Parameter Effects on MR Relation; Hybrid vs Twin

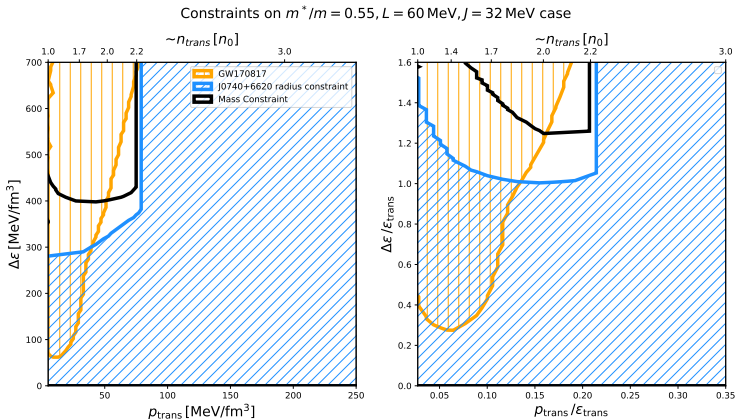
- ρ_{trans} determines the first branch's maximum and the shape of the second branch.
- $\Delta\epsilon$ strongly influences the second's maximum by determining the position of the second branch.



[Christian 2023]

Constraints on Stiff Equation of State

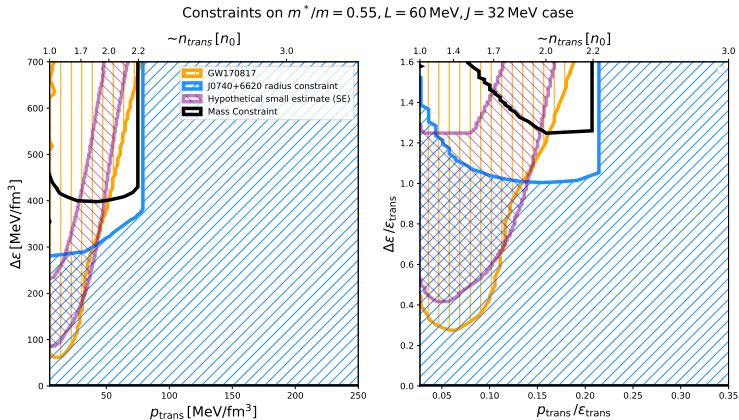
- The GW170817 constraint can be met with a phase transition.



[Christian, Schaffner-Bielich and Rosswog, Phys. Rev. D, 2023]

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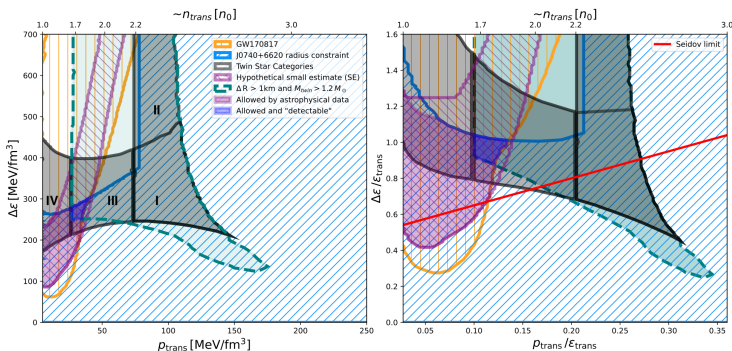
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[Christian, Schaffner-Bielich and Rosswog, Phys. Rev. D, 2023]

Constraints on Stiff Equation of State

- The GW170817 constraint can be met with a phase transition.
- A hypothetical well determined "small" star does not constrain a stiff EoS further.

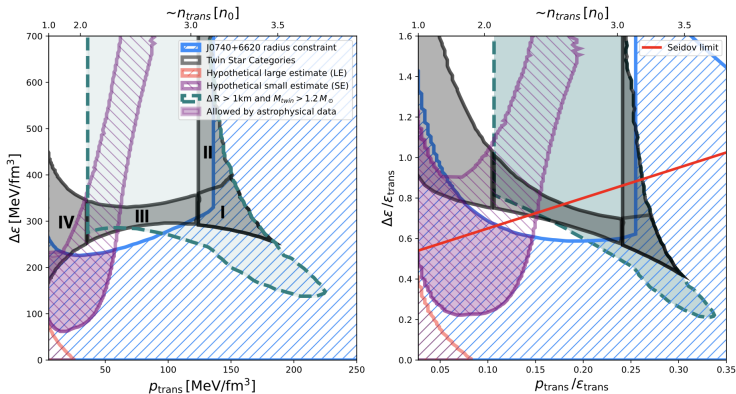


[Christian, Schaffner-Bielich and Rosswog, Phys. Rev. D, 2023]

Constraints on Softer Equation of State

- Large parameter space allowed by constraints.
- No significant ΔR in allowed parameter space.

Constraints on $m^*/m = 0.65, L = 60 \text{ MeV}, J = 32 \text{ MeV}$ case

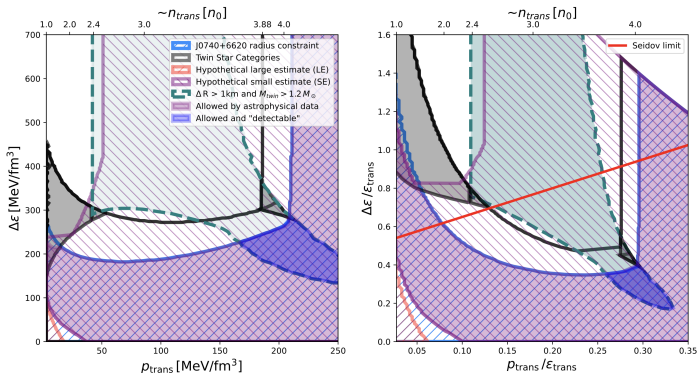


[Christian, Schaffner-Bielich and Rosswog, Phys. Rev. D, 2023]

Constraints on Even Softer Equation of State

- Even soft EoSs do not allow for 'real' twin stars.
- Recent works find similar results (e.g. Blomqvist et al. 2025).

Constraints on $m^*/m = 0.70, L = 60 \text{ MeV}, J = 32 \text{ MeV}$ case



[Christian, Schaffner-Bielich and Rosswog, Phys. Rev. D, 2023]

RG-NJL model overview

For detailed description see: [Gholami et al. Phys. Rev. D, 2024].

- We can construct a Lagrangian:

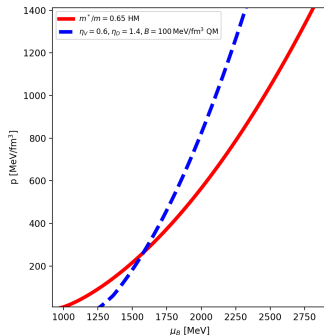
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq} + \mathcal{L}_L$$

Diquark- and vector-coupling parameters η_D and η_V are needed.

- Pressure is determined via the grand potential per volume Ω :

$$p = (\Omega - \Omega_0) - B$$

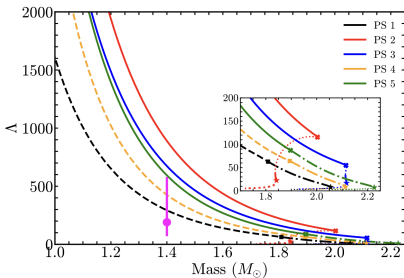
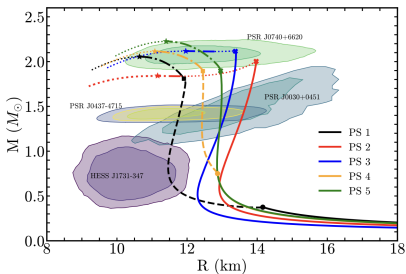
with B as the bag constant.



Determining transition point

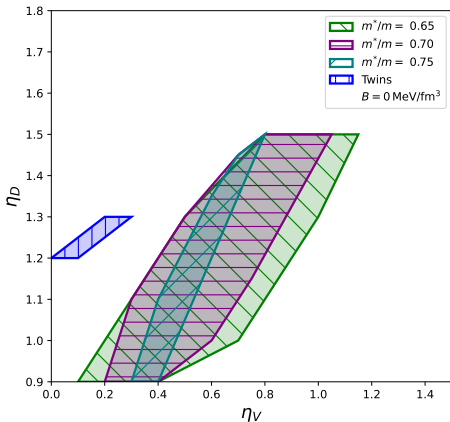
Twin Stars with the RG-NJL model

- Twin stars are possible, if one or more constraint is ignored.
- Tidal deformability is the biggest concern.



Constraints on Microphysical Quarkphase

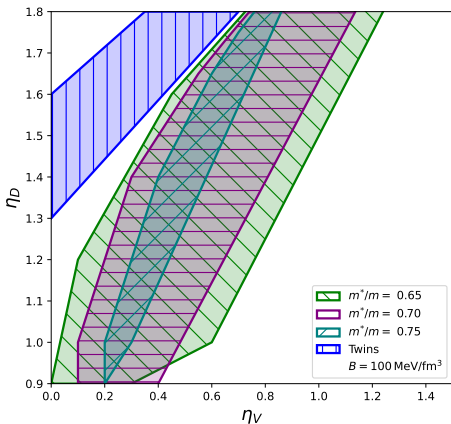
- Stiffer EoSs allow much easier to combine with a quark phase.
- Twin stars and allowed parameter sets are mutually exclusive.



[Christian et al., A&A, 2025]

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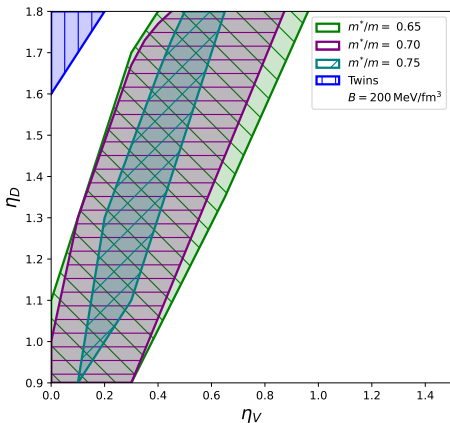
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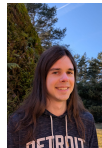


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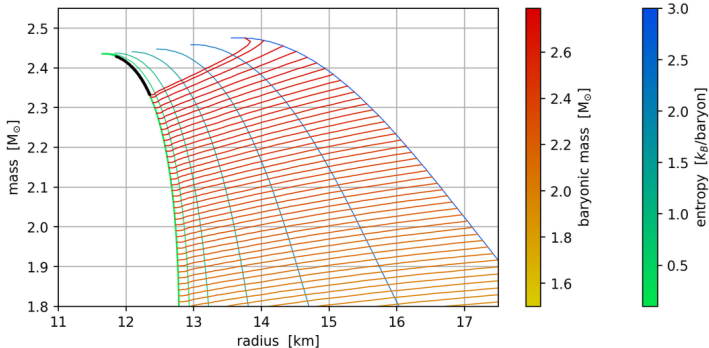
Approximating Thermal Evolution

Assumptions:

- Baryonic mass is constant during cooling.
- Proto-neutron star is isentropic.
- Charge fraction is constant.

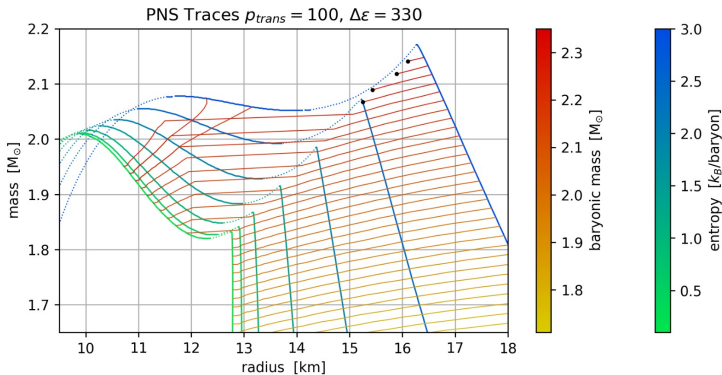


Jannik Milthaler



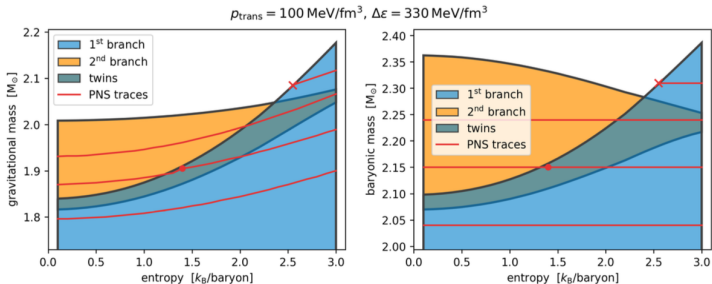
Unrealized Hybridstars

- Maximum masses are not reachable.
- Hybrid stars can start as purely hadronic proto-neutron stars.

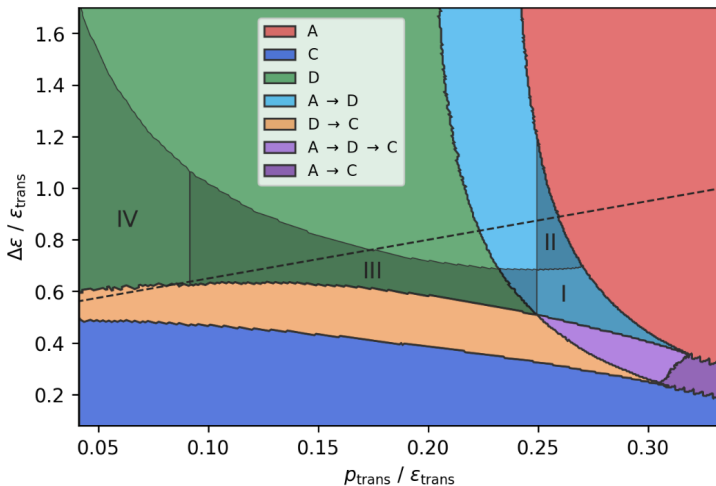


Twin Stars Come and Go

- The mass range of twin stars is dependent on the cooling stage.
- Note: We are assuming an instant conversion from hadronic to quark matter.

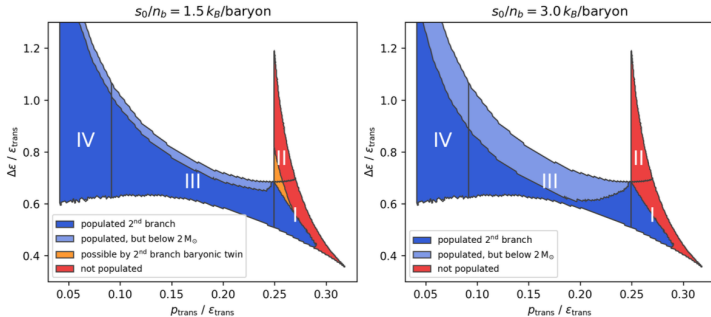


Behavior Based on Parameter Space

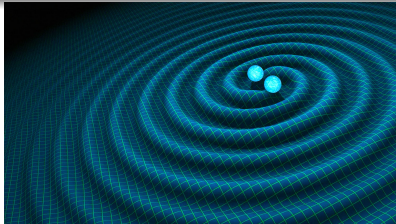


Ruled out Regions

- Based on thermal behavior category II can be ruled out.
- Category I is still possible in some cases.



Summary



What we discussed:

- What are twin stars and why do we care?
They are very strong indicators of phase transitions in neutron stars.
- Are twin stars even possible?
Considering mass, radius and tidal deformability: only for very few EoS with extreme conditions.
- Are twin stars even possible (now with temperature)?
Considering proto-neutron star evolution: Even less EoSs allow twin stars.

⇒ **Twin stars are unlikely, but a quark core is not.**

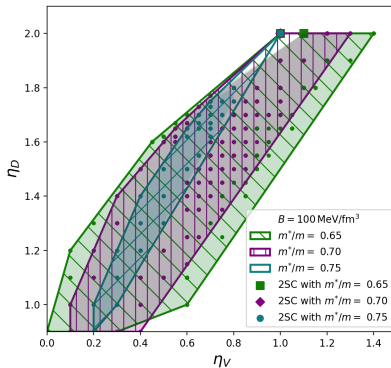
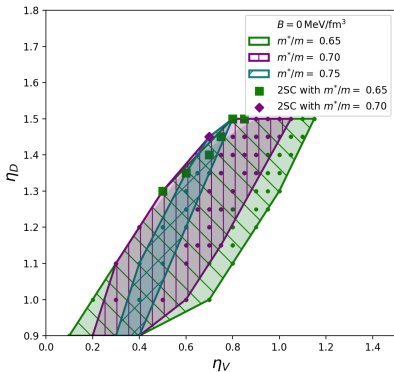
Lagrangian

- Relativistic mean field Lagrangian (Hornick et al. 2018):

$$\begin{aligned}
 \mathcal{L} = & \sum_B \bar{\psi}_B \left(i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\vec{\rho} B} \gamma_\mu \vec{\tau} \cdot \rho^\mu \right) \psi_B \\
 & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
 & - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} + \frac{1}{2} m_{\vec{\rho}}^2 \vec{\rho}_\mu \cdot \rho^\mu - \frac{1}{3} b m_n (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 \\
 & + \Lambda_\omega (g_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu) (g_\omega^2 \omega_\mu \omega^\mu) + \frac{\zeta}{4!} (g_\omega^2 \omega_\mu \omega^\mu)^2
 \end{aligned}$$

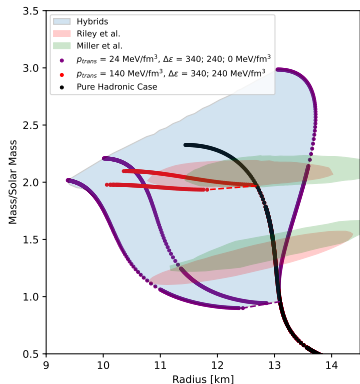
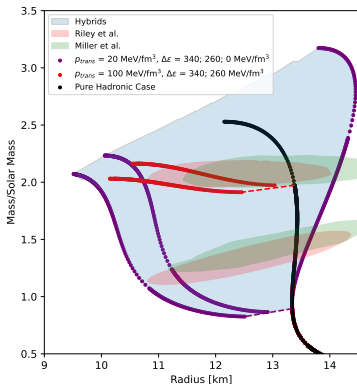
- At high baryon densities: $\bar{\omega} = \langle \omega \rangle$, $\bar{\sigma} = \langle \sigma^0 \rangle$ and $\bar{\rho} = \langle \rho_3^0 \rangle \Rightarrow$ mean field theory.

CSC Phase in Neutron Stars



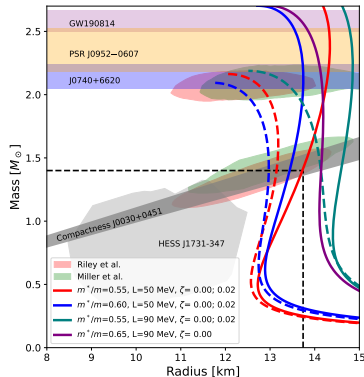
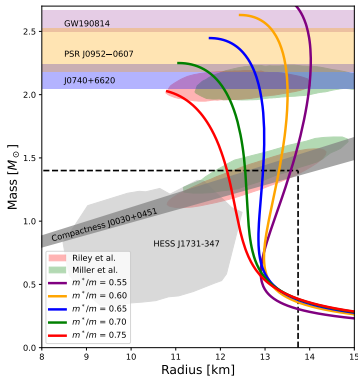
[Christian et al. (2025)]

Hybrid stars NICER constraints



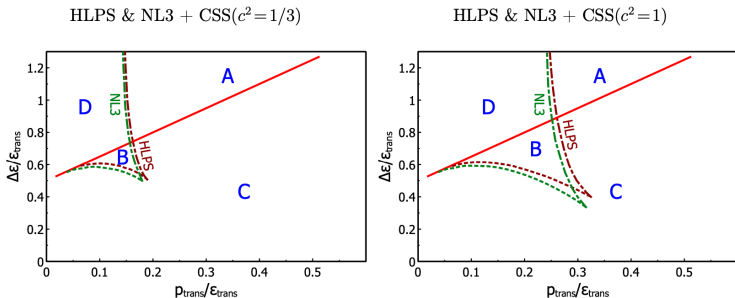
[Christian and Schaffner-Bielich (2021), Phys. Rev. D]

MR constraints for more RMF models



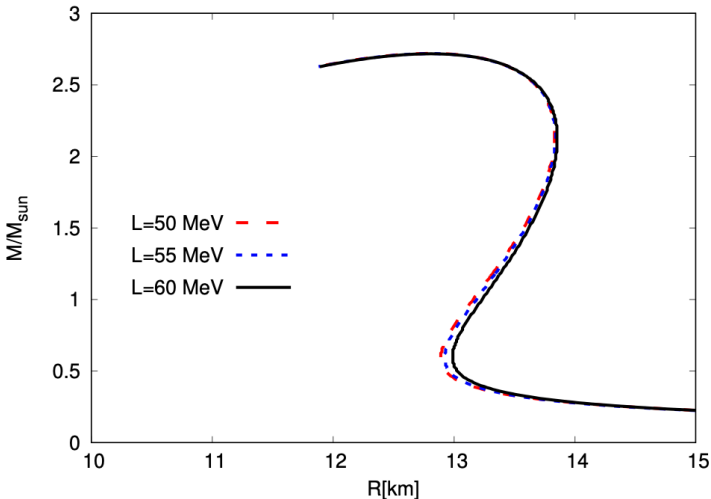
[Christian 2023]

Influence of c_{QM} and hadronic EoS on parameter space



Alford et al. 2013, Phys. Rev. D

Mass-Radius Change for L Variation



[Hornick et al. 2018, Phys. Rev. C]

Relativistic Mean Field Approach

- Nucleon interactions are modeled with σ , ω and ρ mesons
- At high baryon densities: $\bar{\omega} = \langle \omega \rangle$, $\bar{\sigma} = \langle \sigma^0 \rangle$ and $\bar{\rho} = \langle \rho_3^0 \rangle \Rightarrow$ mean field theory
- Fix coupling constants from experiments:

Saturation density	n_0	0.16 fm^{-3}	g_σ, g_ω, b, c
Binding Energy	E/A	-16.3 MeV	
Compressibility	K	240 MeV	
Effective mass	m^*/m	$0.55 - 0.75$	
Symmetry energy	J	$30 - 32 \text{ MeV}$	g_ρ, Λ_ω
Slope parameter	L	$40 - 60 \text{ MeV}$	

- In this case $J = 32 \text{ MeV}$ and $L = 60 \text{ MeV}$ via chiral effective field theory

Gravitational Wave Event GW170817

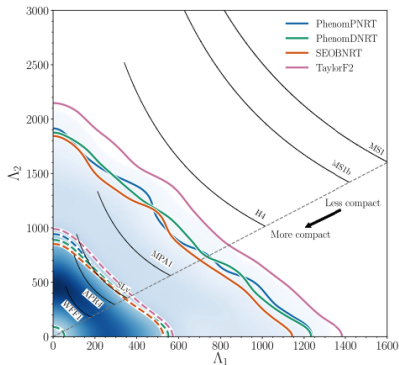
- In a binary system the companion's tidal field induces a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

- Obtain dimensionless form:

$$\Lambda = \frac{\lambda}{m^5}$$

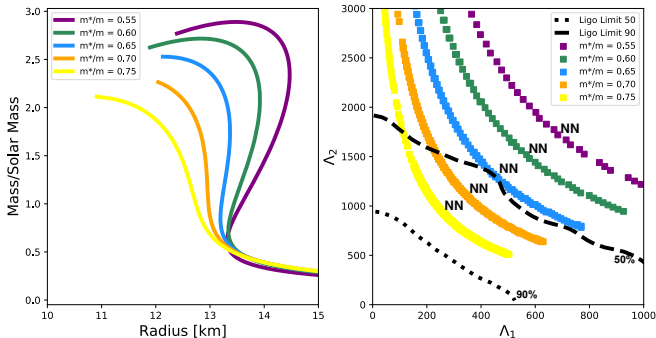
- Upper limit for combined value:



[Abbott et al. 2019, Phys. Rev. X]

$$\tilde{\Lambda} = \tilde{\Lambda}(\Lambda_1, m_1, \Lambda_2, m_2) \leq 720$$

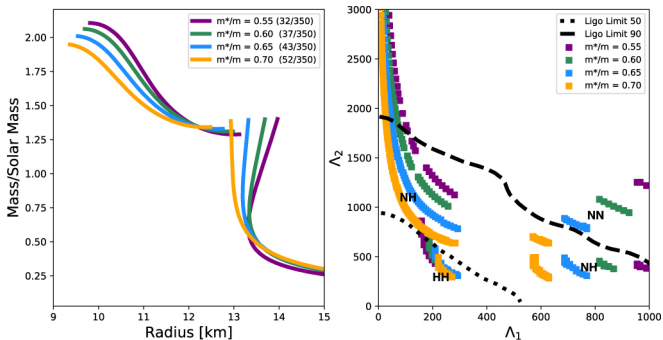
Closer Look: Tidal Deformability Constraint



[Christian and Schaffner-Bielich (2019), ApJL]

- Only EoSs with $m^*/m \geq 0.65$ are soft enough to fit the data.

Tidal Deformability of Hybrid Stars



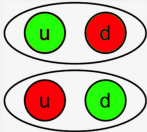
[Christian and Schaffner-Bielich (2019), ApJL]

- Hybrid EoSs are more compatible with GW170817.

Color Superconducting Phases

- Quark matter can be separated into two phases.
- Medium μ : Two-flavor supercolor conductive phase.
- High μ : color-flavor-locked phase.

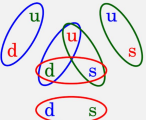
2SC



Intermediate $\mu \lesssim M_s$
1 finite gap parameter
 Δ_{ud}

The diagram shows two horizontal ovals. The top oval contains a green circle labeled 'u' and a red circle labeled 'd'. The bottom oval contains a red circle labeled 'u' and a green circle labeled 'd'.

Color-flavor-locking (CFL)

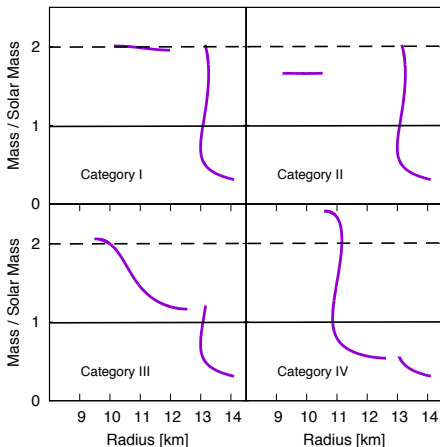


Large $\mu \gg M_s$
3 finite gap parameters
 $\Delta_{ud}, \Delta_{us}, \Delta_{ds}$

The diagram shows several quarks (u, d, s) arranged in a cluster. Three overlapping ellipses represent color singlet states: a blue ellipse containing 'u' and 'd', a green ellipse containing 'u' and 's', and a red ellipse containing 'd' and 's'.

Categories of Twin Stars

- **Category I:** Both maxima meet mass constraint M_{data} .
- **Category II:** Only the hadronic maximum exceeds M_{data} .
- **Category III:** Only the hybrid maximum exceeds M_{data} .
- **Category IV:** Only hybrid stars can be observed.



[Christian, Zacchi and Schaffner-Bielich (2018),
Eur. Phys. J. A]