Superfluidity (in compact stars)

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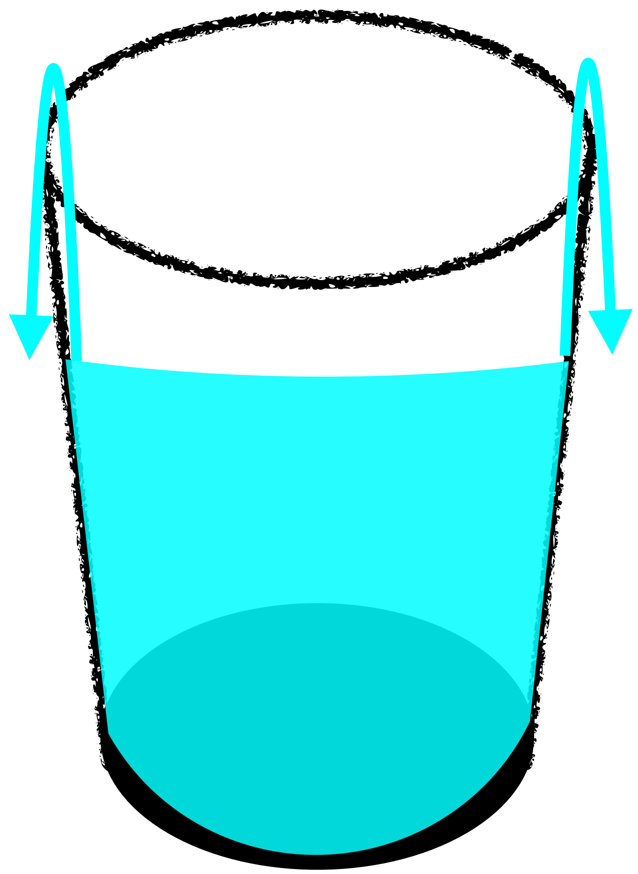
Outline

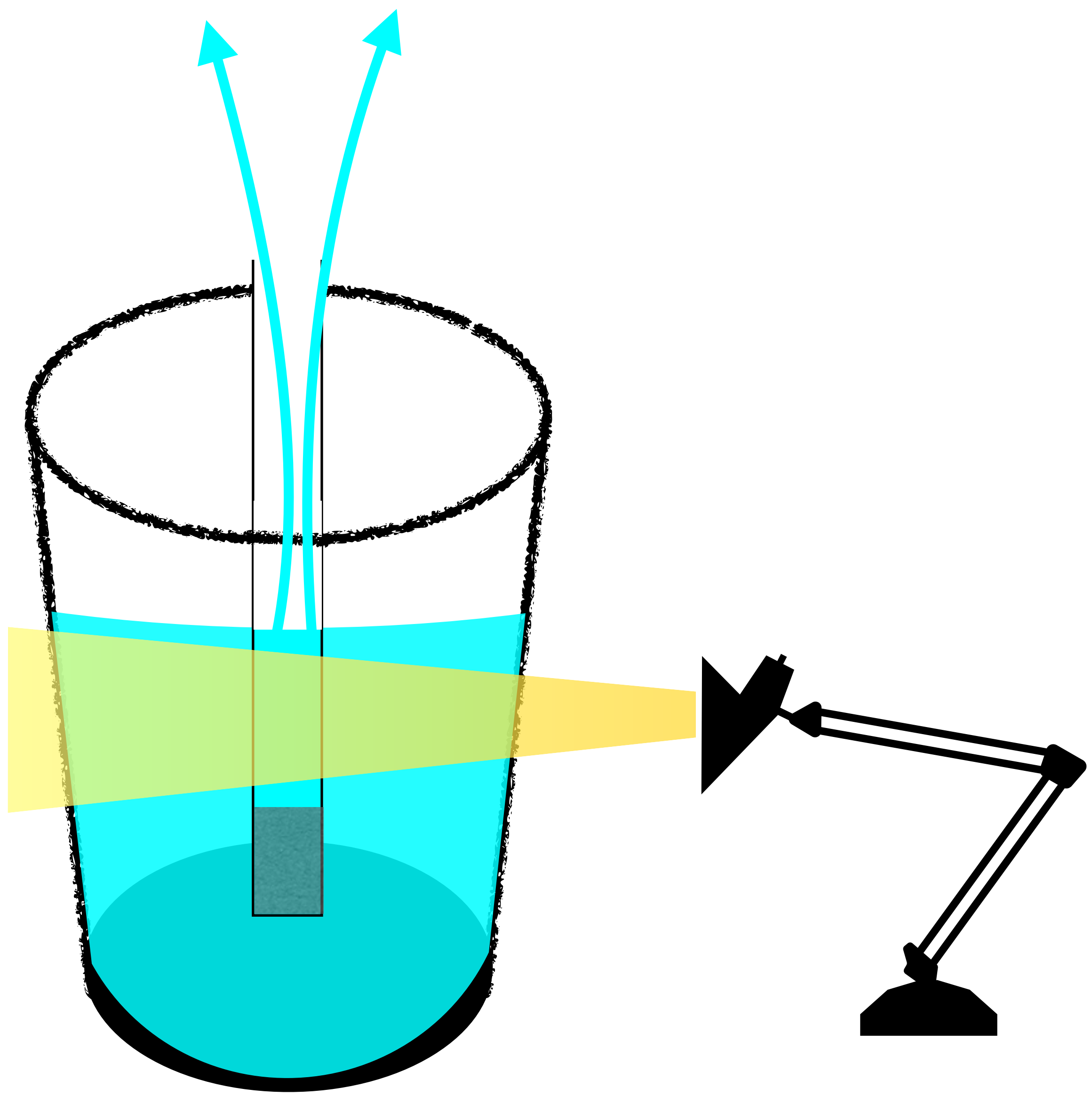
- **Superfluids**
- **Compact stars**
- **Emulations**
- **Glitches**
- **Conclusions**

Superfluids

A fluid with superpowers



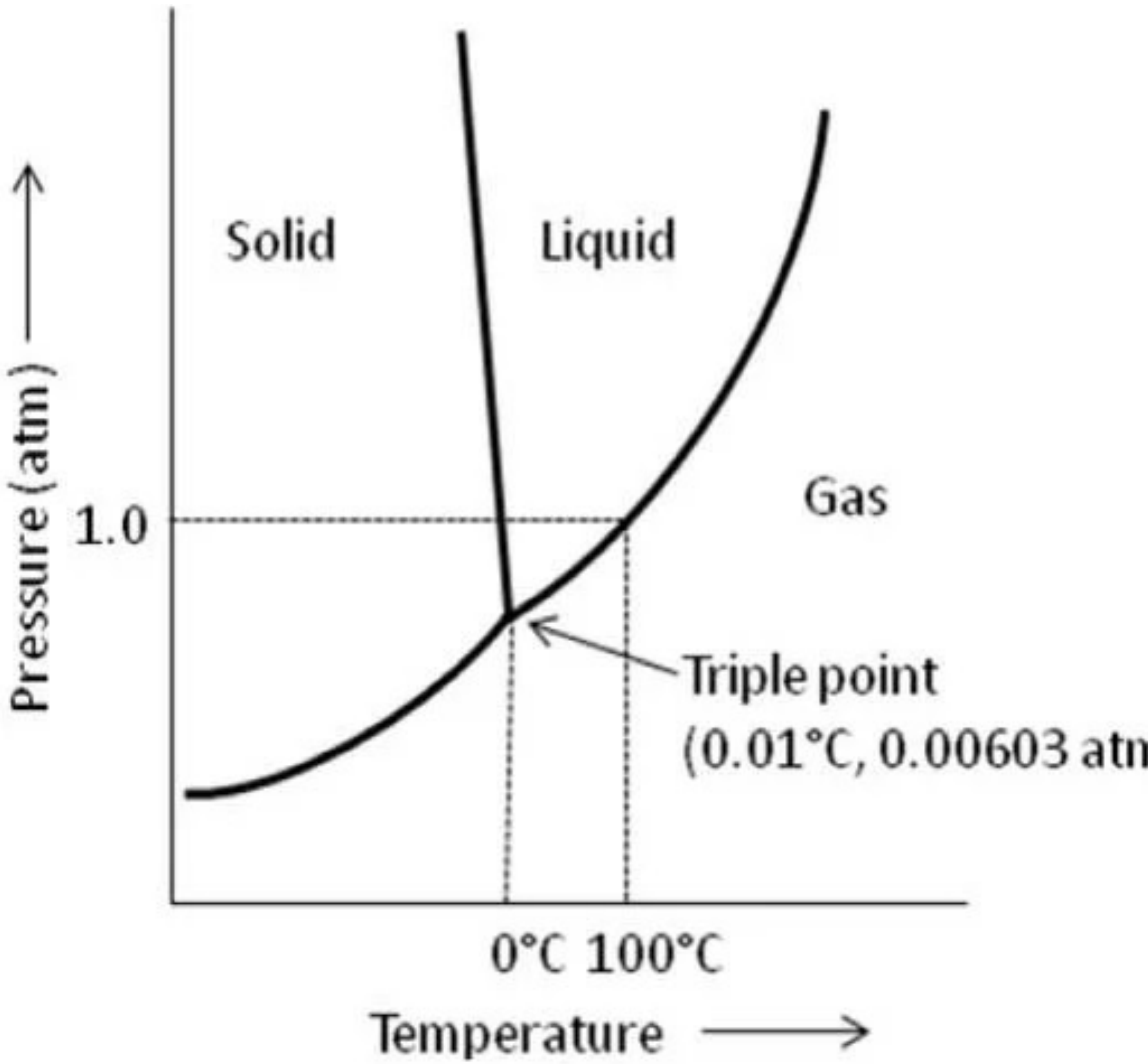
Climbing power



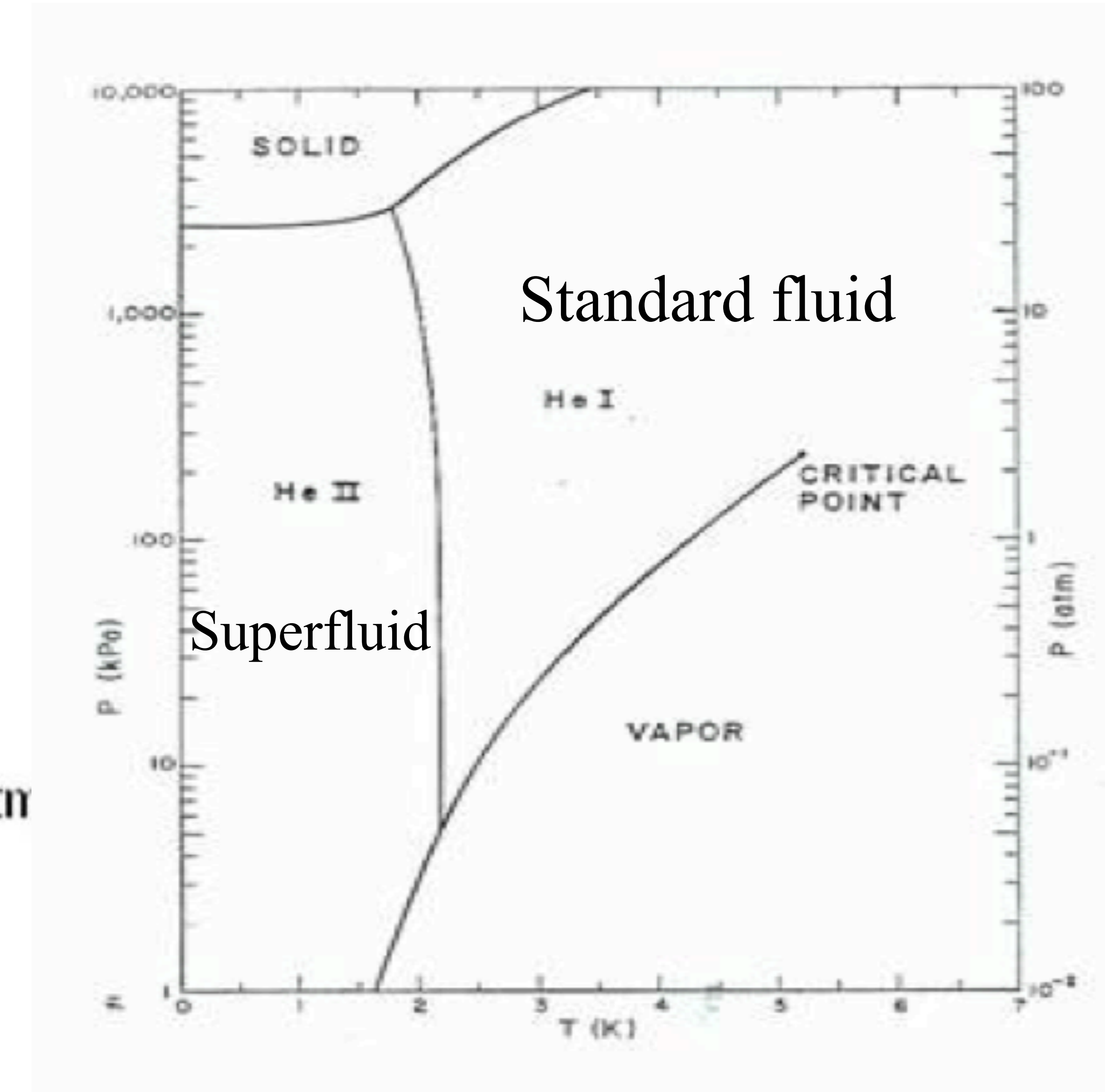
Flying power

Phase diagram

WATER



^{4}He



What is a superfluid?

"A fluid that flows with no viscosity"

What is viscosity?

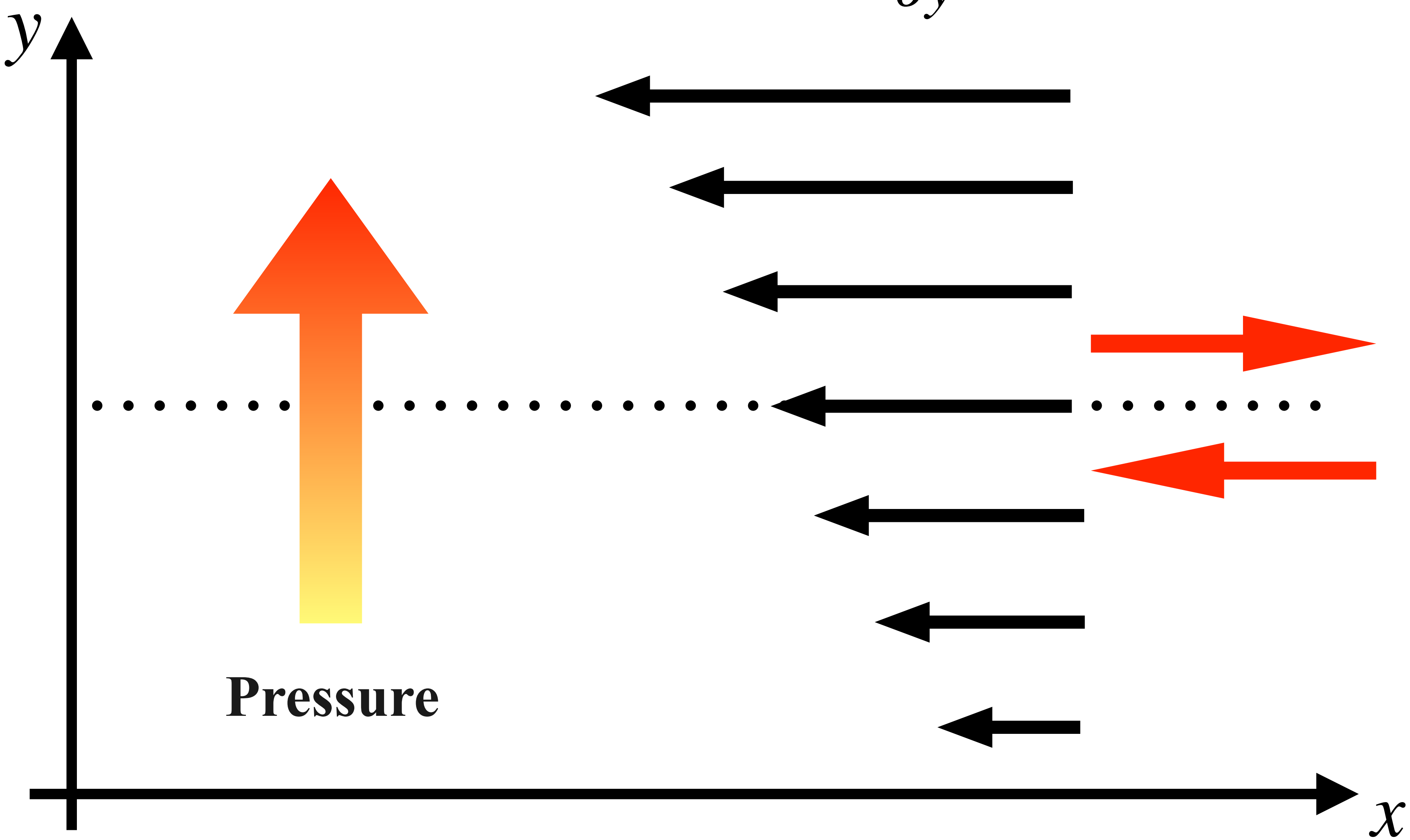
- Measures fluid's resistance to movement
- Measures how "sticky" is a fluid

A good fluid has by definition low viscosity.

However, a fluid with zero viscosity does not stick at all: it becomes dry

Shear viscosity

$$\mathbf{u} = (u_x, 0, 0) \qquad \frac{\partial u_x}{\partial y} \neq 0$$

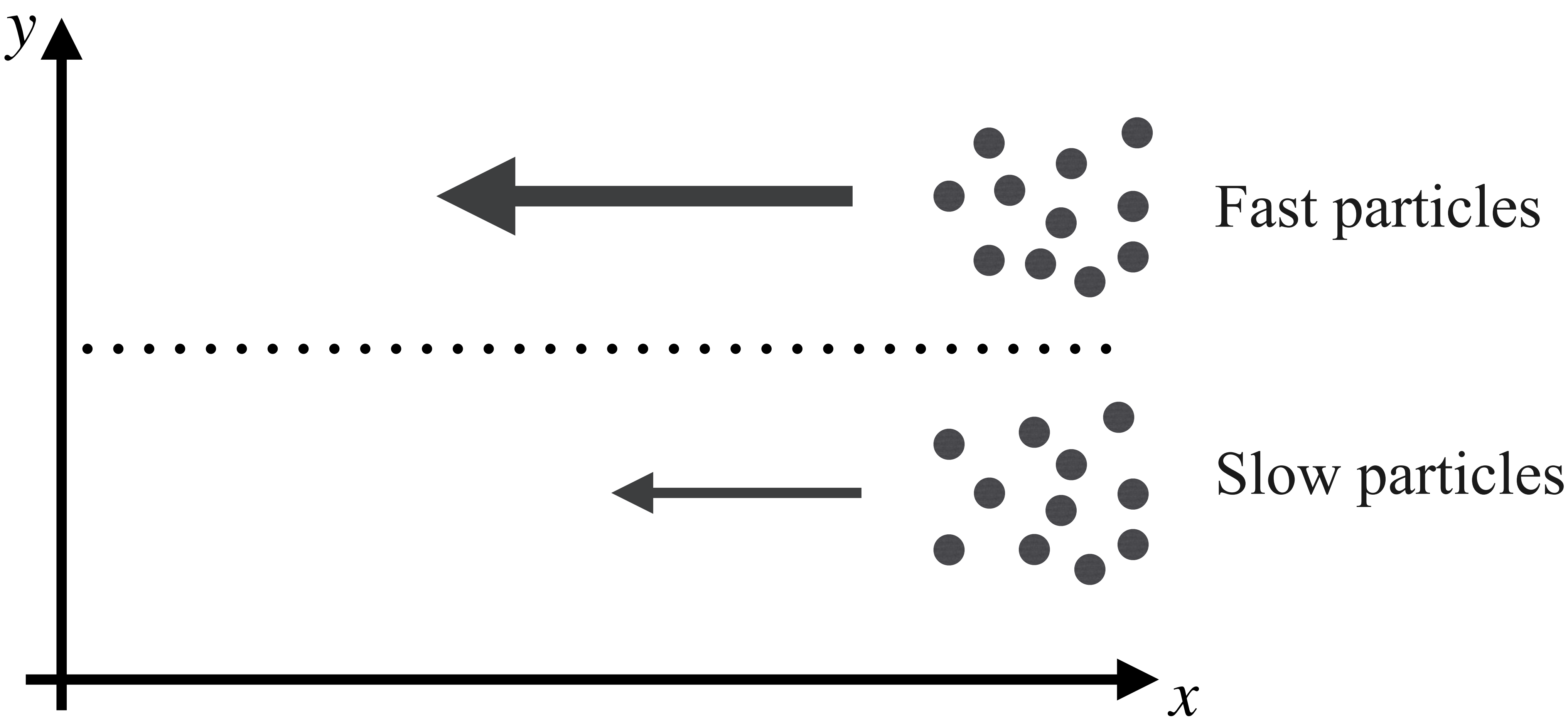


$$F = \left| \nu \frac{\partial u_x}{\partial y} \right|$$

shear viscosity coefficient

Unstable hydrodynamic configuration

Microscopic



Diffusion tends to isotropize the flow

Double role of interactions:

- **Needed to scatter particles between the two layers: produce viscosity**
- **Strong interactions reduce the mean free path: reduce viscosity**

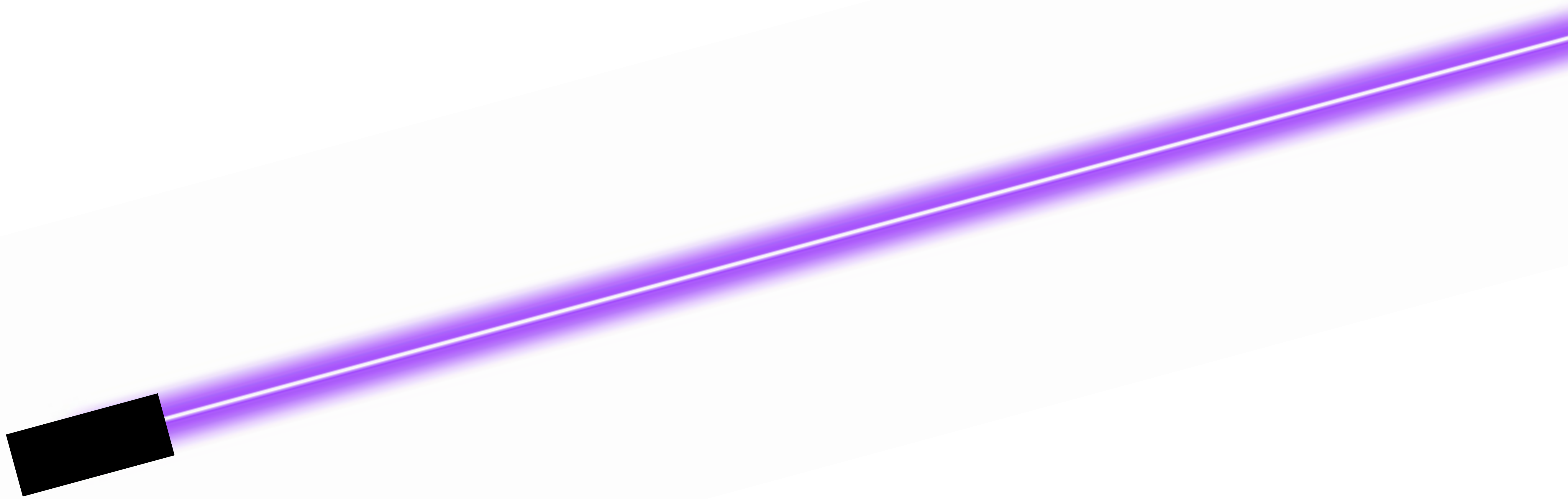
“A fluid that flows with no viscosity”

It has infinite scattering length… or zero scattering length?

In any case the fundamental role is played by excitations

In a superfluid there are very few excitations

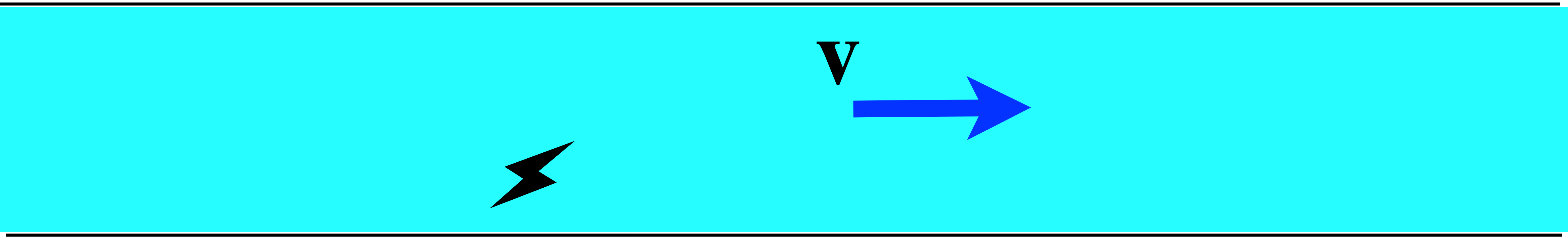
Consider a laser beam in vacuum



Photons do not scatter: no shear viscosity

It is not a superfluid, because it actually does not flow

Superfluid property



$$E = \frac{1}{2}Mv^2$$ Energy of the fluid **no** excitations - lab frame

$$\epsilon \equiv \epsilon(p)$$ Excitation energy - comoving frame

$$E = \frac{1}{2}Mv^2 + \epsilon + \mathbf{p}\cdot\mathbf{v}$$ Energy of the fluid **with** one excitation - lab frame

Excitations appear if $\epsilon + \mathbf{p}\cdot\mathbf{v} < 0 \longrightarrow \epsilon - pv < 0$

Superfluid condition $v < v_{\text{cr}} = \text{Min}\frac{\epsilon(p)}{p}$ Landau's criterion: $\text{Min}\frac{\epsilon(p)}{p} \neq 0$

Which system has this property?

Consider a system with a global $U(1)$ symmetry

Spontaneous symmetry breaking

Nambu Goldstone boson (NGB)

$$\epsilon = c_s p$$

speed of sound!

$$\text{Min}\frac{\epsilon(p)}{p} = c_s$$

SSB and superfluidity

Superfluid

Broken global symmetry

Goldstone theorem → **Easy** transport of the quantum numbers of the broken group

At sufficiently low T a macroscopic quantum state!

Does it always work?

Does the Goldstone theorem always imply a linear dispersion law?

Counter-example: Ferromagnets

Magnons are NGBs with dispersion law $\epsilon \propto p^2$ ➡ $\text{Min}\frac{\epsilon(p)}{p} = 0$

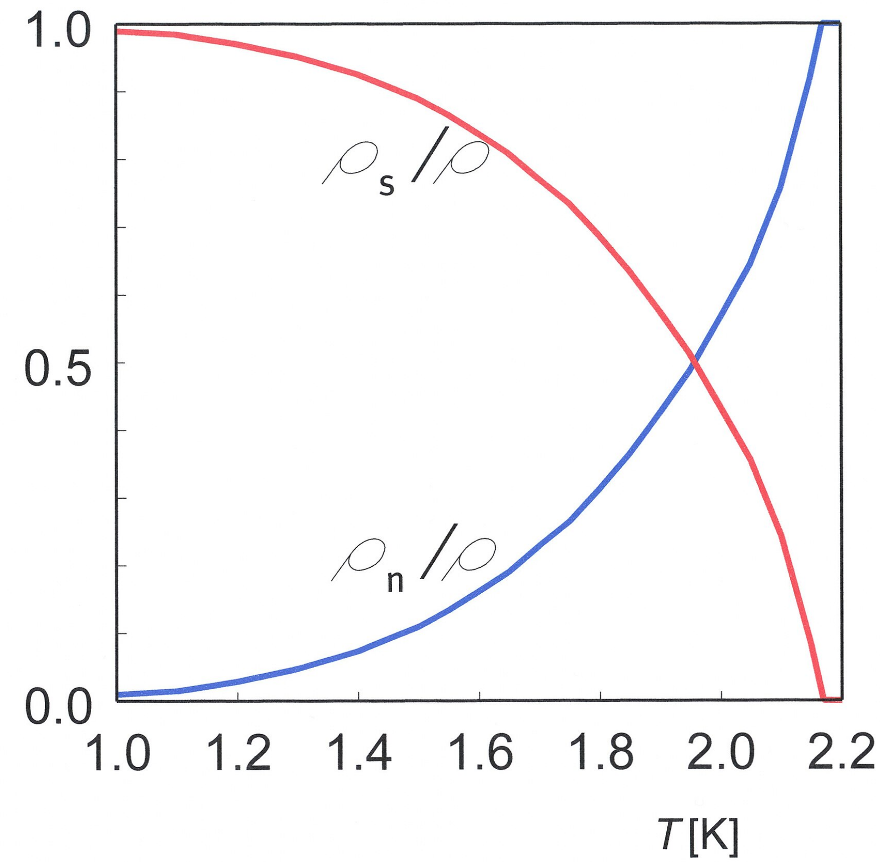
They are not superfluids!

Restrictions:

On How to Count Goldstone Bosons
H.B. Nielsen and S. Chadha
Nucl.Phys.B 105 (1976) 445-453

Landau - Tisza two fluid model

At $T \neq 0$ hydrodynamic
two components description



Superfluid component

Vanishing viscosity
Vanishing entropy

Coherent fluid

"Separable"
by a superleak

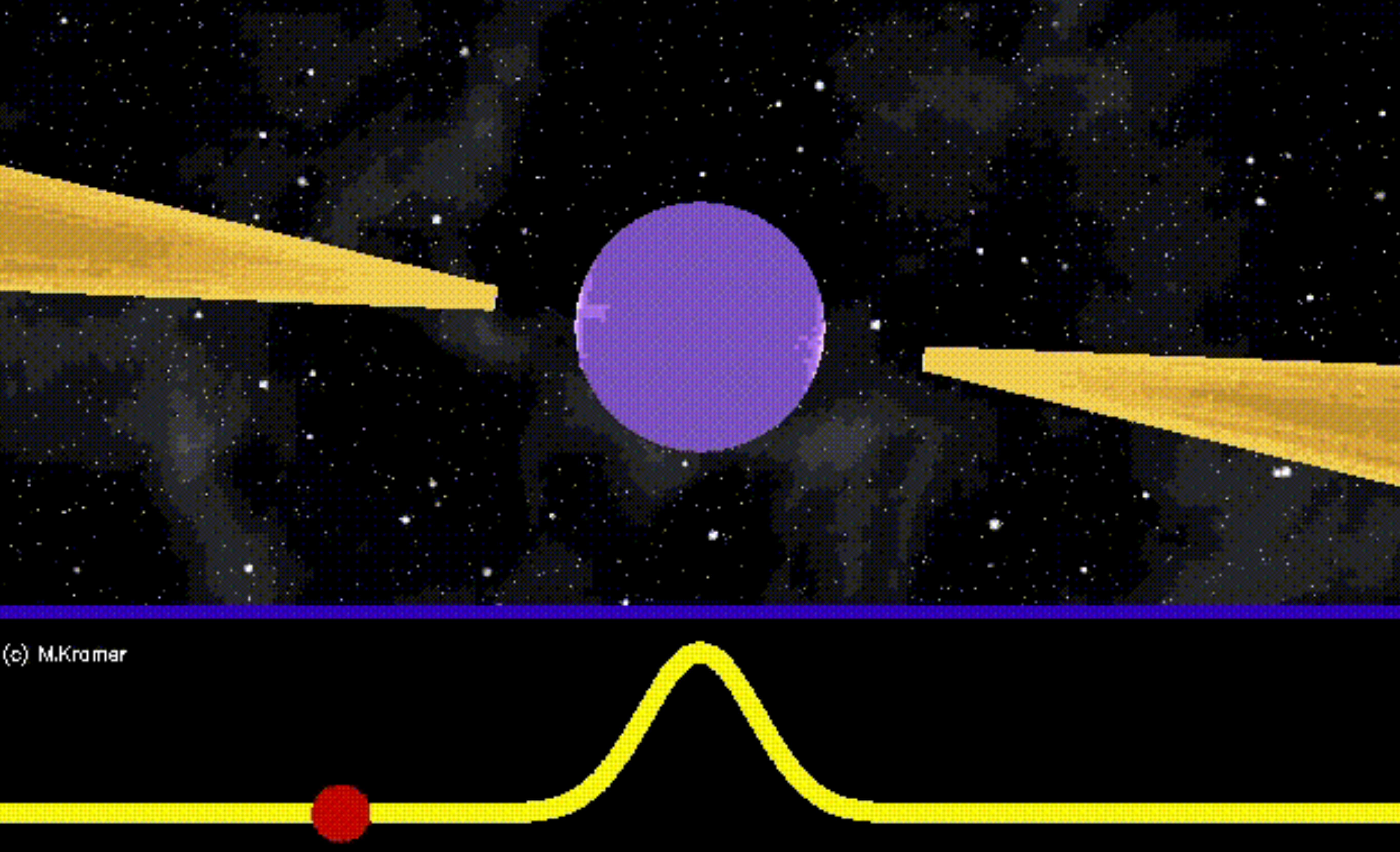
Normal component

Non-vanishing viscosity
Non-vanishing entropy

Excitations

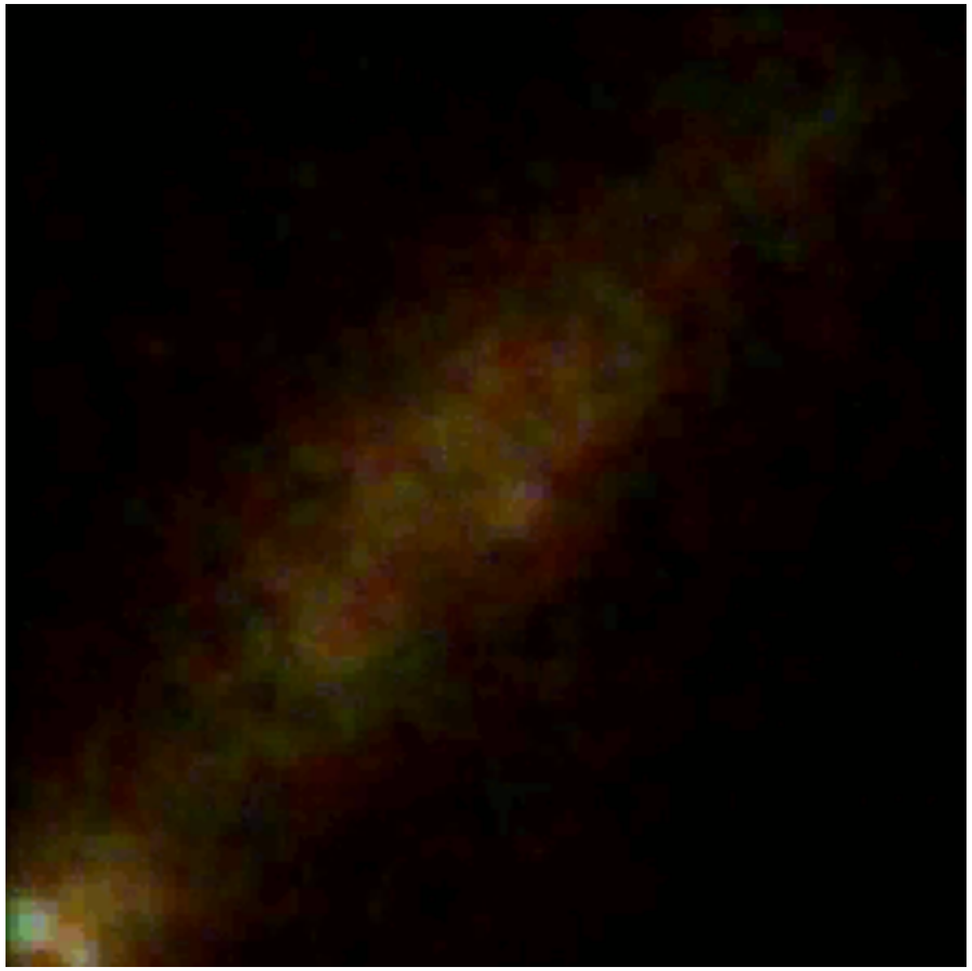
Rotation

How we see Neutron Stars



Pulsars

Vela



[Goddard Space Flight Center]

PSR B0329
P~0.7s

Vela
P~89ms

Crab
P~33ms

PSR J1748 –2446ad
P~1.4ms

Centrifuge 1600 rounds/minute ~ 30 rounds/s thus P~33 ms

Ferrari engine: P~3.16 ms

Spin down

Neutron stars lose angular momentum by photon emission

The process is **extremely** slow

$$\dot{P} \sim 10^{-22} - 10^{-9}\, ss^{-1}$$

loses 10^{-13} **seconds per century**

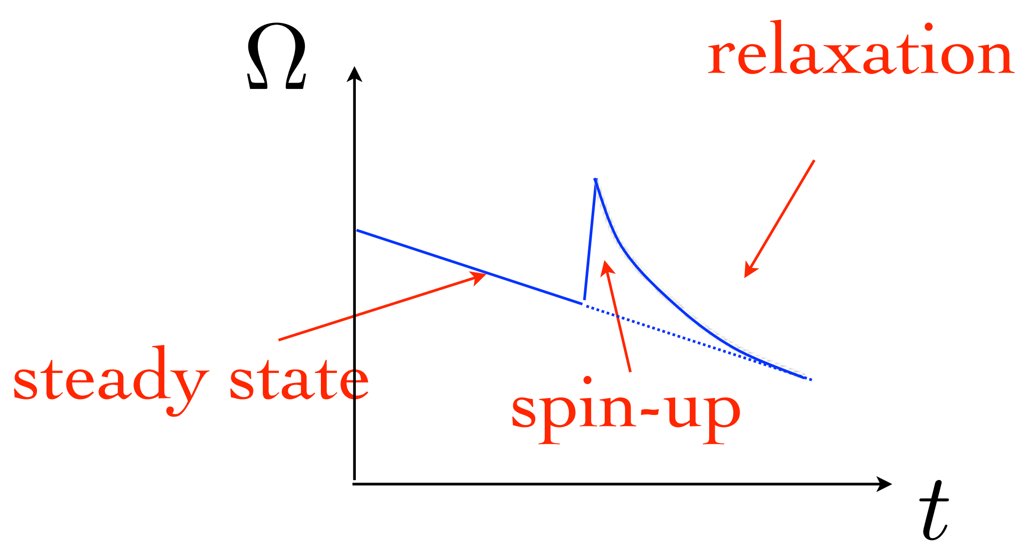
loses one second per century

One century $\sim 10^9 s$

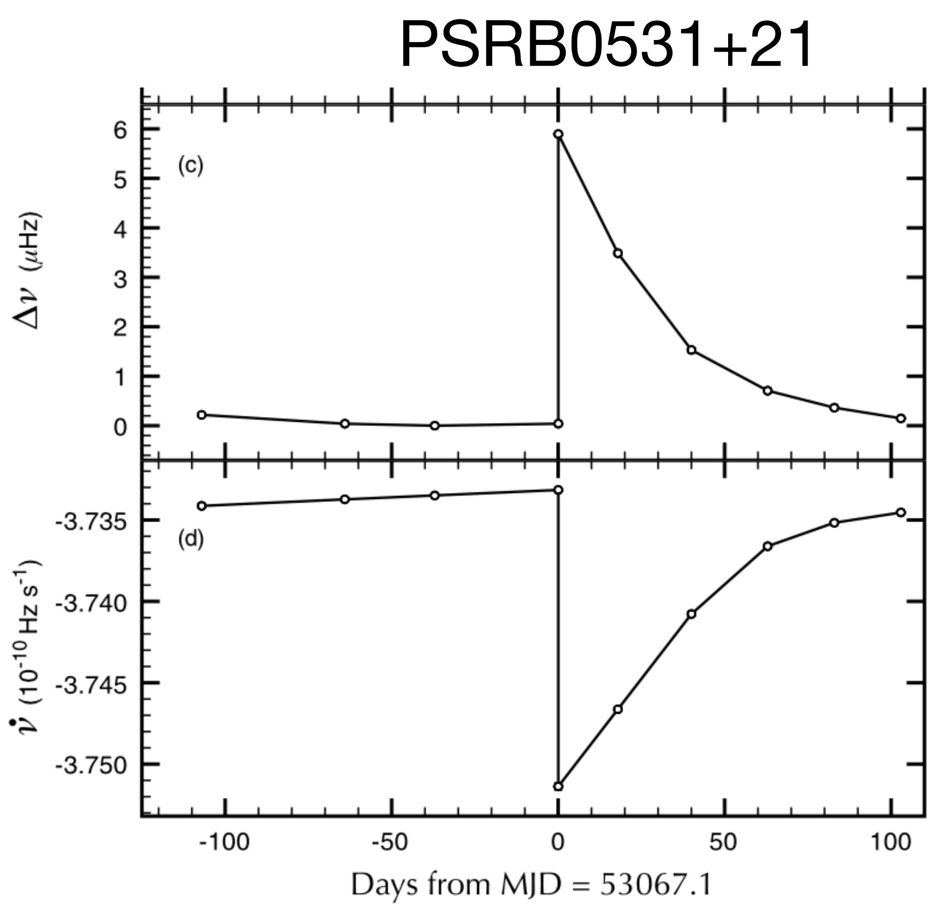
Glitches

Sudden speed-up in the rotational frequency

1. They occur without warning
2. Large variety of time scales



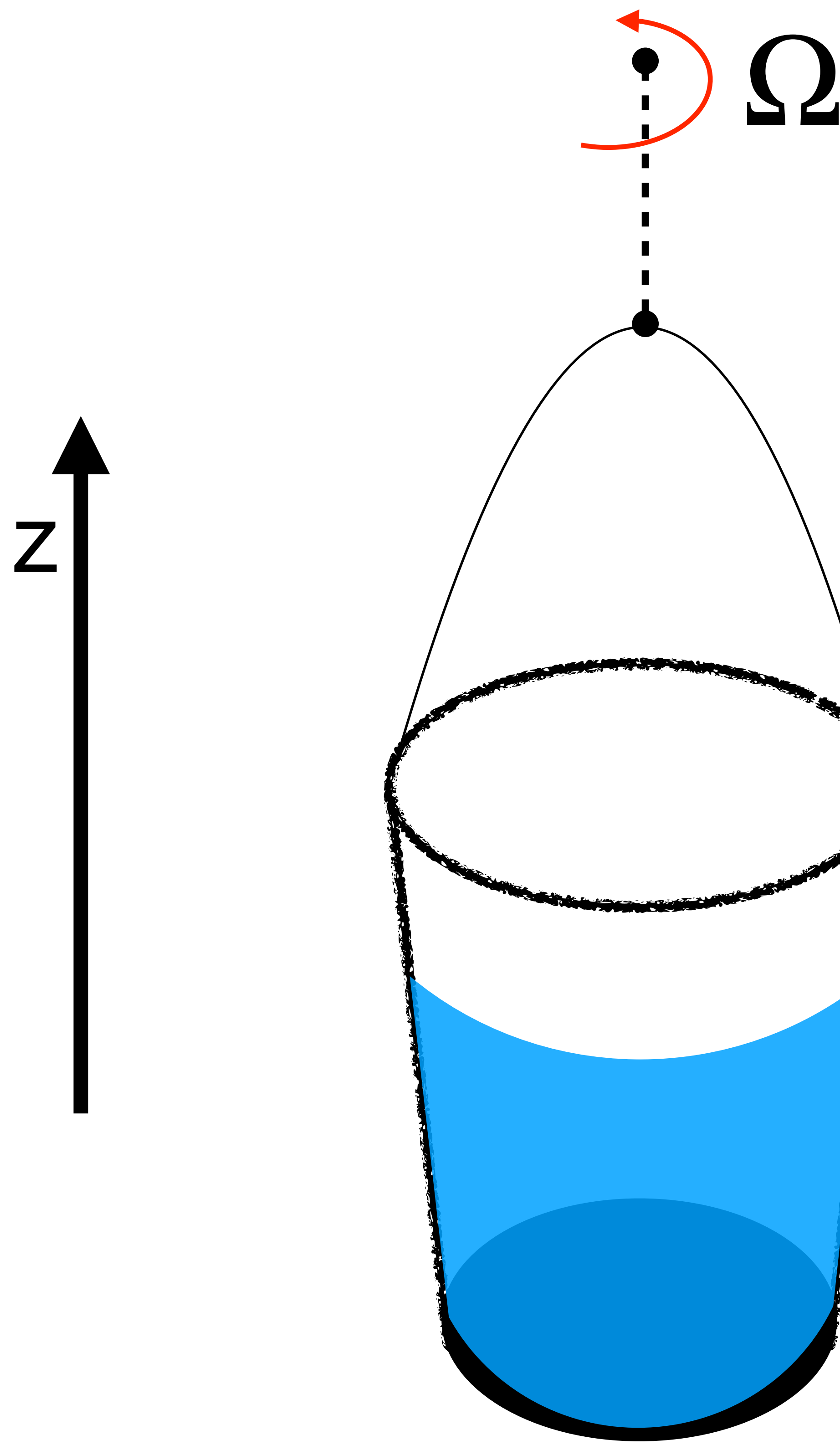
$$\Delta\Omega/\Omega \sim 10^{-12} - 10^{-3}$$



Espinoza et al., Mon. Not. R. Astron. Soc. 414, 1679–1704 (2011)

Rotating superfluid

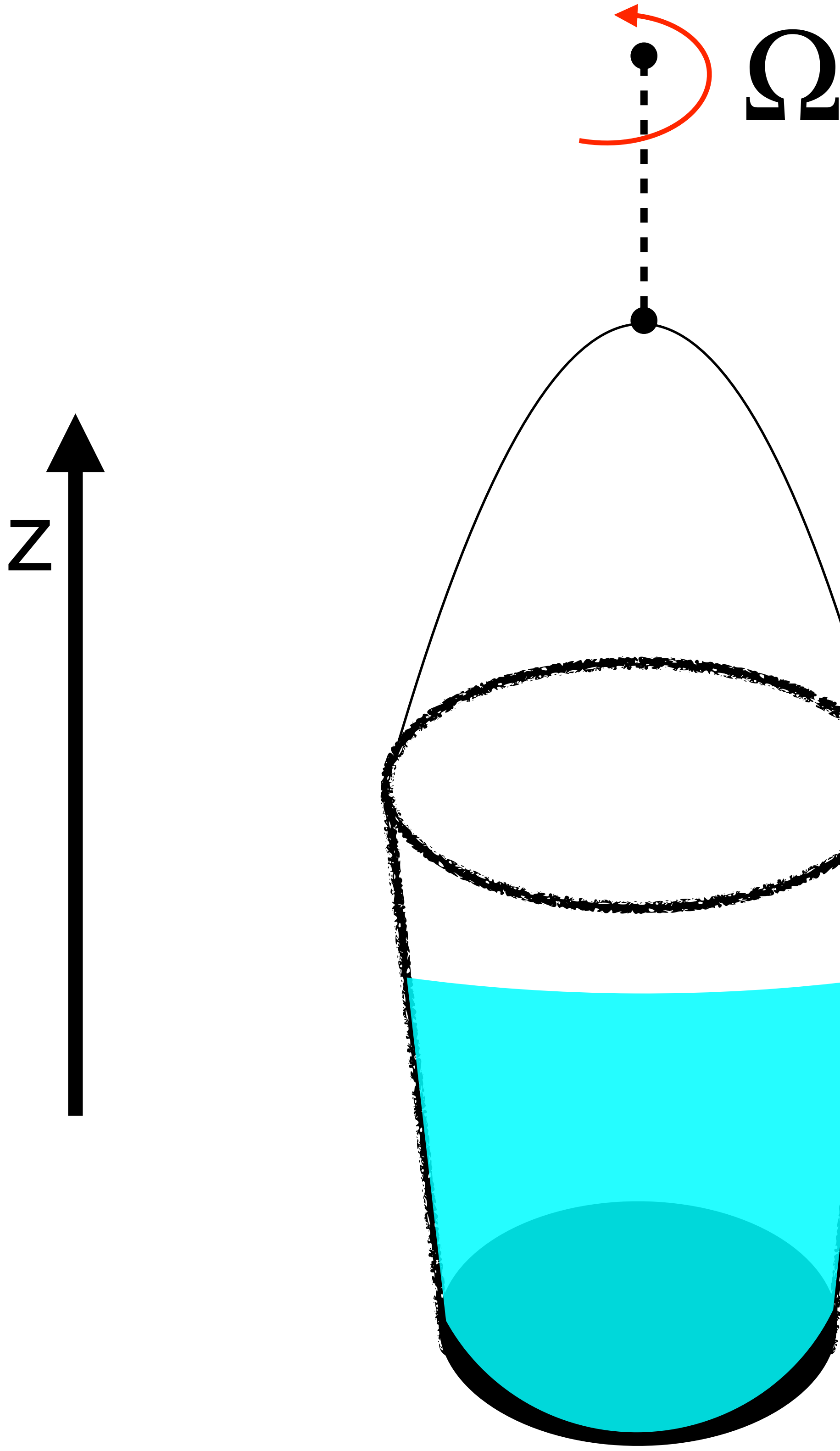
Rotating fluid



$$z_s = \frac{1}{2g}\Omega^2 r^2$$

$$L_z = I\Omega$$

Rotating supefluid

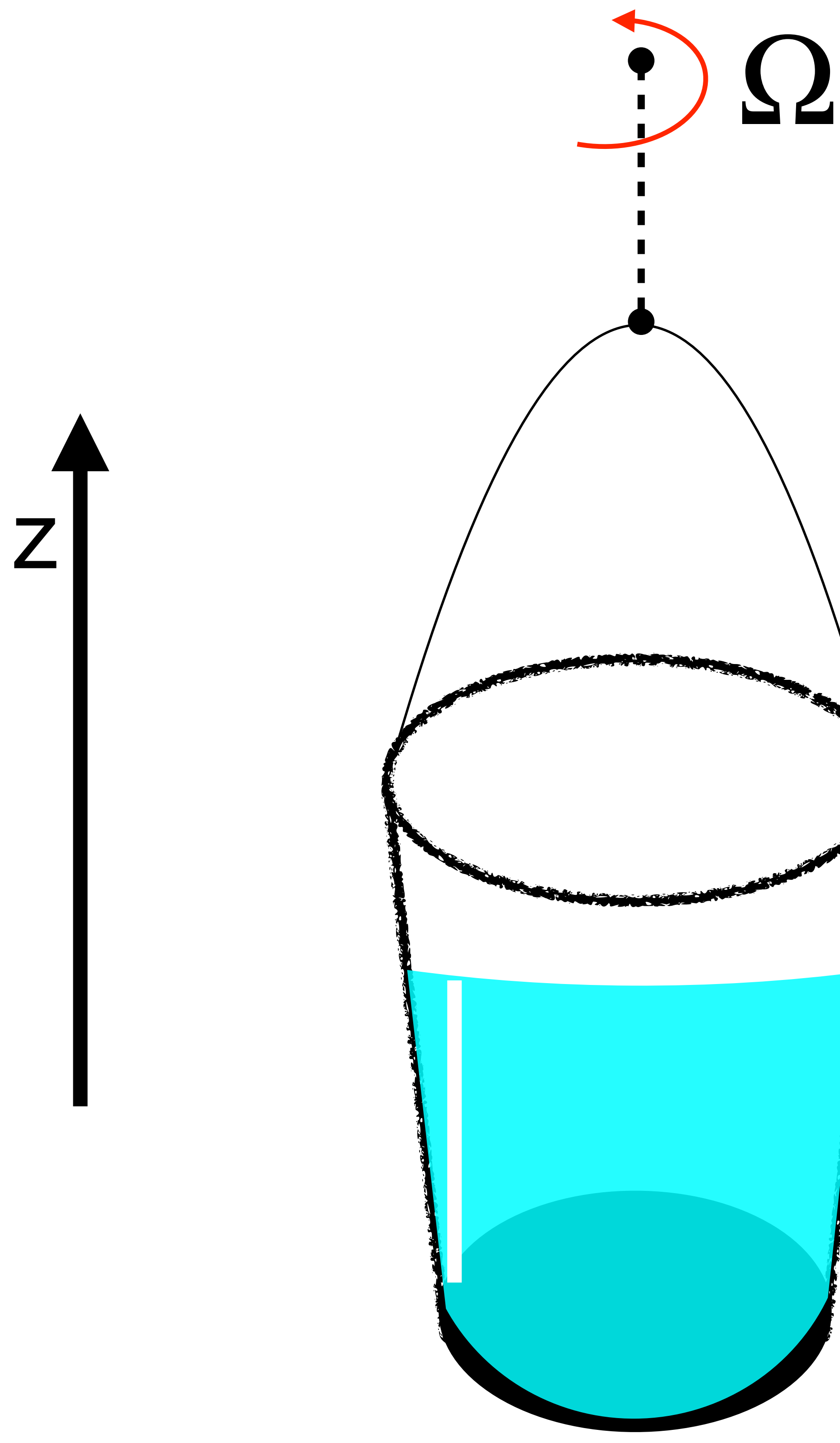


$\Omega < \Omega_c$

$z_s = 0$

$L_z = 0$

Rotating supefluid

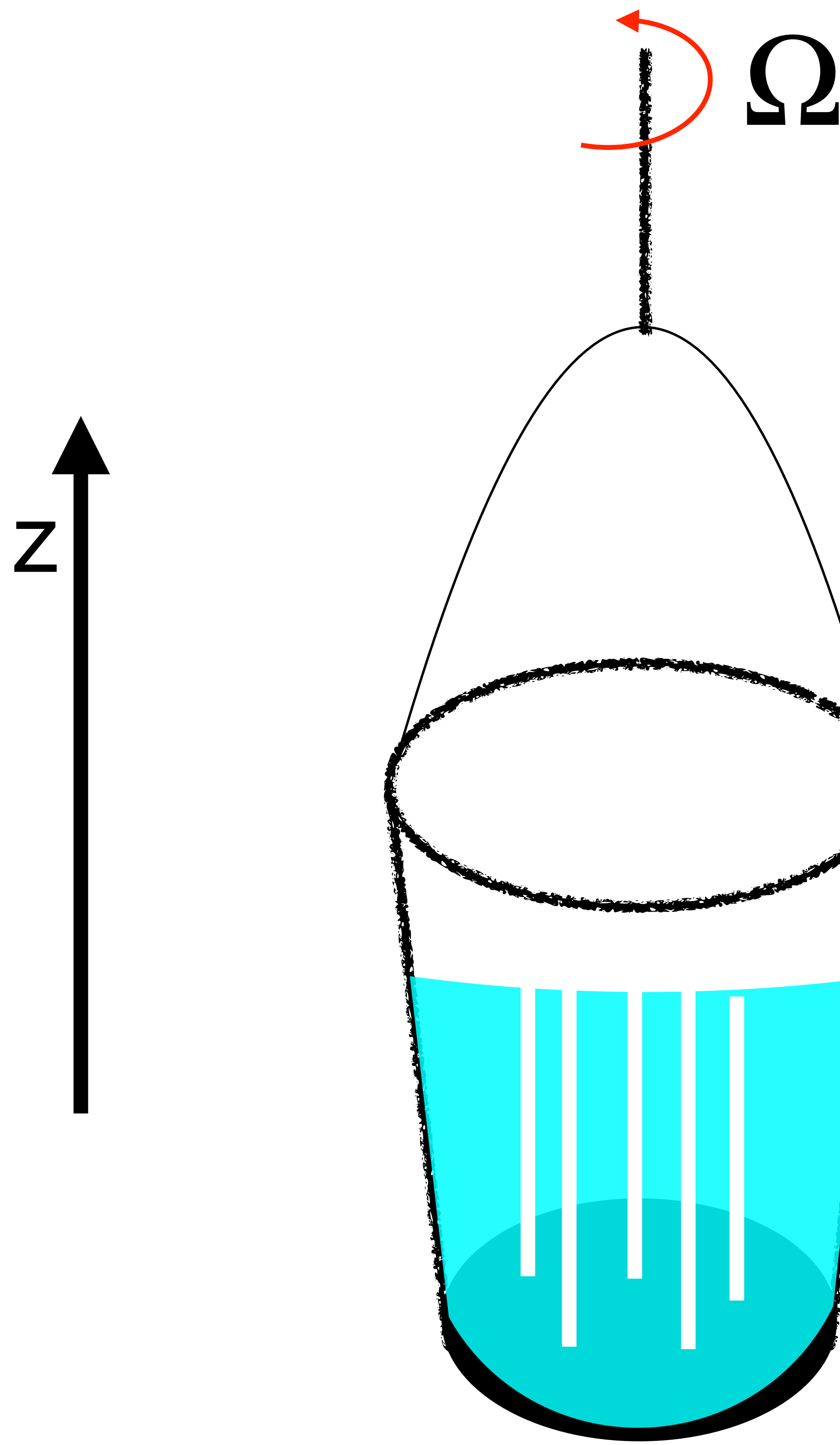


$$\Omega \gtrsim \Omega_c$$

$$z_s \simeq 0$$

$$L_z \simeq N\hbar$$

Rotating supefluid



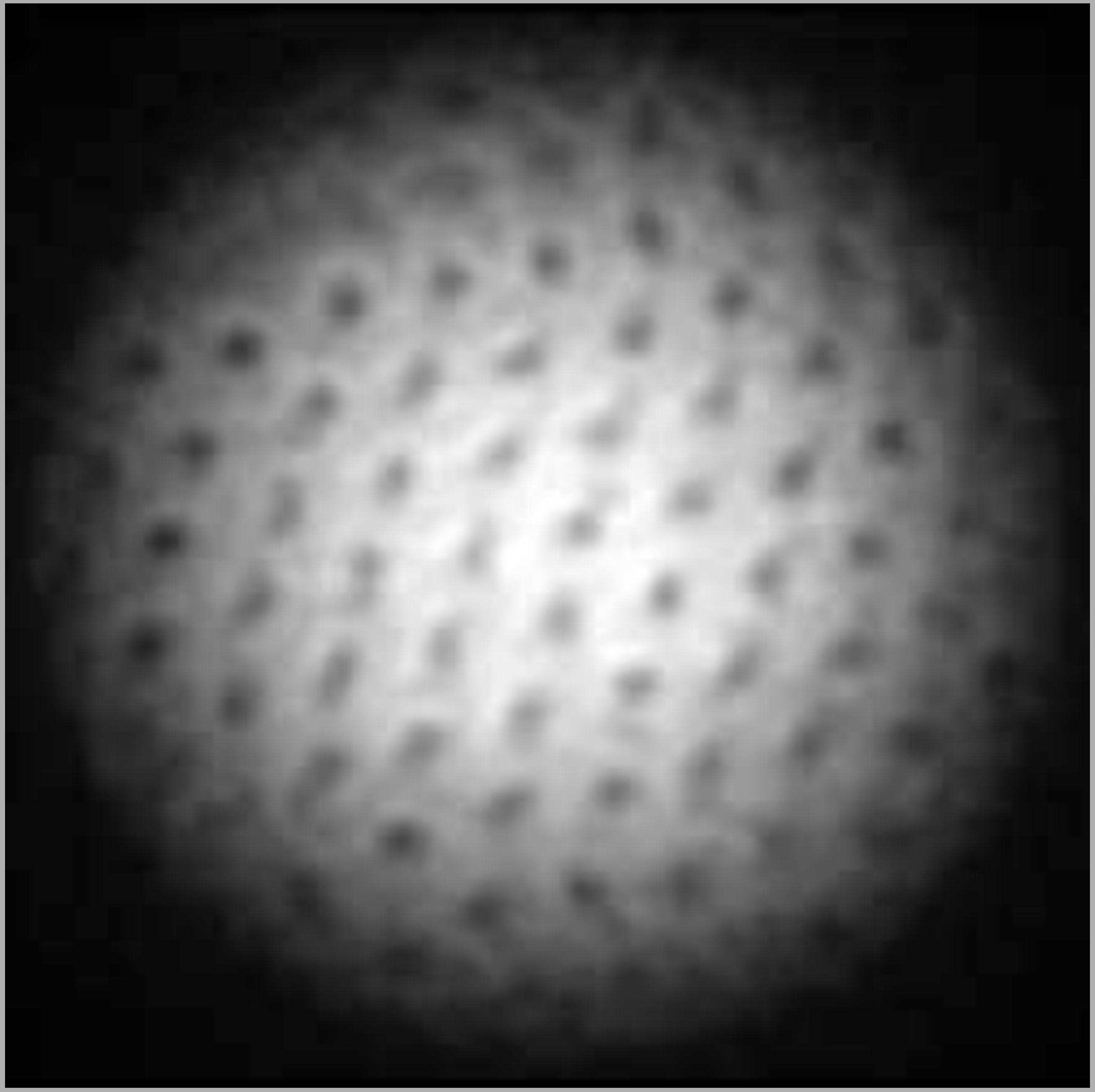
$$\Omega \gg \Omega_c$$

$$z_s \simeq 0$$

$$L_z \simeq nN\hbar$$

total winding number

vortices rotate with angular velocity Ω



Rotation to characterize superfluids

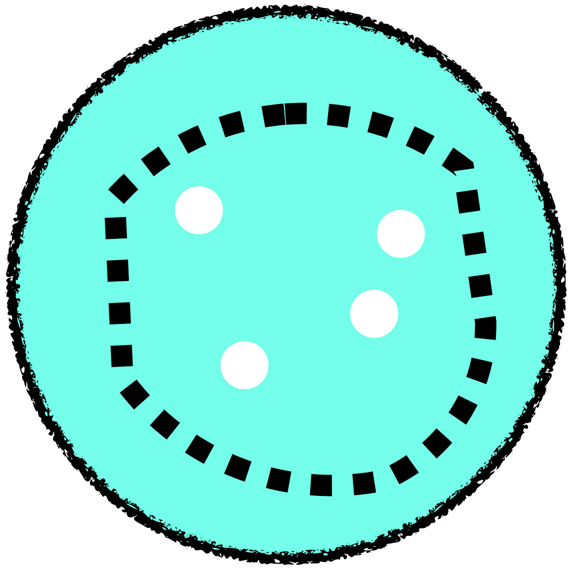
A superfluid is irrotational: $\mathbf{v} = \frac{\nabla \varphi}{m}$

Almost everywhere $\nabla \times \mathbf{v} = \mathbf{0}$

It rotates when vortex singularities appear

$$\oint \mathbf{v} \cdot \mathbf{dl} = \frac{2\pi n}{m}$$

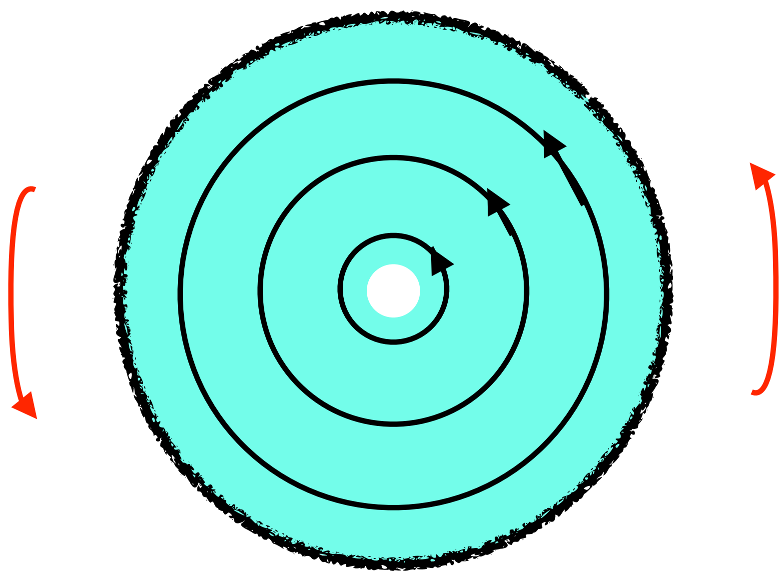
Cylinder top view



Angular momentum quantization

For simplicity: central vortex

CYLINDER TOP VIEW



Cylindrical coordinates (r, φ, z)

$\rho \equiv \rho(r)$ $\quad\quad$ $\mathbf{v} \equiv v\,\hat{\varphi}$

Circulation quantization

$$\oint \mathbf{v} \cdot \mathbf{dl} = \frac{2\pi n}{m}$$

Continuity equation

$$\nabla \cdot (\rho \mathbf{v}) = 0$$

Angular momentum quantization

Circulation quantization

$$\oint \mathbf{v} \cdot \mathbf{dl} = \frac{2\pi n}{m}$$

Stoke's theorem

$$\oint \mathbf{v} \cdot \mathbf{dl} = \int (\nabla \times \mathbf{v}) \cdot \mathbf{dS}$$

$$\int (\nabla \times \mathbf{v}) \cdot \mathbf{dS} = \frac{2\pi n}{m}$$

$$(\nabla \times \mathbf{v})_z = \frac{2\pi n}{m} \delta^2(\mathbf{r})$$

Angular momentum quantization

Continuity equation

$$\nabla \cdot (\rho \mathbf{v}) = 0$$

$$\mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0$$

Cylindrical coordinates. (r, φ, z)

$$\rho \equiv \rho(r) \qquad \mathbf{v} \equiv v\,\hat{\varphi}$$

$$\mathbf{v} \cdot \nabla \rho = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{v} = 0$$

$$\mathbf{v} = \nabla \times \mathbf{w}$$

$$\mathbf{w} = (0,0,w)$$

$$w \equiv w(r)$$

Angular momentum quantization

$$(\nabla \times \mathbf{v})_z = \frac{2\pi n}{m}\delta^2(\mathbf{r})$$

$$\mathbf{w} = (0,0,w)$$

$$\mathbf{v} = \nabla \times \mathbf{w}$$

$$w \equiv w(r)$$

$$\nabla \times (\nabla \times \mathbf{w}) = \nabla(\nabla \cdot \mathbf{w}) - \nabla^2 \mathbf{w}$$

$$\nabla \cdot \mathbf{w} = \mathbf{0}$$

$$\nabla^2 w = -\frac{2\pi n}{m}\delta^2(\mathbf{r})$$

$$w = -\frac{n}{m}\log r$$

$$\mathbf{v} = \frac{n\hbar}{mr}\hat{\varphi}$$

Angular momentum quantization

$$\mathbf{v} = \frac{n\hbar}{mr}\hat{\varphi} \qquad L_z = \int \rho(\mathbf{r}\times\mathbf{v})_z\, dV = \int \rho r v_\varphi\, dV$$

$$L_z = \int \rho r v_\varphi\, dV = \frac{n\hbar}{m}\int \rho dV = nN\hbar$$

The state with $n = 1$ is energetically favored $L_z = N\hbar$

Superfluid

Irrespective of its position, every particle contributes with $\hbar$ to the total angular momentum

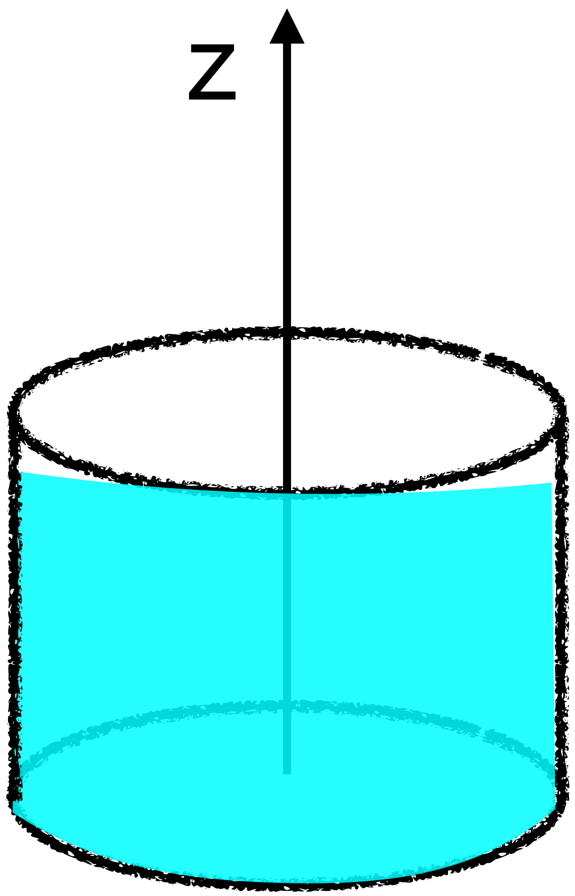
Superfluid

When stirred does not rotate, unless quantized vortices are created.

In that case each particle carries angular momentum $\hbar$

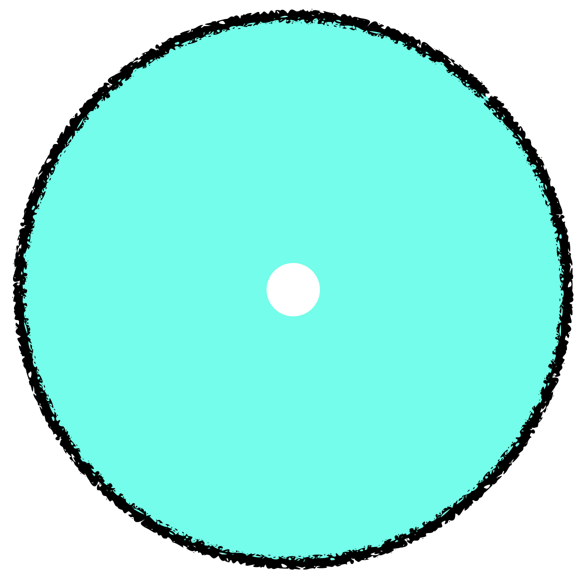
It almost works

Rotating deformed supefluid

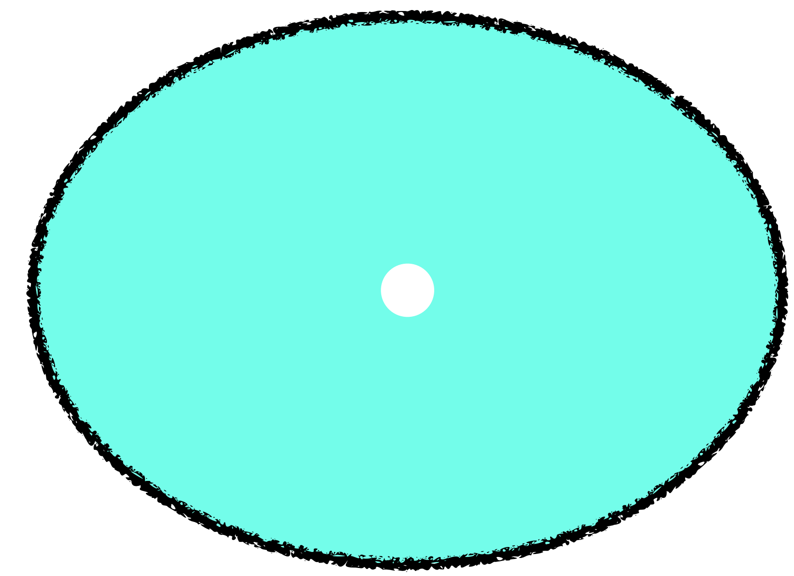


TOP VIEW

cylinder



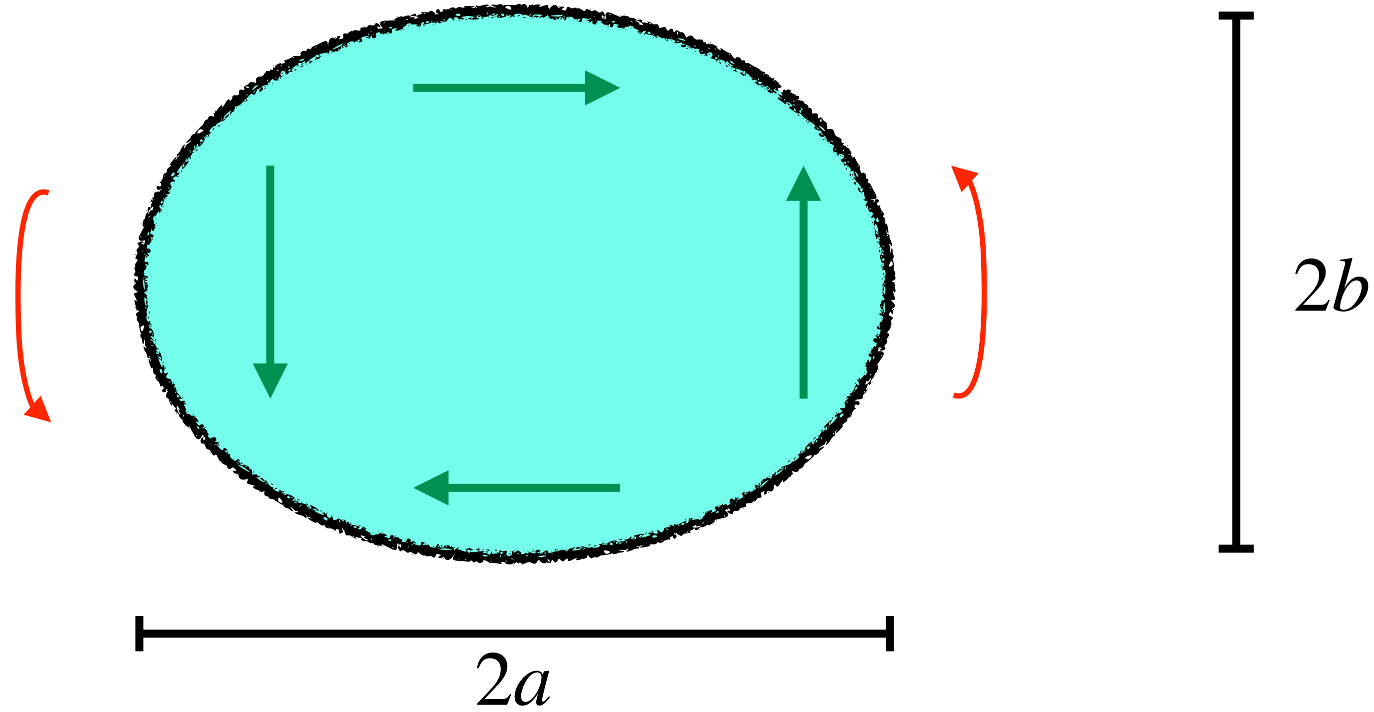
elliptic cylinder



$\Omega < \Omega_c$	$L_z = 0$	$L_z = I_s \Omega$
$\Omega \gtrsim \Omega_c$	$L_z \simeq N\hbar$	$L_z \simeq I_s \Omega + N\hbar$

Elliptic cylinder

TOP VIEW



$$\Omega < \Omega_c \qquad \oint \mathbf{v} \cdot d\mathbf{l} = \mathbf{0} \qquad \nabla \times \mathbf{v} = \mathbf{0}$$ everywhere

It rotates, but it is irrotational: counterflow is present!

$$L_z \simeq I_s \Omega \qquad I_s = \alpha \, I_{RB} \qquad \alpha = \left(\frac{a^2 - b^2}{a^2 + b^2} \right)^2$$

$$\Omega \gtrsim \Omega_c \qquad L_z \simeq I_s \Omega + N\hbar$$

A.L. Fetter
Journal of Low Temperature Physics, Vol. 16 (1974)

Superfluid effective definition

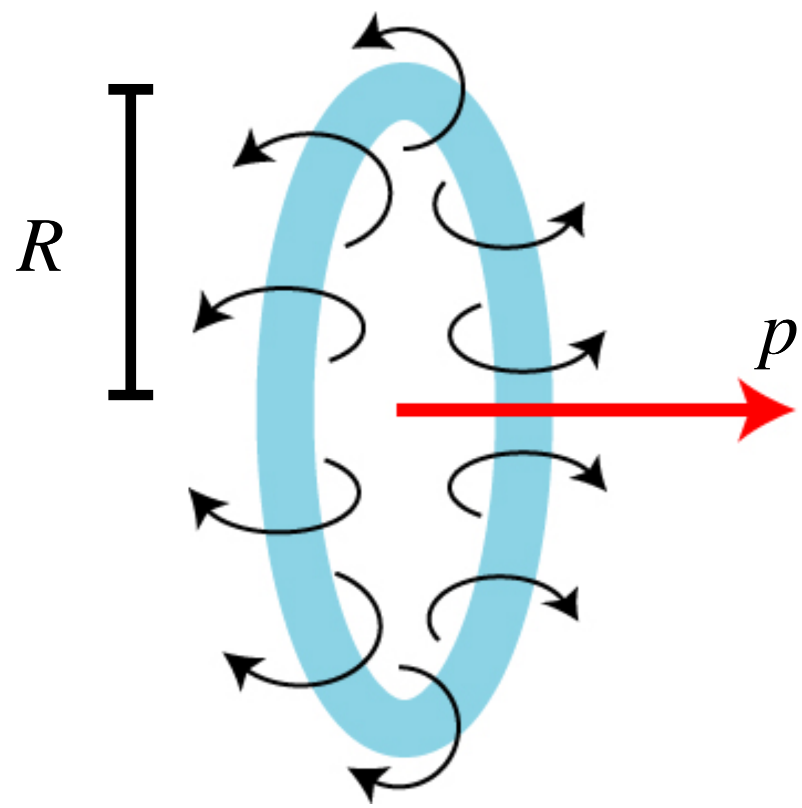
When stirred can host quantized vortices

Vortex induces $\sim \hbar$ angular momentum change per particle

Critical velocity

The estimate $v = c_s$ too optimistic: other low energy excitations may exist

Vortex rings



$$\epsilon = \frac{2\pi^2\rho^2 R}{m^2}\log R$$

$$p = \frac{2\pi^2\rho R^2}{m}$$

Vortex ring
critical velocity

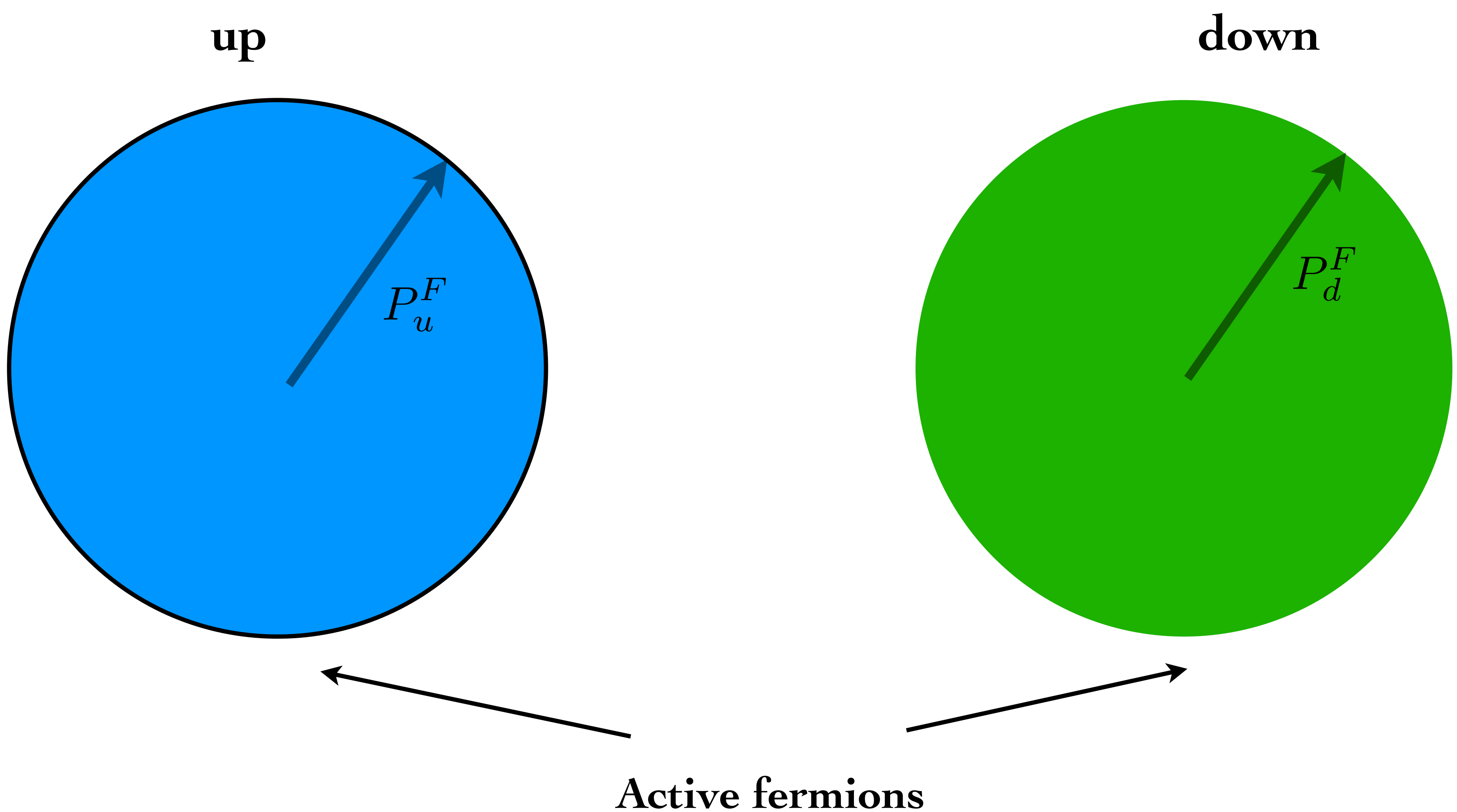
$$v_{\text{cr}} = \frac{\epsilon}{p} = \frac{\log R}{mR}$$

Fermionic superfluids

(Neutron Stars are made of fermions)

Fermions

Spin up and spin down fermions at $T = 0$

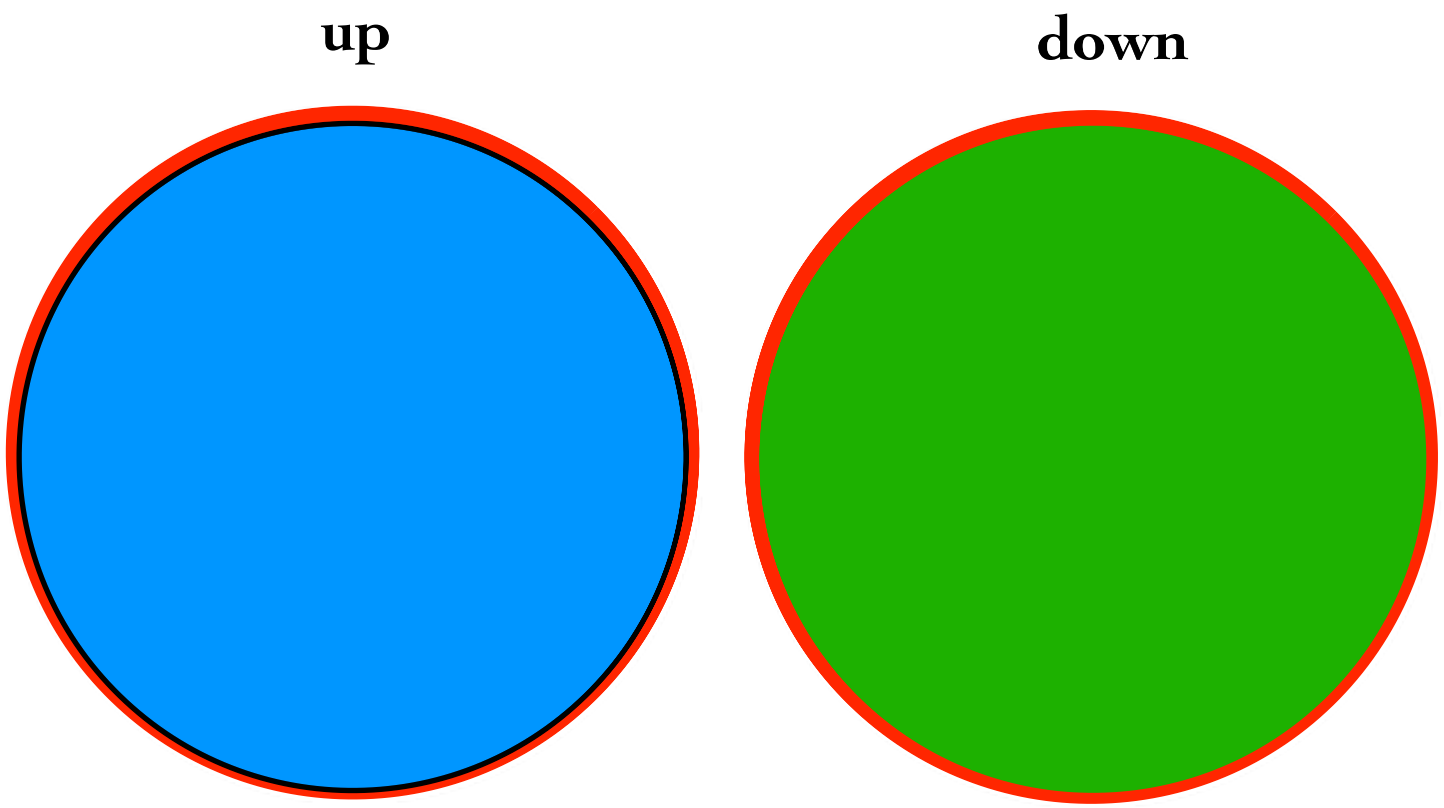


$$\epsilon = \frac{p^2}{2m} - \mu \quad \Rightarrow \quad \text{Min}\frac{\epsilon(p)}{p} = 0$$

NOT SUPERFLUID

Need to quench excitations

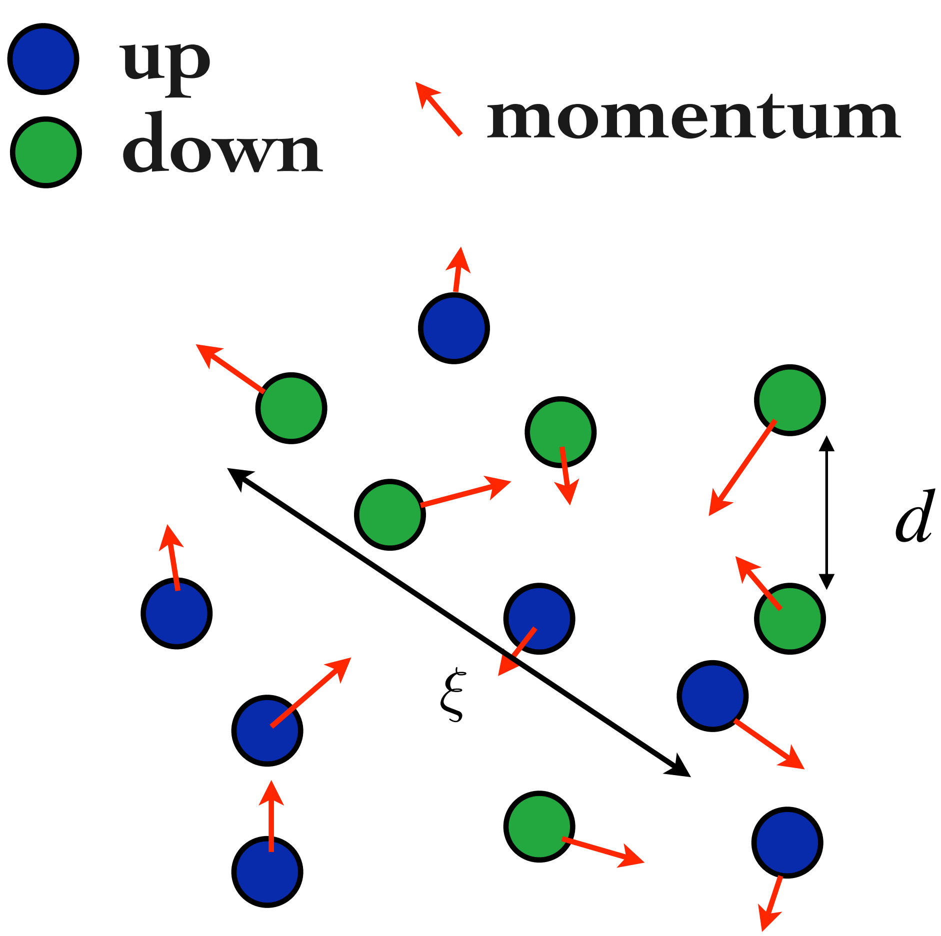
There are fermions that can be easily excited and scatter. Need to stop them!



Any attractive interaction triggers pairing

BCS qualitative description

Cooper pair: “difermion” with energy gap Δ

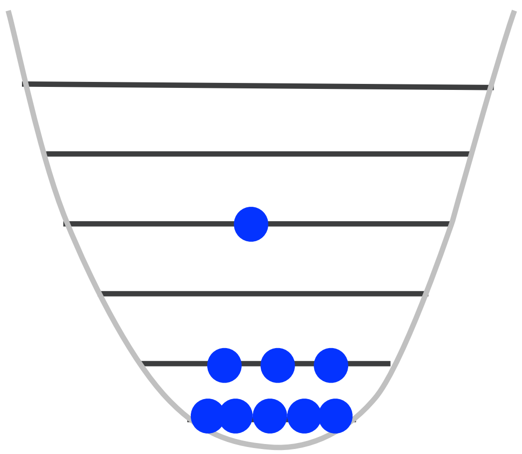


BCS: loosely bound pairs $\xi \gg d$

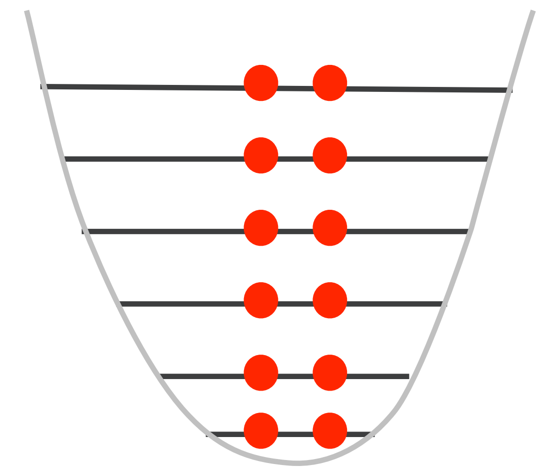
BEC: tightly bound pairs $\xi \sim d$

Fermionic vs bosonic superfluids

BOSONS



FERMIONS



Bosons “like” to stay together

^{4}He becomes superfluid at
$T_c \simeq 2.17$ K, Kapitsa (1937)

Any weak attraction: Cooper pairing

^{3}He becomes superfluid at
$T_c \simeq 0.0025$ K, Osheroff (1971)

BEC

BCS

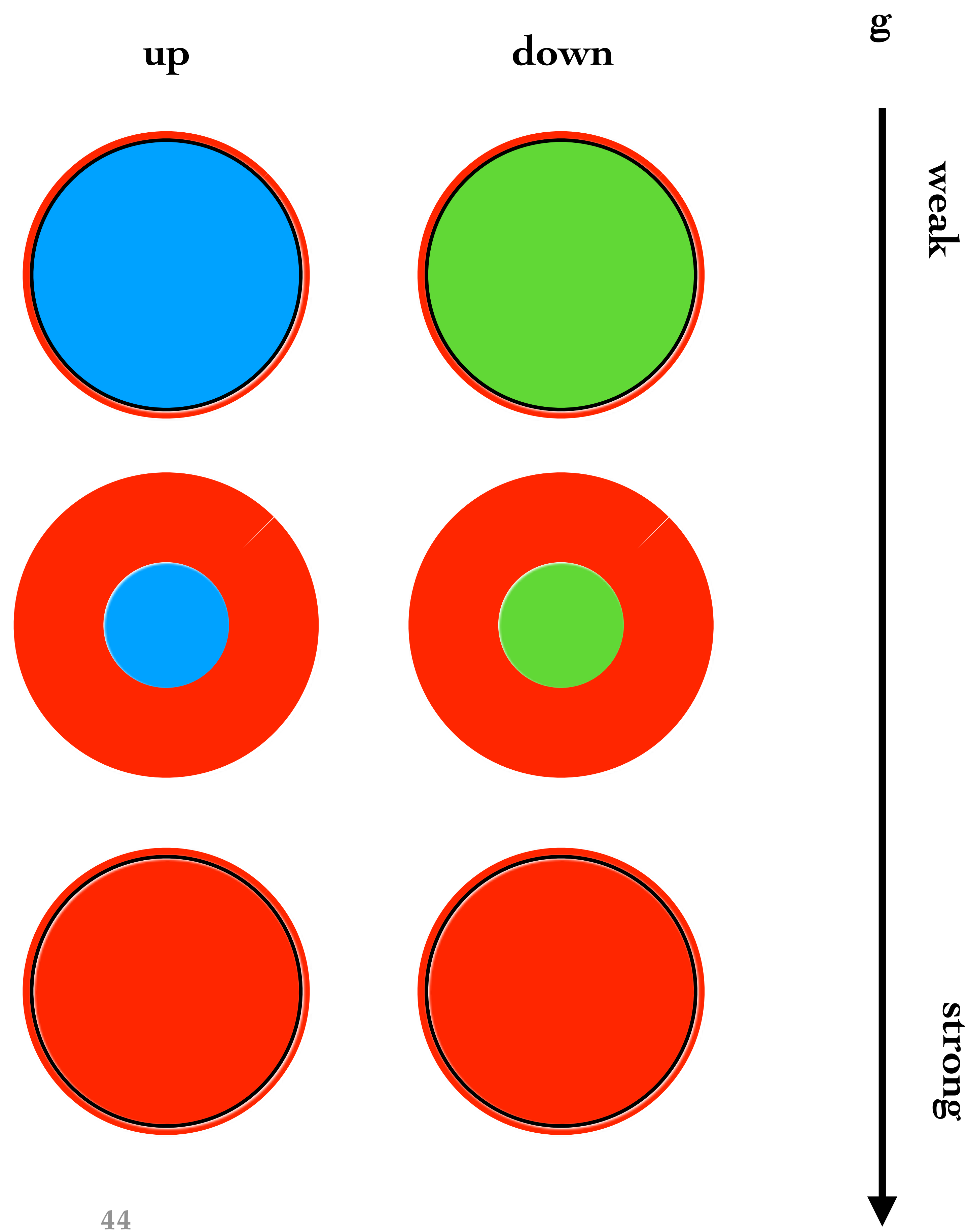
interaction strength

BCS-BEC crossover

BCS
fermi surface phenomenon

BCS-BEC crossover
bound pairs

BEC
tightly bound pairs



Neutron superfluid

We note that the superfluidity of nuclear matter can lead to interesting macroscopic phenomena if stars with neutron cores exist. Such a star would be in a superfluid state with a transition temperature corresponding to 1 MeV.

A. B. Migdal, SOVIET PHYSICS JETP VOLUME 37 (10) **1960**

BCS-like pairing in Stars?

Critical temperature		Star (surface) temperature	
^{4}He	$1K \sim 10^{-4}$ **eV**		
		$10^5 K \sim 10$ **eV**	Sun
^{3}He	$10^{-3} K \sim 10^{-7}$ **eV**		
nucleons	$10^{10} K \sim 1$ **MeV**	$10^6 K \sim 0.1$ **keV**	Neutron Star

Nucelons in a NS can be superfluid!

Which pairing channel ?

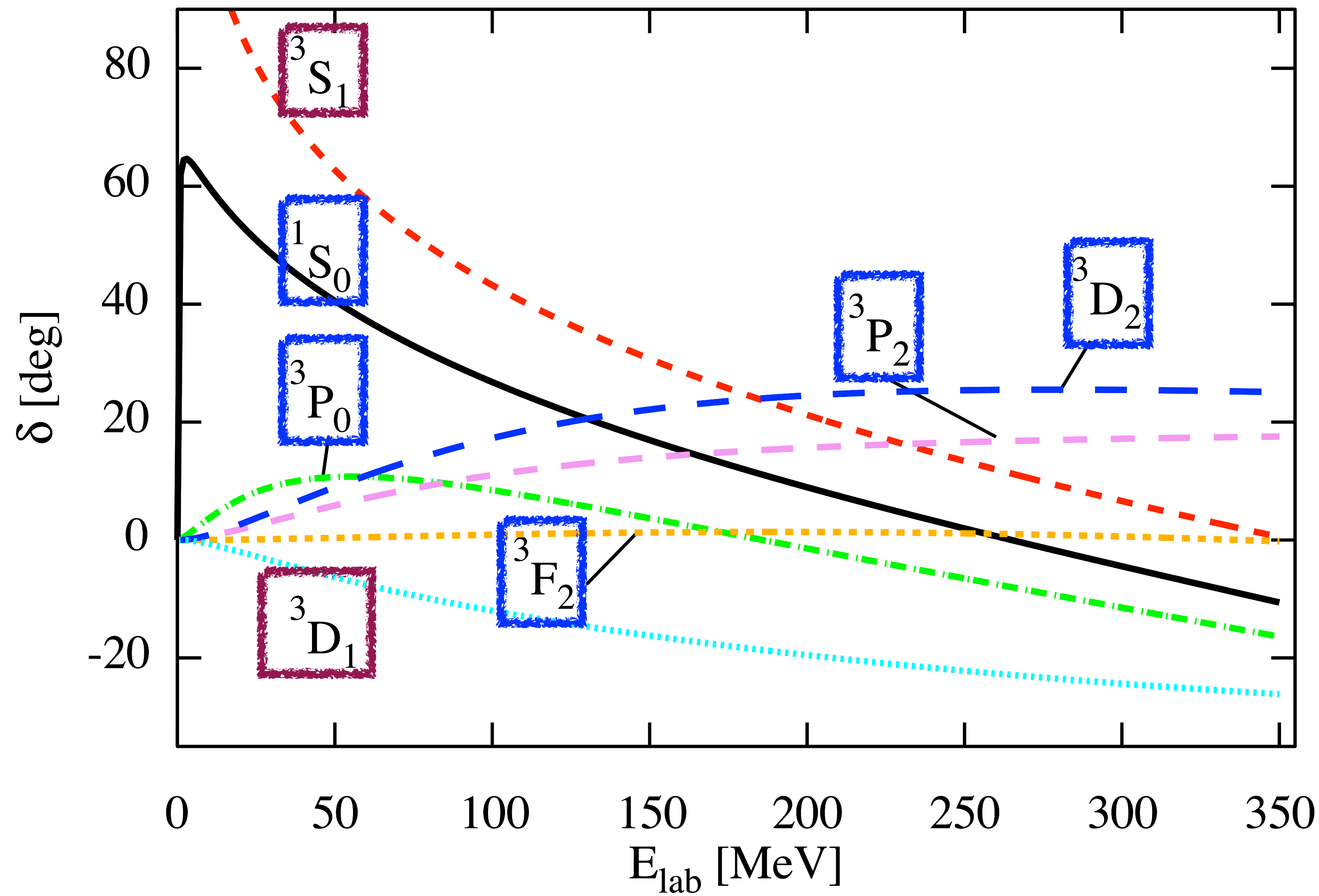
We have spin, isospin and angular momentum to accomodate $I + \underbrace{L + S}_{J}$ odd

Notation: $^{2S+1}L_J$

Information from phase shifts

$I = 0$

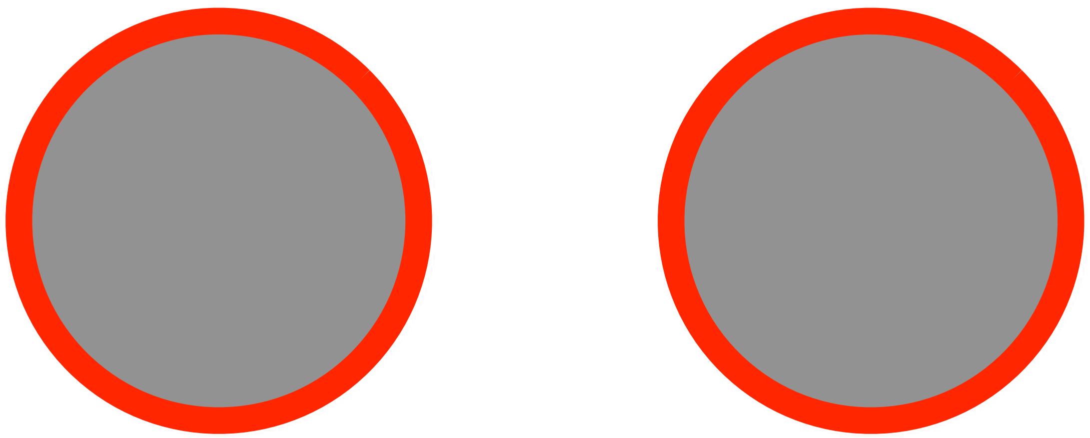
$I = 1$



A. Sedrakian and J. W. Clark, EPJ A (2019) 55: 167

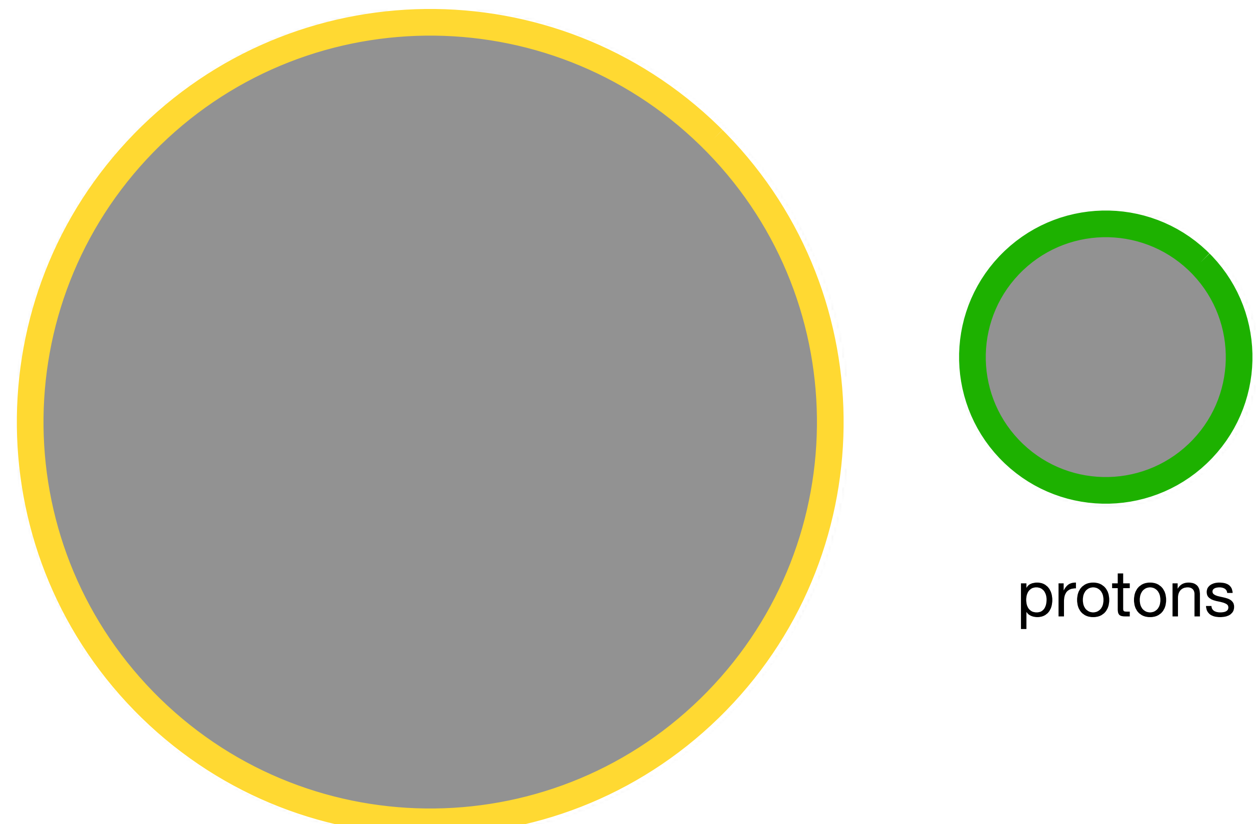
Which pairing channel ?

Symmetric nuclear matter



$I = 0$ **Dominant**

Asymmetric nuclear matter



$I = 1$ **Dominant**

Which pairing channel ?

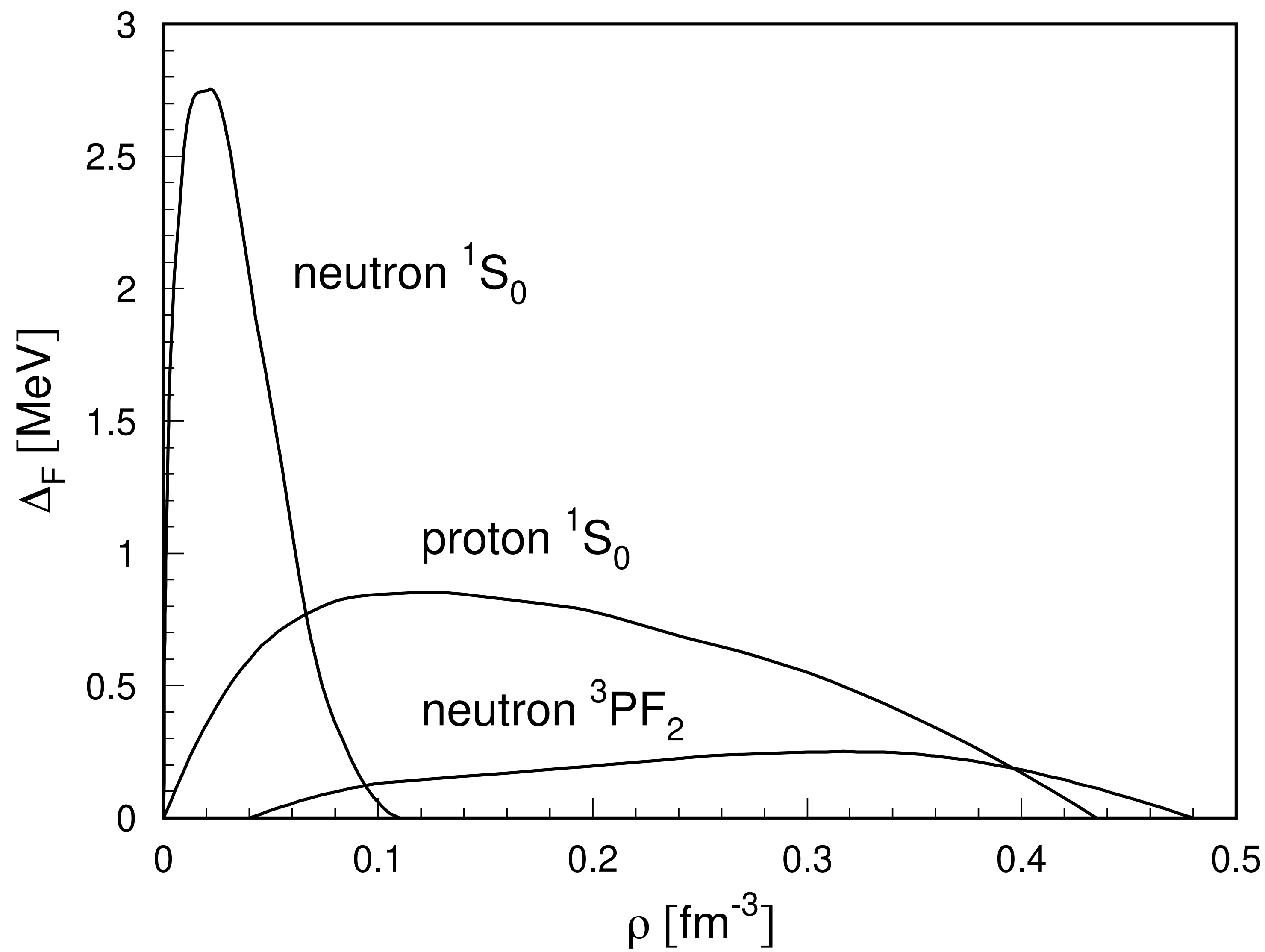
$^3S_1 - ^3D_1$ deuton $I = 0$

Interaction channels can be **coupled**

$^3P_2 - ^3F_2$ di-neutron $I = 1$

$I = 1$

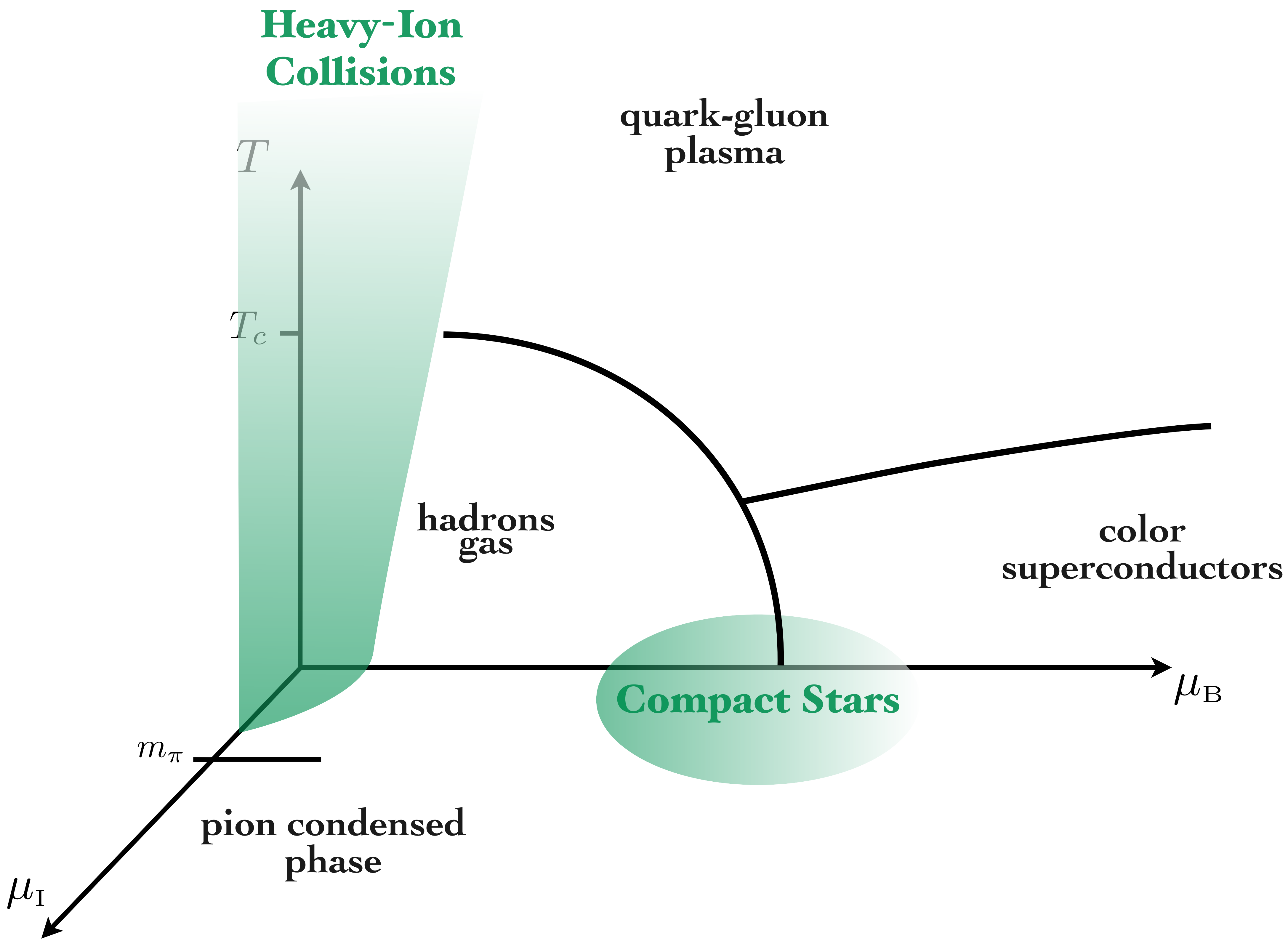
Density dependence



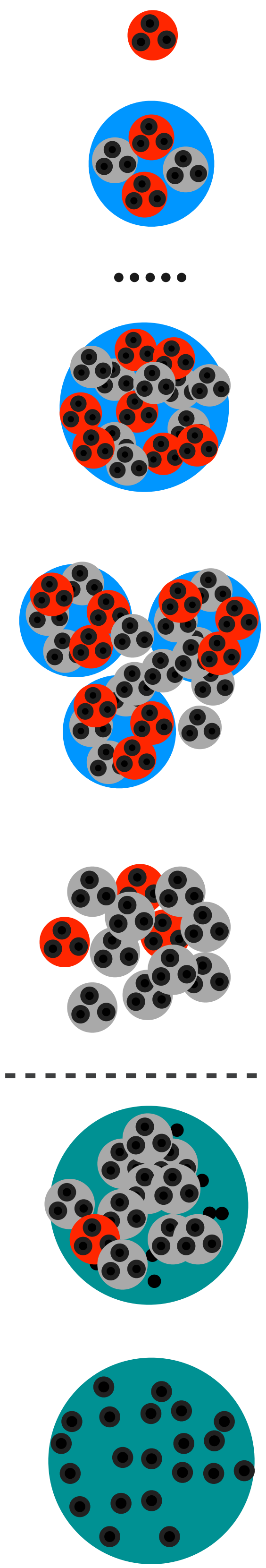
U. Lombardo, H.-J. Schulze, Lect.Notes Phys.578:30-53,2001

Phases of quark matter

Phases of hadronic matter



Increasing baryonic density

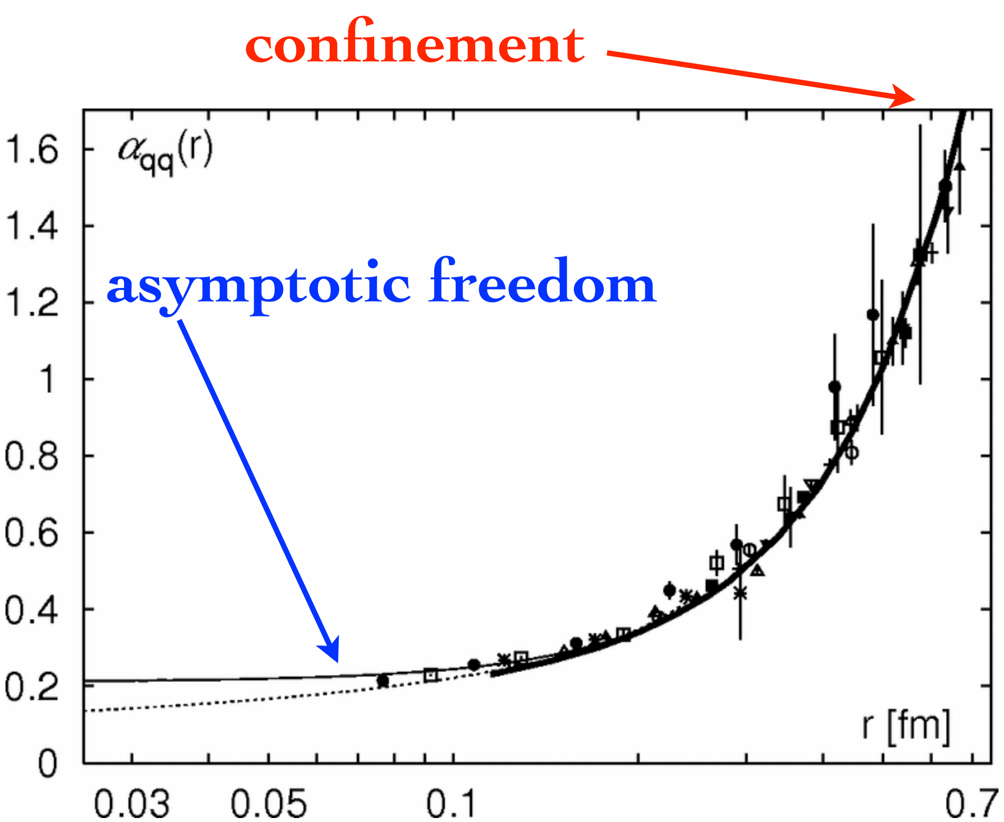


	Density	CS part	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H				
He	......	atmosphere		light nuclei
......				.....
Fe	10 g cm^{-3}	outer crust	confining	heavy nuclei
neutron drip	10^{11} g cm^{-3}			
neutron proton soup	10^{14} g cm^{-3}	inner crust		neutrons and protons
		core		
quark drip	extreme		strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
quark soup	more than extreme		weak coupling	Cooper pairs of quarks NGBs

ρ

The dawn of color superconductors

QCD is an asymptotic free theory: in the UV interactions are perturbative



"Running" of the QCD interaction strength

Kaczmarek and Zantow
Physical Review D 71(11):114510 (2005)

Also we might except [sic] superfluidity and superconductivity, since the interquark forces are attractive in at least some channels.

J. C. Collins and M.J. Perry Phys.Rev.Lett. 34 (1975) 1353

Symmetries

Three flavor QCD

$$\mathscr{L} = \sum_{i=1,2,3} \bar{\psi}_i (i\gamma^\mu D_\mu - m)_{ij} \psi_j - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

Neglecting u, d and s quark masses

global chiral symmetry

$$U(1)_B \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)} \times SU(3)_C$$

global baryonic number

electromagnetism

gauge color

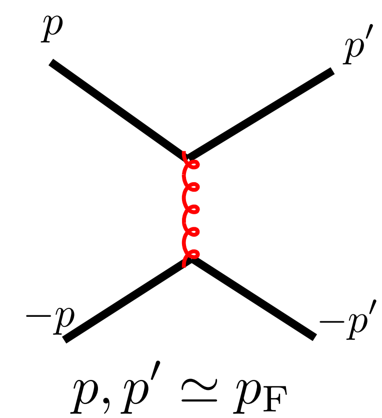
A large symmetry group can be broken in a zoo of possible phases

Pairing

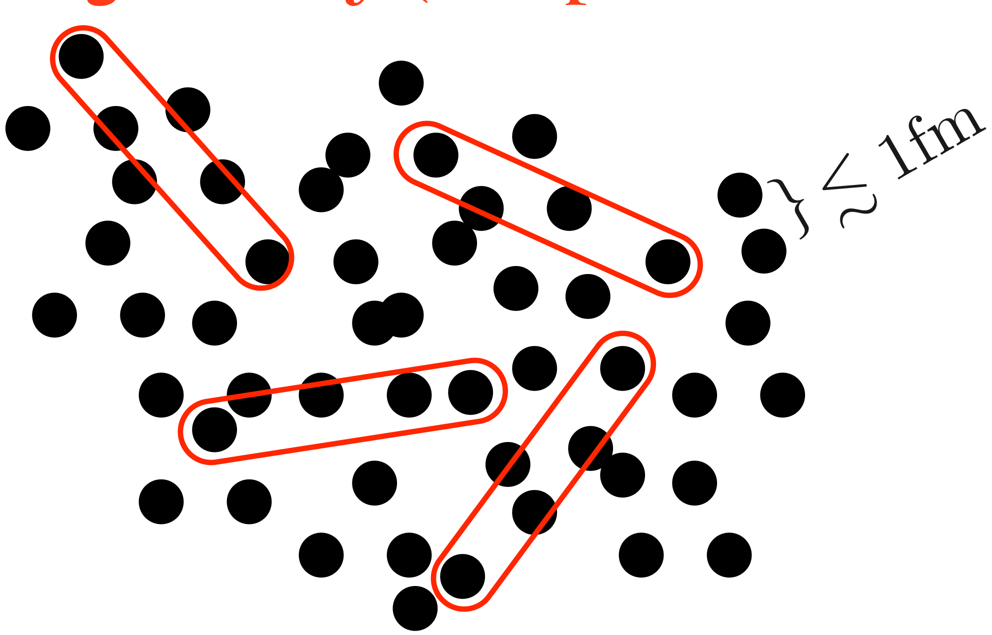
Attractive interaction (perturbative)

$$3 \times 3 = \bar{3}_A + 6_S$$

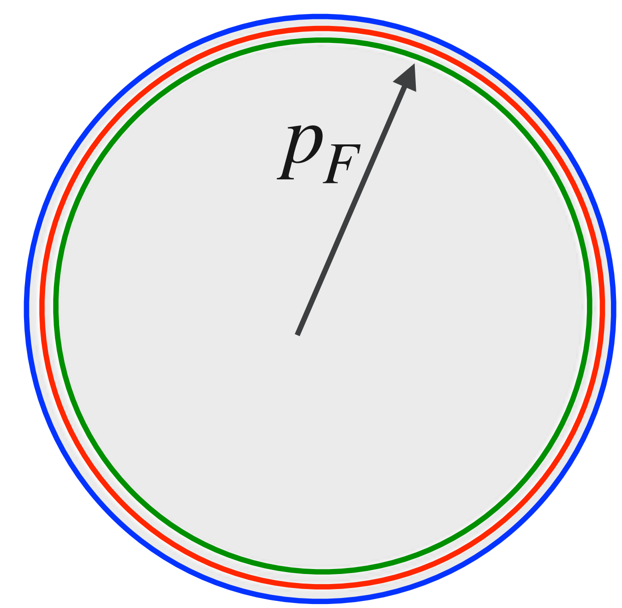
attractive channel



Very high density (Compact Star inner core)



Liquid of quarks with correlated diquarks



Fermi spheres of u,d, s quarks

Superfluid vs Superconductor

Superfluid

Broken global symmetry

Goldstone theorem → Transport of the quantum numbers of the broken group with almost no dissipation

Superconductor

"Broken gauge symmetry"

Higgs mechanism → **Gauge fields with mass, M penetrate for a length** $\lambda \propto 1/M$

Color Flavor Locked phase

Pairing of quarks of all flavors and colors

Alford, Rajagopal, Wilczek Nucl.Phys. B537 (1999) 443

Symmetry breaking

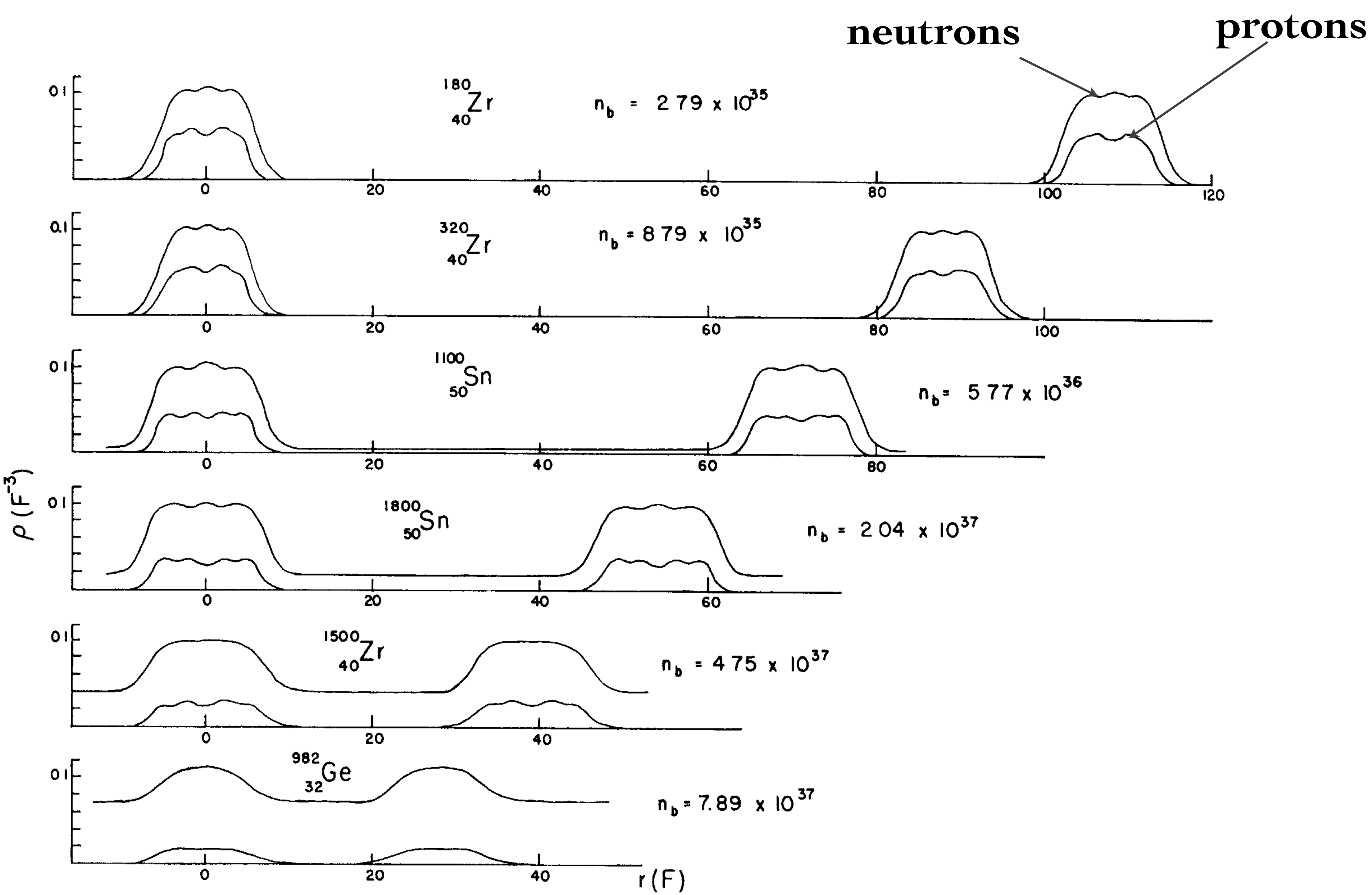
$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\rm Q}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\rm Q}}} \times Z_2$$

- Breaking of $SU(3)_c$: 8 gauge bosons become massive
- χSB: 8 (pseudo) Nambu-Goldstone bosons (NGBs)
- $U(1)_B$ breaking: 1 NGB. A genuine superfluid mode

The system is at the same time a (color) superconductor and a (baryonic) superfluid

Emulations

Superfluid and inhomogeneous



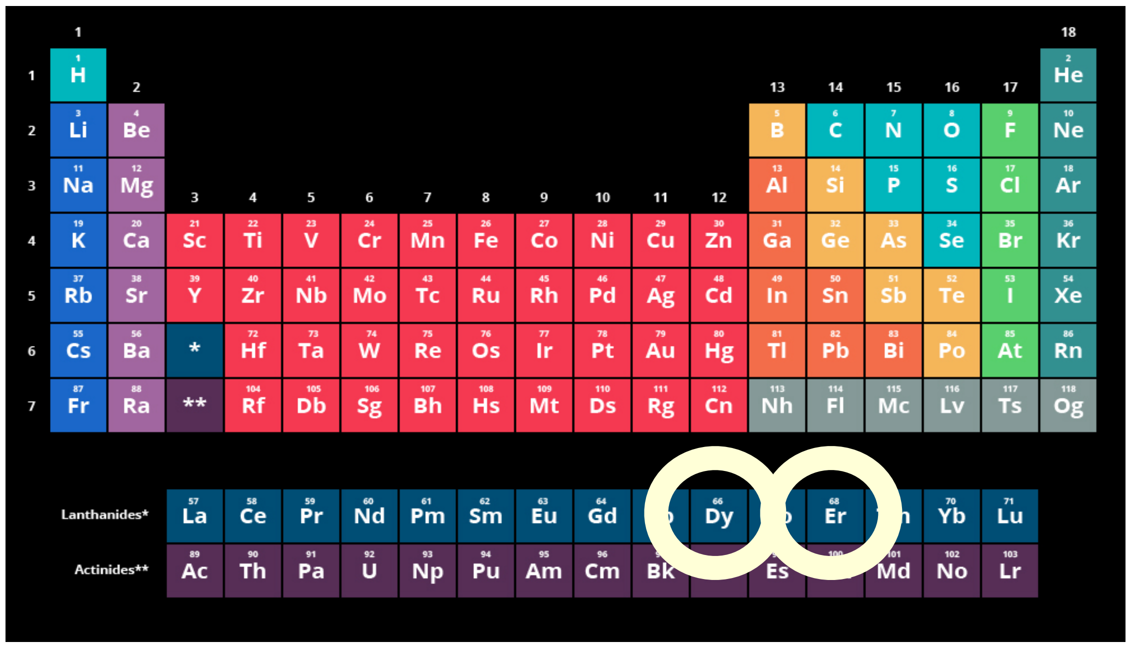
J. W. Negele and D. Vautherin, *Nucl. Phys. A207, 298 (1973)*

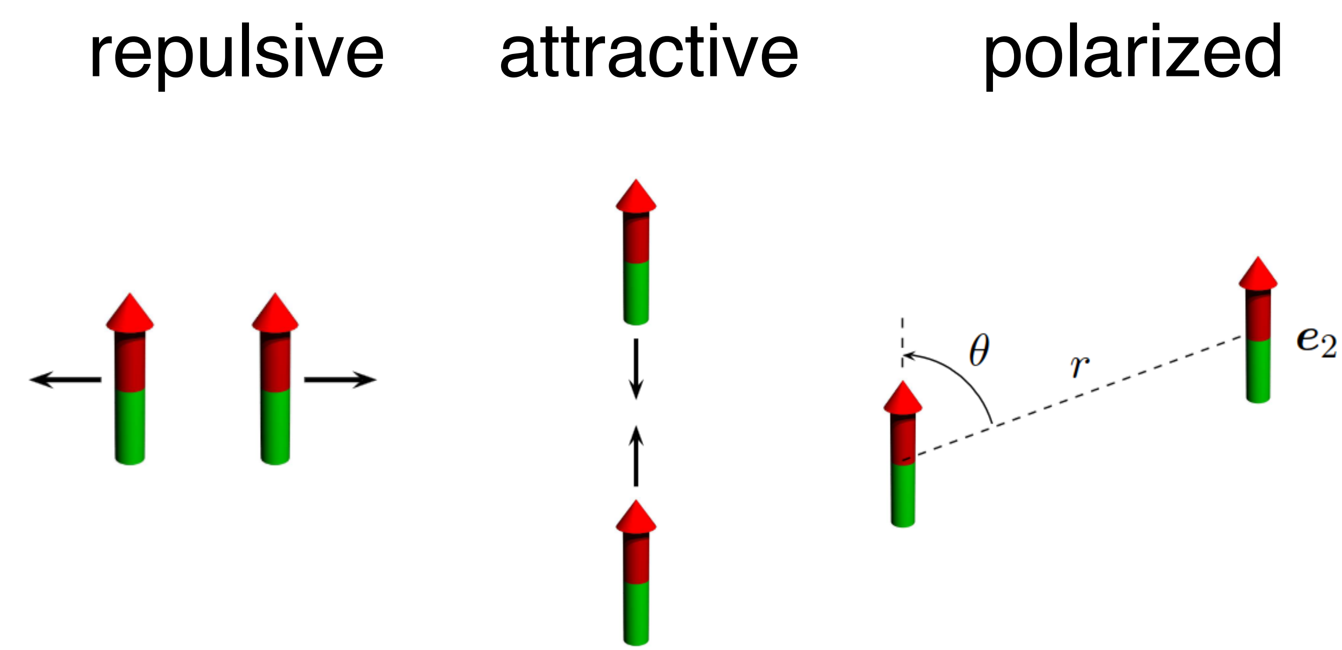
Supersolids

Superfluid **Rigid**

M. Boninsegni and N. V. Prokof'ev, *Rev. Mod. Phys. 84, 759 (2012)*

Ultracold dipolar atoms





"Long-range" dipolar interaction

$$U_{\rm dd}(\boldsymbol{r}) = \frac{3\hbar^2 a_{\rm dd}}{m}\,\frac{1-3\cos^2\theta}{r^3}$$

$$a_{dd} = \frac{\mu_0\,\mu_m^2\,m}{12\pi\hbar^2}$$

^{165}Dy, $a_{dd} \simeq 131 a_0$

Short-range repulsion (Feshbach resonance)

$$U_c(\mathbf{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\mathbf{r})$$

a_s tunable

Relevant parameters

Relative interaction strength $\epsilon_{dd} = \frac{a_{dd}}{a_s}$ ← a_{dd}: fixed, a_s: tunable

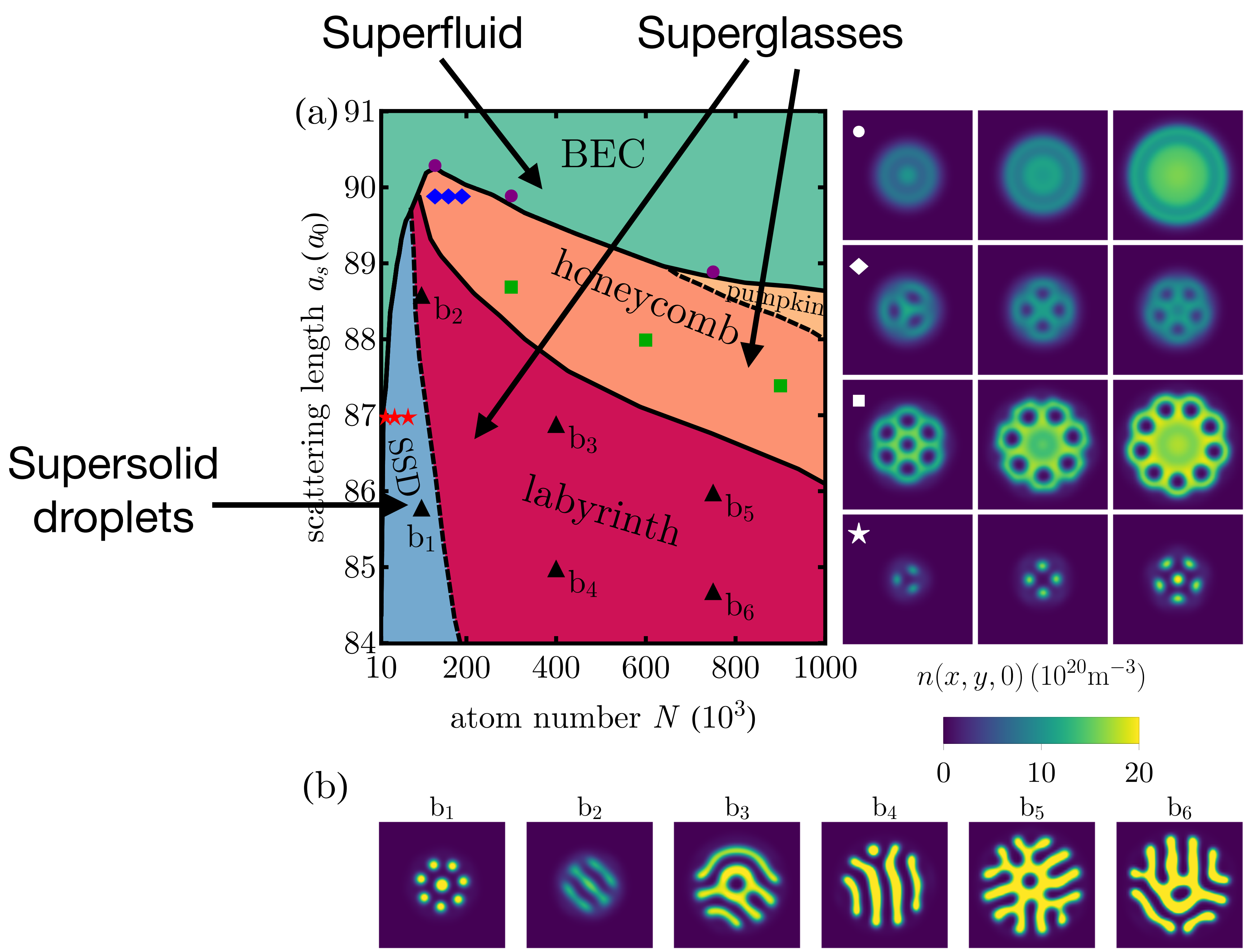
Number density $n = \frac{N}{V}$ tunable

Tuning the relative interaction strength

The Fano-Feshbach resonance allows to change a_s and thus ϵ_{dd}

Superfluid	Supersolid or Superglass	Crystal or glass
Contact interaction dominates	Competition between interactions **Inhomogeneous Superfluid**	Dipolar interaction dominates **Solid**
$\epsilon_{dd} \ll 1$	$\epsilon_{dd} \sim 1$	$\epsilon_{dd} \gg 1$

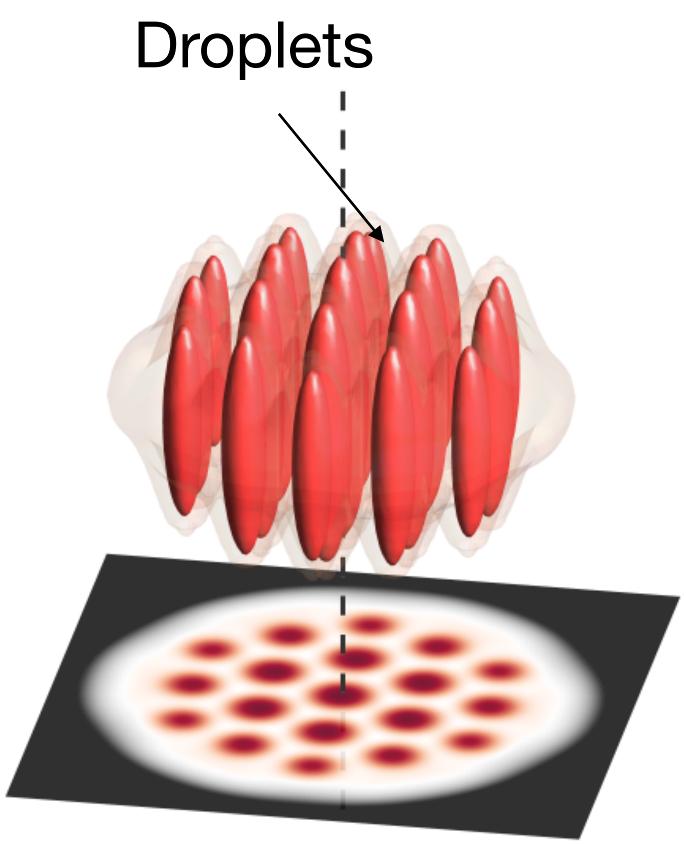
Phase diagram



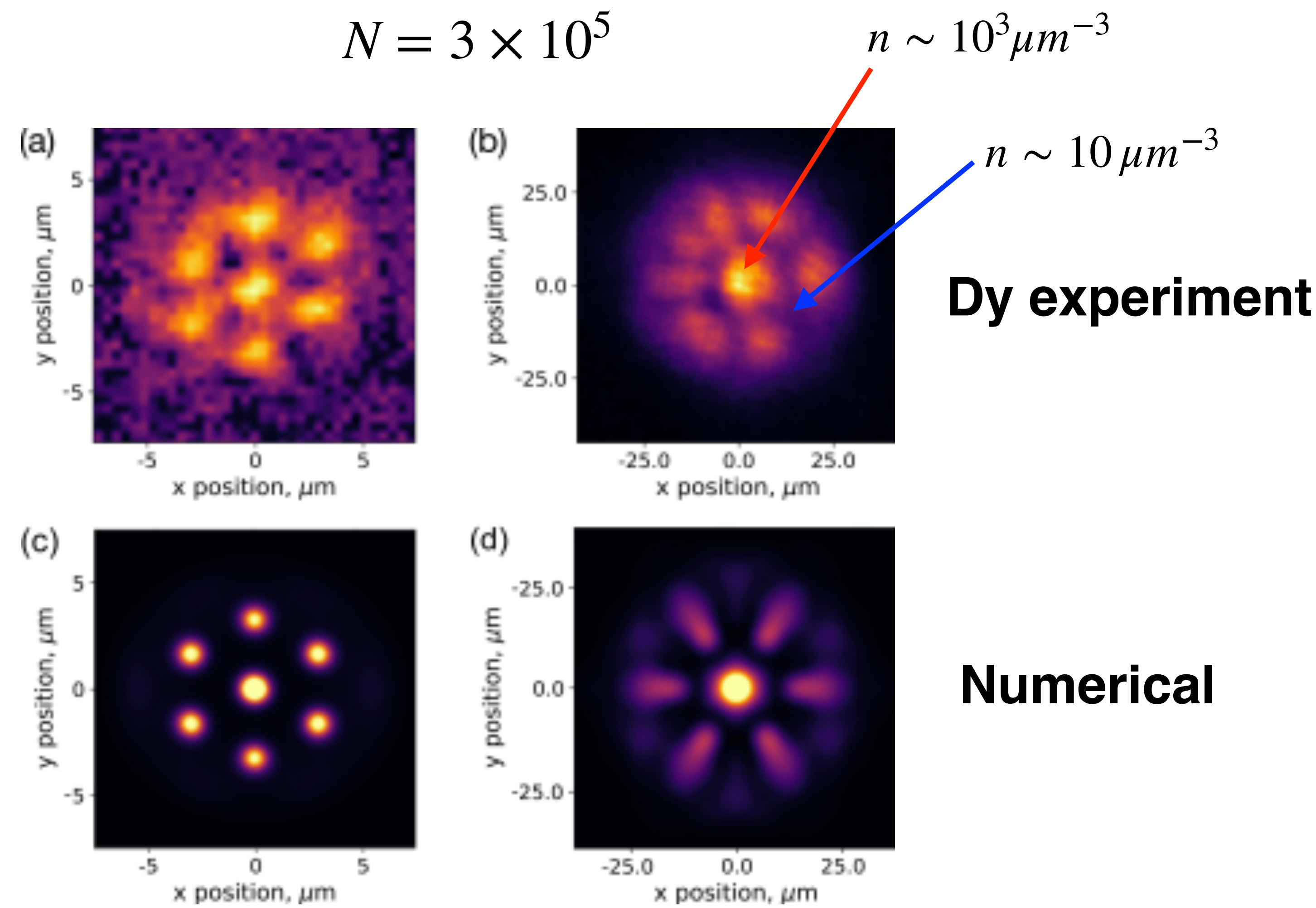
J. Hertkorn et al.,
Phys. Rev. Research 3, 033125 (2021)

Supersolid droplets

Observations of supersolids@ MIT, Pisa/LENS, Stuttgart, Innsbruck



$N = 3 \times 10^5$



Bland *et al* *PRL 128, 195302 (2022)*

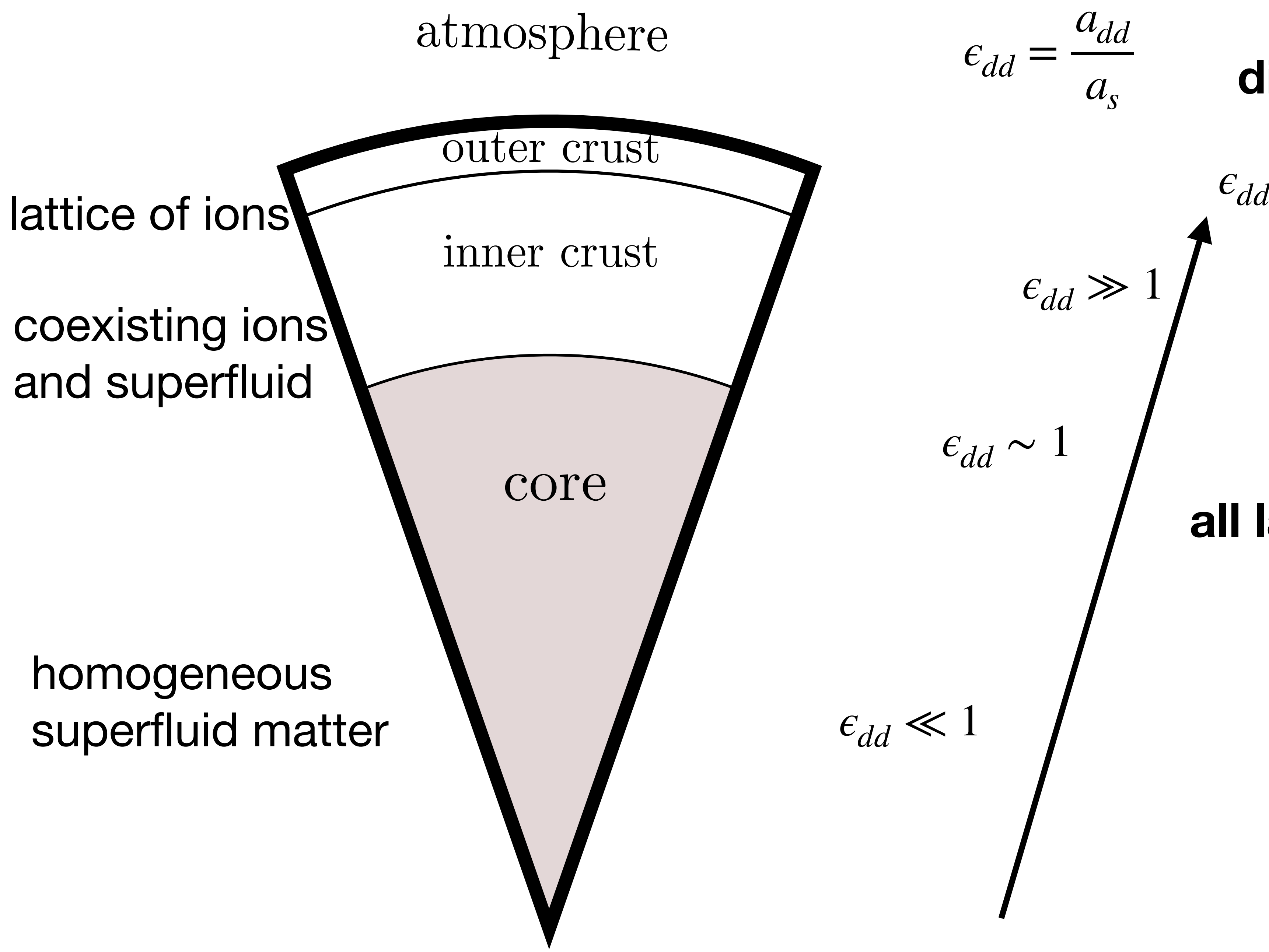
Compact stars vs Supersolids

Nuclear matter	*Supersolids*
Fermions	**Bosons (fermions can in principle be used)**
Long range attraction **Short range repulsion**	**Long range attraction** **Short range repulsion**
Scalar, vector and tensor forces	**Dipole-dipole + s-wave scattering**
High density $\rho \sim 10^{14}$**g/cm**^{3}	**Diluted** $\rho \sim 10^{-5}$**g/cm**^{3}
Density and interactions given by nature	**Density and interactions tunable**

Neutron star vs dipolar superfluids

$$\epsilon_{dd} = \frac{a_{dd}}{a_s}$$

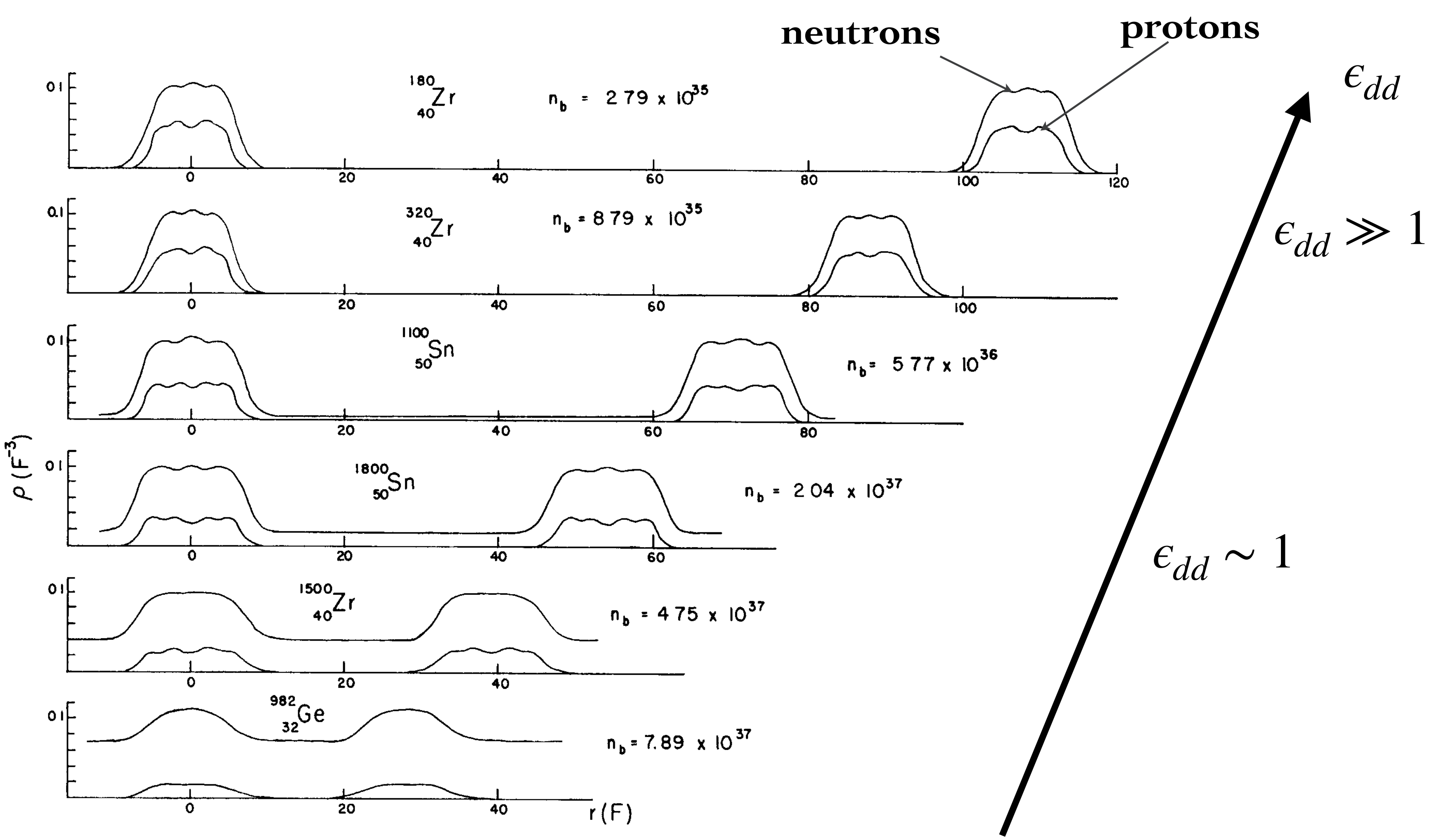
dipolar superfluids



In principle
all layers could be emulated !

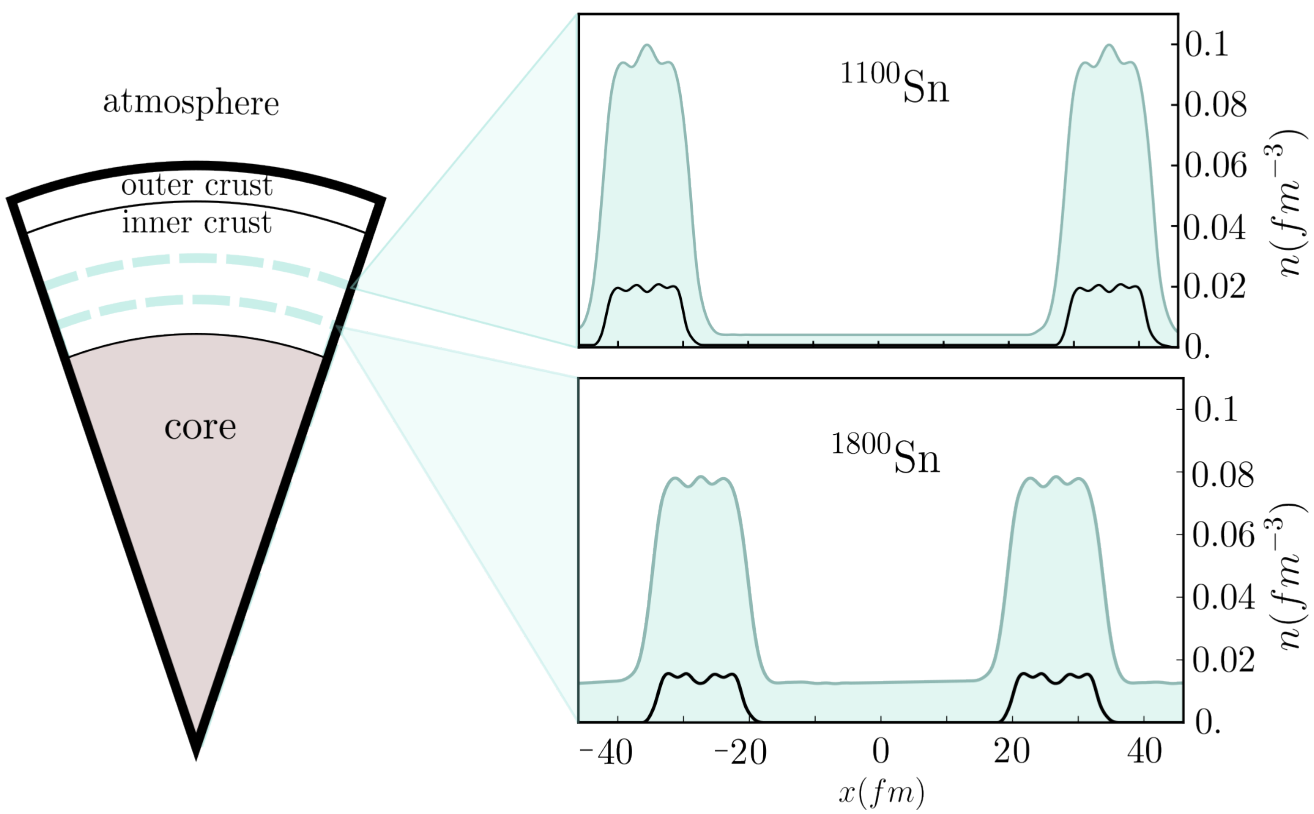
Inner crust

Particularly relevant for glitches: inner crust provides the pinning of superfluid vortices

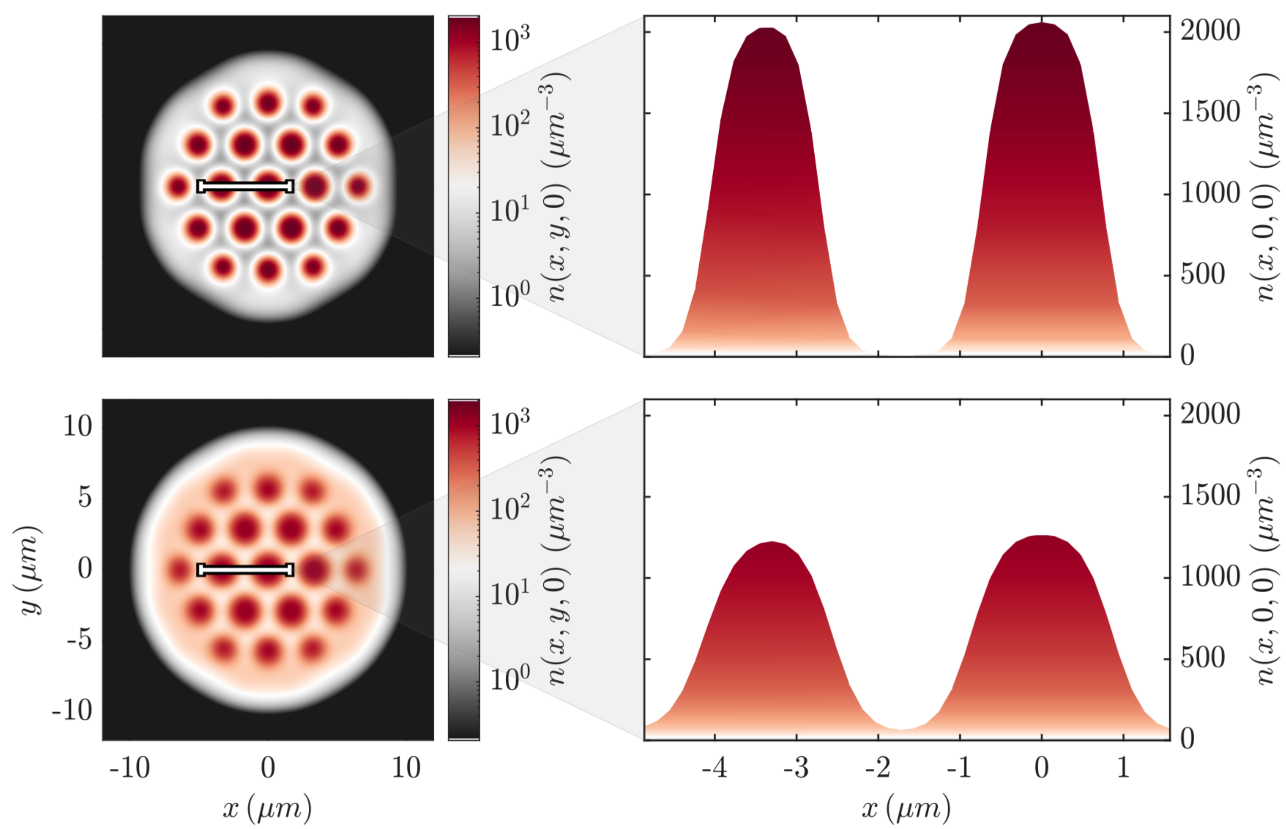


J. W. Negele and D. Vautherin, *Nucl. Phys. A207, 298 (1973)*

Neutron Star inner crust



Dipolar Supersolid



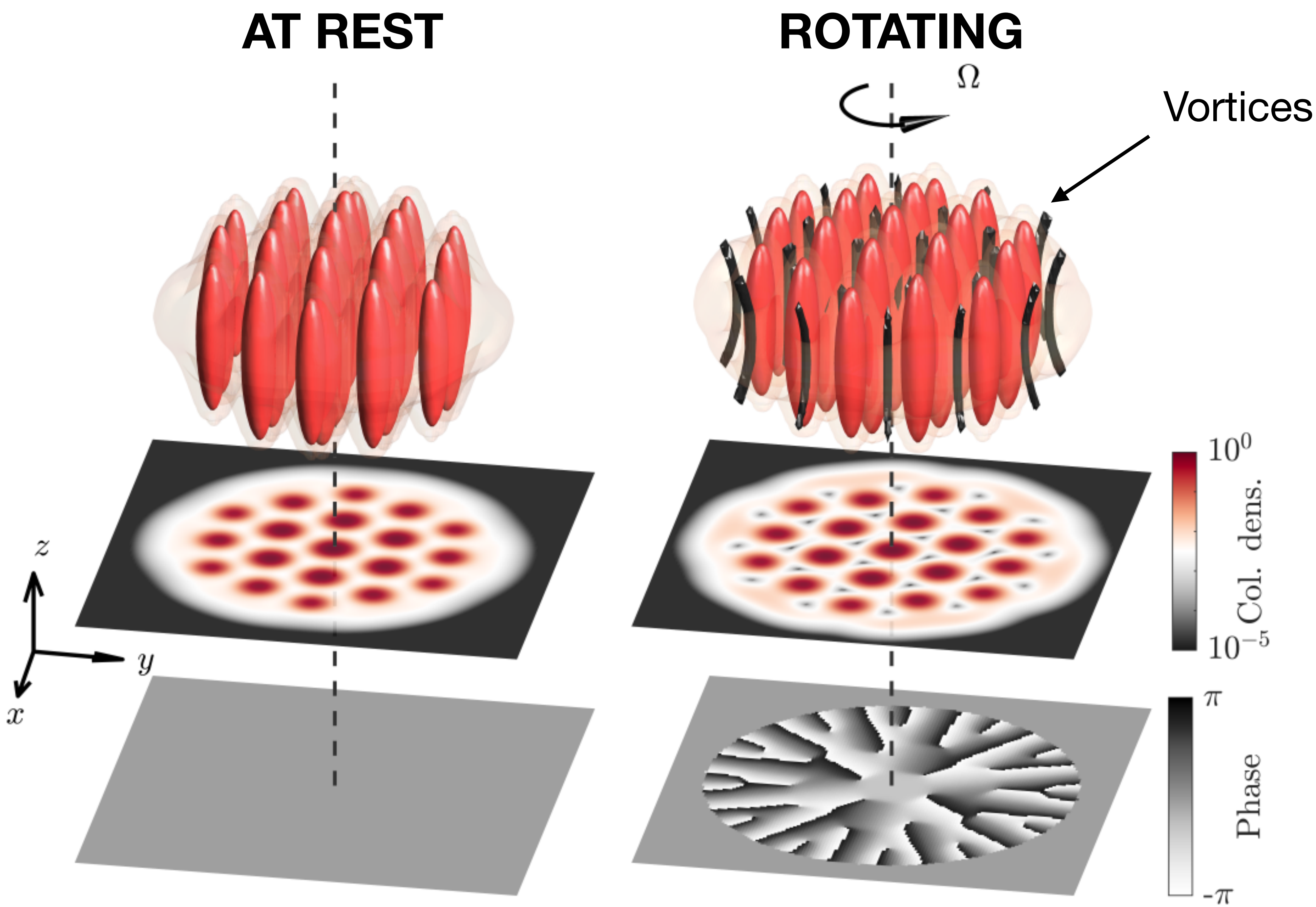
$\epsilon_{dd} \sim 1$

Poli, Bland, White, Mark, Ferlaino, Trabucco, MM

Phys.Rev.Lett. 131 (2023) 22, 223401

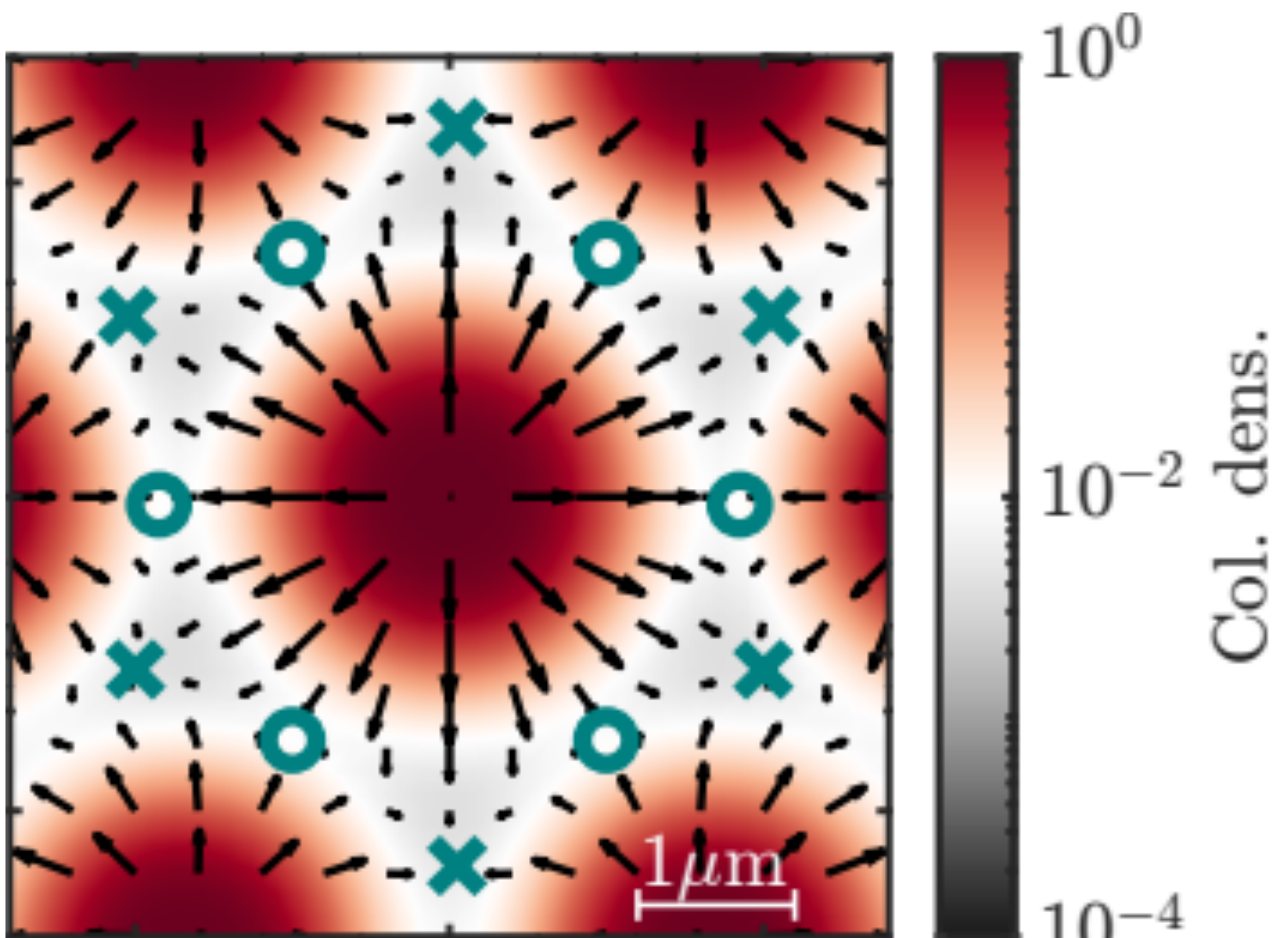
Emulating neutron star glitches

Supersolids



Vortex pinning

X = stable
O= metastable



$a_s = 90a_0\,, \quad \boldsymbol{\omega}_{\mathrm{trap}} = 2\pi \times (50, 130)\mathrm{Hz}$

Angular momentum

total winding number

As in oblate superfluids $$L_z \simeq I_s \Omega + nN\hbar$$

number of particles

$$0 \le \alpha \le 1$$

$$I_s = \alpha\, I_{RB}$$

$\alpha = 1$ completely solid

$\alpha = 0$ completely superfluid

} Depends on ϵ_{dd} and N

Generalization

$$L_z \simeq I_s \Omega + L_{\text{vort}}$$

Takes into account that noncentral vortices contribute less than $N\hbar$

Evolution

Optical trap → Outer crust

Dipolar atoms → Inner crust

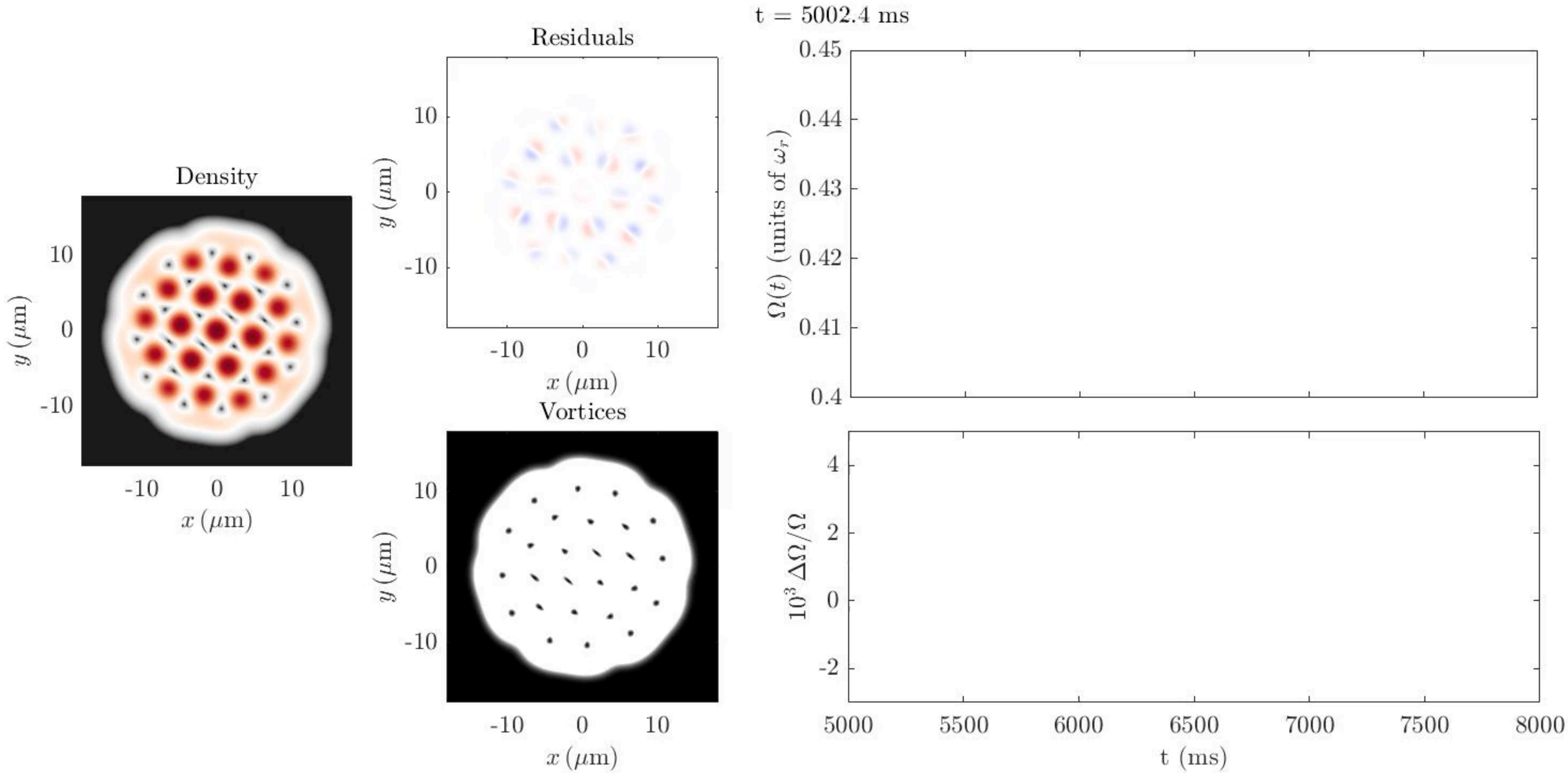
To emulate the NS spin down, we put a “break”

$$\dot{L}_{total} = -N_{em}$$

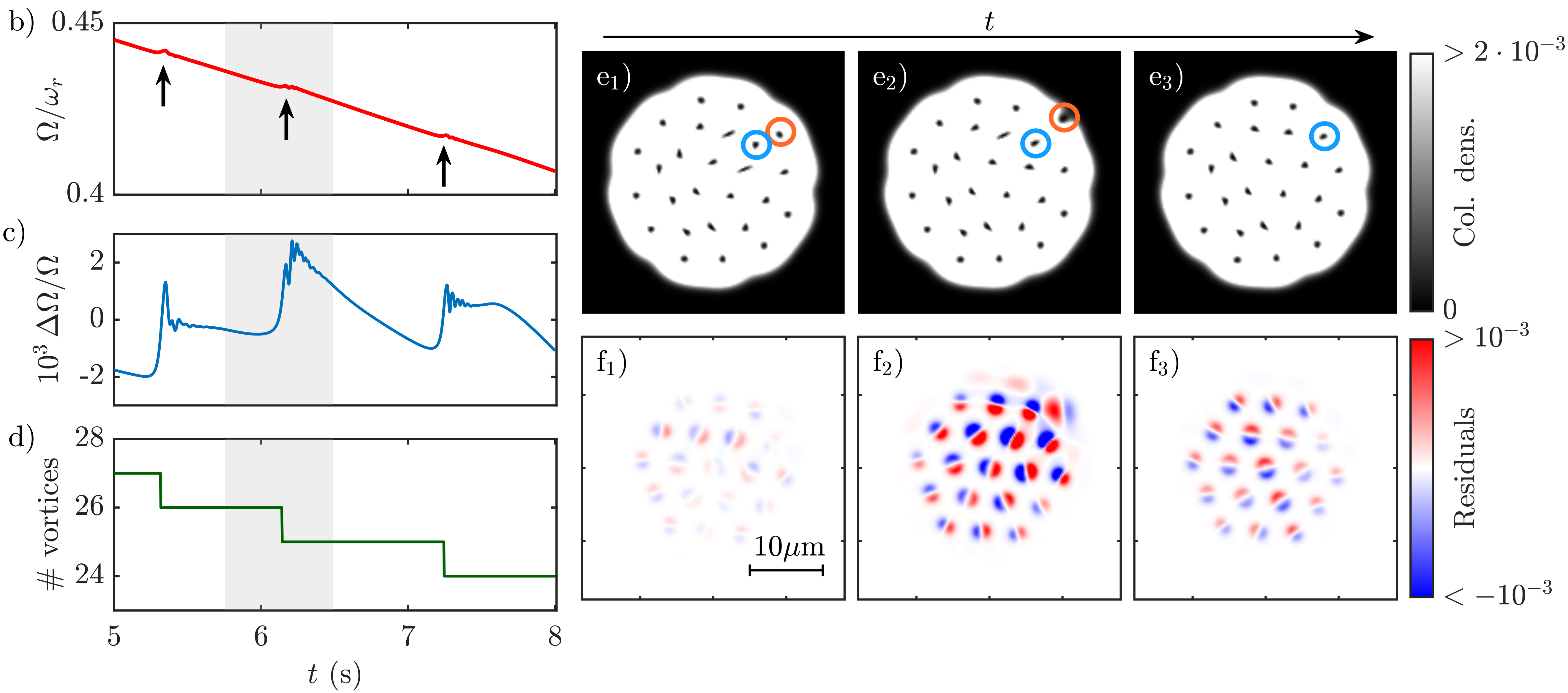
Supersolid glitches

Corotating frame

Time evolution



Supersolid glitches



More hadronic matter phases

Supersolids are **versatile.**
Possible applications

Crystalline color superconductors

Pion crystals

Other inhomogeneous superfluids

Conclusion

- Superfluids are intriguing **macroscopic quantum states**
- Hadronic matter in compact stars can be **superfluid**
- Ultracold atoms could be used to **emulate** some aspects of NSs

Thank you!

Backup

Numerical simulation

Evolution of the macroscopic wavefunction

Dissipation

Angular rotation of the trap

$$i\hbar\frac{\partial\Psi}{\partial t} = (1 - i\gamma)\left[\mathscr{H}[\Psi; a_s, a_{\rm dd}, \omega] - \Omega(t)\hat{L}_z\right]\Psi$$

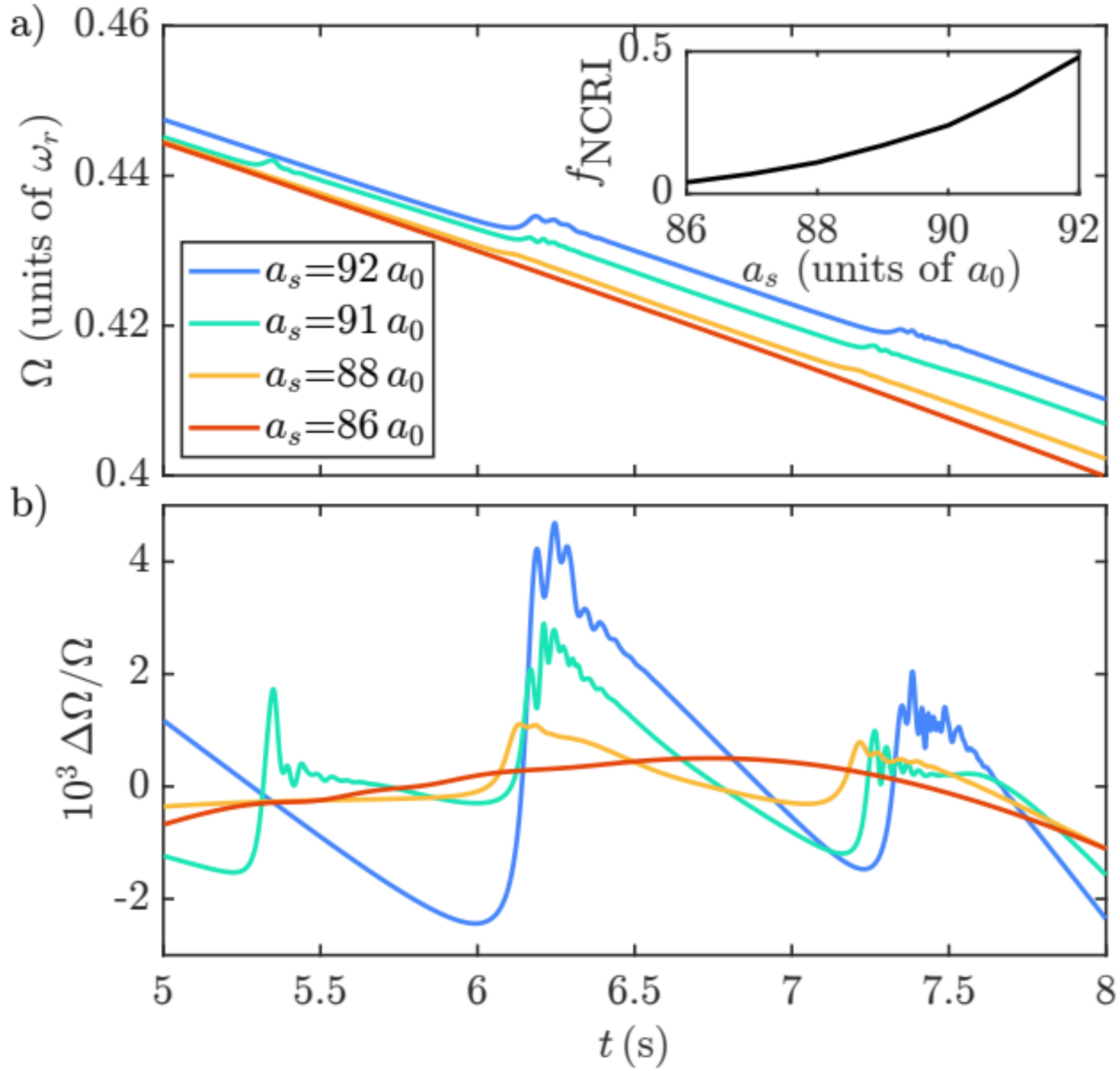
Hamiltonian

$$\mathscr{H}[\Psi; a_s, a_{\rm dd}, \boldsymbol{\omega}] = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\left[\omega_r^2(x^2+y^2) + \omega_z^2 z^2\right] + \int \mathrm{d}^3\mathbf{r}'\, U(\mathbf{r}-\mathbf{r}')\left|\Psi(\mathbf{r}',t)\right|^2 + \gamma_{\rm QF}|\Psi(\mathbf{r},t)|^3 - \mu$$

Trapping potential (pancake-like)

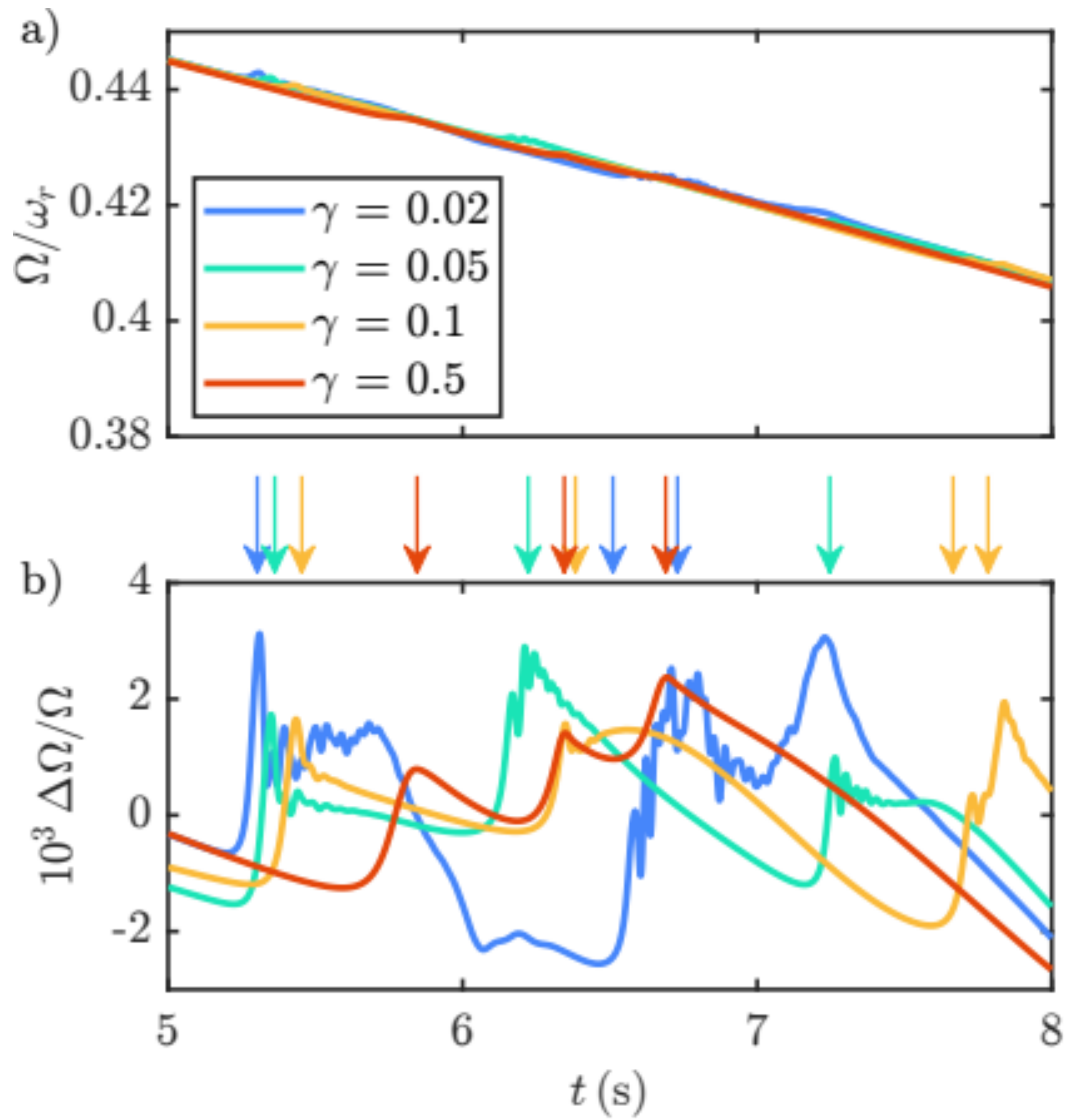
Self-interaction

LHY correction



$$\alpha = 1 - f_{\mathrm{NCRI}}$$

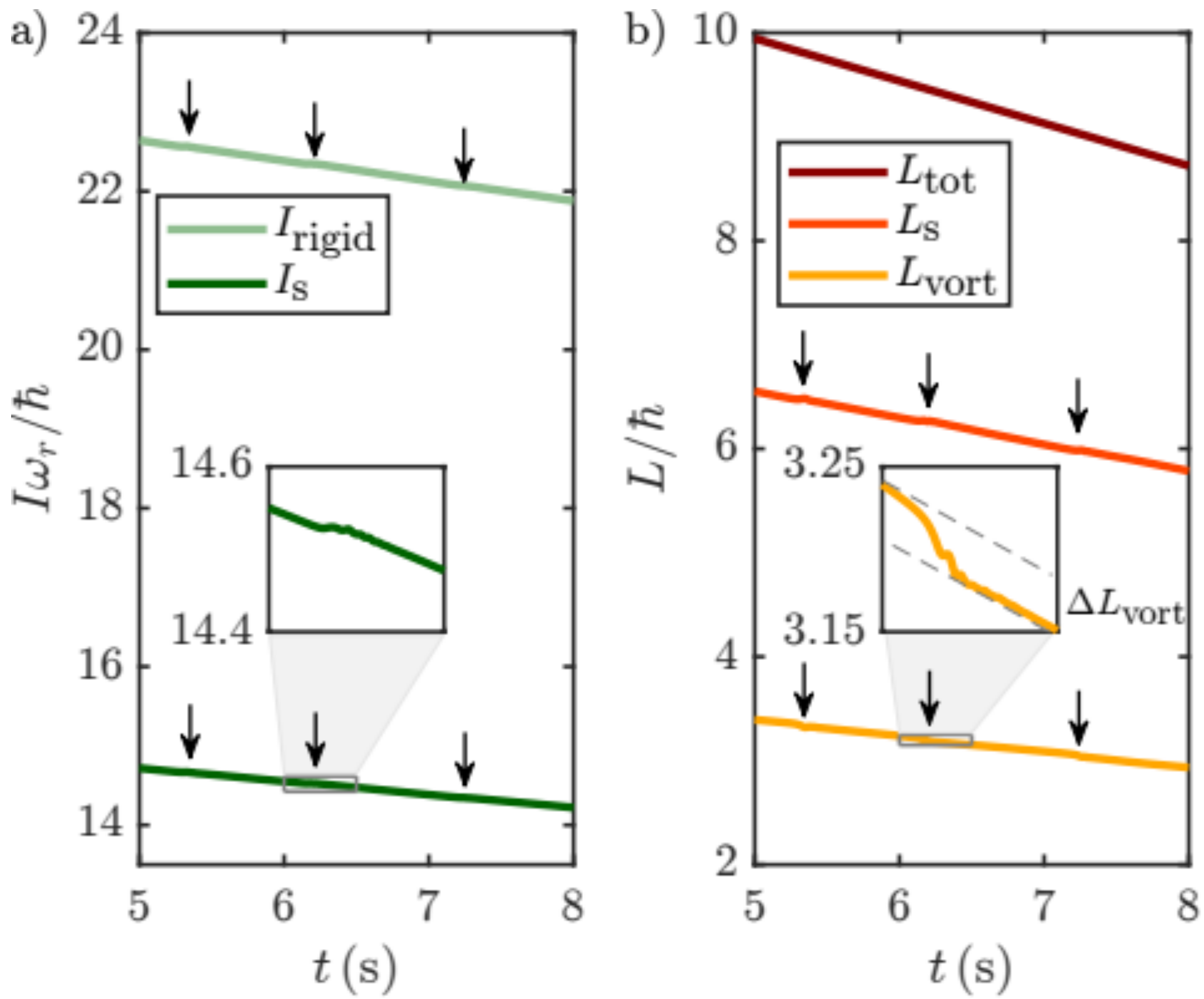
- Parameters $N_{\mathrm{em}} = 4.3 \times 10^{-35}\mathrm{kg\ m^2/s^2}\,,\quad \gamma = 0.05\,,\quad \Omega_{\mathrm{init}} = 0.5\omega_r\,.$

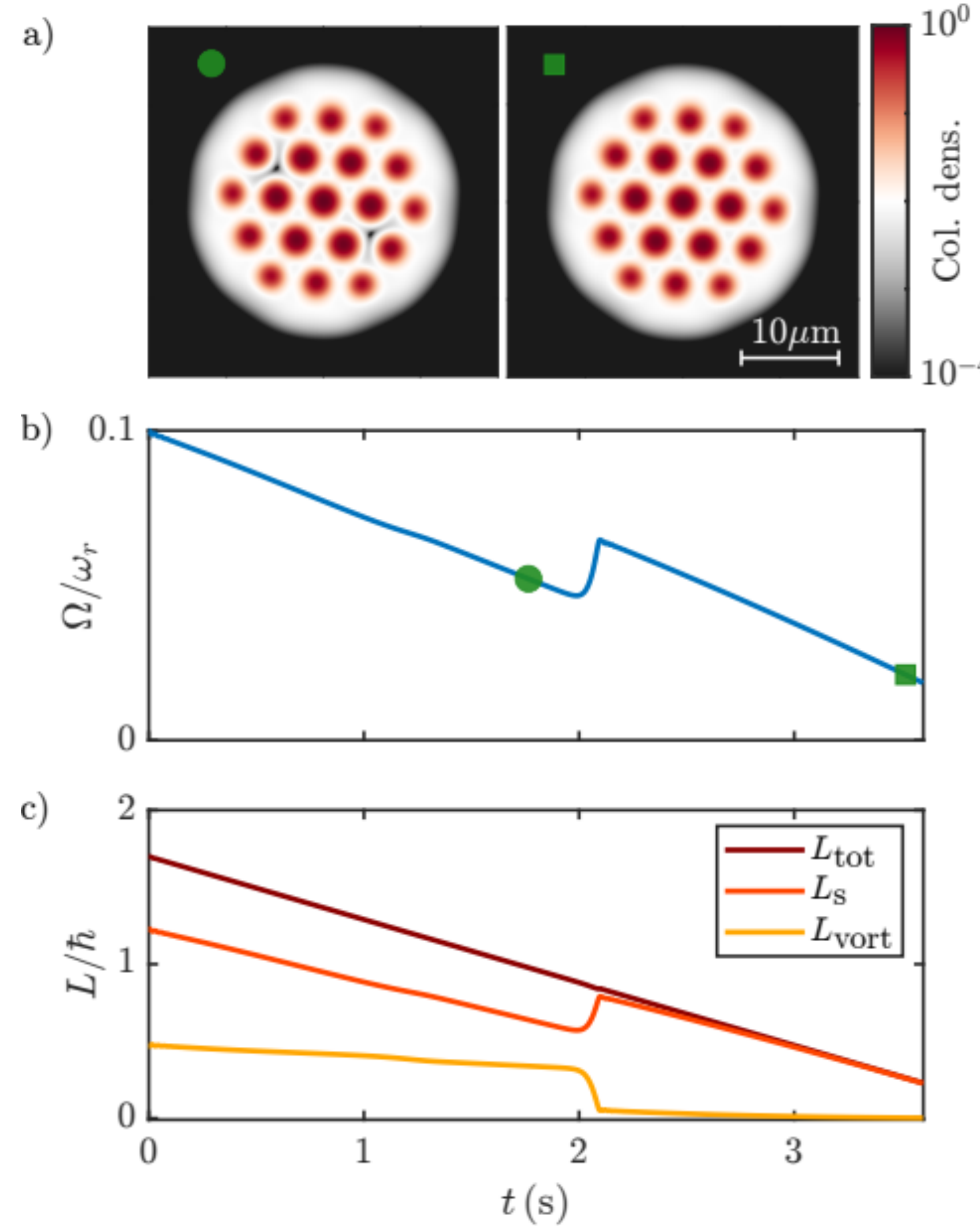


- Different values of γ mimic different coupling with the outer crust

- Parameters $N_{\rm em} = 4.3 \times 10^{-35}\text{kg m}^2/\text{s}^2\,,\ a_s = 91a_0\,,\quad \Omega_{\rm init} = 0.5\omega_r\,.$

Testing the angular momentum decomposition





Outer crust

$$\rho \lesssim 4.3 \times 10^{11} \text{ g cm}^{-3}$$

Weak equilibrium works to produce the most stable isotope

Isotope	Z/A	ρ_t(g/cm^3)	μ_e (MeV)
^{56}Fe	0.464	7.96×10^6	0.95
^{62}Ni	0.452	2.71×10^8	2.61
^{64}Ni	0.437	1.3×10^9	4.31
^{66}Ni	0.424	1.48×10^9	4.45
^{86}Kr	0.419	3.12×10^9	5.66
^{84}Se	0.405	1.10×10^{10}	8.49
^{82}Ge	0.390	2.80×10^{10}	11.4
^{80}Zn	0.375	5.44×10^{10}	14.1
^{78}Ni	0.359	9.64×10^{10}	16.8
^{126}Ru	0.350	1.29×10^{11}	18.3
^{124}Mo	0.339	1.88×10^{11}	20.6
^{122}Zr	0.328	2.67×10^{11}	22.9
^{120}Sr	0.317	3.79×10^{11}	25.4
^{118}Kr	0.305	4.31×10^{11}	26.2

Neutron rich matter

many electrons

neutron drip

Haensel and Pichon
Astron.Astrophys. 283 (1994) 313

Density matters!

Energy density scaling

kinetic (single particle)

$\mathcal{E}_{sp} \propto n$

Interaction (mean field)

$\mathcal{E}_{contact} \propto n^2$ $\mathcal{E}_{dipolar} \propto U_{dd} n^2$

Quantum fluctuations

$\mathcal{E}_{LHY} \propto n^{5/2}$

Repulsive

Long-range inhomogeneous

"Interaction channels" change with density

Evolution

Optical trap → Outer crust

Dipolar atoms → Inner crust

To emulate the NS spin down, we put a “break” on the optical trap

$$\dot{L}_{total} = -N_{em}$$

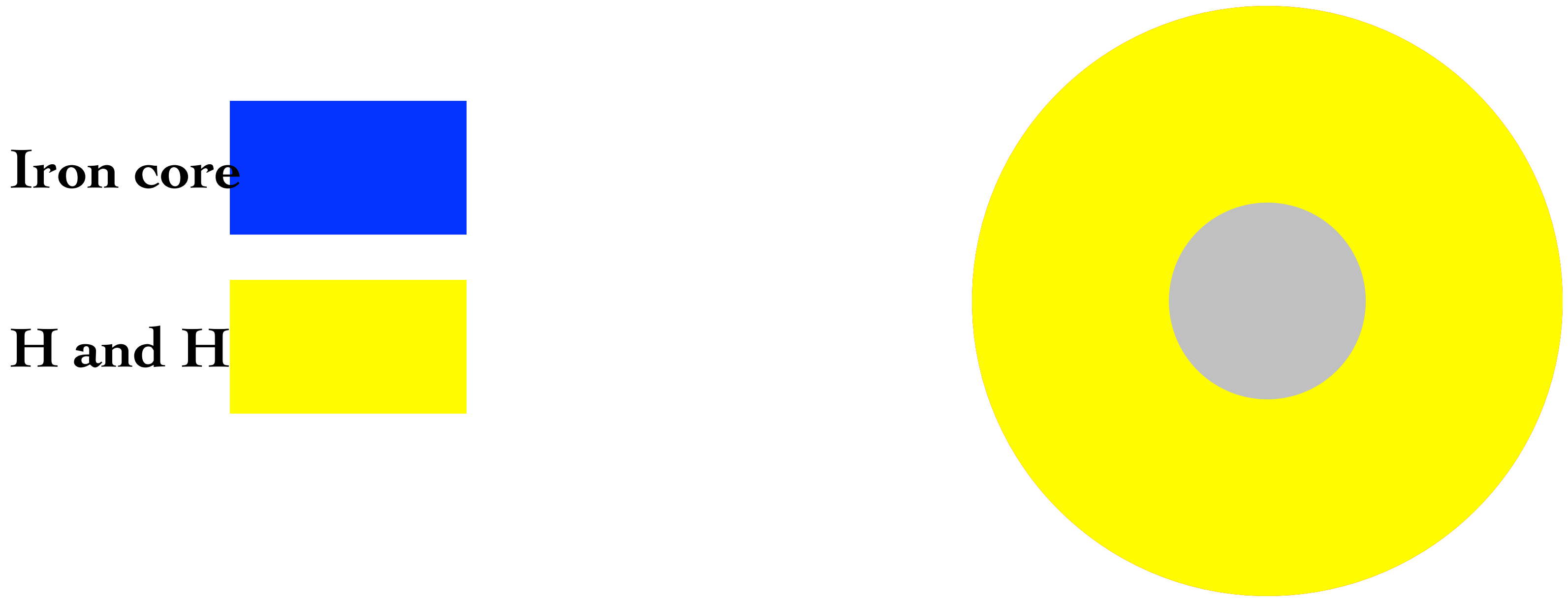
System of equations solved recursively

$$I_{solid}\dot{\Omega} = -N_{em} - \dot{L}_{vortices} - \dot{I}_{solid}\Omega$$

$$i\hbar\frac{\partial \Psi}{\partial t} = (1 - i\gamma)\left[\mathscr{H}[\Psi; a_s, a_{\mathrm{dd}}, \omega] - \Omega(t)\hat{L}_z\right]\Psi$$

Dawn of neutron

$8M_{\odot} \lesssim M \lesssim 20M_{\odot}$



During the stellar contraction ions capture electrons forming neutrons

Matter falling towards the stellar center bounces back, because neutrons cannot be easily squeezed.
A shockwave is produced: a supernova!

Shear viscosity η

Diffusion between layers results in an effective **friction**

$$\eta \sim n\, p\, \lambda$$

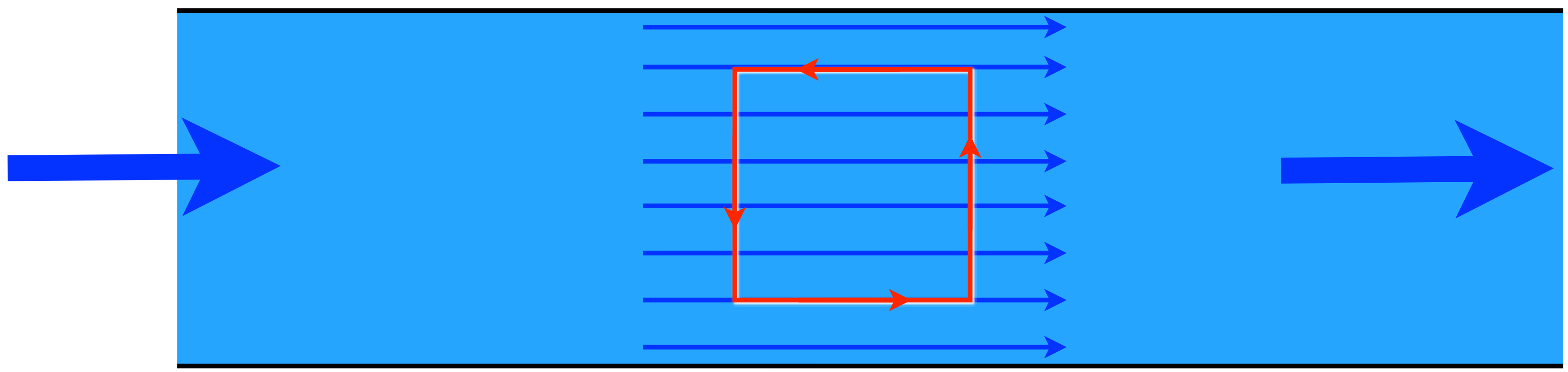
λ	mean free path
p	average momentum
n	number density

From $p\lambda \geq \hbar$ it follows that $\dfrac{\eta}{n} \geq \hbar$

In relativistic systems **entropy** works better. Entropy density $s \propto k_B n$

$$\frac{\eta}{s} \sim p\lambda \geq \frac{\hbar}{k_B}$$

Unviscid (dry) fluid



Continuity equation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler equation $$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla \mathbf{p}}{\rho} - \nabla \phi$$

Vorticity $\boldsymbol{\Omega} = \nabla \times \mathbf{v}$

Unviscid fluid $\Omega = 0$

The flow is permanently irrotational $\mathbf{v} = \nabla \varphi$

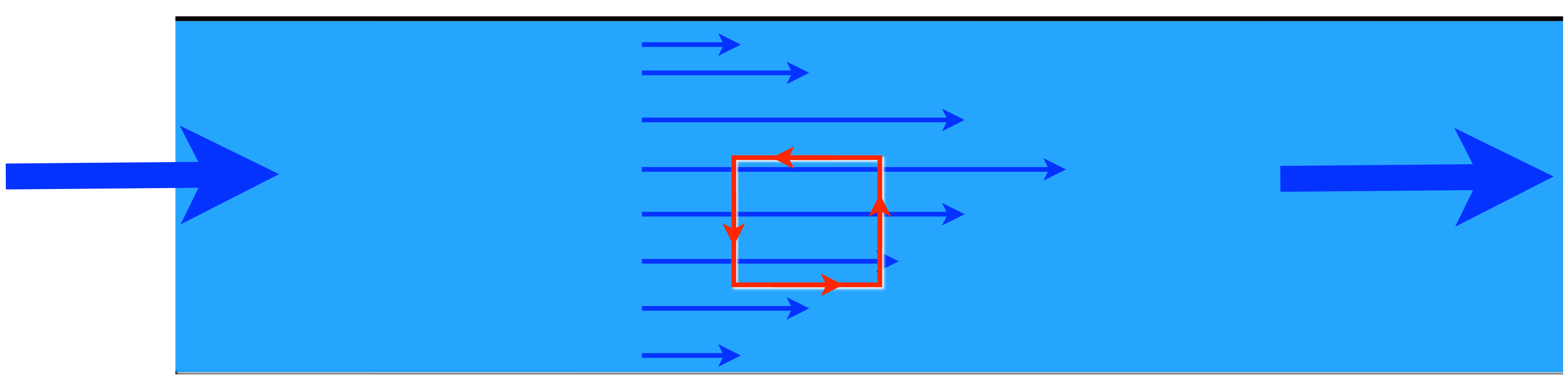
Viscous fluid

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)=-\nabla p-\rho\nabla\phi+\eta\nabla^2\mathbf{v}+\zeta\nabla(\nabla\cdot\mathbf{v})$$

shear viscosity **bulk viscosity**

Using the vorticity $$\frac{\partial \mathbf{\Omega}}{\partial t}+\nabla\times(\mathbf{\Omega}\times\mathbf{v})=\frac{\eta}{\rho}\nabla^2\mathbf{\Omega}$$

Vorticity is generated by the shear viscosity



Univiscid flow is not the characterizing property of a superfluid.

Consider a ballistic gas. It "flows" without dissipation. But it is certainly not a superfluid.

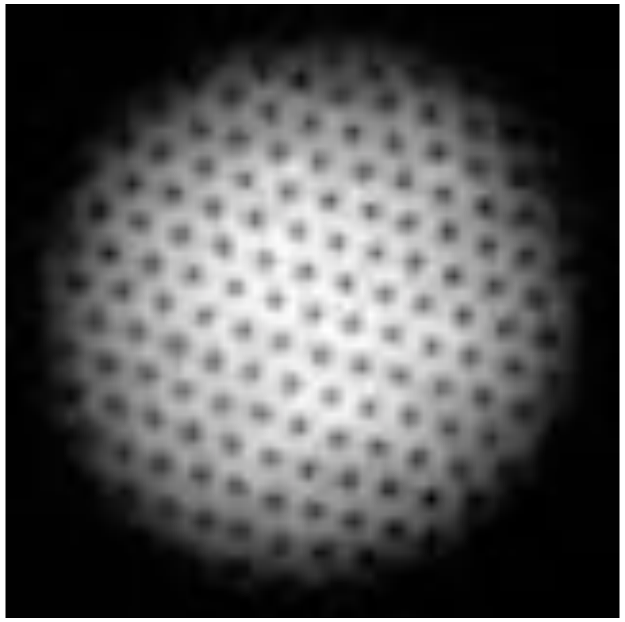
The characterizing property of a superfluid is potential flow

with quantized vortices when in rotation

$$\boldsymbol{v} = \nabla\varphi \rightarrow \nabla \times \boldsymbol{v} = 0 \text{ almost everywhere}$$

$$\oint_C d\boldsymbol{l} \cdot \boldsymbol{v} = \Delta\varphi = n\pi \text{ because } \varphi \text{ is a phase}$$

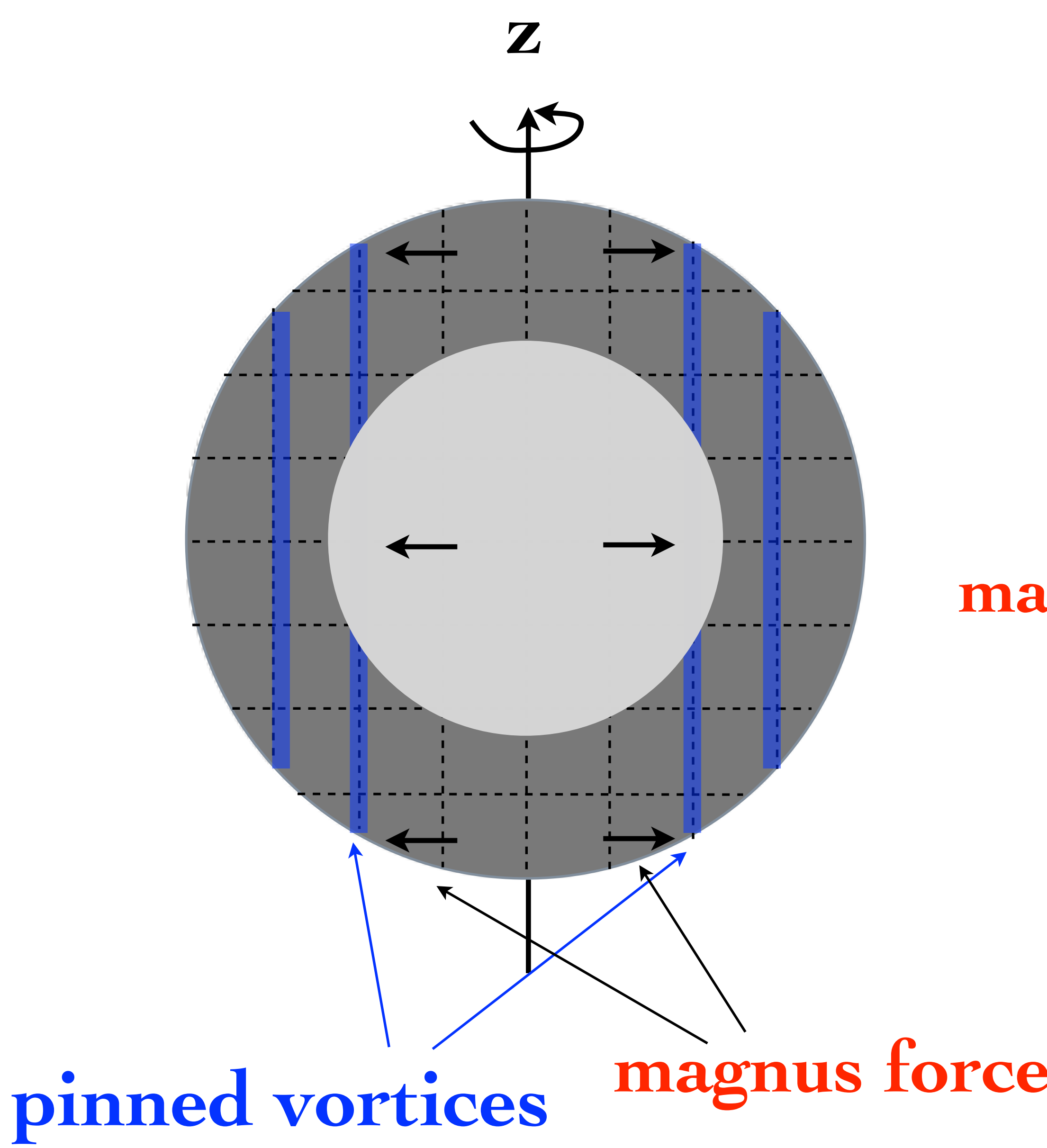
Since vortices repel they form an Abrikosov lattice



Possible glitch mechanism

1) Superfluid vortices are initially pinned

2) Superfluid vortices unpin and transfer angular momentum to the crust

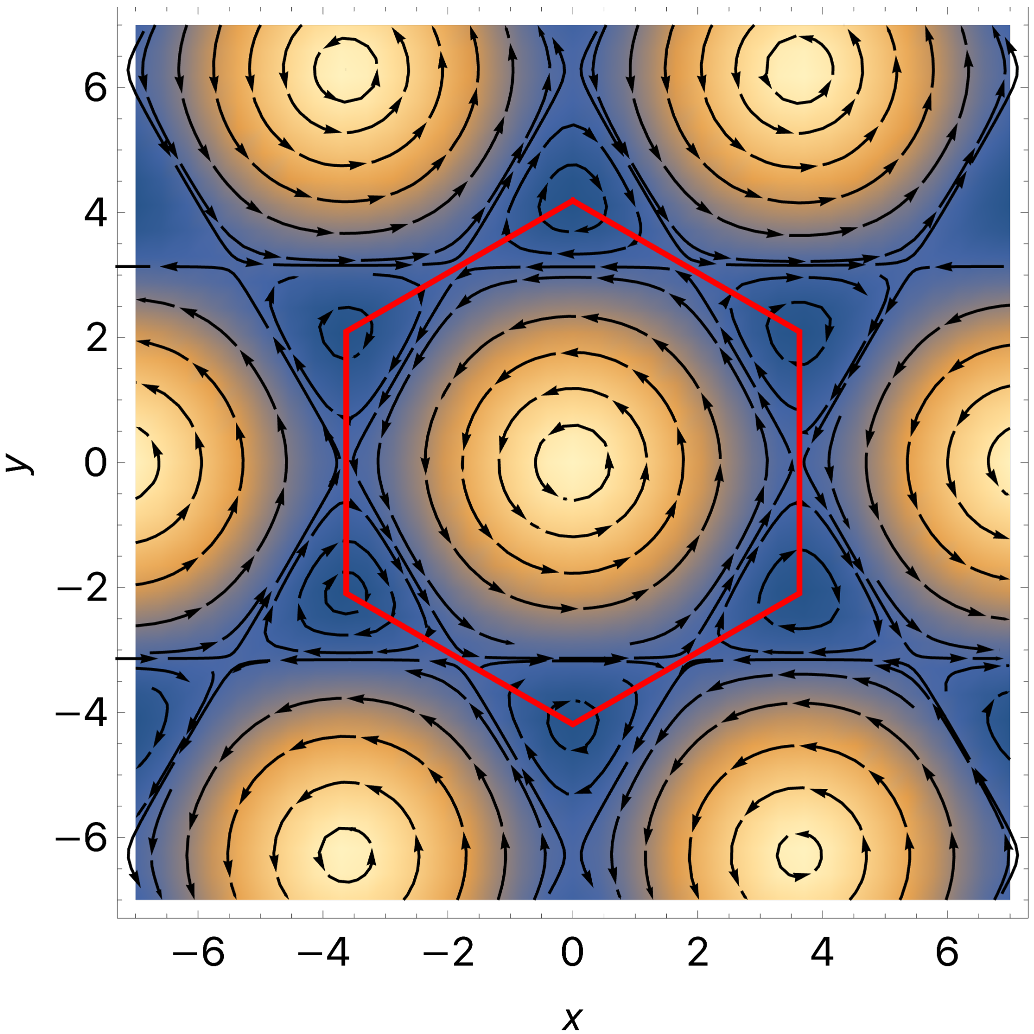


vortices move if
magnus force > pinning force

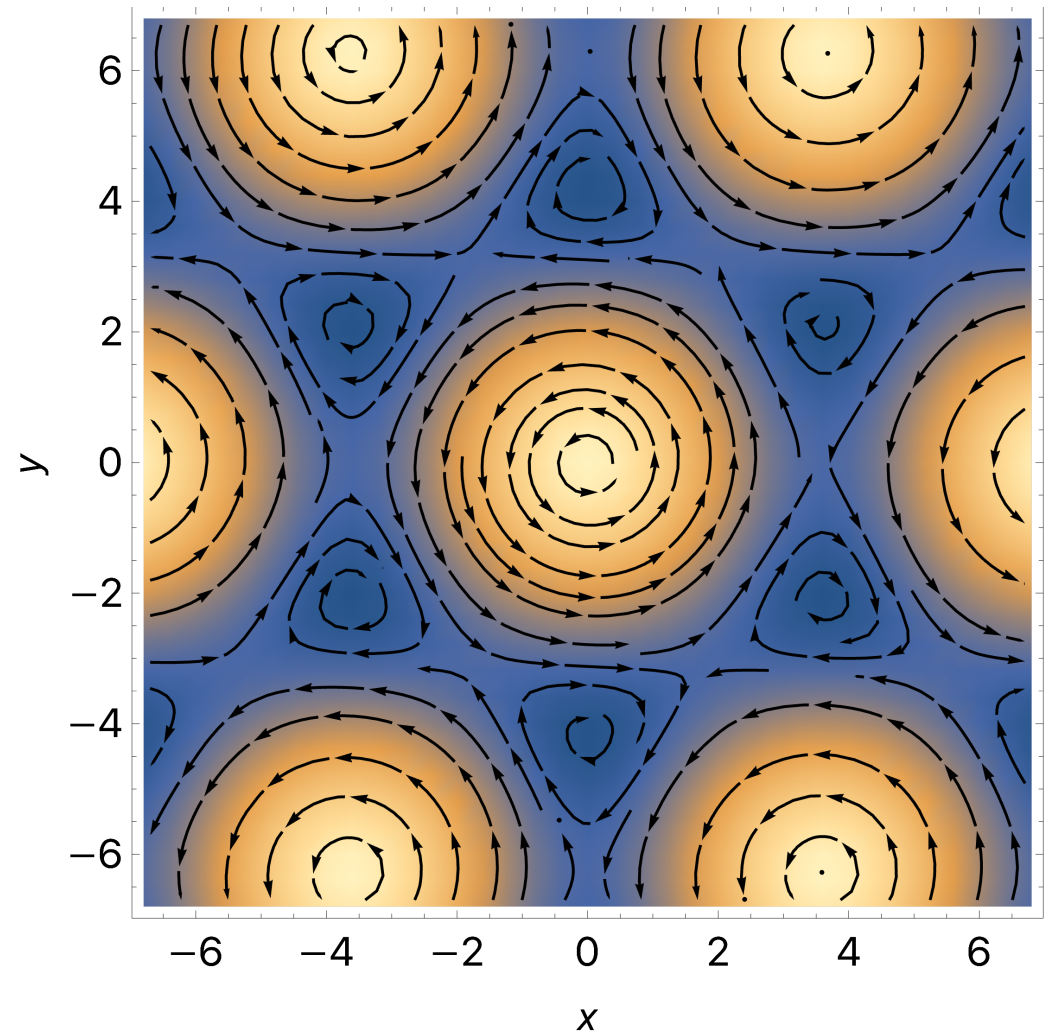
Vortex pinning

Vortex streamlines

Without dissipation



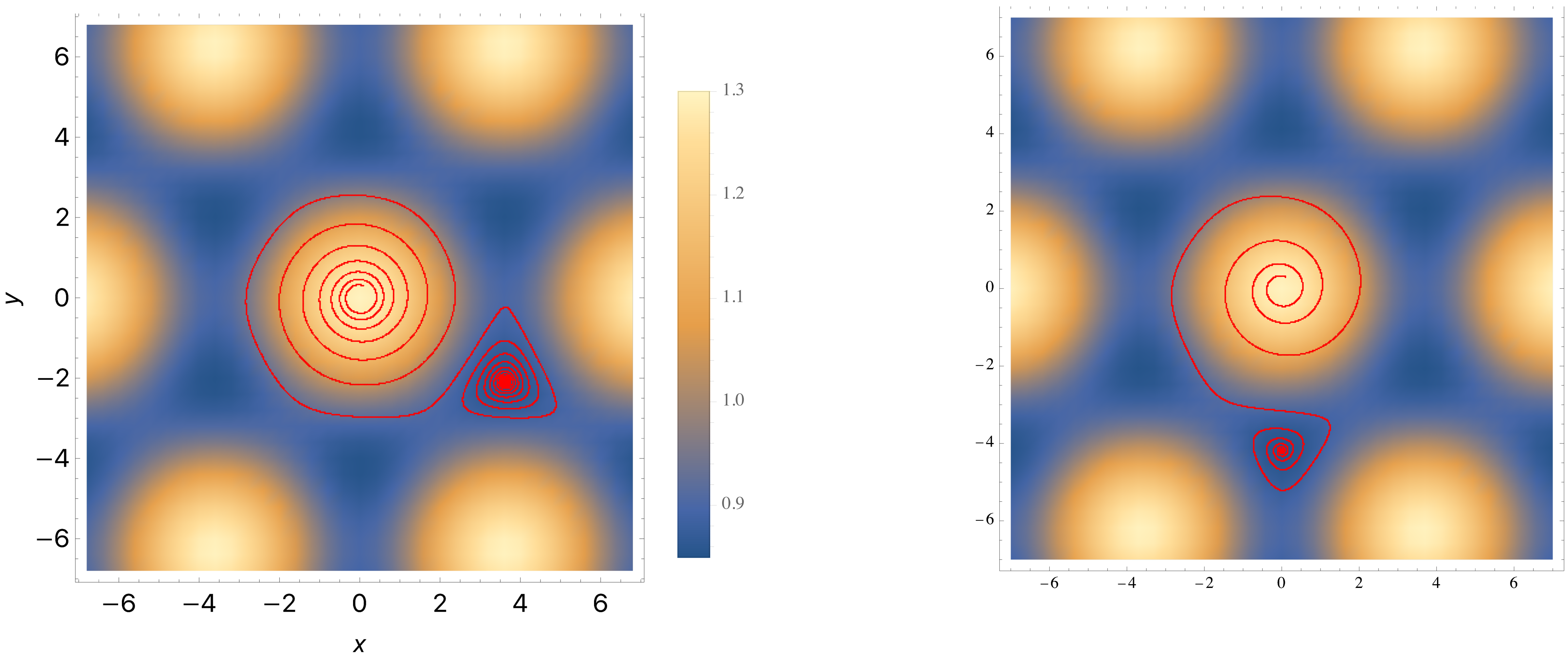
With dissipation



$$\mathbf{v}_v \simeq -\frac{\hbar}{m}\left\{\gamma\nabla\log\sqrt{\rho}+\hat{\mathbf{z}}\times\nabla\log\sqrt{\rho}\right\}=\mathbf{v}_\perp+\mathbf{v}_\parallel$$

MM et al. Few Body Syst. 65 (2024) 81

Vortex evolution with dissipation



$$\mathbf{v}_v \simeq -\frac{\hbar}{m}\left\{\gamma \nabla \log \sqrt{\rho} + \hat{\mathbf{z}} \times \nabla \log \sqrt{\rho}\right\} = \mathbf{v}_\perp + \mathbf{v}_\parallel$$

MM et al. Few Body Syst. 65 (2024) 81