Color Superconductivity, Proto-Neutron Stars and Dark Matter in Compact Stars

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Compact Stars in the QCD Phase Diagram (CSQCD2026)
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CRC-TR 211 Strong-interaction matter under extreme conditions

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HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Content

Introduction: Compact Objects

Compact Objects

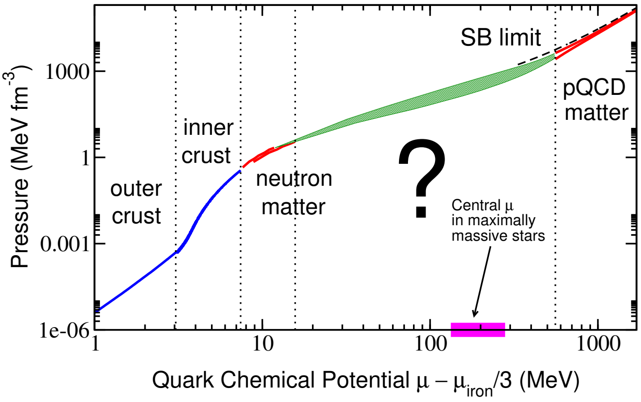
canonical compact objects: endpoints of stellar evolution i.e. white dwarfs, neutron stars, and black holes
extension to exotic matter (and mixtures thereof):

- hyperon star: neutron star with nucleons and hyperons
- quark star: made of quark matter
- hybrid star: mixture of baryonic and quark matter
- boson star: made of (dark) bosons
- fermion star: made of (dark) fermions

generic classes of bound compact stars in GR:

- gravitationally bound: stabilized by gravity and matter pressure
- selfbound: stabilized by vacuum pressure of matter

Interpolating Equation of State (EoS)

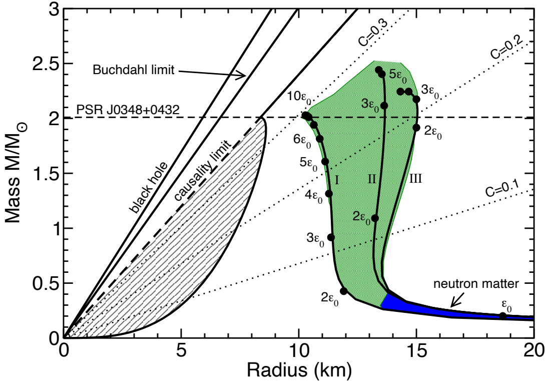


(Kurkela, Fraga, JSB, Vuorinen 2014)

- smooth interpolation by piece-wise polytropes (green band)
- demand causality $c_s^2 \leq 1$ and $M_{\max} \geq 2M_\odot$
- no information on what the matter within the green band is made of!

Mass-radius relation (smooth interpolation)

(JSB, Compact Star Physics 2020)

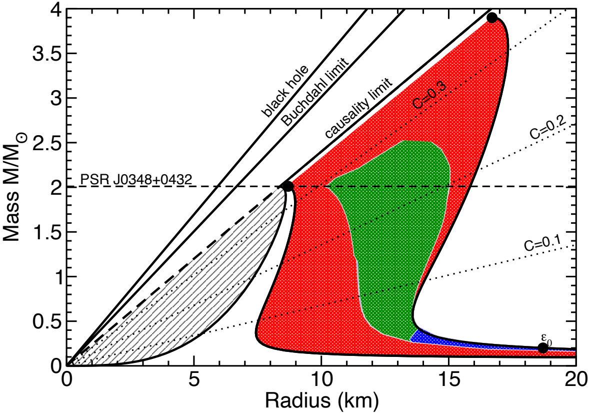


- Buchdahl limit: incompressible fluid ($C = GM/R = 4/9$)
- causality limit: $P = c_s^2 \cdot (\epsilon - \epsilon_0)$

  with a speed of sound of $c_s = c = 1$ ($C = 0.354$)
- interpolated EOS: $R = 10 - 15$ km ($M = 2M_\odot$)

Mass-radius relation (with jumps)

(JSB, Compact Star Physics 2020)



- causal EOS $p = \epsilon$ $(\Delta\epsilon = 0)$: right bound in mass-radius relation
- causal EOS with jump $\Delta\epsilon$: left bound in mass-radius relation with $M_{\text{max}} = 2M_\odot$

Minimal Mass of Proto-Neutron Stars

based on Selina Kunkel, Stephan Wystub, JSB 2024

Proto-Neutron Star Evolution

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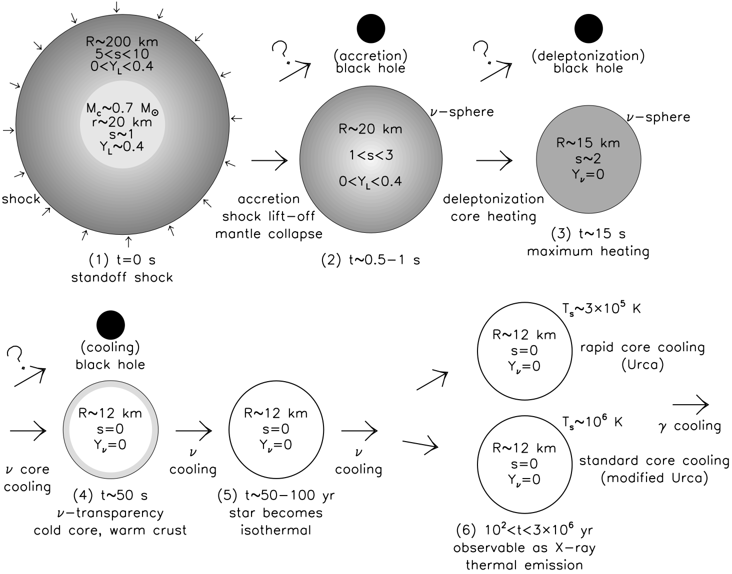
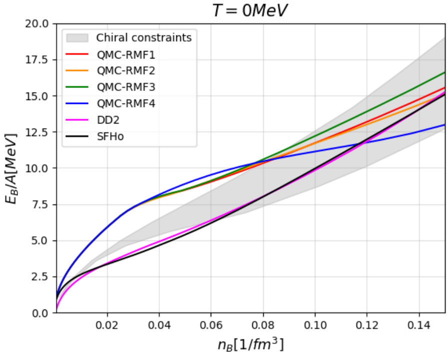


(Prakash, Lattimer, Pons, Steiner, Reddy 2001)

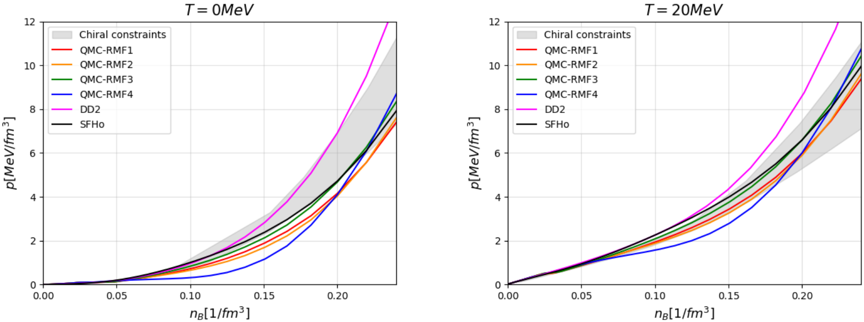
Constraints from chiral EFT for $T = 0$



(Kunkel, Wystub, JSB 2025)

- chiral constraints on the binding energy for pure neutron matter at $T = 0$ (Krüger, Tews, Hebeler, Schwenk 2013)
- QMC-RMF1 to QMC-RMF4: new parameter sets from Alford, Brodie, Haber, Tews 2023

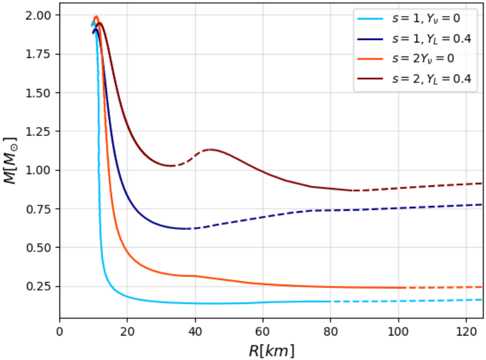
Constraints from chiral EFT for $T = 20$ MeV



(Kunkel, Wystub, JSB 2025)

- chiral constraints on the pressure for pure neutron matter at $T = 20MeV$ (Keller, Hebeler, Schwenk 2023)
- chiral EFT has a much weak temperature dependence

Mass-Radius Relation for Proto-Neutron Stars



(Kunkel, Wystub, JSB 2025)

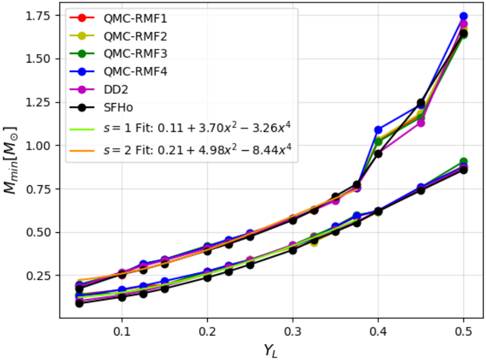
- mass-radius relation for set DD2
- first stage: lepton-rich ($Y_L = 0.4$), third stage: no neutrinos ($Y_\nu = 0$)
- minimum mass increases from $0.1M_\odot$ ($T = 0$) to $0.6M_\odot$ (for entropy per baryon $s = 1$)

Minimum Mass for Proto-Neutron Stars

$Y_L = 0.4,\ s = 1$			
EOS	$M_{\text{min}}[M_\odot]$	R[km]	$n_B[\text{fm}^{-3}]$
QMC-RMF1	0.617	37.9	0.149
QMC-RMF2	0.615	36.6	0.163
QMC-RMF3	0.618	38.0	0.145
QMC-RMF4	0.624	36.1	0.164
DD2	0.620	38.3	0.134
SFHo	0.613	37.0	0.153

- rather common properties of minimal mass configurations: mass $M_{\text{min}} \approx 0.62 M_\odot$, radius $R \approx 37$ km, maximum density around saturation density

Minimal Mass for Different Lepton Fractions



(Kunkel, Wystub, JSB 2025)

- minimal mass increases strongly with lepton fraction Y_L
- rather independent on parameter sets
- results for $Y_L > 0.4$ involve the liquid-gas phase transition (need full thermodynamical potential instead of tables)

Color-Superconductivity in Neutron Stars

based on Hosein Gholami, Ishfaq Rather, Marco Hofmann, Michael Buballa, JSB 2025
and Jan-Erik Christian, Ishfaq Rather, Hosein Gholami, Marco Hofmann 2025

RG-Consistent NJL Model

Consider the extended NJL-model of the form

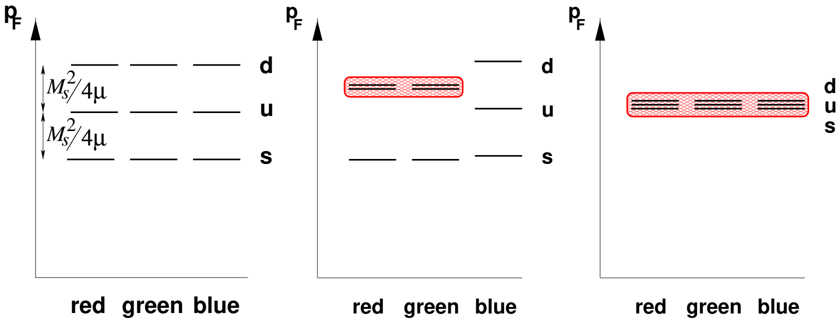
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_V + \mathcal{L}_{qq} + \mathcal{L}_\mathsf{L} \tag{1}$$

with

- $\mathcal{L}_0$: kinetic terms
- $\mathcal{L}_{\bar{q}q}$: pseudoscalar and scalar contact interactions
- $\mathcal{L}_V$: repulsive vector interaction (η_V: vector coupling)
- $\mathcal{L}_{qq}$: diquark interactions (η_D: diquark coupling)
- $\mathcal{L}_\mathsf{L}$: (free) lepton terms

Describes interacting quark matter including pairing of quarks
Two free parameters: diquark coupling η_D and vector coupling η_V
RG-consistent treatment via counterterms removing cutoff artefacts
(Gholami, Hofmann, Buballa 2025)

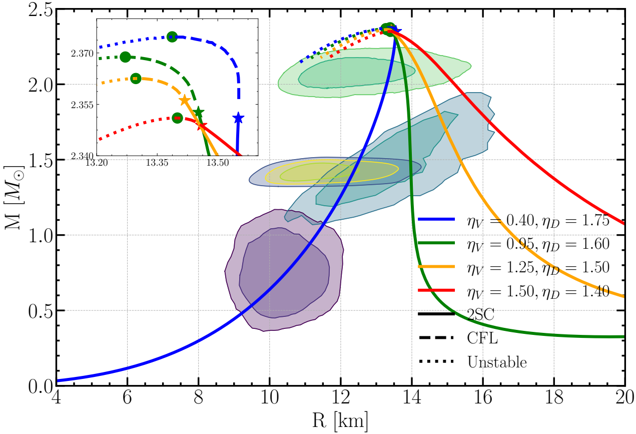
Colour-Superconductivity



(Alford, Rajagopal, Schaefer, Schmitt 2007)

- quarks will pair in dense matter (Cooper theorem)
- 2SC phase: pairing of two flavours and two colours, one colour and one flavour unpaired
- CFL phase: colour-flavour-locked phase, pairing of three flavours and three colours

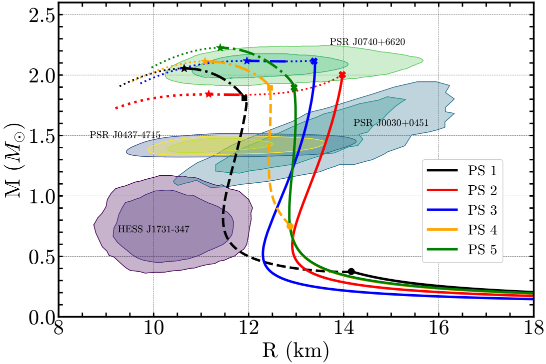
Mass-Radius Relation for Quark Stars



(Gholami, Rather, Hofmann, Buballa, JSB 2025)

- different choices of diquark coupling η_D and vector coupling η_D
- same maximum mass but different curves
- stable CFL phase appears for massive star configurations

Mass-Radius Relation for Hybrid Stars



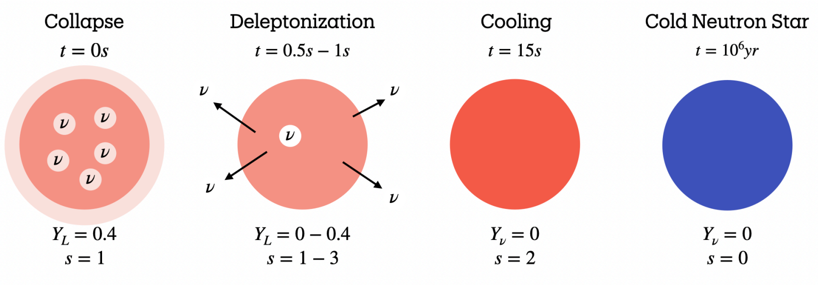
(Christian, Rather, Gholami, Hofmann 2025)

- added hadronic EoS: RMF parameter sets FB (Frankfurt-Barcelona)
- solid lines: hadronic EoS, dashed: 2SC phase, dash-dotted: CFL phase
- separate stable branches appear: twin star solutions
- hadron matter to quark matter transition: to 2SC or directly to CFL phase

Proto-Neutron Stars with Color-Superconducting Phases

based on Selina Kunkel, Ishfaq Rather, Hosein Gholami, Marco Hofmann, JSB 2026

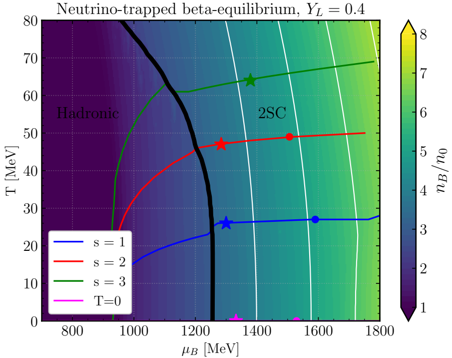
Evolution of a Proto-Neutron Star



(Kunkel, Rather Gholami, Hofmann, JSB 2026)

- initially lepton-rich (lepton fraction $Y_L = 0.4$ from initial progenitor star's iron core)
- deleptonization within a second (neutrino-trapped), core heating due to compression
- cooling down with one minute (neutrino-transparent $Y_\nu = 0$)
- late time cooling for a million years at low $T < 100keV$

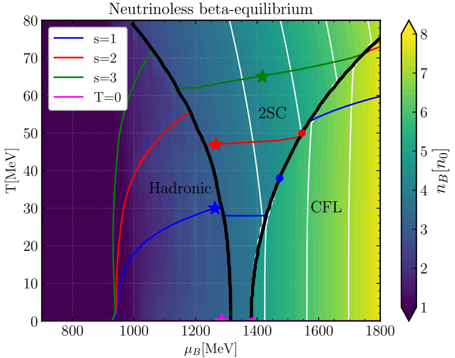
Phase Diagram for Neutrino-Trapped Matter



(Kunkel, Rather Gholami, Hofmann, JSB 2026)

- RG-consistent three-flavor NJL model matched to DD2 EoS
- lines at constant entropy per baryon s, dots: maximally stable configuration, stars: maximum baryon number for initially $s = 3$
- first order phase transition to 2SC quark phase, no CFl phase

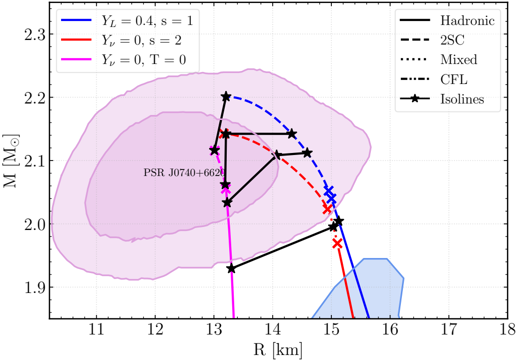
Phase Diagram for Neutrinoless Matter



(Kunkel, Rather Gholami, Hofmann, JSB 2026)

- first order phase transition lines to 2SC and CFL phases
- isentropes follow phase coexistence lines
- along isentropes matter is cooler in the 2SC phase than in hadronic phase
- along isentropes the CFL phase is hotter compared to the 2SC phase

Mass-Radius Diagram of Hybrid Stars and Isolines



(Kunkel, Rather Gholami, Hofmann, JSB 2026)

- mass-radius of hybrid stars: compatible with constraints from NICER
- isolines along constant baryon number from neutrino-trapped to cold stage
- four different scenarios: collapse to black hole, CSC phase survives, CSC phase disappears, CSC phase exists only temporarily

Neutron Stars with Bosonic Dark Matter

based on Sarah Pitz and JSB 2024, 2025

Selfinteracting Scalar (Bosonic) Matter

Start with a classical selfinteracting field ϕ:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - V \tag{2}$$

with a selfinteraction potential

$$V = \frac{\lambda}{2^{n/2}} \left(\phi^* \phi\right)^{n/2} \qquad (n > 2) \tag{3}$$

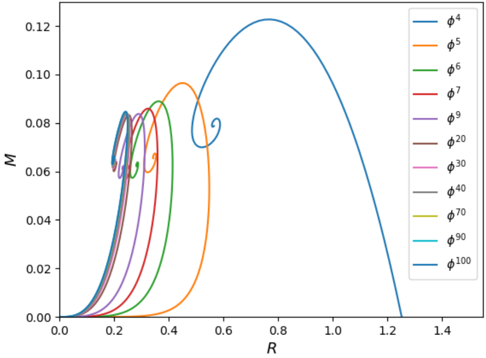
Results in an equation of state of the form

$$\varepsilon' = p'^{2/n} + \frac{n+2}{n-2}\, p'. \tag{4}$$

with dimensionless pressure and energy density

$$p' = \frac{p}{\varepsilon_0} \qquad \varepsilon' = \frac{\varepsilon}{\varepsilon_0} \tag{5}$$

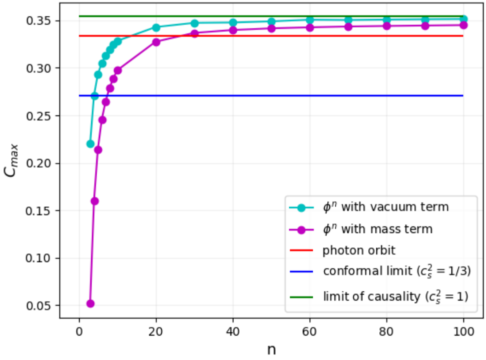
Mass-Radius Curves for Boson Stars



(Pitz and JSB 2024)

- case $n = 4$: canonical boson stars with constant radius (Colpi, Shapiro, Wasserman 1986)
- case $n > 4$: mass-radius curves start at origin
- note: these latter cases are not selfbound stars!

Maximal Compactness of Boson Stars



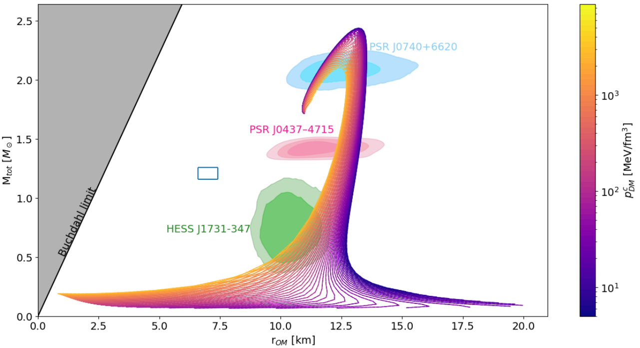
(Pitz and JSB 2024)

- maximal compactness C_{max} increases with power law index n
- for $C > 1/3$: radius lies within the photon orbit like a black hole (black hole mimicker)
- limit of causality: $C = 0.354$ for a causal equation of state $p = \varepsilon$

Neutron Stars with Bosonic Dark Matter

- add neutron star matter with piece-wise polytropic equation of state
- use limiting equations of states (softest: EoS1 and stiffest: EoS2) from Kurkela, Fraga, JSB, Vuorinen 2014
- vary parameters of bosonic dark matter: mass and power index n
- choose case $n = 4$: canonical case and $n = 40$: ultracompact case
- solve two-fluid TOV equations
- check for stability using the method of Hippert, Dillingham, Tan, Curtin, Noronha-Hostler, Yunes 2023

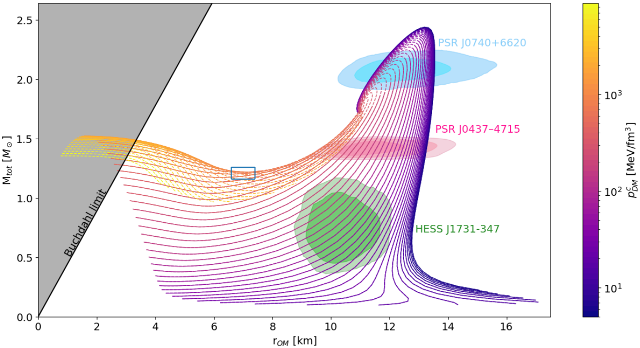
Neutron Stars with Regular Dark Matter



(Pitz and JSB 2025)

- power index $n = 4$, boson mass $m_b = 1000$ MeV
- colour code: dark matter central pressure
- mass-radius constraints from NICER (blue and pink), HESS (green, atmosphere modelling), and XTE (rectangle, from thermonuclear burst)

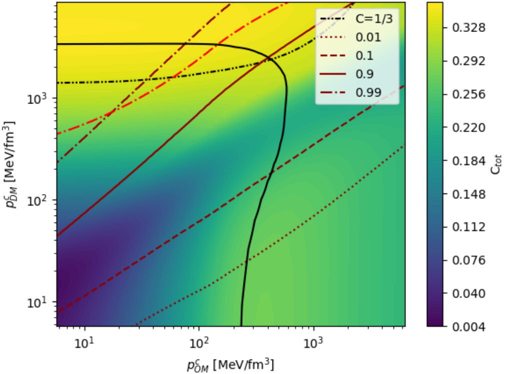
Neutron Stars with Compact Dark Matter



(Pitz and JSB 2025)

- power index $n = 40$, boson mass $m_b = 300$ MeV
- solid lines: stable configurations
- stable mass-radius configurations 'outside the box' of pure neutron stars
- violates the Buchdahl limit (by taking the visible radius)

Ultracompact Neutron Stars with Dark Matter



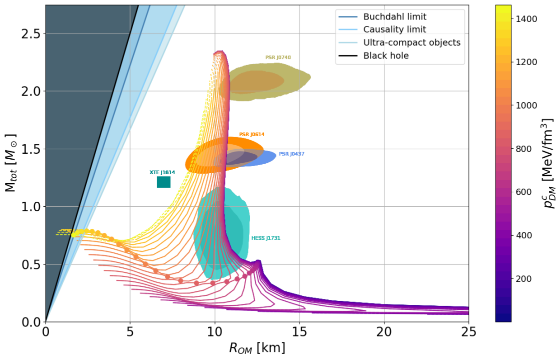
(Pitz and JSB 2025)

- contour plot for different central pressures of neutron matter and dark matter
- within solid black line: stable configurations, legend: fraction of dark matter
- ultracompact configurations for neutron stars with dark matter ($C > 1/3$)
- red dash-dotted line: Buchdahl limit (by taking the visible radius)

Hybrid Stars with Bosonic Dark Matter

based on Sarah Pitz, Ishfaq Rather, JSB 2026

Neutron Stars with Quark and Dark Matter



(Pitz, Rather, JSB 2026)

- power index $n = 4$, boson mass $m_b = 500$ MeV, constant speed of sound for quark matter ($c_s^2 = 1$)
- solid lines: stable configurations, filled circles: onset of phase transition
- violates the Buchdahl limit (by taking the visible radius)

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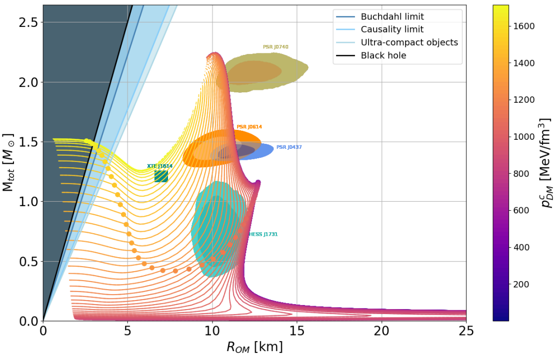

(Pitz, Rather, JSB 2026)

- power index $n = 40$, boson mass $m_b = 300$ MeV, constant speed of sound for quark matter ($c_s^2 = 1$)
- phase transition can be present for low-mass neutron stars
- violates the Buchdahl and the black hole limit (by taking the visible radius)

Tidal Deformability of Dark Hybrid Stars

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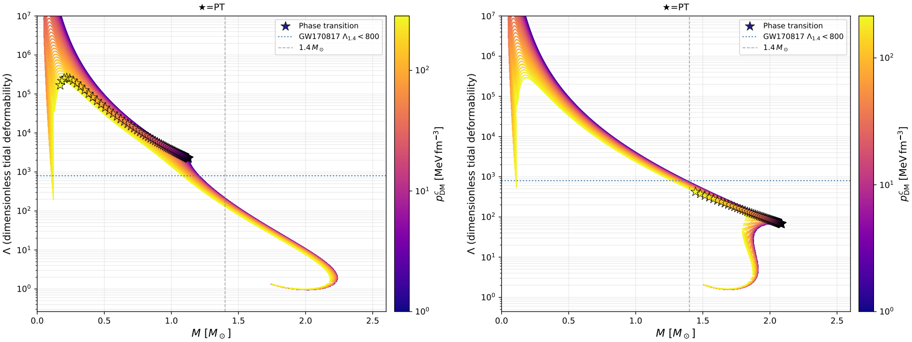

(Pitz, Rather, JSB 2026)

- power index $n = 4$, boson mass $m_b = 1000$ MeV, different onset of phase transition
- phase transition can be present for low masses (high tidal deformability) or high masses (low tidal deformability)

Summary

Summary

- Study of proto-neutron star evolution with chirally constrained EoS:
- Established lower mass limit for neutron stars of $M = 0.6 M_\odot$

- Study of CSC phases in neutron stars with the RG-consistent NJL model:
- Stable neutron star configurations with 2SC and CFL phases possible being compatible with astrophysical data

- Study of CSC phases in proto-neutron stars:
- four different scenarios: collapse to black hole, CSC phase survives, CSC phase disappears, CSC phase exists only temporarily

Summary II

- Study of strongly selfinteracting boson stars:
- Boson stars can be ultracompact (close to causality limit)

- Study of bosonic dark matter in neutron stars:
- Ultracompact configurations of neutron stars with dark matter possible with a light ring (black hole mimicker)

- Study of hybrid stars with dark matter:
- Phase transition to quark matter can be present for low mass stars with violation of Buchdahl limit