

# Using Bayesian inference to determine the equation of state of baryonic matter

Constança Providência

Universidade de Coimbra, Portugal

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# Outline

- ▶ What is the composition of NS?
- ▶ Using Bayesian inference to constrain the microphysics
- ▶ Testing CEDF models
- ▶ Including hyperons: implications
- ▶ Constraining hybrid stars
- ▶ Conclusions

# Motivation

- ▶ Why and how to use Bayesian inference as a tool to constrain the microphysics that describe neutron stars?  
determination of the entire parameter space consistent with the imposed constraints
- ▶ What are the advantages of Bayesian analysis?  
the output is a probability, comparison of models
- ▶ What are the limitations of Bayesian analysis?  
Prior choice, computationally intensive, determination of limits

# Collaborators

The work I will discuss has been developed with my collaborators

- ▶ Tuhin Malik, Helena Pais, Márcio Ferreira (UCoimbra)
- ▶ Milena Albino, João Cartaxo, Tiago Custódio (UCoimbra)
- ▶ Chun Huang, Laura Tolos, Anna Watts, Verónica Dexheimer

Publications:

- ▶ Malik ApJ930 (2022) 17; PRD106 (2022) 063024; PRD107 (2023)103018
- ▶ Albino PRD110 (2024) 083037; PRD113 (2026) 083019
- ▶ Huang MNRAS 529 (2024) 4650; MNRAS 536 (2025) 3262
- ▶ Cartaxo ApJS 282 (2026) 33

# What is the composition of NS? Is it hadron matter?

- ▶ **Hadron structure:** compact 'hard' valence quark core surrounded by a 'soft' cloud of quark-antiquark pairs
- ▶ **isoscalar and isovector electric form factors of the nucleon:** central core with a root mean square radius of  $\sim 0.5$  fm (Kaiser & Weise PRD110, 015202)
- ▶ **hadron cores overlap if  $\rho \geq 6\rho_0$**
- ▶ **close packing of hard spheres (R= 0.5 fm):  $\rho \sim 8\rho_0$**
- ▶ **Maximum  $\rho$  considered:  $1.2 \text{ fm}^{-3} \sim 7.5$  to  $8 \rho_0$**
- ▶ **Covariant energy density functional:** easy inclusion of non-nucleonic degrees of freedom, temperature, finite nuclei, nuclear matter and astrophysical constraints.
- ▶ **Important as a reference to compare with other scenarios**
- ▶ **What is the effect of including exotic degrees of freedom?**

# CEDF: covariant energy density functional

- ▶ **Lagrangian density:** causal Lorentz-covariant Lagrangian (baryon densities and meson fields) (Mueller & Serot NPA606, 508; Horowitz & Piekarewicz PRL86,5647)

$$\mathcal{L}_{RMF} = \sum_{B=\text{baryons}} \mathcal{L}_B + \mathcal{L}_{\text{mesons}} + \mathcal{L}_l,$$

- ▶ **Baryonic contribution:**  $\mathcal{L}_B = \bar{\psi}_B [\gamma_\mu D_B^\mu - M_B^*] \psi_B$ ,  
 $D_B^\mu = i\partial^\mu - g_{\omega B} \omega^\mu - \frac{g_{\rho B}}{2} \boldsymbol{\tau} \cdot \mathbf{b}^\mu - g_{\phi B} \phi^\mu$   
 $M_B^* = M_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*$

- ▶ **Meson contribution**

$$\mathcal{L}_{\text{mesons}} = \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\sigma^*} + \mathcal{L}_\phi + \mathcal{L}_{\text{non-linear}}$$

- ▶ **Generalization:** Inclusion of the scalar isovector  $\delta$ -meson (Lui PRC65, 045201; Gaitanos NPA732, 24; Santos PRC111, 035805; Scurto 2503.18889)

- ▶ **Lepton contribution:**  $\mathcal{L}_l = \sum_l \bar{\psi}_l [\gamma_\mu i\partial^\mu - m_l] \psi_l$

# EOS: relativistic mean field description

## Density dependence of the EOS determined by introducing

- ▶ **non-linear meson terms**: FSU2, FSU2H (Chen& Piekarewicz PRC90,044305; Tolos, Centelles& Ramos ApJ834,3)

$$\mathcal{L}_{non-linear} = -\frac{1}{3} b g_{\sigma}^3(\sigma)^3 - \frac{1}{4} c g_{\sigma}^4(\sigma)^4 + \frac{\xi}{4!} (g_{\omega} \omega_{\mu} \omega^{\mu})^4 + \Lambda_{\omega} g_{\rho}^2 \boldsymbol{\rho}_{\mu} \cdot \boldsymbol{\rho}^{\mu} g_{\omega}^2 \omega_{\mu} \omega^{\mu},$$

- ▶ **Parameters**:  $g_i (i = \sigma, \omega, \rho)$ ,  $b, c, \xi, \Lambda_{\omega}$  (Malik PRD 107,103018)
- ▶ **density dependent couplings**:
  - ▶  $\Gamma_i(x) = \Gamma_{i0} h_i(x)$ ,  $x = \rho / \rho_0$
  - ▶ **Parameters**:  $n_0, \Gamma_{i0}, a_i, b_j, c_i, d_i$   $i = \sigma, \omega, \rho$  (not all independent)
- ▶ **Bayesian estimation of model parameters**

# Density dependence of the couplings

►  $h_i^{DDH}(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2} \quad i = \sigma, \omega,$

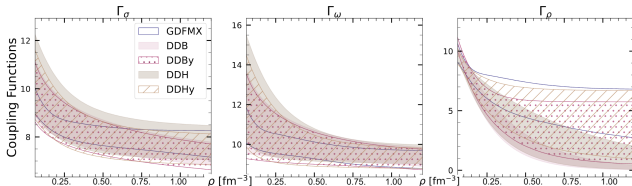
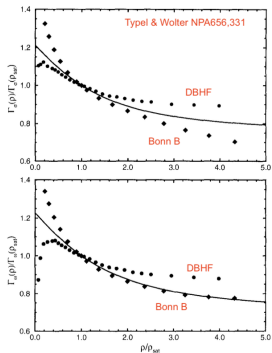
$h_\rho^{DDH}(x) = \exp[-a_\rho(x-1)]$

DD2, DDME2 (Typel NPA656 331, PRC81 015803; Lalazissis PRC71 024312)

►  $h_{i=\sigma,\omega}^{DDB}(x) = \exp[-(x^{a_i} - 1)],$  (Malik ApJ 930, 17)

►  $\Gamma_i^{GDFMX}(\rho) = a_i + (b_i + d_i x^3) e^{-c_i x}, \quad i = \sigma, \omega, \rho$   
(Gogelein PRC77, 025802; Char PRD108, 103045)

- Can we generalize the density dependence of the couplings without imposing a function?



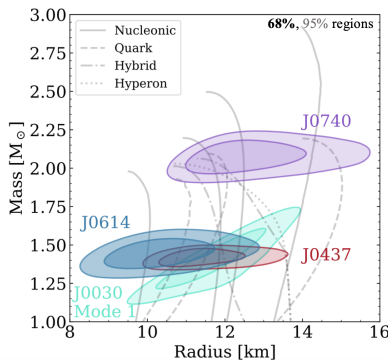
# Nuclear constraints

- ▶ Nuclear matter properties

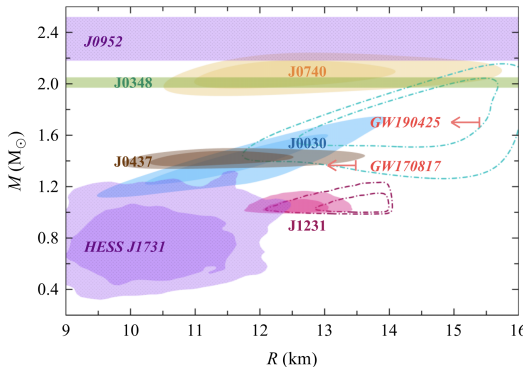
Quantity	Constraints	
		Value/Band
NMP [MeV]	$\rho_0$	$0.153 \pm 0.005$
	$\epsilon_0$	$-16.1 \pm 0.2$
	$K_0$	$230 \pm 40$
	$J_{\text{sym},0}$	$32.5 \pm 1.8$

- ▶ Finite nuclei properties: charge radii, binding energies, neutron skin (PREX, CREX)

# Astrophysical constraints



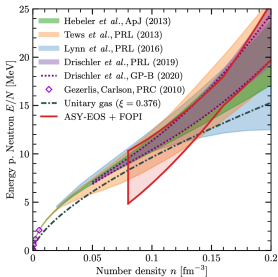
NICER /D. Choudhury



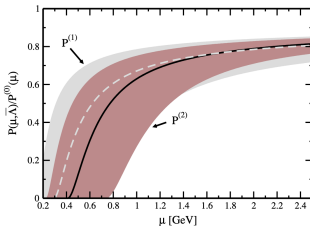
Li, Tian, Sedrakian PRC111 055804

- ▶ **Future missions and observatories** will initiate an era of unprecedented observational precision.

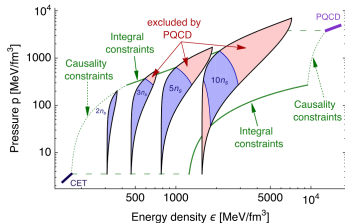
# Theoretical calculations



Huth Nat 606, 276



Kurkela PRD81,105021



Komoltsev PRL128,202701

- ▶ **Low densities:** Chiral effective field theoretical calculations of pure neutron matter (Hebeler ApJ 773, 11; Tews PRL110, 032504; Drischler PRL122, 042501)
- ▶ **High densities:** perturbative QCD with causality and thermodynamic consistency (Komoltsev PRL128, 202701 )

# Bayesian inference

- ▶ **Bayes theorem**: determination of parameters

$$\mathcal{P}(\theta|D) = \frac{\mathcal{L}(D|\theta)\pi(\theta)}{\mathcal{Z}}$$

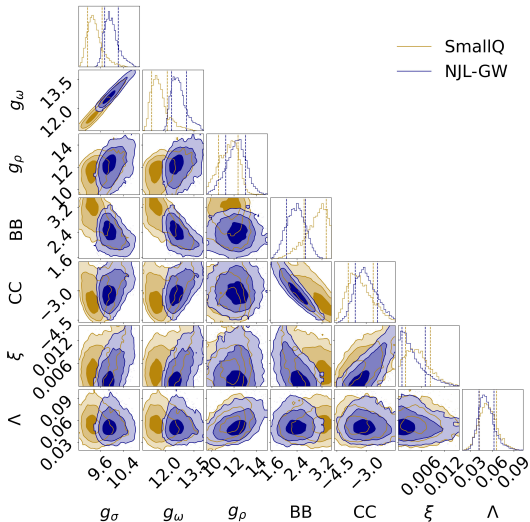
- ▶  $\pi(\theta)$  - **prior probability** for model with parameters  $\theta$
- ▶  $\mathcal{L}(D|\theta)$  - **likelihood probability** of model with  $\theta$  satisfies data
- ▶  $\mathcal{P}(\theta|D)$  **posterior probability**: prob. model with  $\theta$  is true given the data
- ▶  $\mathcal{Z}$  - **evidence probability**: normalization factor

$$\mathcal{Z} = \int \mathcal{L}(D|\theta_i)\pi(\theta_i)d\theta_i \equiv \mathcal{P}(D|M_1)$$

- ▶ Needs **sampling techniques** to select parameters  $\theta$  with high posterior probability ( MCMC, Nested sampling...)

# Bayesian inference: Prior

- ▶ **Prior**: contains information on the parameters of the model already known
- ▶ **Priors used**: uniform priors
- ▶ **Limits of priors**
  - ▶ adjusted by posterior probabilities if necessary
  - ▶ tested with a small number of live points



# Bayesian inference: Likelihood

► **Nuclear matter properties ( $\mathcal{L}_{\text{NMP}}$ )**

$$\log(\mathcal{L}_{\text{NMP}}) = -\frac{1}{2} \sum_j \left[ \left( \frac{d_j - m_j(\theta)}{\sigma_j} \right)^2 + \log(2\pi\sigma_j^2) \right].$$

► **Pure Neutron Matter ( $\mathcal{L}_{\chi\text{EFT}}$ ):** from  $\chi\text{EFT}$  at  $\rho = 0.08, 0.12, 0.16\text{fm}^{-3}$

$$\log(\mathcal{L}_{\text{PNM}}) = \log \left[ \prod_j \frac{1}{2\sigma_j} \frac{1}{\exp \frac{|d_j - m_j(\theta)| - \sigma_j}{0.015} + 1} \right].$$

► **X-ray NICER Data ( $\mathcal{L}_{\text{NICER}}$ ):** J0030+0451, J0740+6630, J0437+4715

$$\mathcal{L}_{\text{NICER}} = \int_{M_{\min}}^{M_{\max}} dm P(m|EOS) \times P(d_{X\text{-ray}}|m, R(m, EOS))$$

$$P(m|EOS) = \begin{cases} \frac{1}{M_{\max} - 1M_{\odot}}, & \text{if } M_{\min} \leq m \leq M_{\max}, \\ 0, & \text{otherwise,} \end{cases}$$

Gaussian KDE to get probability from NICER data  $d_{X\text{-ray}}$

# Bayesian inference: Likelihood

- ▶ **Gravitational wave data ( $\mathcal{L}_{\text{GW}}$ ):** GW170817

$$\mathcal{L}_{\text{GW}} = \int_{M_{1,\text{min}}}^{M_{1,\text{max}}} dm P(m_1 | \text{EOS}) \times P(d_{\text{GW}} | m_1, m_2(m_1) \Lambda(m_1, \text{EOS}), \Lambda(m_2, \text{EOS}))$$

$$P(m | \text{EOS}) = \begin{cases} \frac{1}{M_{\text{max}} - 1M_{\odot}}, & \text{if } M_{\text{min}} \leq m \leq M_{\text{max}}, \\ 0, & \text{otherwise,} \end{cases}$$

$m_2$  from  $\mathcal{M}_c = 1.186 M_{\odot}$ , Gaussian KDE for probability GW data  $d_{\text{GW}}$

- ▶ **Range of Phase Transition ( $\mathcal{L}_{\text{LphT}}$ ):** ( $\rho_0 \leq \rho_{\text{trans}} \leq 0.40$ .)

$$\log \mathcal{L}(\text{phT}) = \left[ \frac{(\rho_{\text{trans}} - 0.275)^2}{2(0.08)^2} \right]^5$$

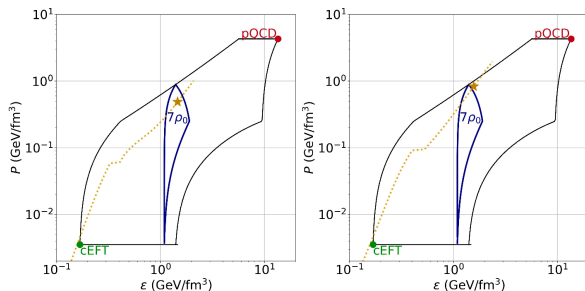
# Bayesian inference: Likelihood

- **Perturbative QCD ( $\mathcal{L}_{\text{pQCD}}$ ):** simple, with most restrictive  $X$

$$\mathcal{L}_{\text{pQCD}}^S = \begin{cases} 1, & \text{if } [P(7\rho_0), \varepsilon(7\rho_0)] \text{ inside region} \\ 0, & \text{if } [P(7\rho_0), \varepsilon(7\rho_0)] \text{ outside region} \end{cases}$$

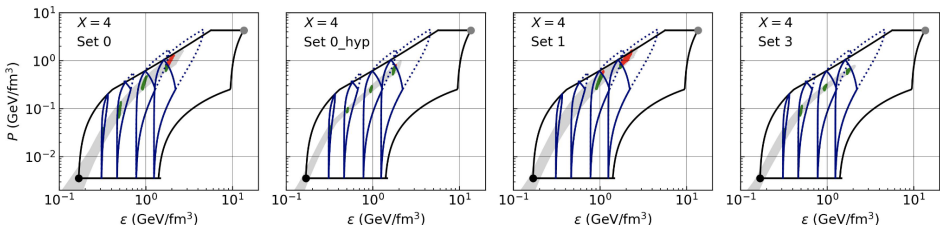
or (Komoltsev, Gorda, Kurkela. zenodo.7781233.)

$$\mathcal{L}_{\text{pQCD}}^{\text{MC}} = \frac{1}{1000} \sum_{j=0}^{1000} \mathcal{L}_{\text{pQCD}}^S(X_j)$$



see talk O Komoltsev

# How is pQCD EoS imposing extra constraints



## pQCD constraints of Komoltsev&Kurkela PRL128,202701

- ▶ stability, causality, thermodynamic consistency (Komoltsev & Kurkela PRL128, 202701).
- ▶ solid black lines: pQCD constraints in  $\varepsilon - p$  domain
- ▶ solid blue lines: constraints at  $n = 2, 3, 5, 8n_s$
- ▶ Excluded models: small  $\xi$  (set 1), large maximum mass and radii ( $\mathcal{L}_{NL} = \frac{\xi}{41} (\omega_\mu \omega^\mu)^2$ )
- ▶ Hyperonic EOS: pQCD has no effect

# Inference framework

- ▶ **Nuclear Matter Properties (NMP):**  $n_0 = 0.153 \pm 0.005 \text{ fm}^{-3}$ ,  $\varepsilon_0 = -16.0 \pm 0.2 \text{ MeV}$ ,  $K_0 = 230 \pm 40 \text{ MeV}$ ,  $J_{\text{sym},0} = 32.5 \pm 2.5 \text{ MeV}$ .
- ▶ **Tidal Deformability from GW170817:** gravitational wave data (LVC, Abbott 2018)
- ▶ **Masses and Radii from NICER:** PSR J0437~4715 (Choudhury 2024) PSR J0030+0451 (Riley 2019, Miller 2019, ST+PDT hotspot model Vinciguerra 2024); PSR J0740+6620 (Salmi 2024)
- ▶ **Theoretical constraints:**
  - ▶ pQCD constraints imposed at  $1.2\text{fm}^{-3}$  (likelihood function Komoltsev 2023)
  - ▶  $\chi$ EFT-derived constraints: energy per neutron from Huth 2022, at  $\rho = 0.08, 0.12, 0.16, 0.2\text{fm}^{-3}$
- ▶ **Total likelihood**

$$\mathcal{L}_{\text{total}}(D|\boldsymbol{\theta}) = \mathcal{L}_{\text{NMP}}(\boldsymbol{\theta}) \times \mathcal{L}_{\text{GW170817}}(\boldsymbol{\theta}) \\ \times \mathcal{L}_{\text{NICER}}(\boldsymbol{\theta}) \times \mathcal{L}_{\text{pQCD}}(\boldsymbol{\theta}) \times \mathcal{L}_{\chi\text{EFT}}(\boldsymbol{\theta}).$$

# Bayesian inference: nested sampling

- ▶ **Nested sampling**: calculates both posteriors and the Bayesian evidence.
- ▶ **MultiNest (PyMultiNest)**:
  - ▶ it is generally faster for smooth, low-to-moderate dimensional posteriors and is well-tested in astrophysics. Can sometimes miss fine details or fail in extremely high-dimensional spaces.
- ▶ **UltraNest**:
  - ▶ uses MLFriends/region sampling and tends to be more robust for posteriors with sharp features, complex degeneracies, or where the ellipsoidal approximation in MultiNest can underestimate the prior volume (and bias the evidence).
- ▶ use **MultiNest** for speed when the problem is well-behaved
- ▶ use **UltraNest** when you need confidence the evidence isn't biased, or as a cross-check.

# Bayesian inference in RMF

- ▶ [Traversi, Char & Pagliara \(ApJ897, 165\)](#): Bayesian analysis to constrain RMF EOS of dense nucleonic matter considering NMP and astrophysical observations
- ▶ [Beznogov & Raduta \(PRC107, 045803\)](#): different roles played by pressure and energy per particle of PNM in constraining the isovector behavior of nuclear matter, studied both covariant density functionals and Skyrme models
- ▶ [Jia-Jie Li & A Sedrakian \(PLB865 139501; PRC111 055804\)](#): Bayesian inferences on covariant density functionals from multimessenger astrophysical data, studied the effect of different likelihoods
- ▶ [P. Char, C. Mondale, F. Gulminelli, M. Oertel \(PRD108, 103045\)](#): Generalized description of neutron star matter with a nucleonic relativistic density functional
- ▶ [X Sun, Z Miao, B Sun, Ang Li \(ApJ942, 55\)](#): Astrophysical Implications on Hyperon Couplings and Hyperon Star Properties with RMF EOS
- ▶ see talk [Bao-An Li](#)

# Prior versus posterior: constrained by observations

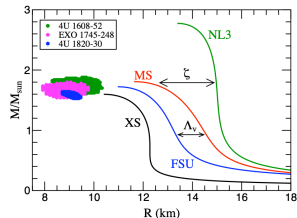
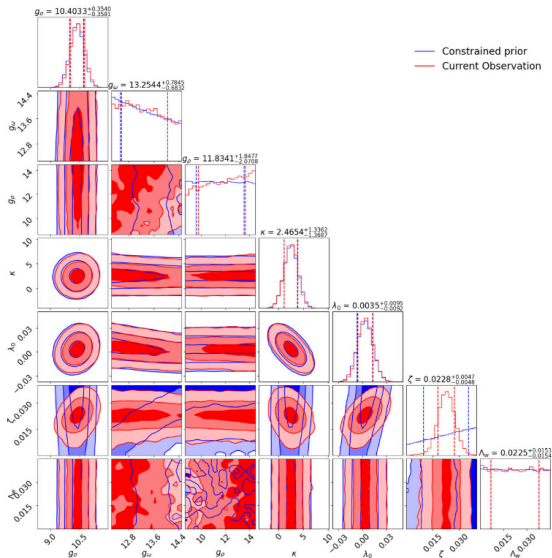
Huang et al MNRSA 529, 4650

## Current observation astrophysical inputs

- ▶ PSR J0740+6620 from radio timing (Fonseca 2021);
- ▶ tidal deformabilities for mergers GW170817 and GW190425 (LVC 2017, 2020)
- ▶ masses and radii from the NICER observations of PSR J0030+0451 and PSR J0740+6620 (Riley et al. (2019, 2021).)

# Prior versus posterior: constrained by observations

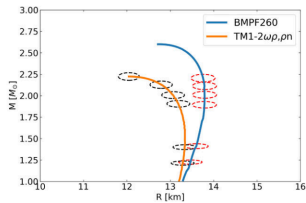
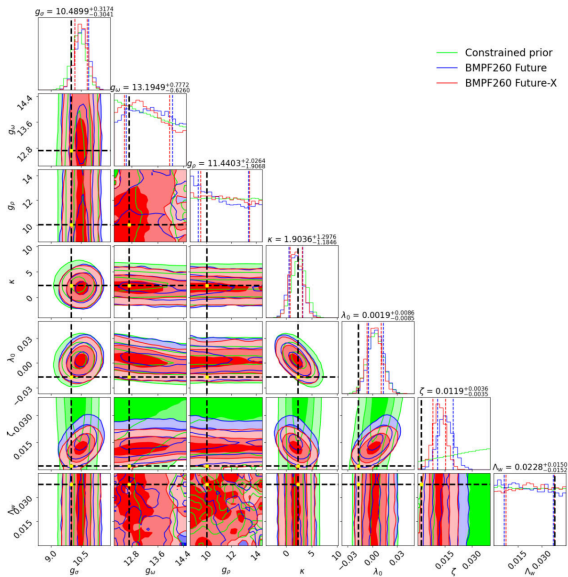
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Fattoyev PRC82 025805

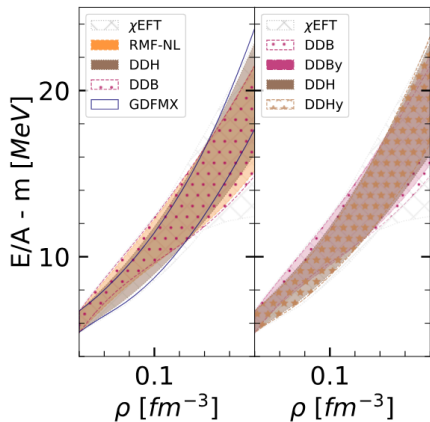
# Prior versus posterior: Future observations (2%)

Huang et al MNRSA 529, 4650



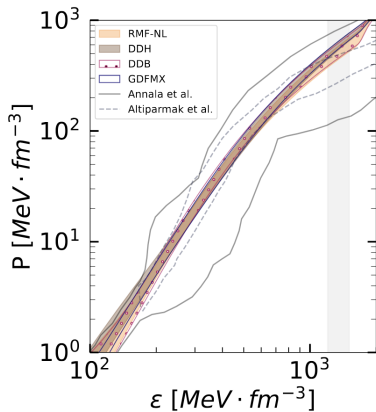
# NS properties: testing CEDF models

Cartaxo ApJS 282:33,2026



90%CI, pure neutron matter

( $\chi$ EFT: Huth 2022)

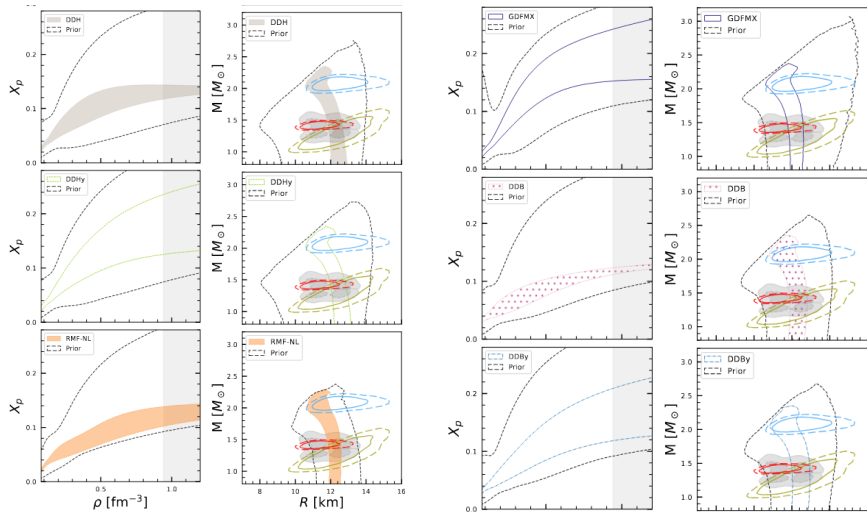


90%CI CEDF;

dotted  $\text{PDF} \geq 0.08$  (Altiparmak 2022)

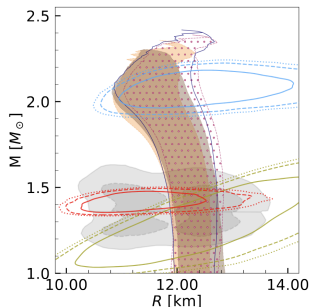
- **CEDF Models:** DD couplings (DDH, DDB, GDFMX), nonlinear meson terms (RMF-NL)

# Priors: $X_p$ vs $\rho$ , $M$ vs $R$



# NS properties: testing CEDF models

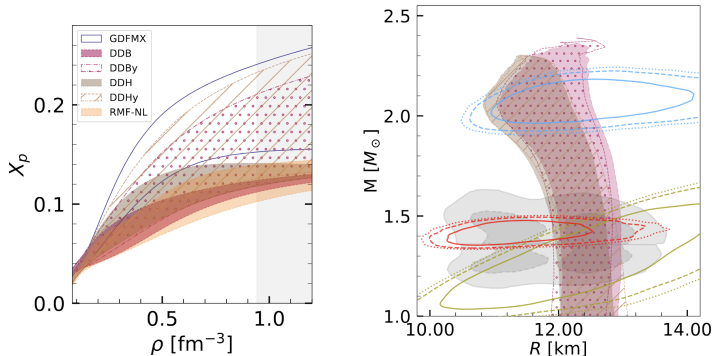
CEDF: DDH, DDB, GDFMX, RMF - NL (Cartaxo ApJS 282:33,2026)



- ▶ **Mass-radius region:** compatible among models, a realistic distribution is defined by the envelope
- ▶ **CompactObject package:** <https://zenodo.org/records/14181695>
- ▶ **CompactObject package:** graphical user interface (GUI) application, allowing extracting, filtering, and visualizing targeted subsets of data.

# Proton fraction

Cartaxo ApJS 282:33,2026

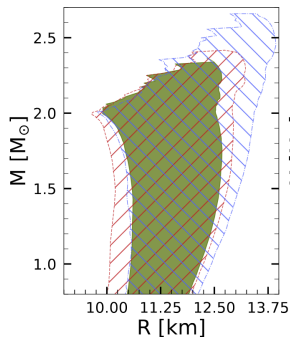
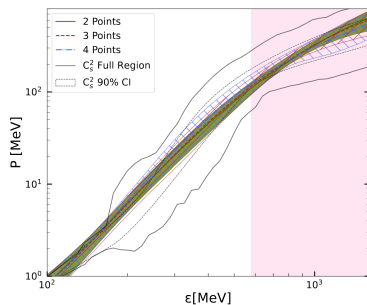


- ▶ **proton fraction:** strongly correlated with symmetry energy
- ▶ **DDH and DDB:** the exponential decrease of  $\Gamma_\rho \rightarrow$  no direct Urca
- ▶ **Generalization:**  $\Gamma_\rho(\rho) \rightarrow \Gamma_{\rho,0} (y \exp[-a_\rho(x-1)] + (1-y))$ ,  $0 < y \leq 1$
- ▶ **MR with extra  $y$  parameter:** larger radii, opens nucleonic direct Urca processes
- ▶ **GDFMX:** large  $Y_\rho$ , favors direct Urca in low mass stars.

results sensitive to prior and constraints see Char & Mondal PRD111,103024)

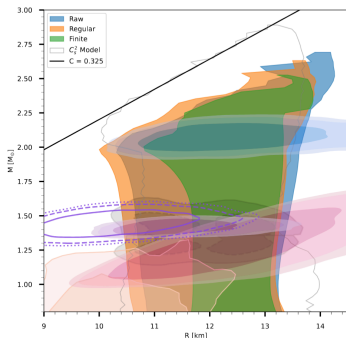
# Generalization of CEDF

- ▶ **Generalize the density dependence of the coupling constants:** couplings defined in terms of Bezier curves (technique allows representing any smooth function using a small number of control points.)
- ▶ **Increasing the number of control points:** increases the flexibility of the model
- ▶ **Objective:** impose observational, theoretical, finite nuclei and nuclear matter constraints.

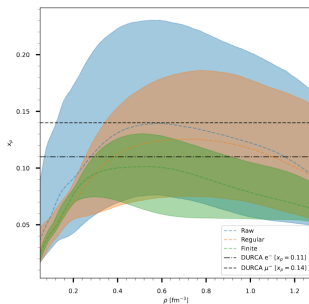


# Effect of nuclear properties on posterior

- ▶ **Raw dataset:** causality, monotonic behaviour of the pressure and coupling density-dependence, chEFT at low densities, pQCD at high densities,  $M_{max} > 1.97M_{\odot}$ .
- ▶ **Regular dataset** plus NMP constraints
- ▶ **Finite dataset** plus NMP and finite nuclei calculation constraints.



(a) 95% confidence interval.



(b) Proton fraction  $\alpha(\rho)$ .

# Effect of nuclear properties on posterior

Quantity	Units	Raw	Regular	Finite
$M_{\max}$	$M_{\odot}$	$2.086^{+0.115}_{-0.080}$	$2.078^{+0.121}_{-0.074}$	$2.138^{+0.106}_{-0.104}$
$\rho_c$	$\text{fm}^{-3}$	$0.847^{+0.199}_{-0.160}$	$0.990^{+0.135}_{-0.203}$	$0.883^{+0.116}_{-0.127}$
$\rho_0$	$\text{fm}^{-3}$	$0.148^{+0.018}_{-0.014}$	$0.152^{+0.003}_{-0.006}$	$0.156^{+0.002}_{-0.001}$
$e_0$	MeV	$-16.232^{+2.273}_{-4.065}$	$-16.095^{+0.182}_{-0.155}$	$-16.467^{+0.107}_{-0.084}$
$K_0$	MeV	<u><math>431.397^{+350.628}_{-196.522}</math></u>	<u><math>233.990^{+43.037}_{-27.984}</math></u>	<u><math>274.341^{+16.381}_{-27.954}</math></u>
$J_{\text{sym},0}$	MeV	$35.927^{+8.137}_{-4.345}$	$32.793^{+1.997}_{-1.054}$	$33.888^{+1.071}_{-0.674}$
$L_{\text{sym},0}$	MeV	$45.844^{+22.630}_{-22.711}$	$46.159^{+16.532}_{-16.579}$	$51.538^{+8.086}_{-5.875}$
$K_{\text{sym},0}$	MeV	$-1178.780^{+1179.270}_{-2263.246}$	$-525.456^{+784.846}_{-756.263}$	$-611.441^{+337.398}_{-353.104}$
$\Lambda_{1.4}$	—	$335.427^{+139.547}_{-149.150}$	$258.831^{+150.261}_{-140.811}$	$313.304^{+115.592}_{-111.765}$

TABLE III: Median values with  $1\sigma$  uncertainties for the three cases.

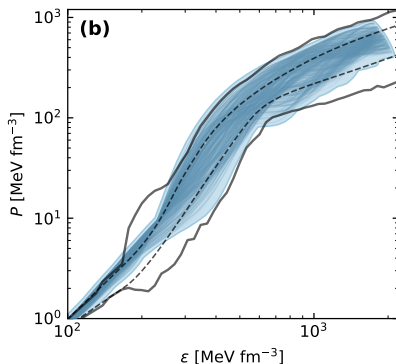
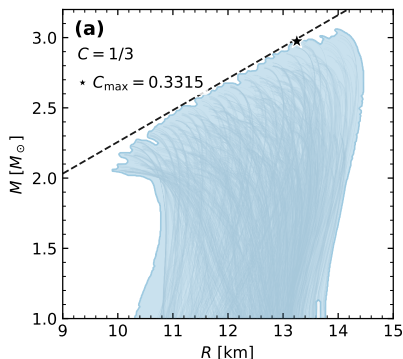
# Effect of nuclear properties on posterior: nuclei fitted

Nucleus	Z	N	Model BE	Ref BE	$\Delta$ BE	Model $R_c$	Ref $R_c$	$\Delta R_c$
<sup>16</sup> O	8	8	-127.9410	-127.620	-0.321	2.7040	2.699	+0.005
<sup>24</sup> O	8	16	-162.0071	-168.384	+6.377	2.8050	2.915	-0.110
<sup>40</sup> Ca	20	20	-342.0593	-342.050	-0.009	3.4300	3.478	-0.048
<sup>48</sup> Ca	20	28	-415.9517	-416.001	+0.049	3.5130	3.477	+0.036
<sup>56</sup> Ni	28	28	-463.6275	-483.992	+20.365	3.7610	3.737	+0.024
<sup>68</sup> Ni	28	40	-592.9625	-590.408	-2.554	3.8990	3.966	-0.067
<sup>90</sup> Zr	40	50	-783.1138	-783.898	+0.784	4.2660	4.269	-0.003
<sup>100</sup> Sn	50	50	-825.6718	-825.297	-0.375	4.5050	4.480	+0.025
<sup>112</sup> Sn	50	62	-950.9069	-953.526	+2.619	4.5890	4.585	+0.004
<sup>120</sup> Sn	50	70	-1019.7752	-1020.539	+0.764	4.6450	4.643	+0.002
<sup>132</sup> Sn	50	82	-1095.5426	-1102.840	+7.297	4.7320	4.714	+0.018
<sup>208</sup> Pb	82	126	-1636.3700	-1636.430	+0.060	5.5090	5.501	+0.008
<sup>294</sup> Og	118	176	-2083.8664	-2085±10	+1.134	6.2390	6.25	-0.011

Rows in red correspond to fitted nuclei, while rows in black correspond to model predictions for unfitted nuclei

# Getting the extremes

- ▶ Using a **Covariance matrix adaptation evolution strategy (CMA-ES)**: designed to return a single best candidate at the end of the optimization.
- ▶ **Maximum compactness with pQCD 0.333** (Annalla et al PRX12 011058; Rezzolla & Ecker arxiv: 2510.12870)



# What is the effect of hyperons?

→ see talk Isaac Vidaña

- ▶ **Full baryon octet:**  $n, p, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$
- ▶ **Hyperon couplings:** ratios relative to the N couplings

$$g_{iY}(\rho) = x_{iY} \cdot g_{iN}(\rho), \quad i = \sigma, \omega, \rho, \phi, \quad Y = \Lambda, \Sigma, \Xi.$$

- ▶ **The *vector couplings*** fixed by SU(6) quark-model sym:

$$\begin{aligned} x_{\omega\Lambda} = x_{\omega\Sigma} &= \frac{2}{3}, & x_{\omega\Xi} &= \frac{1}{3}, & x_{\rho Y} &= 1 \text{ (all } Y) \\ x_{\phi\Lambda} = x_{\phi\Sigma} &= -\frac{\sqrt{2}}{3}, & x_{\phi\Xi} &= -\frac{2\sqrt{2}}{3}, & x_{\phi N} &= 0. \end{aligned}$$

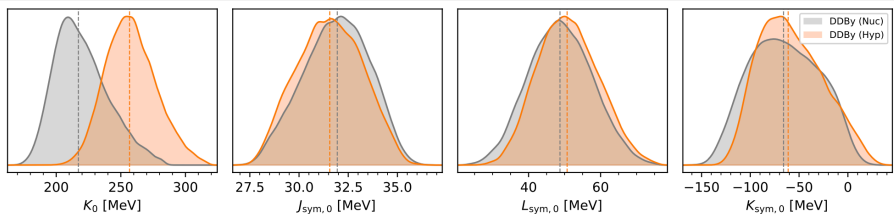
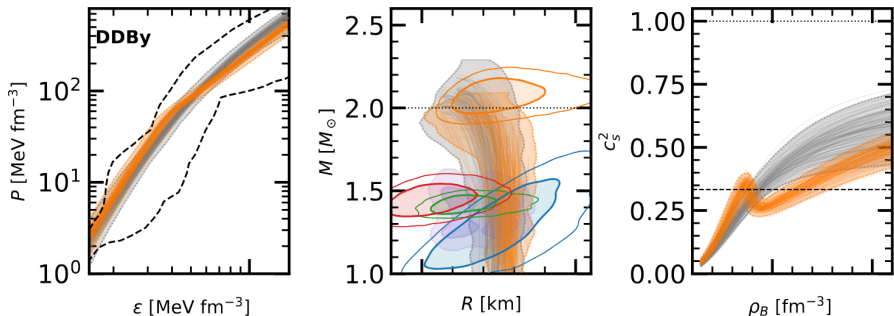
- ▶ **Scalar couplings:** prior

	Min	Max
$x_S$	0.600	0.625
$x_{S\Sigma}$	0.430	0.510
$x_{S\Xi}$	0.295	0.335



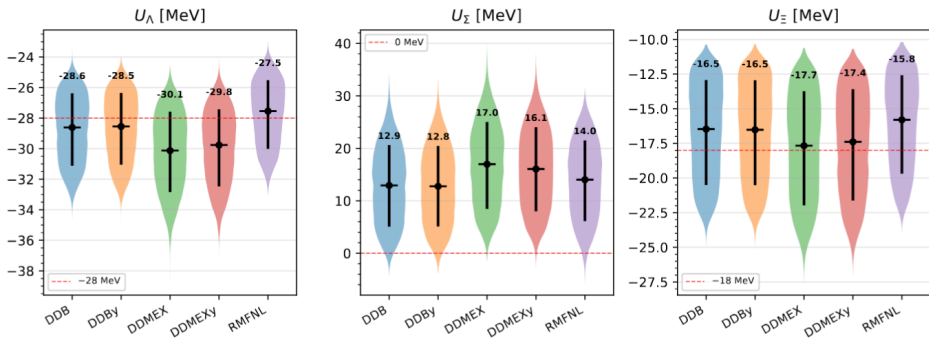
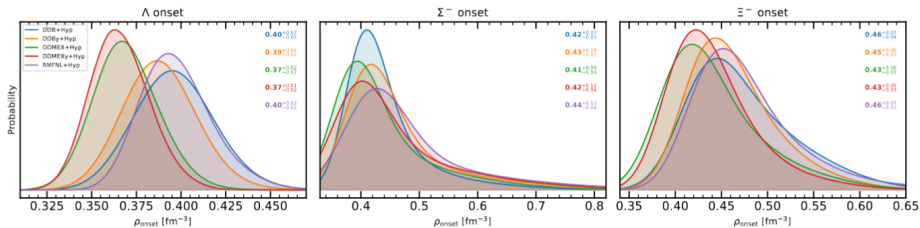
# DDB RMF EOS: what is the effect of hyperons?

Malik, Sanson, CP preliminary



# what is the effect of hyperons?

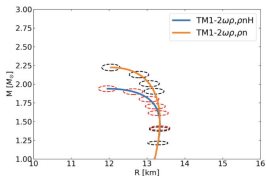
Malik, Sanson, CP preliminary



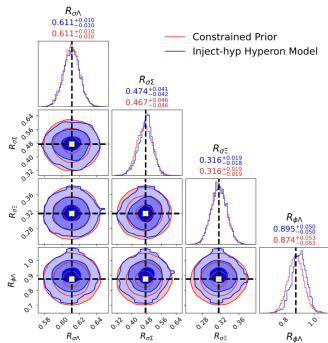
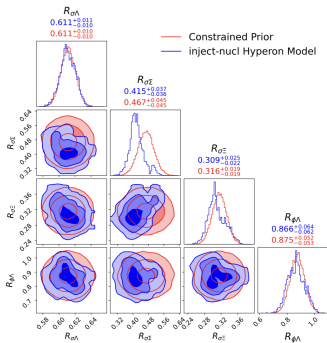
# Presence of hyperons from Observations with CEDF

Huang et al, MNRAS 536, 3262–3275 (2025)

- ▶ 2% uncertainty on M-R: Bayes factor decisively favors hyperonic over nucleonic model, but not enough to constrain parameters

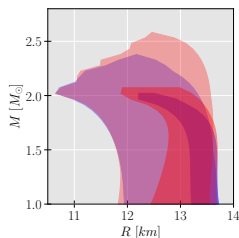
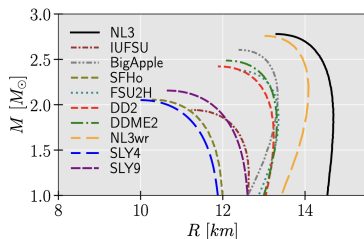


Injected data + Model	$\ln(Z)$	Bayes' factor
Nucleonic + Nucleonic	-210.05	...
Nucleonic + Hyperonic	-211.85	N/H = 6.04
Hyperonic + Nucleonic	-116.68	...
Hyperonic + Hyperonic	-111.45	H/N = 186.79



# Can the slope of the MR curve tell us something?

M Ferreira & CP, PRD112, 083058)



$M/M_{\odot}$	1.2		1.4		1.8		Total	
$dM/dR$	-	+	-	+	-	+	< 0	$\neq 0$
hyp	95	16,501	193	15,953	16,146	0	77	16,069
nucl	13,025	4,511	16,023	1,514	17,537	47	11,495	6,042

## RMF - NL models: nucleonic versus hyperonic

- ▶ **negative** slope: nucleonic star
- ▶  $1.2M_{\odot}$ : **negative** slope  $\rightarrow$  nucleonic stars?
- ▶  $1.4M_{\odot}$ : **positive** slope  $\rightarrow$  hyperonic stars?

# Hybrid Stars

Albino PRD110 083037 & 2511.02653

- ▶ hadronic EOS: RMF with NL terms
- ▶ Quark EoS:
  - ▶ NJL with 4 and 8 quark terms

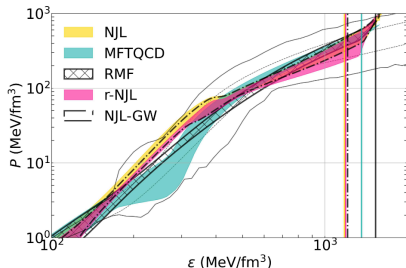
$$\mathcal{L} = \mathcal{L}_{\text{NJL}} + \mathcal{L}_{\text{t Hooft}} + \mathcal{L}_V + \mathcal{L}_{VV} + \mathcal{L}_V'$$

- ▶ MFTQCD: MIT bag model with vector term
- ▶ Phase transition: Maxwell construction
- ▶ Parameters of RMF & quark models:
  - ▶ nuclear: SNM ( $E/A$ ,  $K_0$ ,  $J_{\text{sym}}$ ), PNM ( $E/A$  at 0.05, 0.15 fm<sup>-3</sup>)
  - ▶ Maxwell construction
  - ▶ pQCD constraints
  - ▶ NICER + GW170817

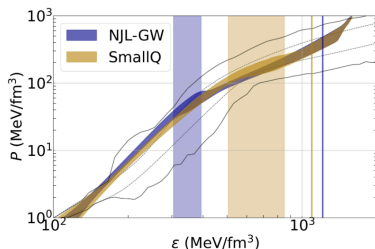
# Hybrid Stars

Albino PRD110 083037 & 2511.02653

- ▶ large Q cores:
  - ▶  $\Delta p \geq 100 \text{ MeV/fm}^{-3}$
  - ▶ H-Q transition:  $[0.15:0.4] \text{fm}^{-3}$
- ▶ Small Q cores
- ▶ **Comments:**
  - ▶ Compatible with gray: Annala Nat Phys 16, 907
  - ▶ MFTQCD model: deconfinement at much lower baryon densities than NJL model.
  - ▶ NJL: requires hard hadronic EOS



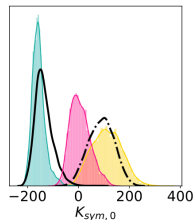
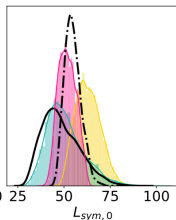
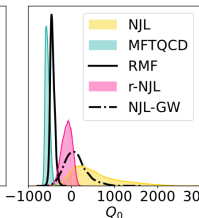
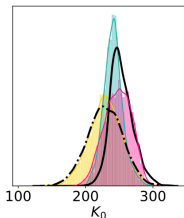
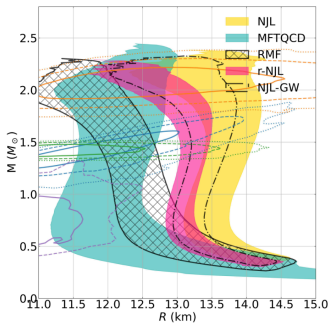
Albino PRD113, 083019



Albino preliminary

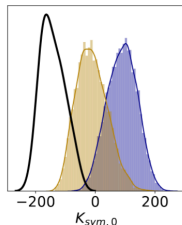
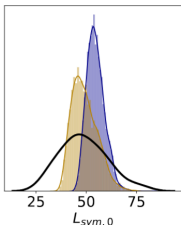
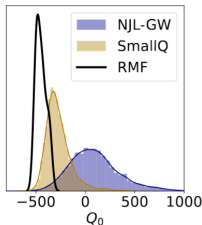
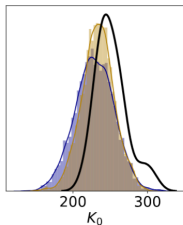
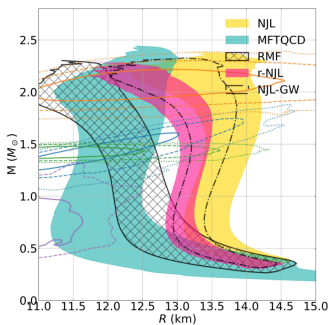
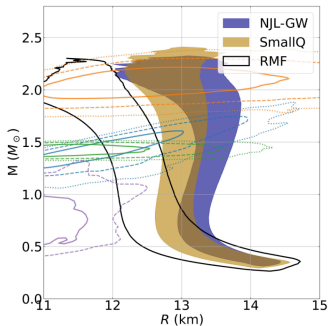
# Hybrid Stars: Nuclear Matter Properties

Albino PRD113 083019 & PRD110 083037

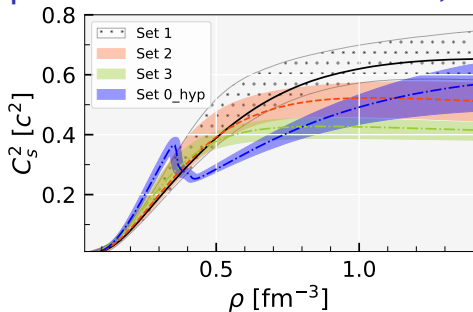


# Hybrid Stars: Nuclear Matter Properties

Albino PRD113 083019 & PRD110 083037

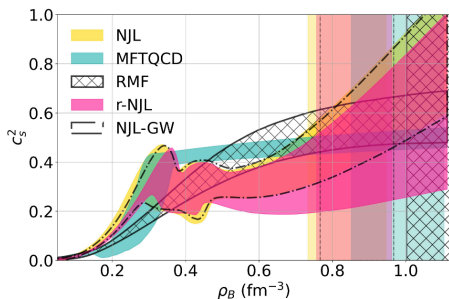


# Speed of sound: nucleonic, hyperonic, hybrid EOS



set 1:  $\xi \in [0, 0.004]$   
 set 2:  $\xi \in [0.004, 0.015]$   
 set 3:  $\xi \in [0.015, 0.04]$

Malik PRD106 063024

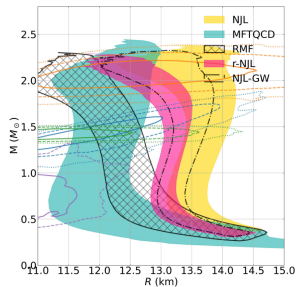


Albino PRD113 083019

see poster Rafael Cardoso

# Composition from $dM/dR$ - hybrid stars

Albino PRC113 083019



$M$	$1.2M_{\odot}$		$1.4M_{\odot}$		$1.6M_{\odot}$		$1.8M_{\odot}$	
	$dM/dR$ +	-	+	-	+	-	+	-
NJL	6306	208	6008	506	4896	1618	2509	4005
NJL-GW	7010	248	6423	835	4605	2653	1483	5775
r-NJL	5180	147	4562	765	2358	2969	206	5121
MFTQCD	4392	487	4326	553	3459	1420	1119	3760
RMF	822	5215	175	5862	70	5967	38	5999

see poster P Kalita

# Some conclusions

- ▶ **Regions span by the CEDF EOS:**
  - ▶ the pressure vs energy density and the mass-radius curves from different RMF models are compatible
  - ▶ but different models span different proton fractions
  - ▶ What are the NS properties that provide information on composition? proton fraction? hyperon content? Cooling? GW? NS quasinormal modes?
  - ▶ Smaller uncertainties with observations are necessary to constrain models.
- ▶ **Bayesian inference and other statistical methods:** define the compatibility of data; connect data to microphysics; identify the most favorable theories given the data.

# Future perspectives and open problems

- ▶ How do additional experimental constraints from finite nuclei and hyperons in matter restrict agnostic predictions?
- ▶ **CEDF**: includes causality, Lorenz covariance, generalizable to include new species, to extend to finite temperature
- ▶ **CEDF**: if incompatible with data, justifies the presence of "exotic matter"
- ▶ **The slope and higher derivatives of the MR curve**: what information does it provide on the existence of non nucleonic degrees of freedom? **does  $R_{2.0} > R_{1.4}$  indicate the presence of exotic matter?**
- ▶ **How many NS observations and how small should the uncertainties be** to constrain the high density EOS?

# Gràcies!



Credits: @UC| Sérgio Brito

## Acknowledgements:

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# GW constraints: GW170817

LVC PRL119 161101; PRL121 (2018) 161101

- ▶ Binary NS system:  $\Lambda_1, \Lambda_2$  and  $M_1, M_2$
- ▶ binary mass ratio  $q = M_2/M_1 < 1$ : GW170817  $0.73 < q < 1$
- ▶ Chirp mass:  $M_{\text{chirp}} = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5} = 1.186 M_{\odot}$ ,
- ▶ Effective tidal deformability

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5},$$

- ▶  $q$  versus  $\tilde{\Lambda}$  observational data for GW170917 ( LIGO/Virgo collaboration) the solid green contours (50% CI), dashed contours (90% CI) and dotted contours (99% CI)

# GW constraints: PRL121 161101

