Speed of Sound in Neutron Stars

Thermodynamic Structure and Implications for Dense Matter

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Outline of the Presentation

Motivation: Neutron Star Observations

Neutron stars probe strongly interacting matter at densities far beyond those accessible in terrestrial experiments. Recent multimessenger observations have opened a new era of precision neutron-star physics:

- $\sim 2\,M_\odot$ pulsars: accurate mass measurements challenge soft equations of state.
- NICER observations: mass–radius measurements of PSR J0030+0451 and PSR J0740+6620 provide direct constraints on the stiffness of dense matter.
- Gravitational waves: binary neutron-star mergers constrain tidal deformability and the high-density equation of state.
- Exotic compact objects: sources such as HESS J1731–347 raise questions about nonstandard degrees of freedom inside neutron stars.

These observations demand theoretically consistent models of dense matter that satisfy both astrophysical and thermodynamical constraints.

Speed of Sound, Trace Anomaly and EOS Stiffness

- At **zero temperature** ($T = 0$), the speed of sound is defined as:
  $c_s^2 = \frac{dp}{d\varepsilon} \quad d\varepsilon = \mu d\rho, \quad P = \rho^2 \frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right)$
- Here, p is pressure, ε is energy density, ρ is baryon number density, and μ is the baryon chemical potential.

Decomposition of c_s^2: $= 2\frac{\rho}{\mu}\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right) + \frac{\rho^2}{\mu}\frac{d^2}{d\rho^2}\left(\frac{\varepsilon}{\rho}\right) = \alpha + \beta$

- The **trace anomaly** $\Delta \equiv \frac{\varepsilon - 3p}{3\varepsilon} = \frac{1}{3} - \frac{p}{\varepsilon}$
- Recent studies **[PRL 129, 252702 (2022)]** highlight the role of Δ in neutron star structure and EOS constraints.
- The squared **speed of sound**, $\boldsymbol{c_s^2}$, is related to Δ and its derivative:
  $c_s^2 = \frac{1}{3} - \Delta - \Delta', \quad \Delta' = \frac{d\Delta}{d\log\varepsilon}$
- The **polytropic index** γ characterizes the EOS stiffness:
  $\gamma \equiv \frac{\partial \log p}{\partial \log \varepsilon} = \frac{\varepsilon}{p} c_s^2 = \frac{c_s^2}{\frac{1}{3} - \Delta}$

Motivation

Common theoretical indicators of phase transitions

Understanding the nature of dense matter—especially in neutron stars and heavy-ion collisions—requires reliable indicators of phase transitions.

- The speed of sound squared: c_s^2
- The trace anomaly: $\varepsilon - 3P$
- Curvature term in the speed of sound: β

Sign Change in β: Physical Implication PRD 109, L041302 (2024)

- **Recent studies** suggest that a sign change in the curvature term β may signal the emergence of strongly coupled conformal matter in neutron star cores.
- This change is often interpreted as a sign of medium composition change at high densities.

- In this work, we explore the behavior of β within **hadronic** equations of state and examine its connection with the speed of sound and thermodynamic properties of dense matter.

Description of the Hadronic Matter

Nonlinear RMF Lagrangian

$$\mathcal{L}_{\rm NL} = \mathcal{L}_{\rm nm} + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\sigma\omega\rho} \tag{1}$$

Nucleon-Meson Interaction

$$\mathcal{L}_{\rm nm} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_N)\psi + (g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi - \tfrac{1}{2}g_\rho\bar{\psi}\gamma_\mu\vec{\tau}\cdot\vec{\rho}^\mu\psi) \tag{2}$$

Meson Self-interactions

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}bm(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4 \tag{3}$$

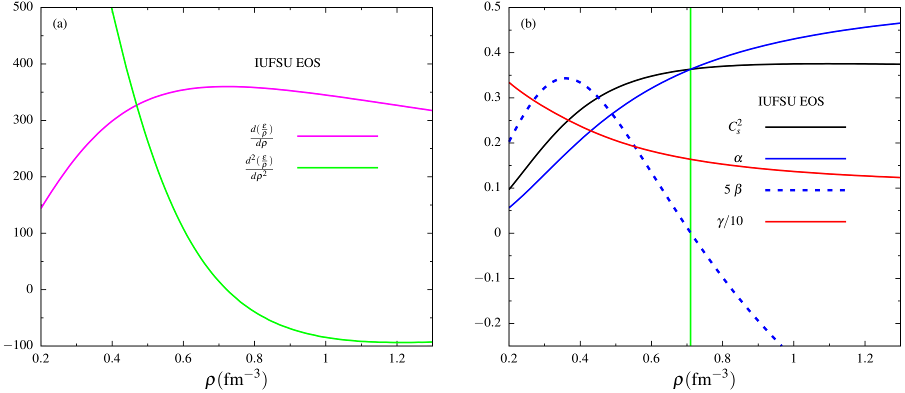
$$\mathcal{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{\zeta}{24}(g_\omega^2\omega_\mu\omega^\mu)^2 \tag{4}$$

$$\mathcal{L}_\rho = -\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu \tag{5}$$

Cross-Couplings

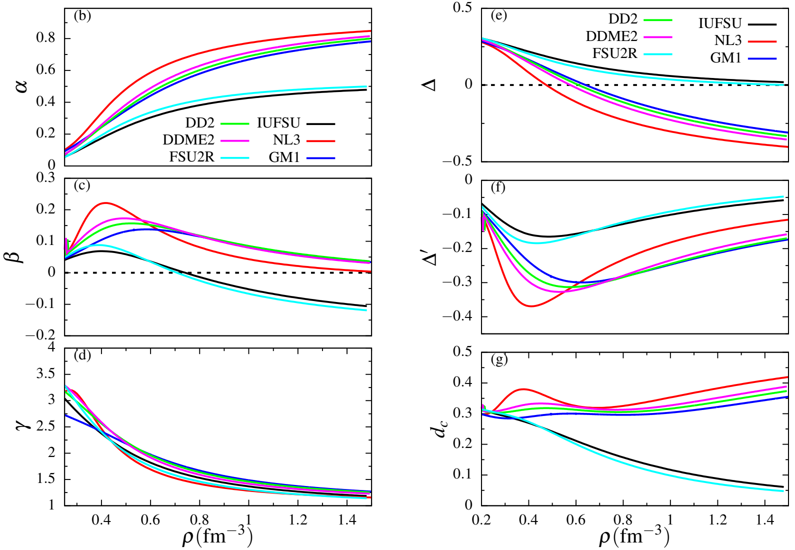
$$\begin{aligned}\mathcal{L}_{\sigma\omega\rho} = {} & g_\sigma g_\omega^2\sigma\omega_\mu\omega^\mu\left(\alpha_1 + \tfrac{1}{2}\alpha_1' g_\sigma\sigma\right) + g_\sigma g_\rho^2\sigma\vec{\rho}_\mu\cdot\vec{\rho}^\mu\left(\alpha_2 + \tfrac{1}{2}\alpha_2' g_\sigma\sigma\right) \\ & + \Lambda_{\omega\rho}g_\omega^2(\omega_\mu\omega^\mu)(g_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu)\end{aligned} \tag{6}$$

Results



- The first derivative reaches a maximum at a characteristic density.
- This leads to a vanishing curvature.
- When α approaches or exceeds c_s^2, the curvature crosses zero and becomes negative.

Behavior of thermodynamic quantities



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Parametrized Speed of Sound and EOS Construction

Speed of Sound Parametrization

$$C_s^2(\varepsilon) = a\left[1 - e^{-b\left(\frac{\varepsilon}{\varepsilon_0}\right)^c}\right]$$

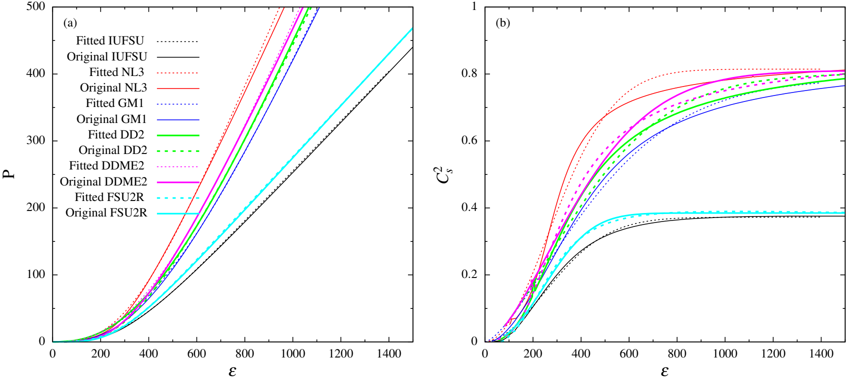
- a : maximum stiffness
- b : growth rate
- c : sharpness parameter
- $\varepsilon_0 = 140\ \mathrm{MeV/fm^3}$

EOS Construction

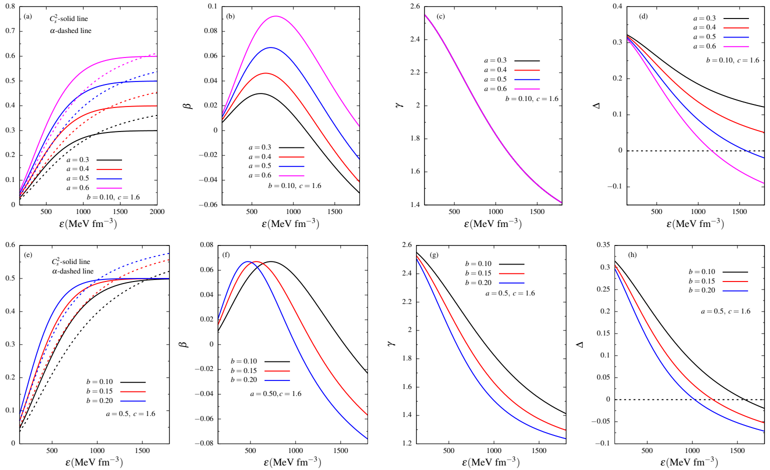
$$P(\varepsilon) = \int_0^{\varepsilon} C_s^2(\bar{\varepsilon})\, d\bar{\varepsilon}$$

- Thermodynamic consistency
- Causality: $C_s^2 \leq 1$
- Correct low- and high-density limits

Merit of Parametrized Equations of State



Thermodynamics : Parametrized Equations of State



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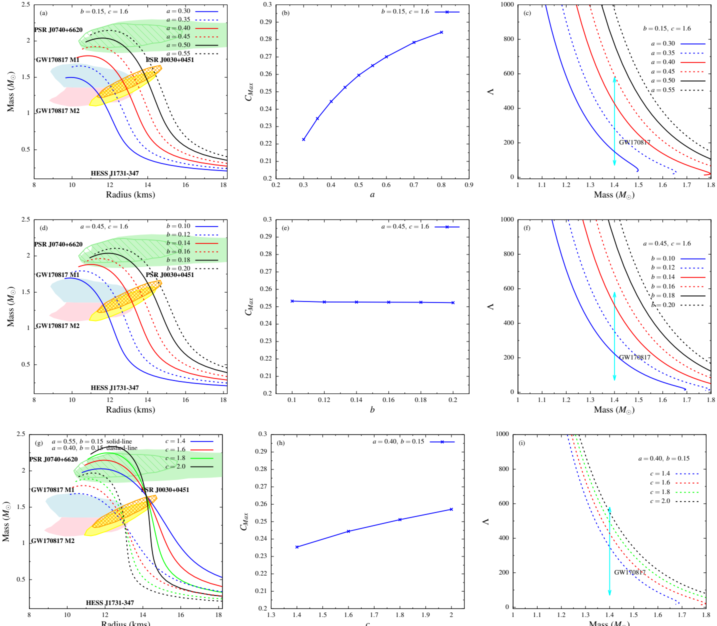
Structural properties: Significance a, b and c



Approximate Universal Relations

Early Quasi-Universal Relation

Lattimer & Prakash [ApJ **550**, 426 (2001)]

- First evidence of quasi-universal behavior in neutron stars.
- Empirical correlation between neutron star radius and pressure near nuclear saturation density.
- The combination $R\,P^{-1/4} \approx \mathrm{constant}$ remains nearly EoS independent.
- Foundation for modern universal relations involving R, C, Λ, and $\bar{I}$.

Recent Universal Relation

A recent study established a new approximate universal relation connecting the ratio of central pressure to central energy density, $\frac{p_c}{\epsilon_c}$, with the compactness C, dimensionless moment of inertia $\bar{I}$, and dimensionless tidal deformability Λ.

- The relation was shown to hold for a wide range of realistic and parameterized neutron star EoSs.

Approximately universal I-Love-$\langle c_s^2 \rangle$ relations for the average neutron star stiffness

Jayana A. Saes [1,*], Raissa F. P. Mendes [2,3,†], and Nicolás Yunes [1,‡]

Show more

Phys. Rev. D **110**, 024011 – **Published 8 July, 2024**

DOI: https://doi.org/10.1103/PhysRevD.110.024011

Universal relation between p_c/ϵ_c and neutron star observables.

Average Speed of Sound

$\langle C_s^2 \rangle = \frac{\int_0^{\varepsilon} C_s^2(\bar{\varepsilon})\, d\bar{\varepsilon}}{\varepsilon} = \frac{P}{\varepsilon} = W$ Ratio of pressure to energy density

$$C_s^2(\varepsilon) = \underbrace{W}_{\text{average}} + \underbrace{W'}_{\text{logarithmic derivative} \;\; \varepsilon \frac{dW}{d\varepsilon}}$$

- W controls **global stellar structure**.
- W' governs **density-dependent evolution of stiffness**.
- Breakdown of quasi-universality in c_s^2–compactness arises from the **derivative term** W'.

Transformation Relations

$$\alpha = \frac{2W}{1+W} \quad \Delta = \frac{1}{3} - W$$

$$\beta = W' + \frac{W(1-W)}{1+W} \quad \Delta' = -W'$$

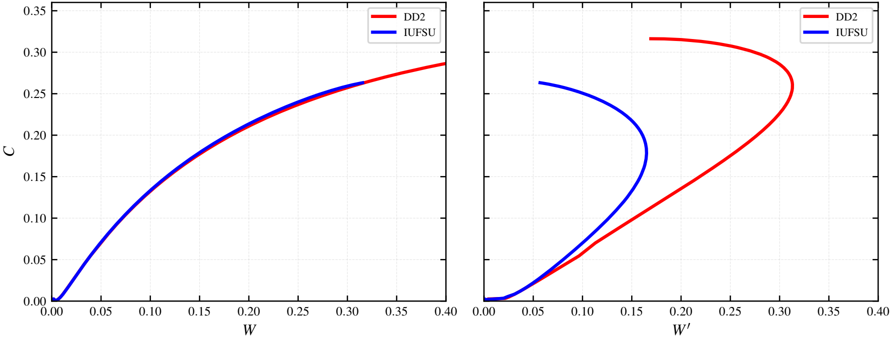
Physical Constraints

Thermodynamic stability ($P > 0$) and causality ($P \leq \varepsilon$) imply:

$$0 \leq W < 1, \qquad 0 \leq \alpha < 1$$

$$-\frac{2}{3} \leq \Delta < \frac{1}{3}$$

Quasi-Universal relation : Neutron star



Decomposition of the Speed of Sound

- The decomposition of the speed of sound clarifies the quasi-universal relation between the compactness C and the average speed of sound W.
- Although the speed of sound c_s^2 is dimensionless, it does **not** exhibit universality with compactness C; this follows directly from its decomposition.
- It is the derivative term that breaks the universality, while the average term preserves it. **Pal & Chaudhuri, in preparation**

Maxwell Construction

Local Charge Neutrality

$$q_H = 0, \qquad q_Q = 0$$

- Sharp phase boundary
- Constant-pressure transition
- Single chemical potential: μ_n
- Favored for: $\sigma_{\rm surf}$ large

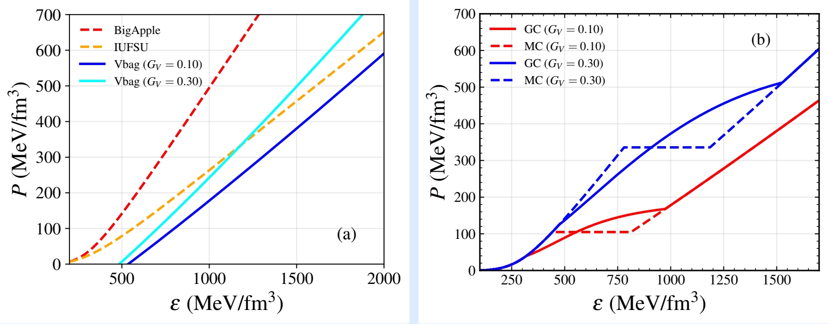
Gibbs Construction

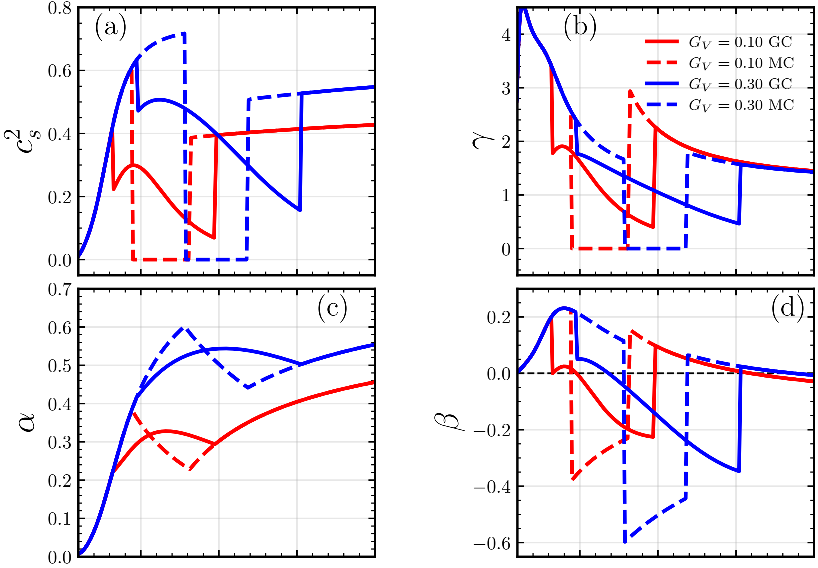
Global Charge Neutrality

$\rho_B = (1-\chi)\rho_H + \chi\rho_Q$

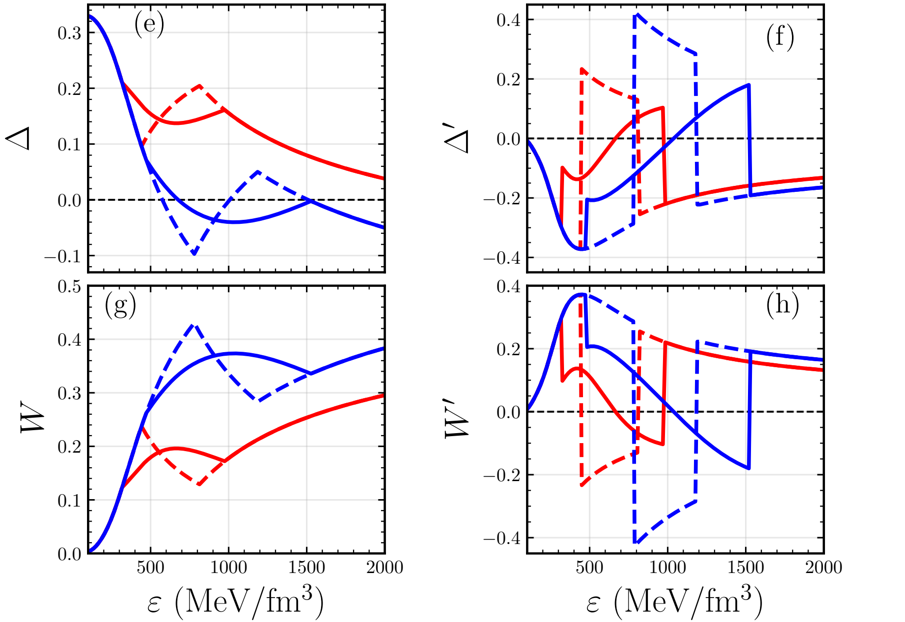
$\rho_{ch} = (1-\chi)q_H + \chi q_Q$

- Extended mixed phase
- Smooth pressure variation
- Two chemical potentials: (μ_n, μ_e)
- Favored for: $\sigma_{\rm surf}$ small




(a)
(b)
(c)
(d)
c_s^2
γ
α
β
$G_V = 0.10$ GC
$G_V = 0.10$ MC
$G_V = 0.30$ GC
$G_V = 0.30$ MC
0.0
0.2
0.4
0.6
0.8
0
1
2
3
4
0.1
0.3
0.5
0.7
−0.2
−0.4
−0.6

Hybrid Star Signature



Thermodynamic Response Functions

Thermodynamic response functions are obtained from the second derivatives of the thermodynamic potential and characterize how the system responds to variations in control parameters such as the baryon chemical potential μ_B.

For cold neutron star matter ($T = 0$), the relevant quantities are:

$$\chi_N, \qquad K_T, \qquad K_B$$

Baryon Susceptibility

$$\chi_N = \left(\frac{\partial \rho_B}{\partial \mu_B}\right) = -\frac{\partial^2 \Omega}{\partial \mu_B^2}$$

Isothermal Compressibility

$$K_T = \frac{1}{\rho_B}\left(\frac{\partial \rho_B}{\partial P}\right)_T = \frac{1}{\rho_B^2}\left(\frac{\partial \rho_B}{\partial \mu_B}\right)_T = -\frac{1}{\rho_B^2}\frac{\partial^2 \Omega}{\partial \mu_B^2}$$

Bulk Modulus

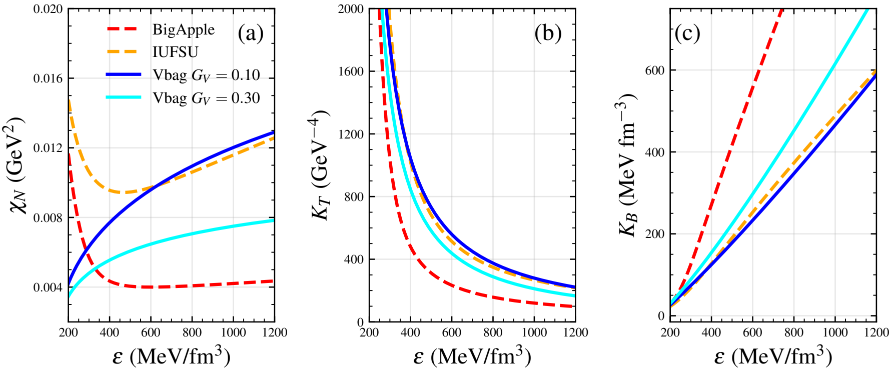
$$K_B = \rho_B \left(\frac{\partial P}{\partial \rho_B}\right)_T = \rho_B^2 \frac{\partial^2 \varepsilon}{\partial \rho_B^2}$$

Key Insight

Since response functions involve second-order derivatives of the thermodynamic potential, they are more sensitive to rapid changes in the EOS than bulk thermodynamic quantities.

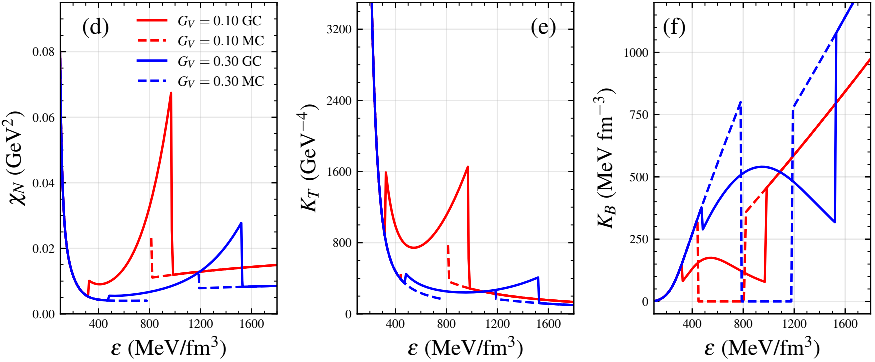
Therefore, they provide a clearer characterization of the quark–hadron phase transition and differences between Maxwell and Gibbs constructions.

Thermodynamic Response Functions in Cold Neutron Star and Quark Star Matter



Thermodynamic Response Functions in hybrid Star Matter

Probing quark–hadron transition through density fluctuations



Pal & Chaudhuri,Under Review

Finite-Temperature Hadronic EOS

Baryon and Scalar Densities

- Vector density:

$$\rho_i = \frac{\gamma_i}{2\pi^2} \int_0^\infty dk\, k^2 \left(f_i - f_{\bar{i}}\right)$$

- Scalar density:

$$\rho_i^s = \frac{\gamma_i}{2\pi^2} \int_0^\infty dk\, k^2 \frac{m_i^*}{\sqrt{k^2 + m_i^{*2}}} \left(f_i + f_{\bar{i}}\right)$$

- Spin degeneracy for baryons:

$$\gamma_i = 2$$

Fermi-Dirac Distribution Functions

$$f_{i/\bar{i}}(k, T) = \left[1 + \exp\left(\frac{\sqrt{k^2 + m_i^{*2}} \mp \mu_i^*}{T}\right)\right]^{-1}$$

$$\mu_i^* = \mu_i - g_\omega \omega_0 - \tau_{3i} \frac{g_\rho}{2} \rho_{03}$$

- Isospin projection:

$$\tau_{3p} = 1, \qquad \tau_{3n} = -1$$

Thermodynamic Quantities of Hadronic Matter

Energy Density :

$$\epsilon_H = \frac{1}{2\pi^2} \sum_i \gamma_i \int_0^\infty dk k^2 \sqrt{k^2 + m_i^{*2}} \left(f_i(k, T) + f_{\bar{i}}(k, T)\right) + \varepsilon_I + \varepsilon_{\text{field}}$$

Pressure :

$$P_H = \frac{1}{6\pi^2} \sum_i \gamma_i \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_i^{*2}}} \left(f_i(k, T) + f_{\bar{i}}(k, T)\right) + P_I + P_{\text{field}}$$

Entropy Density:

$$s_H = \frac{1}{T} \left(P_H + \varepsilon_H - \sum_i \mu_i \rho_i \right) .$$

Variation & Calibration of Hadronic EoS Parameters

PREX-II Constraints *PRL 126, 172503 (2021)*

$$E_{\rm sym} = 38.1 \pm 4.7 \text{ MeV}, \qquad L_{\rm sym} = 106 \pm 37 \text{ MeV}$$

- Construct RMF parameter sets fitted to recent constraints on $E_{\rm sym}$ and $L_{\rm sym}$ for pure neutron matter.

- Study the impact of hadronic parameters on hybrid star structure:
  Effective mass m^*/m, nuclear incompressibility $K_{\rm sat}$, Symmetry energy $E_{\rm sym}$,

Calibration of Couplings *Phys. Rev. C 90, 044305 (2014)*

Coupling constants are calibrated following *Chen & Piekarewicz (2014)*,*Hornick et al. (2018)PRC*:

- g_σ, g_ω, b, c $\longrightarrow$ reproduce B/A, $K_{\rm sat}$, m^* at ρ_0. Norman K. Glendenning
- g_ρ, Λ_ω $\longrightarrow$ determined from J and L.

Determination of Isovector Couplings

The symmetry energy is decomposed as $J = J_0 + J_1$, where the kinetic contribution is $J_0 = \frac{k_F^2}{6E_F}$, $E_F = \sqrt{k_F^2 + m^{*2}}$, and the interaction contribution is $J_1 = J - J_0$.
The slope parameter is similarly written as $L = L_0 + L_1$,
with$L_0 = J_0 \left[1 + \left(\frac{m^*}{E_F}\right)^2 \left(1 - \frac{3\rho}{m^*}\frac{\partial m^*}{\partial \rho}\right)\right]$, and $L_1 = L - L_0$
The nonlinear vector–isovector coupling is obtained as

$$\boxed{\Lambda_{\omega\rho} = \frac{1}{32}\frac{1}{W_0\, J_1}\left(1 - \frac{L_1}{3J_1}\right)\left(\frac{m_\omega^*}{g_\omega}\right)^2}$$

where

$$W_0 = g_\omega \omega_0, \qquad m_\omega^{*2} = m_\omega^2 + \frac{\xi_\omega}{2} W_0^2.$$

Finally, the ρ-meson coupling is

$$\boxed{g_\rho = \frac{m_\rho}{\sqrt{\frac{\rho}{8J_1} - 2\Lambda_{\omega\rho} W_0^2}}}$$

EOS for Quark Matter

$$\Omega = -\sum_{i=u,d,s} \frac{\gamma_i}{6\pi^2} \int_0^\infty \frac{k^4\, dk}{\sqrt{k^2+m_i^2}} \left(f_i^+ + f_i^-\right) + \Omega_l + B_0 - \frac{1}{2} m_V^2 V_0^2 \tag{7}$$

- The vector meson modifies the quark chemical potential:

$$\mu_i = \mu_i^* + g_V V_0$$

- Equation of motion for the vector field:

$$\frac{\partial \Omega}{\partial V_0} = 0 \quad \Rightarrow \quad m_V^2 V_0 = \sum_{i=u,d,s} g_V \rho_i \tag{8}$$

The quark number densities are given by

$$\rho_i = \frac{\gamma_i}{2\pi^2} \int_0^\infty k^2 [f_i^+ - f_i^-] dk \tag{9}$$

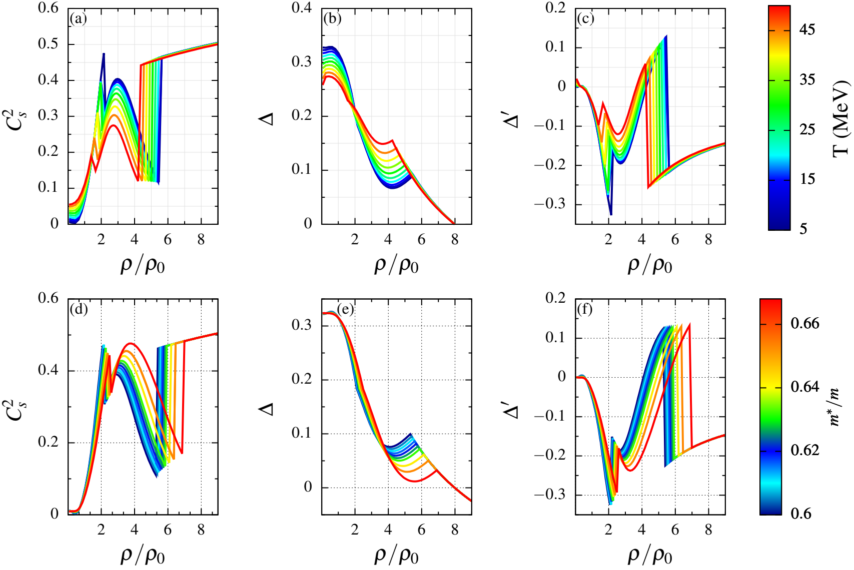
The expression of pressure is given by,

$$P_Q = -\Omega_Q \tag{10}$$

The expression of the energy density is given by,

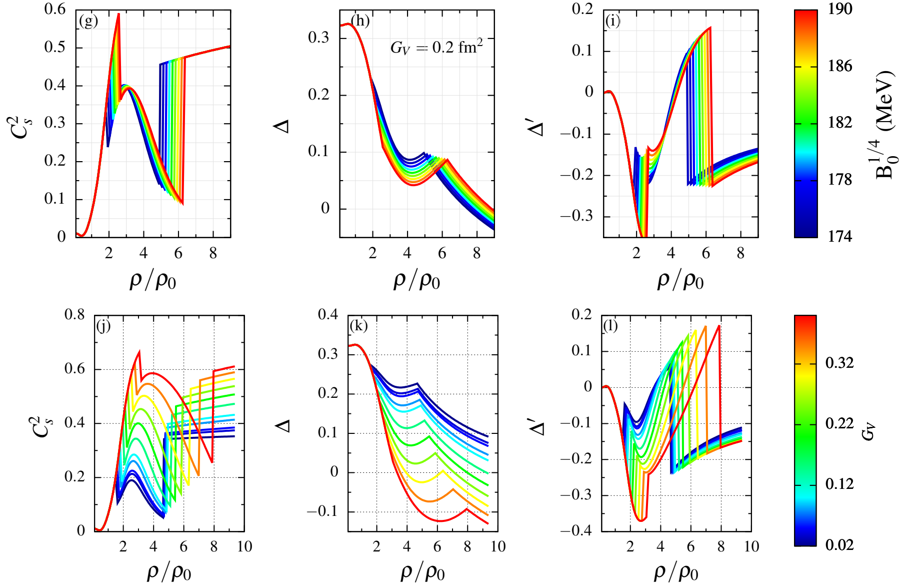
$$\varepsilon_Q = \sum_{i=u,d,s} \frac{\gamma_i}{2\pi^2} \int_0^\infty k^2 \sqrt{k^2 + (m_i^*)^2} \left(f_i^+ + f_i^-\right) dk + B_0 + \varepsilon_e + \frac{1}{2} m_V^2 V_0^2 \tag{11}$$

Speed of Sound Decomposition at Finite Temperature

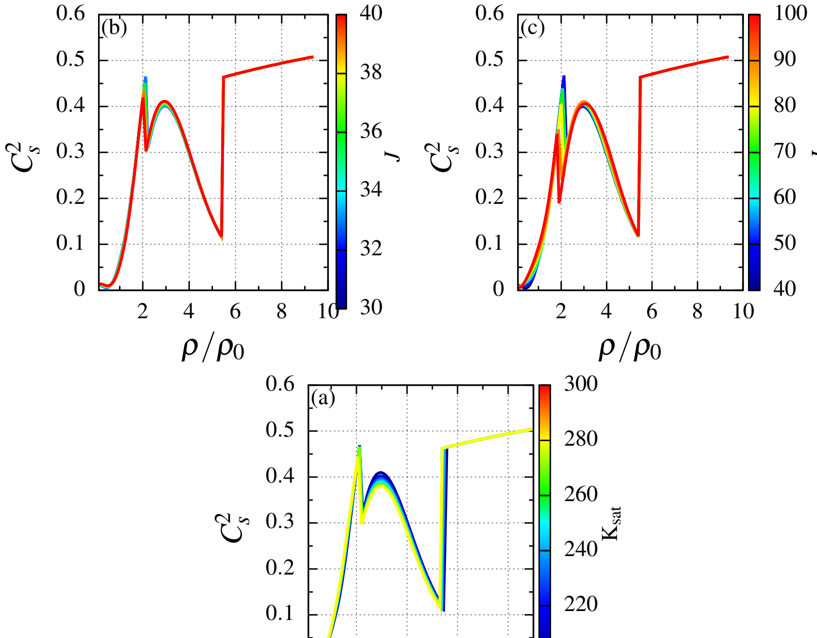


Pal & Chaudhuri, Phys.Rev.D 113 (2026) 10, 103006

Effect of Quark Parameters



Effect of Nuclear Parameters



Summary of the Talk

- Study the behavior of the speed of sound and related thermodynamic quantities in dense matter.
- Investigate signatures of phase transitions using:

$$c_s^2, \qquad \Delta, \qquad \beta,$$

  where Δ is the trace anomaly and β is the curvature term.
- Explore quasi-universal relations in neutron stars.
- Analyze hadronic, quark, and hybrid star equations of state at zero and finite temperature.
- Examine thermodynamic response functions as probes of phase transitions.

Thank You!

Questions?

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