

First-order transitions in dense matter

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Alexander von
HUMBOLDT
STIFTUNG

Outline

- ▶ First-order transitions

 - Sharp (Maxwell) vs Mixed (Gibbs) construction

- ▶ Physics of mixed phases

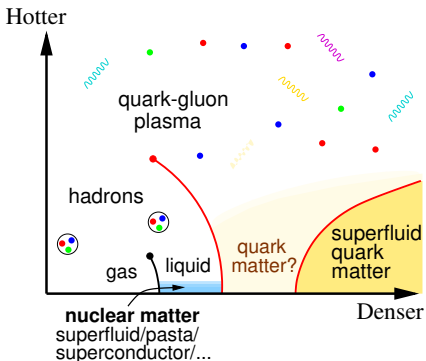
 - Surface tension vs charge separation

- ▶ Sharp transitions and the mass-radius relation

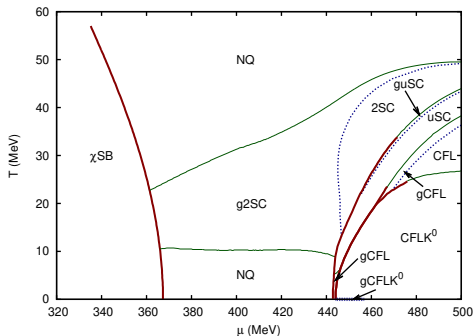
 - Insight from the Constant Sound Speed parameterization

Schematic QCD phase diagram

Conjectured phase diagram



NJL model, uniform phases only



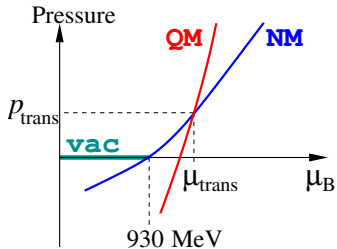
Basler & Buballa, [arXiv:0912.3411](https://arxiv.org/abs/0912.3411)

There are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase.

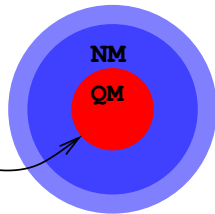
We expect plenty of first-order transitions

Nuclear/quark transition: hybrid stars

First-order transition from nuclear matter to quark matter

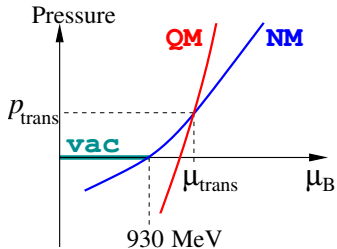


"Maxwell"
sharp interface

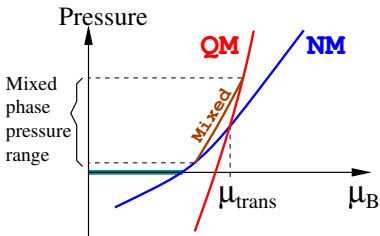
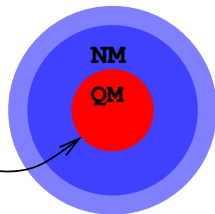


Nuclear/quark transition: hybrid stars

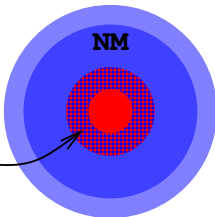
First-order transition from nuclear matter to quark matter



"Maxwell"
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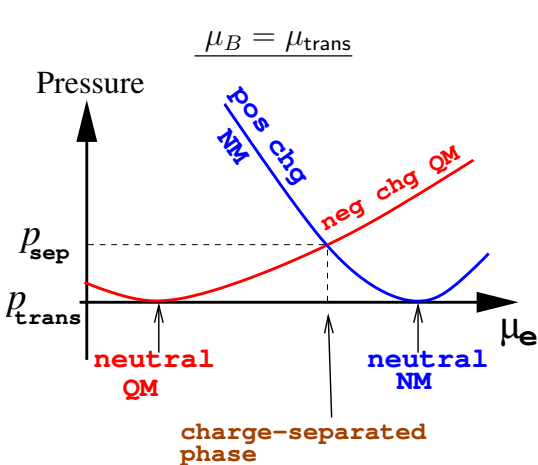


"Gibbs"
mixed phase region



Mixed phase at nuclear/quark transition

If the surface tension is low enough, there will be a mixed *charge-separated* phase at the quark/nuclear interface.



$$\text{charge density } \rho = -\frac{dP}{d\mu_e}$$

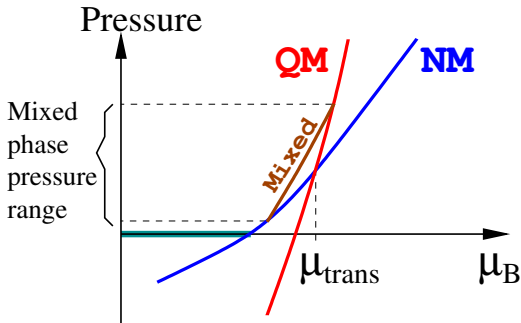
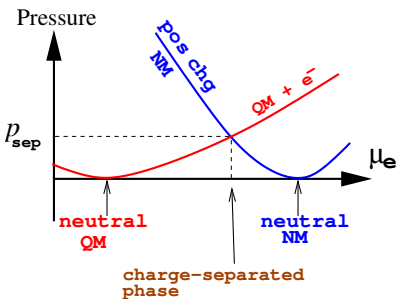
Neutral nuclear matter and neutral quark matter can coexist at zero pressure.

But if they have different electrostatic potentials μ_e then $p_{\text{sep}} > 0$ and it is preferable* to form a charge-separated phase with intermediate μ_e .

* ignoring surface energy costs

Mixed phase instead of interface

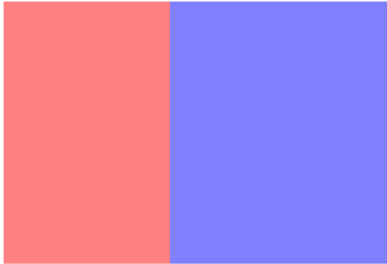
Below and above the critical pressure, there is a range where the mixed phase is favored.



But surface energy costs, e.g. surface tension, may shrink this range.

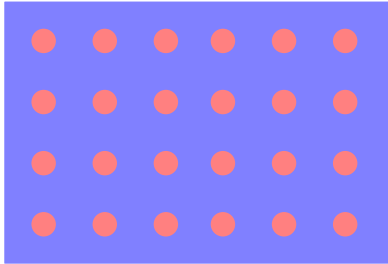
Geometry of the mixed phase

Coarse mixture



surface tension energy: small

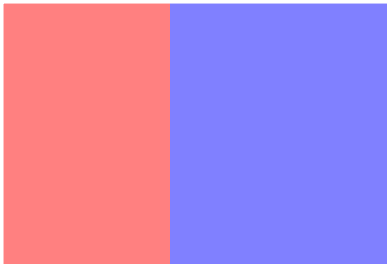
Fine mixture



surface tension energy: large

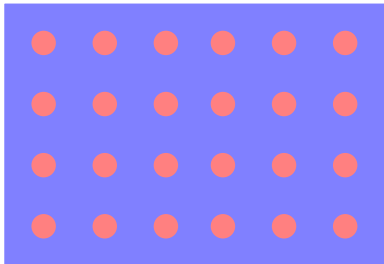
Geometry of the mixed phase

Coarse mixture

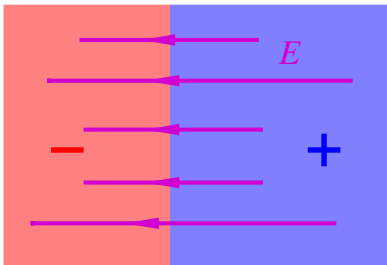


surface tension energy: small

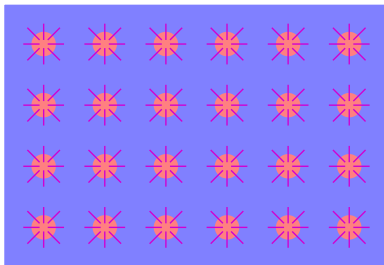
Fine mixture



surface tension energy: large



electric field energy: large

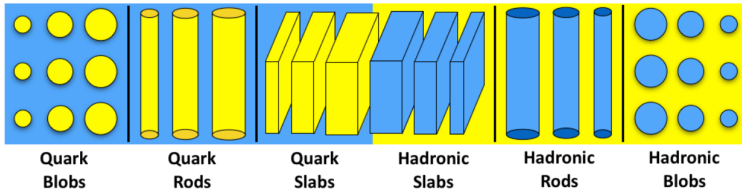


electric field energy: small

Structure of mixed phase

The battle between surface tension and electric field can lead to various geometric structures of a mixed phase

“PASTA”
PHASES



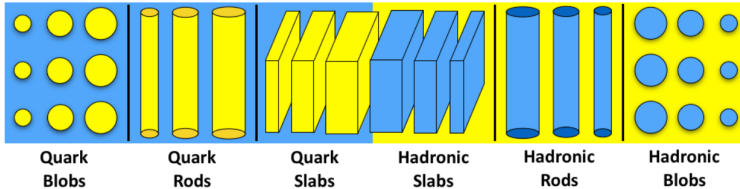
Orsaria et. al.
[arXiv:
1907.04654](https://arxiv.org/abs/1907.04654)

Structure of mixed phase

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The battle between surface tension and electric field can lead to various geometric structures of a mixed phase

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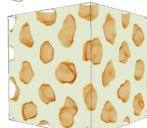
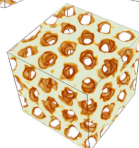
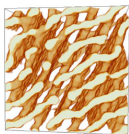
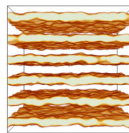
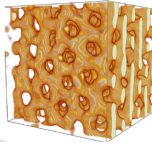
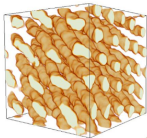
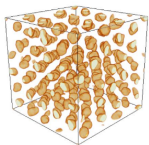


(a) *Gnocchi*

(b) *Spaghetti*

(c) *Waffles*

(d) *Lasagna*



(e) *Defects*

(f) *Antispaghetti*

(g) *Antignocchi*

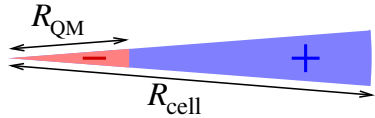
Caplan & Horowitz
[arXiv:1606.03646](https://arxiv.org/abs/1606.03646)

(for purely nuclear
mixed phase)

Charge redistribution in mixed phase

Focus on a unit cell in 1,2,or 3D

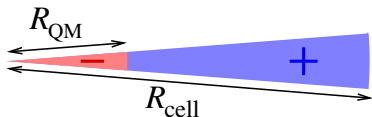
What is the charge distribution?



Charge redistribution in mixed phase

Focus on a unit cell in 1,2,or 3D

What is the charge distribution?



Electron chemical potential determines charge density

Charge density determines electrostatic potential V

But electrostatic potential *is* electron chemical potential

$$\rho = \begin{cases} \rho_{\text{NM}}(\mu_e) & \text{in nuclear matter} \\ \rho_{\text{QM}}(\mu_e) & \text{in quark matter} \end{cases}$$

$$\nabla^2 V(x) = -\rho(x)$$

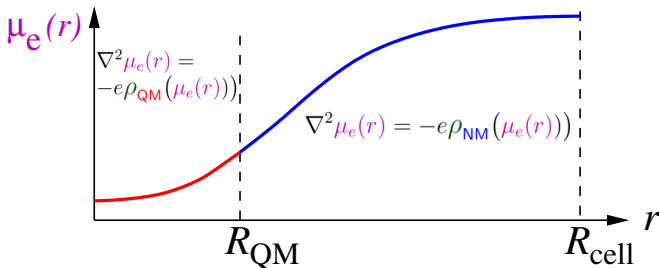
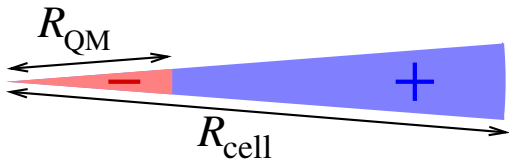
$$\mu_e = eV$$

Need to solve

$$\nabla^2 \mu_e(r) = -e\rho(\mu_e(r))$$

Boundary conditions: No electric field at center or edge of cell

Electrostatic potential in one cell

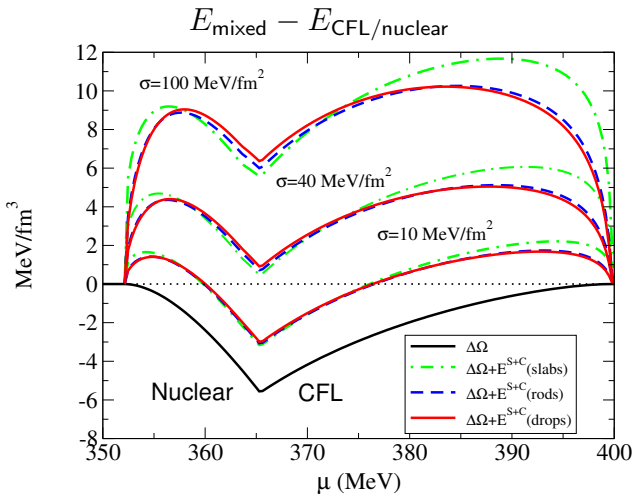


Obtain $\mu_e(r)$ for this configuration.

Vary R_{QM} , R_{cell} , and dimension (slab, rod, sphere) to find which configuration has lowest total energy per baryon.

$$E_{tot} = \int_0^{R_{QM}} \varepsilon_{QM}(\mu_e(r)) dr + \int_{R_{QM}}^{R_{cell}} \varepsilon_{NM}(\mu_e(r)) dr + E_{elec} + E_{surface}(R_{QM})$$

CFL/nuclear mixed phase vs. surface tension



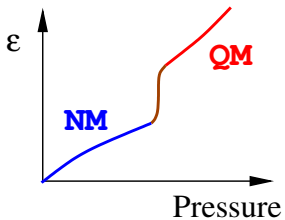
For comparison:
nuclear-vacuum
surface tension (from
liq drop model) is
about 0.8 MeV/fm^2 .

(Alford, Rajagopal, Reddy, Wilczek, [hep-ph/0105009](https://arxiv.org/abs/hep-ph/0105009))

It takes a surface tension of about 30 MeVfm^{-2}
to completely eliminate the mixed phase.

Smooth crossover vs sharp transition

Energy Density Incompressibility $\frac{dp}{d\varepsilon} = c_s^2$ Hybrid star profile

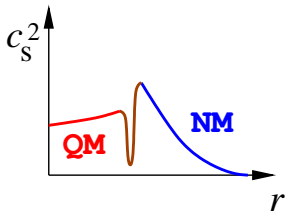
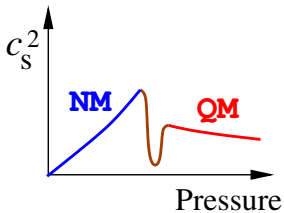
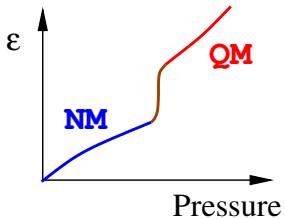


Smooth crossover vs sharp transition

Energy Density

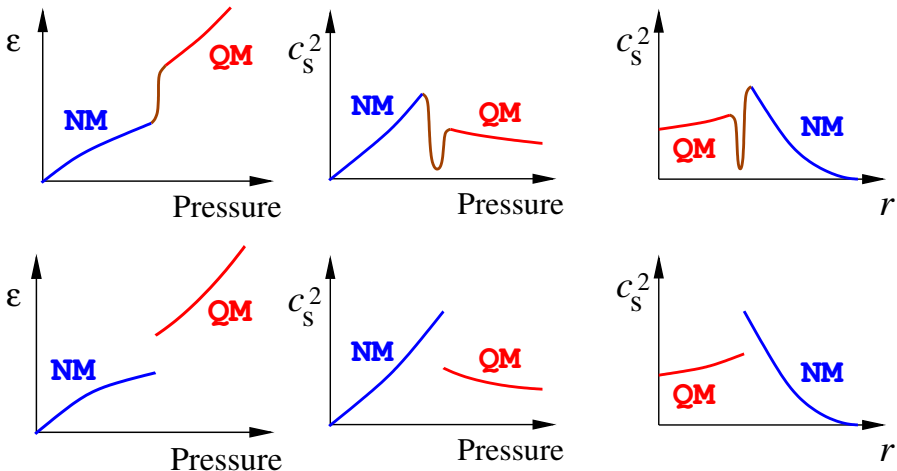
Incompressibility $\frac{dp}{d\varepsilon} = c_s^2$

Hybrid star profile



Smooth crossover vs sharp transition

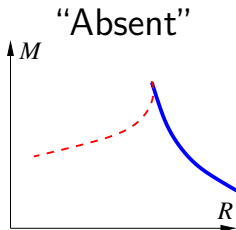
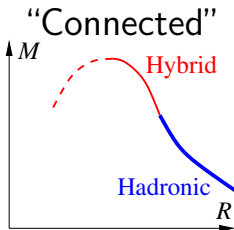
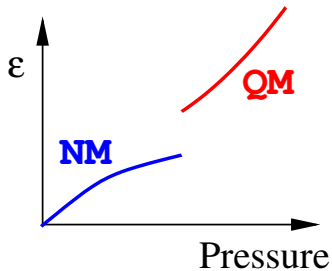
Energy Density Incompressibility $\frac{dp}{d\varepsilon} = c_s^2$ Hybrid star profile



How does a sharp first-order transition affect the mass-radius relation for a hybrid star?

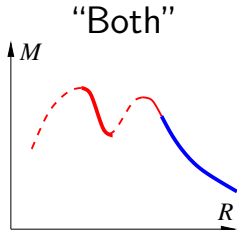
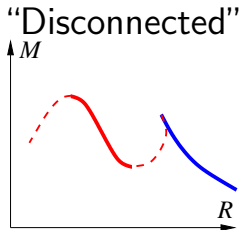
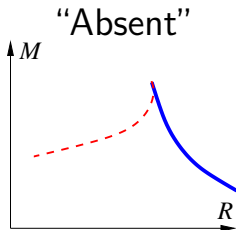
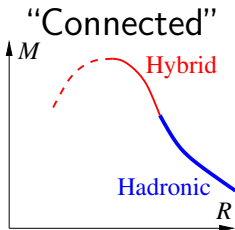
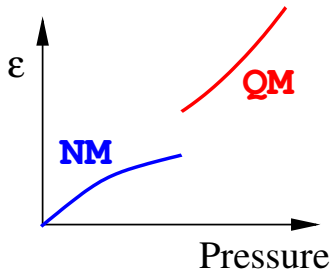
Maxwell-type 1OPT: hybrid star $M(R)$

Mass-radius relation has $\begin{cases} \text{Hadronic branch (no QM core)} \\ \text{Hybrid branch(es) (QM core present)} \end{cases}$



Maxwell-type 1OPT: hybrid star $M(R)$

Mass-radius relation has $\left\{ \begin{array}{l} \text{Hadronic branch (no QM core)} \\ \text{Hybrid branch(es) (QM core present)} \end{array} \right.$

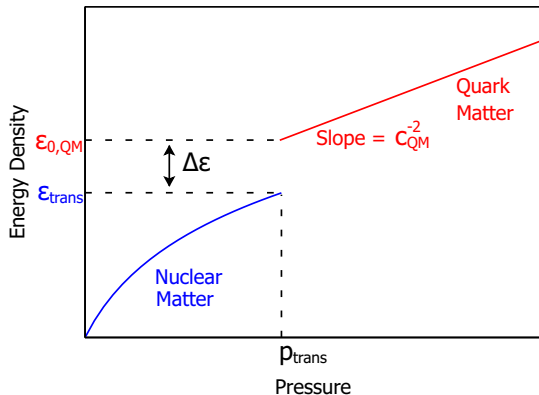


What features of the EoS determine the form?

CSS: a fairly generic QM EoS

Model-independent parameterization of a sharp 1st-order transition to quark matter with Constant [density-indp] Speed of Sound (CSS)

$$\varepsilon_{\text{QM}}(p) = \varepsilon_{\text{trans}} + \Delta\varepsilon + c_{\text{QM}}^{-2}(p - p_{\text{trans}})$$



CSS EoS params:

$$p_{\text{trans}}/\varepsilon_{\text{trans}}$$

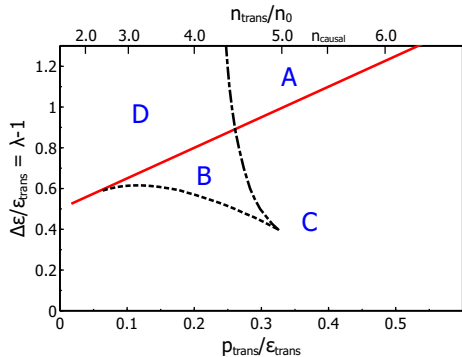
$$\Delta\varepsilon/\varepsilon_{\text{trans}}$$

$$c_{\text{QM}}^2$$

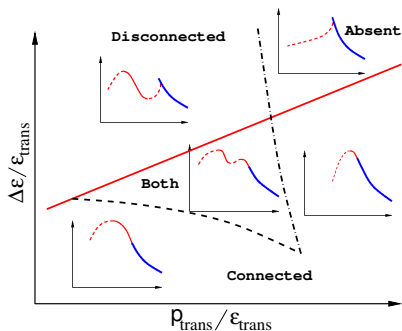
Zdunik, Haensel, [arXiv:1211.1231](https://arxiv.org/abs/1211.1231); Alford, Han, Prakash, [arXiv:1302.4732](https://arxiv.org/abs/1302.4732)
Modeling instead: Talks by David Alvarez Castillo, Jean-Loic Kneur.

CSS “Phase diagram” of hybrid star $M(R)$

Soft NM + CSS ($c_{QM}^2 = 1$)



Schematic



$$\frac{\Delta\epsilon_{\text{crit}}}{\epsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\epsilon_{\text{trans}}}$$

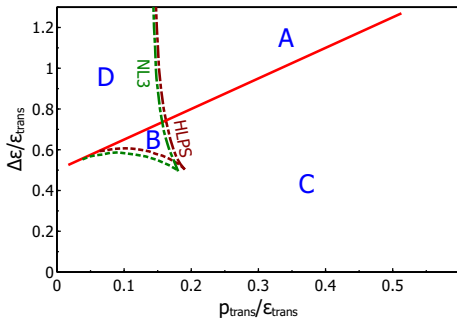
- Below the red line ($\Delta\epsilon < \Delta\epsilon_{\text{crit}}$), there is a connected hybrid branch

(Seidov, 1971; Schaeffer, Zdunik, Haensel, 1983; Lindblom, [gr-qc/9802072](#))

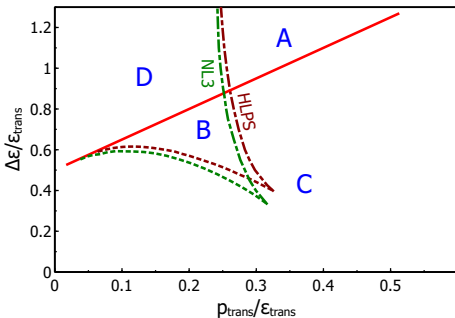
- Inside the dashed “V” there is a disconnected hybrid branch

Sensitivity to nuclear EoS and c_{QM}^2

$$c_{\text{QM}}^2 = 1/3$$



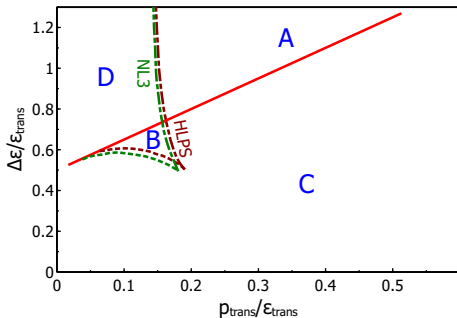
$$c_{\text{QM}}^2 = 1$$



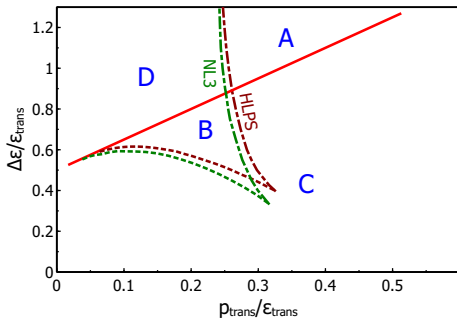
- Nuclear EoS (HLPS=soft, NL3=stiff) does not make much difference.
- Stiffer quark matter (higher c_{QM}^2) favors disconnected branch.

Sensitivity to nuclear EoS and c_{QM}^2

$$c_{\text{QM}}^2 = 1/3$$



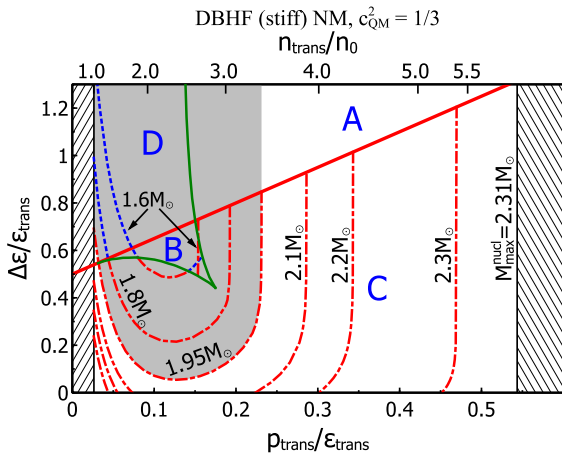
$$c_{\text{QM}}^2 = 1$$



- Nuclear EoS (HLPS=soft, NL3=stiff) does not make much difference.
- Stiffer quark matter (higher c_{QM}^2) favors disconnected branch.

Any observational constraint can be plotted in the CSS parameter space.

Constraints on QM EoS from M_{\max}

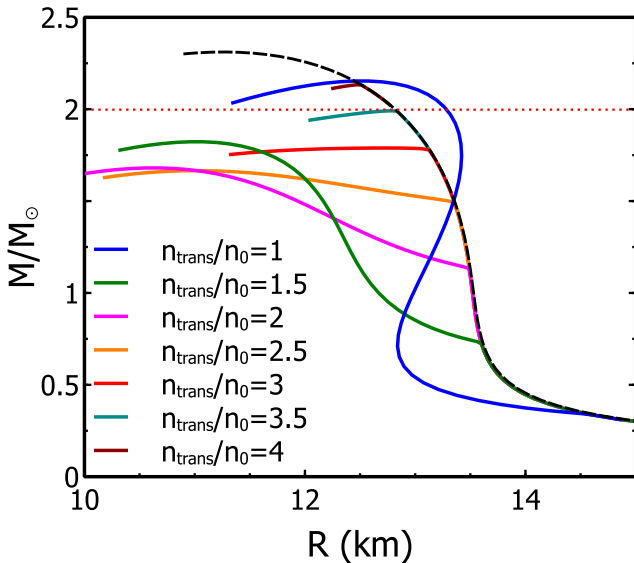


$2 M_{\odot}$ observation allows two scenarios:

- high p_{trans} : very small connected branch
- low p_{trans} : modest $\Delta\varepsilon$, no disconnected branch.

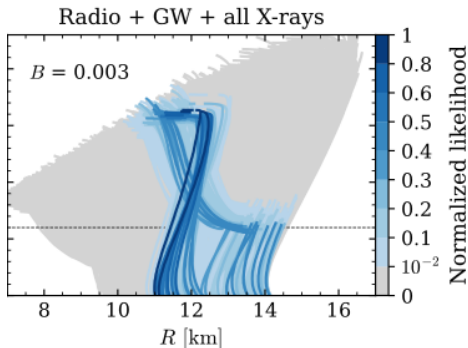
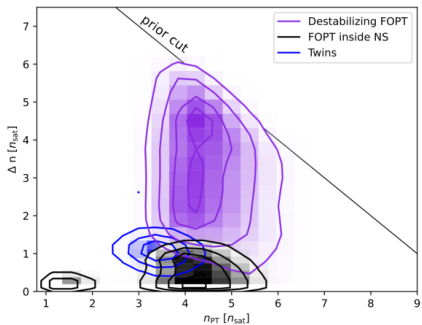
Low p_{trans} and high p_{trans} windows

DBHF (stiff) NM, $c_{\text{QM}}^2 = 1/3$, $\Delta\varepsilon/\varepsilon_{\text{trans}} = 0.4$



Comprehensive surveys

CSS is a simple parameterization. Do its predictions agree with more sophisticated model-independent analyses?



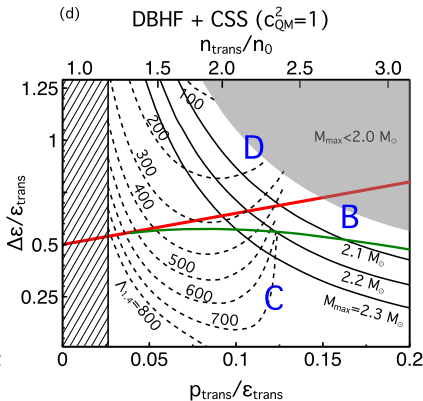
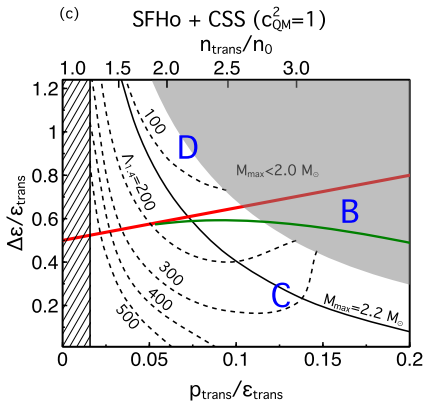
Talks by Oleg Komoltsev, Sofia Blomqvist, Violetta Sagun

Summary: first-order transitions can occur

- at high density, \rightarrow twin branch and possibly destabilizing the star
- at low density, \rightarrow connected hybrid branch

Constraints from other observables: tidal deformability

Any observational constraint can be plotted in the CSS parameter space.
E.g.: tidal deformability



Could we identify hybrid stars via $M(R)$?

Signatures of a phase transition to a high-density phase:

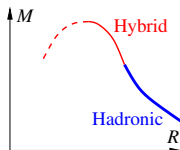
(C) Connected branch:

Masquerade problem

Speed of sound constraints

Talks by Gargi Chaudhuri, Bao-An Li,

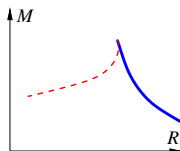
Oleksii Ivanytskyi



(A) No hybrid branch:

Nuclear branch ends with $dM/dR \neq 0$

Can be hard to identify



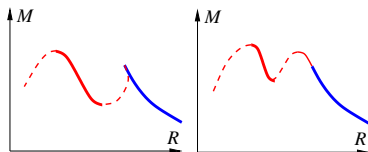
(B,D) Disconnected branch

Twin stars, gap in radii at similar mass

Talks by Dhrubajyoti Rakshit, Mirco Guerrini,

Jan-Erik Christian, Sofia Blomqvist,

Aleksi Vuorinen



Signatures of a sharp first-order transition

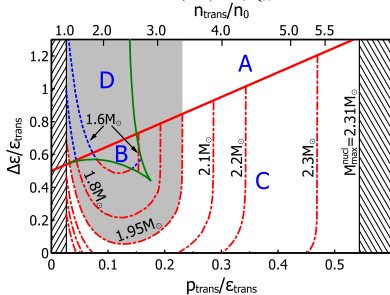
Lots of work being done on this, including many people at this conference.

- ▶ Features of mass-radius relation
 - jump in energy density (“speed of sound”)
 - twin stars
- ▶ Gravitational and electromagnetic signals from mergers:
 - inspiral: tidal deformability
 - post-merger: creation/expansion of new phase
 - Talk by Yongjia Huang, Jorge Solana*
- ▶ Pulsar timing: r -mode spindown, backbending, corequake, quark nova; *Talk by Rachid Ouyed*
- ▶ Cooling (sudden change at a mass threshold)
 - e.g. Wei et. al. [arXiv:2003.08079](https://arxiv.org/abs/2003.08079)
- ▶ Global oscillations of neutron stars; *Talk by Armen Sedrakian*

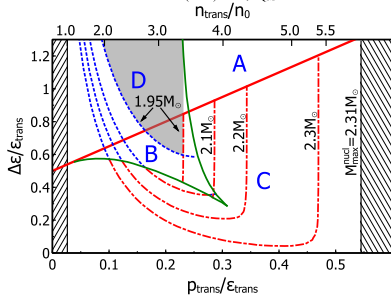
Extra slides

Constraints on QM EoS from M_{\max}

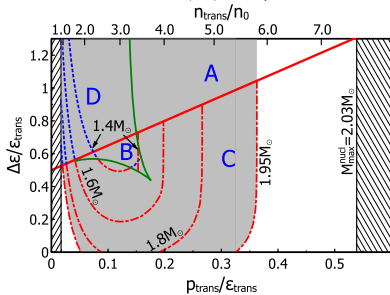
DBHF (stiff) NM, $c_{\text{QM}}^2 = 1/3$



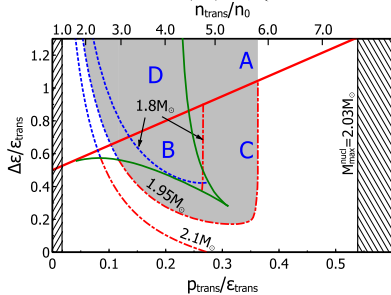
DBHF (stiff) NM, $c_{\text{QM}}^2 = 1$



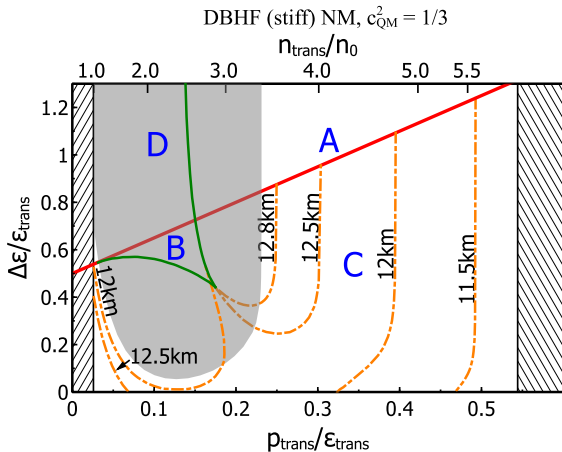
BHF (soft) NM, $c_{\text{QM}}^2 = 1/3$



BHF (soft) NM, $c_{\text{QM}}^2 = 1$



Radius of heaviest star $R_{\max M}$



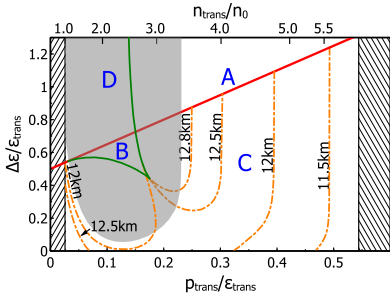
Heaviest star is typically the smallest, so lower limit on $R_{\max M}$ is the minimum radius of compact stars.

High p_{trans} : very short connected hybrid branch, radius like that of heaviest hadronic star.

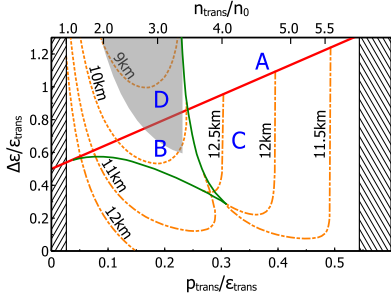
Low p_{trans} : need to zoom in.

Constraints on QM EoS from $R_{\max}M$

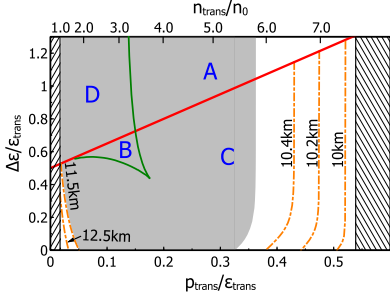
DBHF (stiff) NM, $c_{\text{QM}}^2 = 1/3$



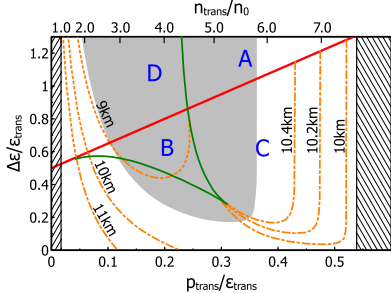
DBHF (stiff) NM, $c_{\text{QM}}^2 = 1$



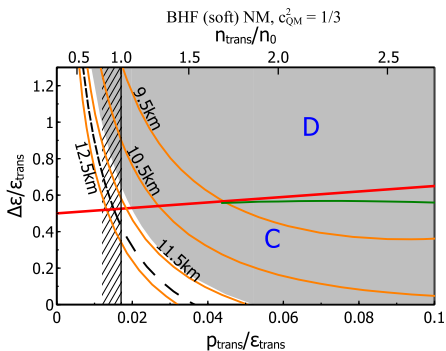
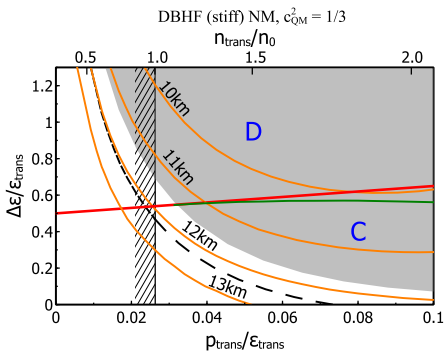
BHF (soft) NM, $c_{\text{QM}}^2 = 1/3$



BHF (soft) NM, $c_{\text{QM}}^2 = 1$



Focus on low p_{trans} and $c_{\text{QM}}^2 = 1/3$



- ▶ $R_{\text{max}M}$ contours closely follow mass contours
- ▶ $M_{\text{max}} > 1.95 M_{\odot}$ requires $R > 11.25$ km
- ▶ dashed line is $M_{\text{max}} = 2.1 M_{\odot}$, requires $R > 12.1$ km
- ▶ Observation of a smaller star \Rightarrow high transition pressure or $c_{\text{QM}}^2 > 1/3$