A More Natural Method to Infer the EOS from Neutron Star Observations

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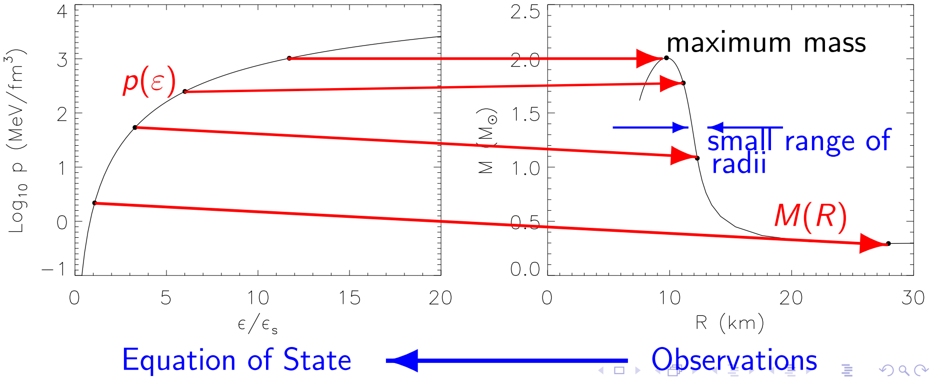
Neutron Star Structure

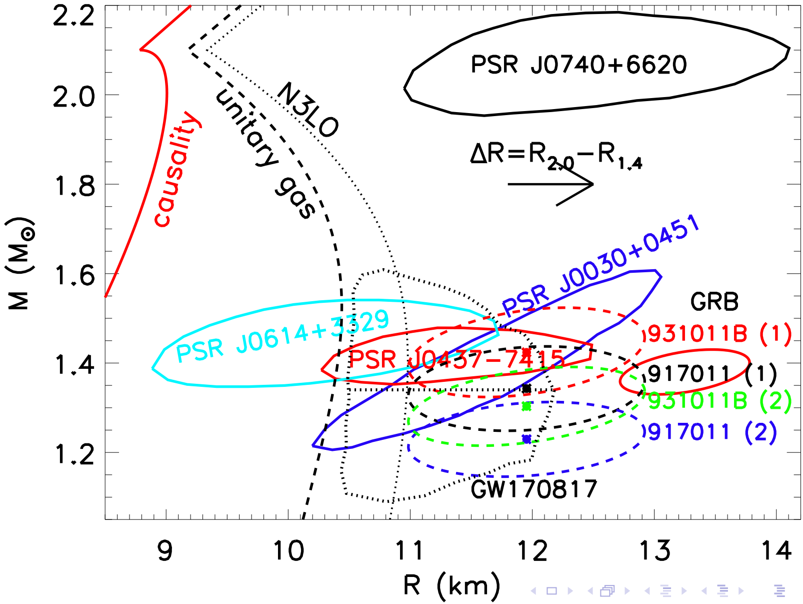
Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$

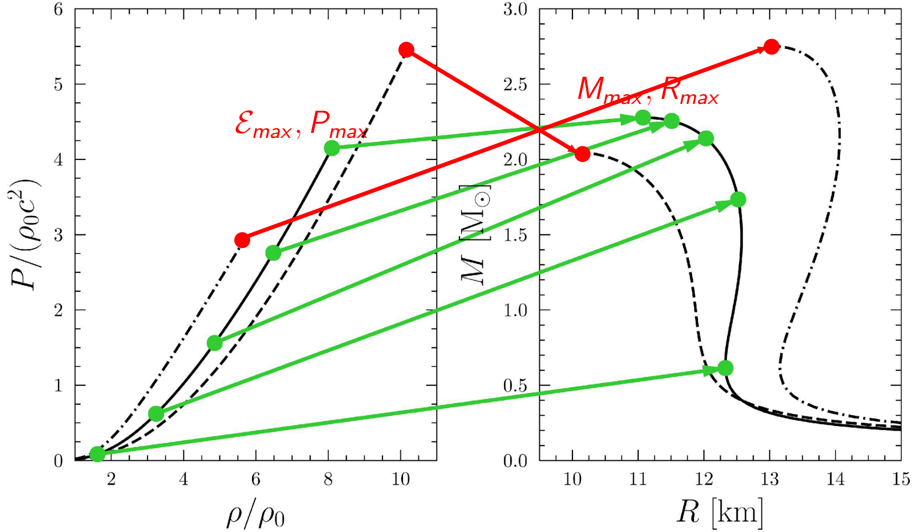
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Summary of Astrophysical Constraints



Maximum Mass As a Unique Scaling Point



$M_{\rm max}$, $R_{\rm max}$, $\mathcal{E}_{\rm max}$, $P_{\rm max}$ Correlation

Ofengeim et al (2023) suggest power-law correlations

$$\mathcal{E}_{\rm c,max} = (1.809 \pm 0.36)\left(\frac{R_{\rm max}}{10{\rm km}}\right)^{-1.98}\left(\frac{M_{\rm max}}{M_\odot}\right)^{-0.171} {\rm GeV~fm}^{-3},$$

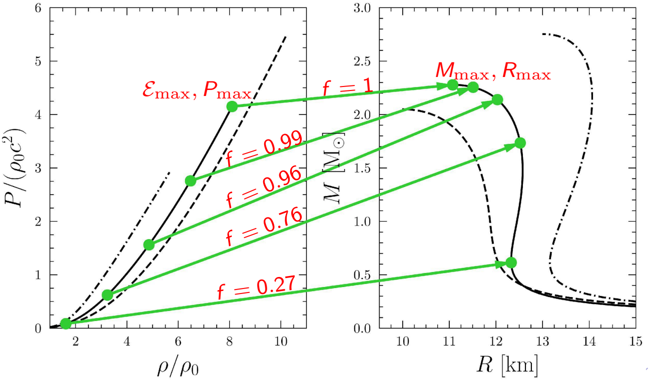
$$P_{\rm c,max} = (118.5 \pm 6.2)\left(\frac{R_{\rm max}}{10{\rm km}}\right)^{-5.24}\left(\frac{M_{\rm max}}{M_\odot}\right)^{2.73} {\rm MeV~fm}^{-3},$$

which are accurate to about 5% in fitting $\mathcal{E}_{c,max}$ and $P_{c,max}$.

Points along $M-R$ curves, at $M = fM_{\rm max}$, have similarly accurate correlations:

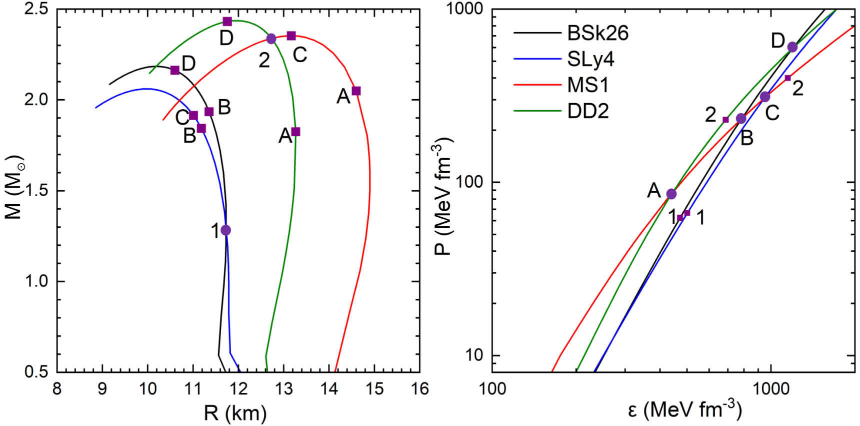
$$\mathcal{E}_{c,f} = a_{\mathcal{E},f} R_{fM_{max}}^{b_{\mathcal{E},f}} M_{max}^{c_{\mathcal{E},f}}$$

$$P_{c,f} = a_{P,f} R_{fM_{max}}^{b_{P,f}} M_{max}^{c_{P,f}}$$



But (M, R) Is Not Equivalent To $(\mathcal{E}_c, P_c)$

While the maximum mass point (M_{max}, R_{max}) predicts $(\mathcal{E}_{c,max}, P_{c,max})$ to about 5%, and similarly for a given fractional maximum mass fM_{max}, the inversion is not unique. Two different equations of state predicting the same (M, R) (numbers in figure) arrive at those values from integration via different paths in $(\mathcal{E}, P)$ space. Similarly, two equations of state with identical values of $(\mathcal{E}_c, P_c)$ (letters) do not have the same (M, R) values.



Correlations at $M = fM_{\rm max}$

Thus, more information than (M, R) needed. We find precision is greatly improved using a 2nd radius from a grid of fractional M_{max} points, *e.g.*, $f \in [1, 0.95, 0.9, 0.85, 4/5, 3/4, 2/3, 0.6, 0.5, 0.4, 1/3]$.

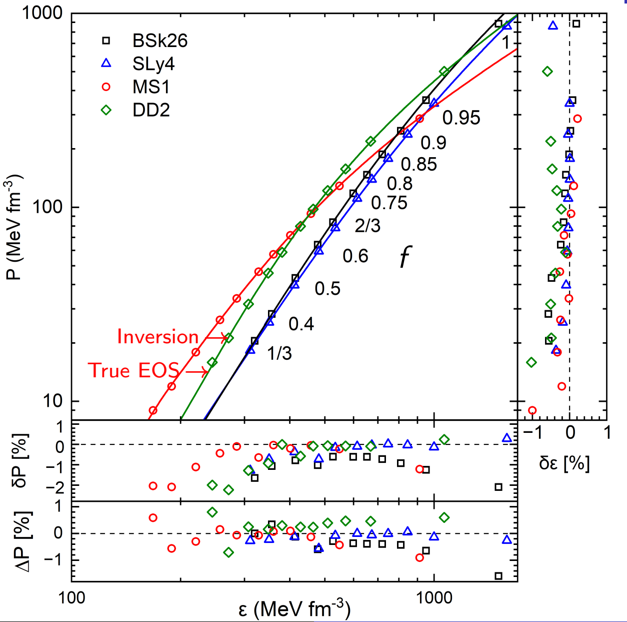
$$\mathcal{E}_f = a_{\mathcal{E},f} \left(\frac{R_{f_1}}{10{\rm km}}\right)^{b_{\mathcal{E},f_1}} \left(\frac{R_{f_2}}{10{\rm km}}\right)^{c_{\mathcal{E},f_2}} \left(\frac{M_{max}}{M_\odot}\right)^{d_{\mathcal{E},f}},$$

$$P_f = a_{P,f} \left(\frac{R_{f_1}}{10{\rm km}}\right)^{b_{P,f_1}} \left(\frac{R_{f_2}}{10{\rm km}}\right)^{c_{P,f_2}} \left(\frac{M_{max}}{M_\odot}\right)^{d_{P,f}},$$

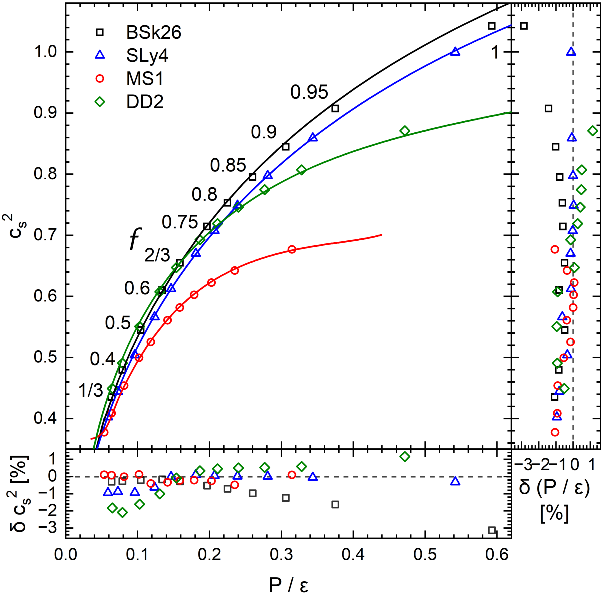
$f = M/M_{\rm max}$	f_1	f_2	$\Delta(\ln \mathcal{E}_f)$	f_1	f_2	$\Delta(\ln P_f)$	$\Delta(\ln \mu_f)$	$\Delta(\ln n_f)$
1	0.95	3/5	0.00289	1	3/5	0.0117	0.00893	0.00401
0.95	0.95	3/4	0.00272	0.95	3/5	0.00701	0.00399	0.00289
0.90	0.95	2/3	0.00226	0.95	2/5	0.00518	0.00299	0.00239
0.85	0.95	1/2	0.00234	0.9	2/5	0.00489	0.00250	0.00234
4/5	0.9	1/2	0.00230	0.85	2/5	0.00462	0.00224	0.00230
3/4	0.85	1/2	0.00239	4/5	2/5	0.00539	0.00206	0.00243
2/3	3/4	1/2	0.00277	2/3	2/5	0.00511	0.00188	0.00257
3/5	3/4	2/5	0.00340	2/3	1/3	0.0172	0.00181	0.00315
1/2	2/3	1/3	0.00477	1/2	2/5	0.00998	0.00175	0.00457
2/5	1/2	1/3	0.00708	1/2	1/3	0.0188	0.00183	0.00672
1/3	1/2	1/3	0.0122	2/5	1/3	0.0259	0.00190	0.0119

greatly reduced uncertaintites!

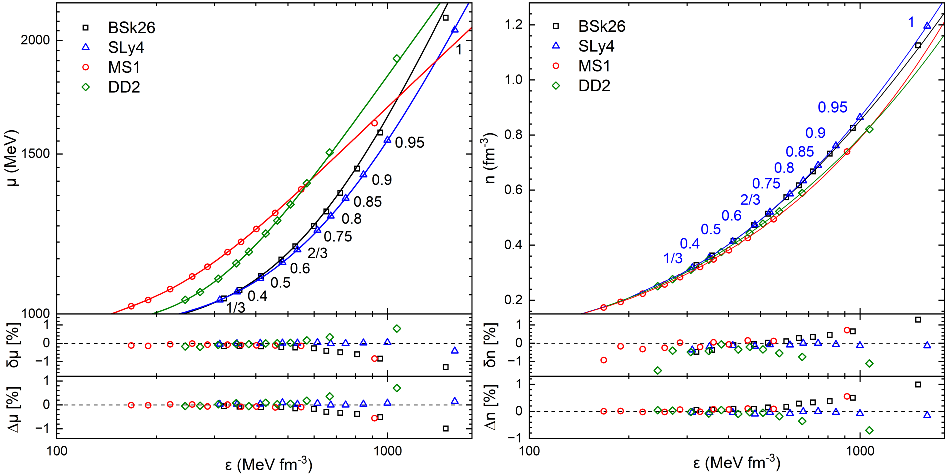
Testing the Inversion



Testing the Inversion for $c_s^2 - P/\mathcal{E}$

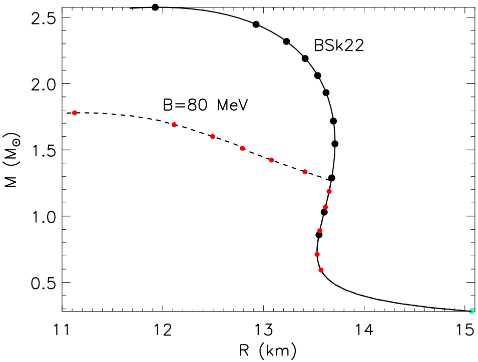


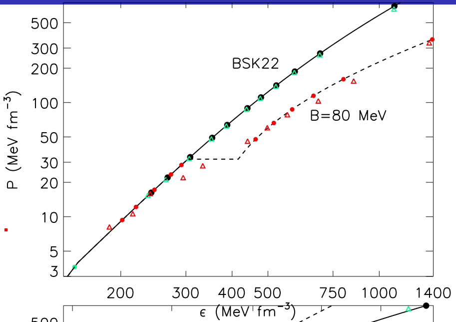
Inversions for μ and n

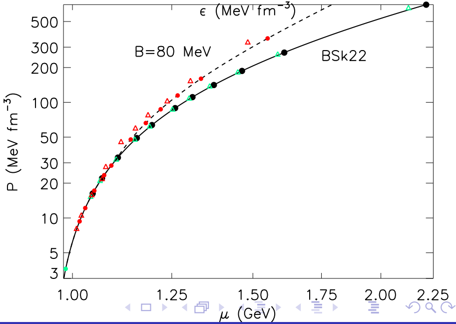


Inversions in Case of First-Order Transitions

Although fitting formulae were established using hadronic EOSs, they also work well in the case a first-order phase transition occurs. In this case, the reconstructed EOS smoothly bridges the phase transition.



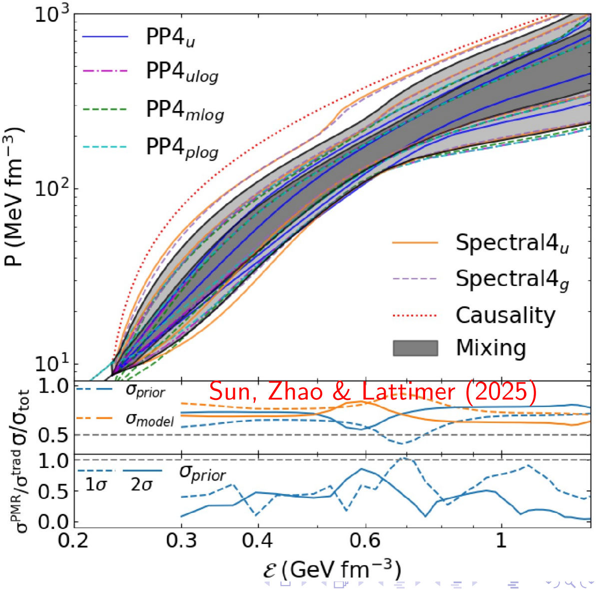




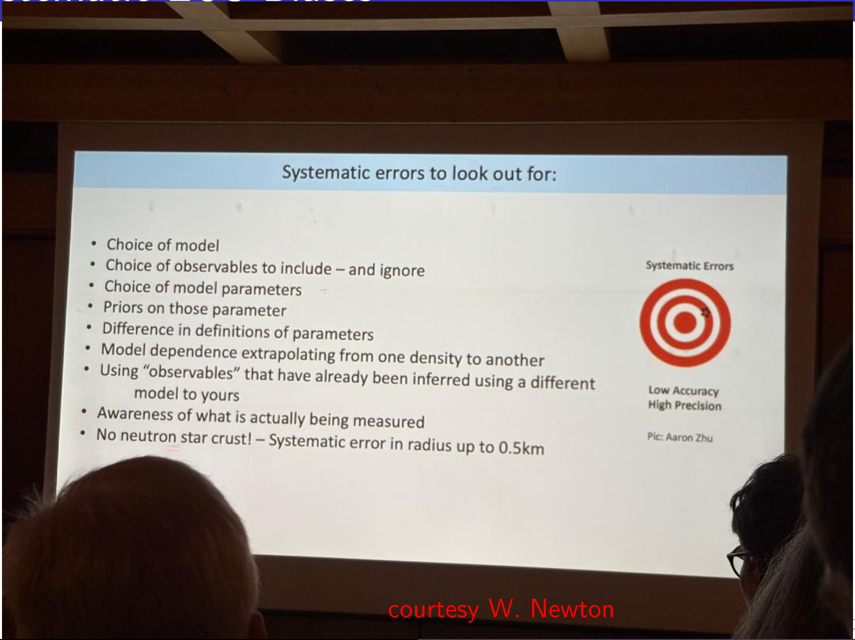
Inversion of *M*-*R* Data and Systematic EOS Biases

To infer the EOS from *M*-*R* data, the traditional approach involves Bayesian methods beginning with *M*-*R* priors generated from millions of EOSs using parameterized EOSs.

But choices of parameterizations and the arbitrary choices of their parameters (bounded by causality, stability and a minimum maximum mass) add significant inference uncertainties, being as large as observational uncertainties.



Systematic EOS Biases



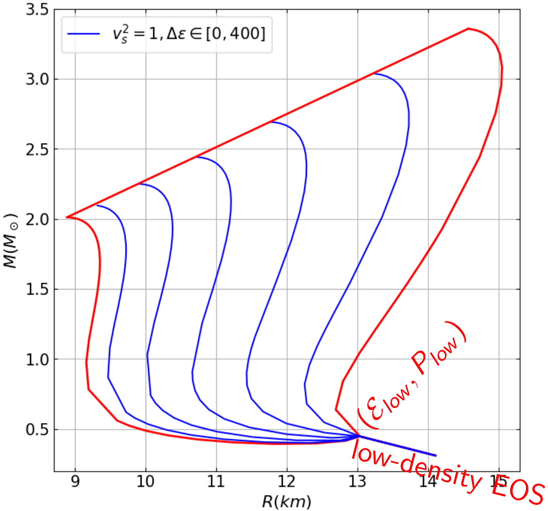
courtesy W. Newton

$M - R$ Limits; Neutron Star Crust and Neutron Matter

Control theory optimization: Mass and radius will be extremized only if $c_s = 0$ or $c_s = c$ for all $\varepsilon > \varepsilon_{low}$ (bang-bang solution).

To maximize mass and R(M), $c_s = c$ for $\varepsilon \geq \varepsilon_{low}$.
$M_{max}^{up} \simeq 4.1(\varepsilon_s/\varepsilon_{low})^{-1/2} M_\odot$
$\sim 3.3 M_\odot$ if $\varepsilon_{low} \simeq 1.5\varepsilon_s$
where $\varepsilon_s \simeq 150$ MeV fm^{-3}.

To minimize R(M), $c_s = 0$ for $\varepsilon_{low} \leq \varepsilon \leq \varepsilon_t$ and $c_s = 1$ for $\varepsilon \geq \varepsilon_t$.
$M_{max}^{lo} \simeq 4.1(\varepsilon_s/\varepsilon_t)^{-1/2} M_\odot$
$\sim 2 M_\odot$ if $\varepsilon_t \simeq 2\varepsilon_s$.



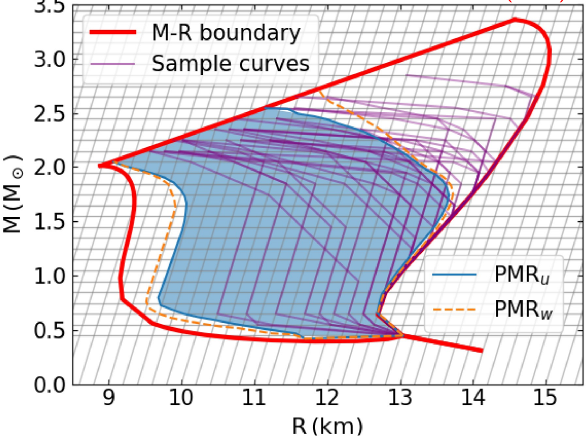
Parametrized M-R (PMR) Method

To avoid prior EOS uncertainties, the PMR method generates a mesh covering the bounded M-R space. The prior consists of M-R curves that satisfy a minimum M_{max} and are produced by connecting ascending mass nodes with segments not violating causality or thermodynamic stability.

A trapezoidal mesh prevents curves from bouncing back-and-forth in R. Bayesian methods assign weights to each surviving M-R curve from M-R observational data. Direct analytic inversion of the M-R curves generates complementary EOS posteriors.

Blue region contains 68% of connected nodes

Sun, Zhao & Lattimer (2025)

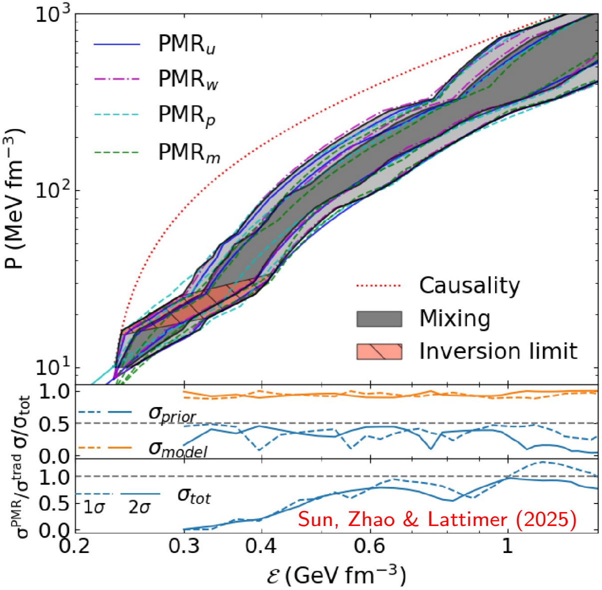


Comparison of EOS Inferences

The PMR method is more natural since its priors and posteriors are both in *M*-*R* space, and observational uncertainties dominate its prior uncertainties at all densities.

In contrast, uncertainties from the EOS priors in the traditional Bayesian framework are about as large as the observational uncertainties.

The PMR method generates a stiffer high-density EOS, and, at all but the highest densities, smaller absolute uncertainties.



Take Aways

- Novel predictions of neutron star properties are on the horizon, including moment of inertia measurements and observations of GRB QPOs.
- Systematic uncertainties affect both observational predictions of masses and radii and inferences of the underlying neutron star EOS.
- At the present time, using traditional Bayesian inference frameworks, these two types of uncertainties have approximately the same magnitude.
- Until precision (~ 0.1 km) measurements of radii are available, it is crucial to reduce systematic uncertainties in EOS inferences.
- The PMR method offers a novel approach that has better control of its systematic prior uncertainties, and, as a result, apparently smaller absolute uncertainties.