Application to NS of RMF predictions and QCD EoS from effective Lagrangians

Passarella Luca, Polytechnic of Turin and INFN Turin

Lavagno Andrea, Polytechnic of Turin and INFN Turin
Drago Alessandro, University of Ferrara and INFN Ferrara
Pagliara Giuseppe, University of Ferrara and INFN Ferrara
Guerrini Mirco, University of Ferrara and INFN Ferrara

Barcellona, 22/5/2026

luca_passarella@polito.it

Istituto Nazionale di Fisica Nucleare

Politecnico di Torino

1859

Table of Contents
Introduction

Outline of the talk

Outline of the talk

- Test the coexistence of neutron stars and strange quark stars with Bayesian inference from astrophysical data:
  - Compare the bayesian evidence of the One Family scenario vs the Two Family scenario

Outline of the talk

- Test the coexistence of neutron stars and strange quark stars with Bayesian inference from astrophysical data:
    - Compare the bayesian evidence of the One Family scenario vs the Two Family scenario

- Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

Outline of the talk

- Test the coexistence of neutron stars and strange quark stars with Bayesian inference from astrophysical data:
    - Compare the bayesian evidence of the One Family scenario vs the Two Family scenario

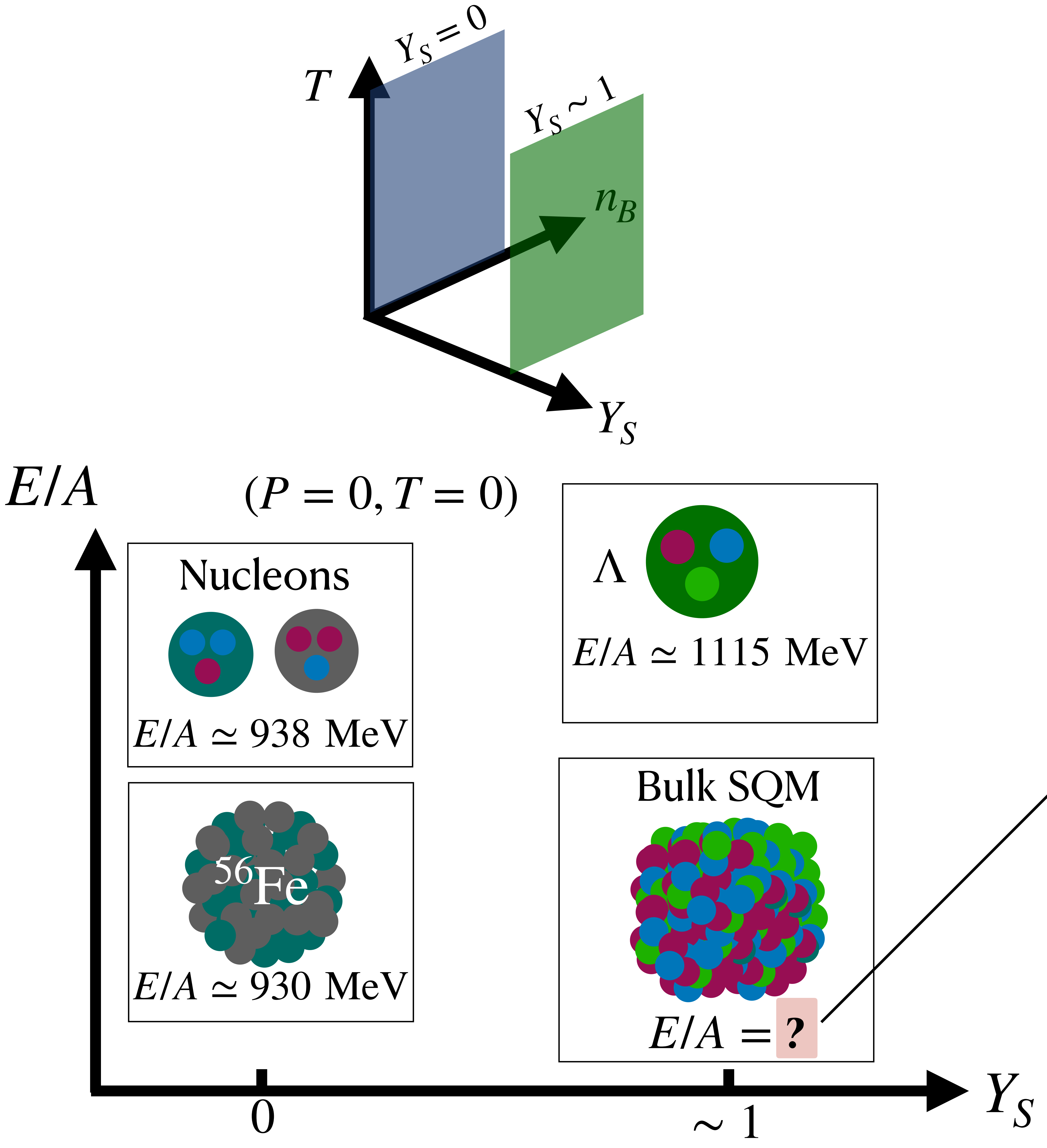
- Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

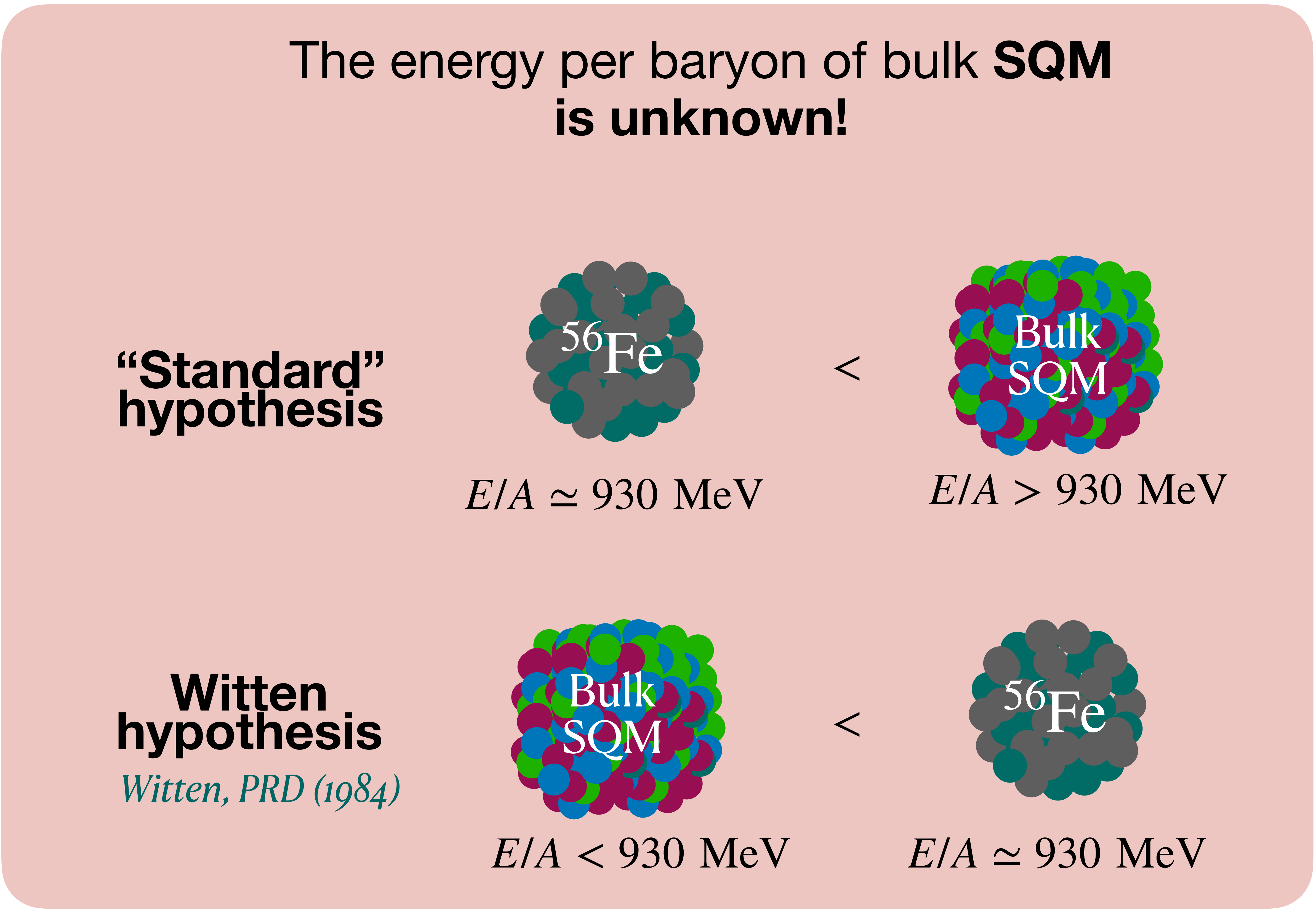
1. Extension of previous work in the pure gauge $SU(3)_c$ sector:
    - Inclusion of thermal fluctuation and excited glueballs
    - Comparison of results with lattice QCD data

2. Contributions meson and baryons sector: σ, π, ω, ρ meson field and nucleons N
    - Investigation of the impact of thermal fluctuations for all fields
    - Exploration of phase diagram in the meson–baryon sector

Table of Contents
Introduction

Two family scenario: Witten hypothesis

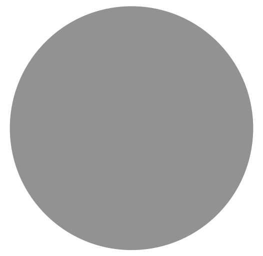




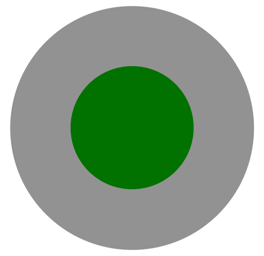
Two family scenario: SQM stability

One-family scenario

- **No** absolutely stable SQM (“**Standard**” hypothesis)
- Some compact stars may be **hybrid stars (HybSs)**

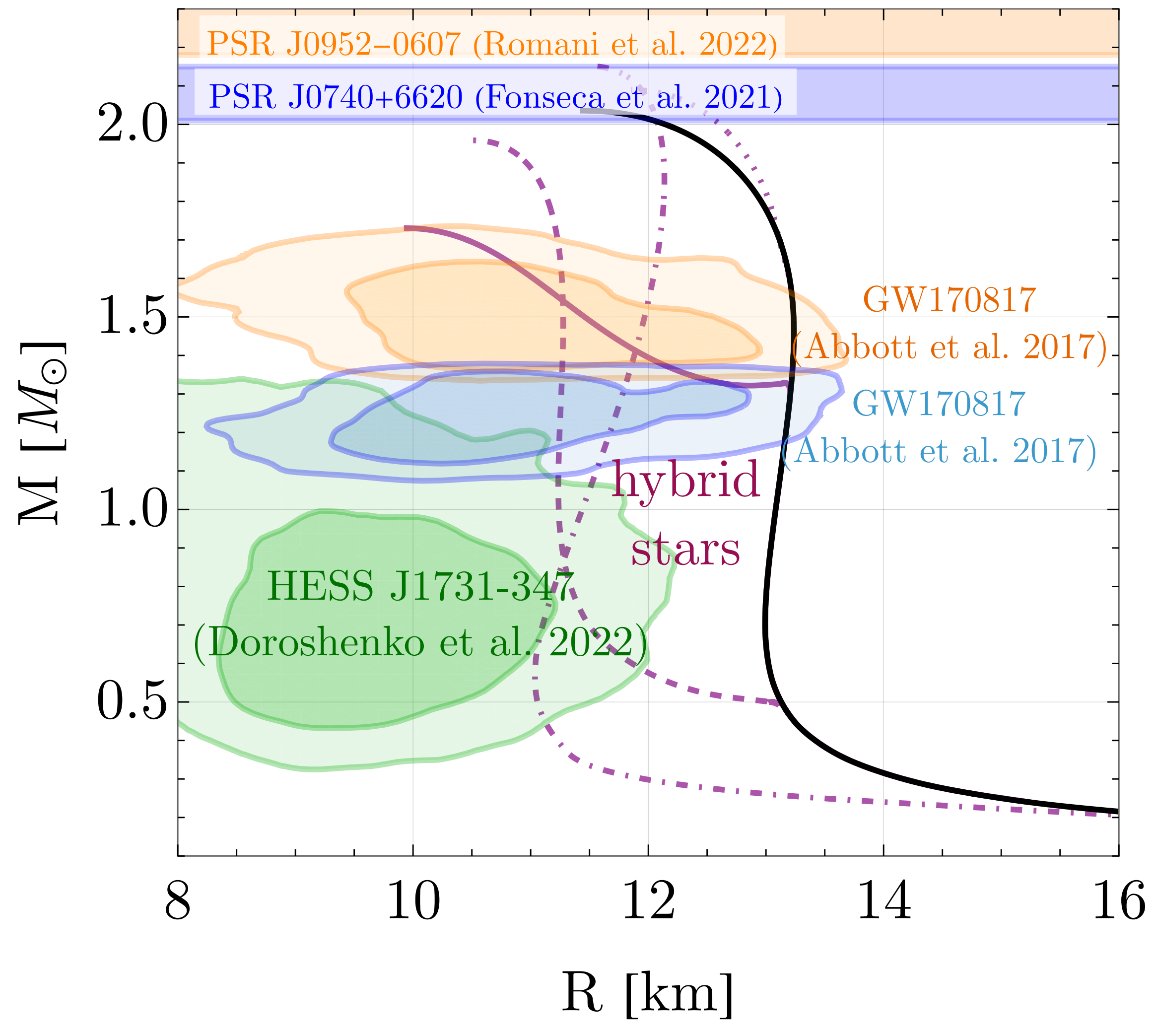


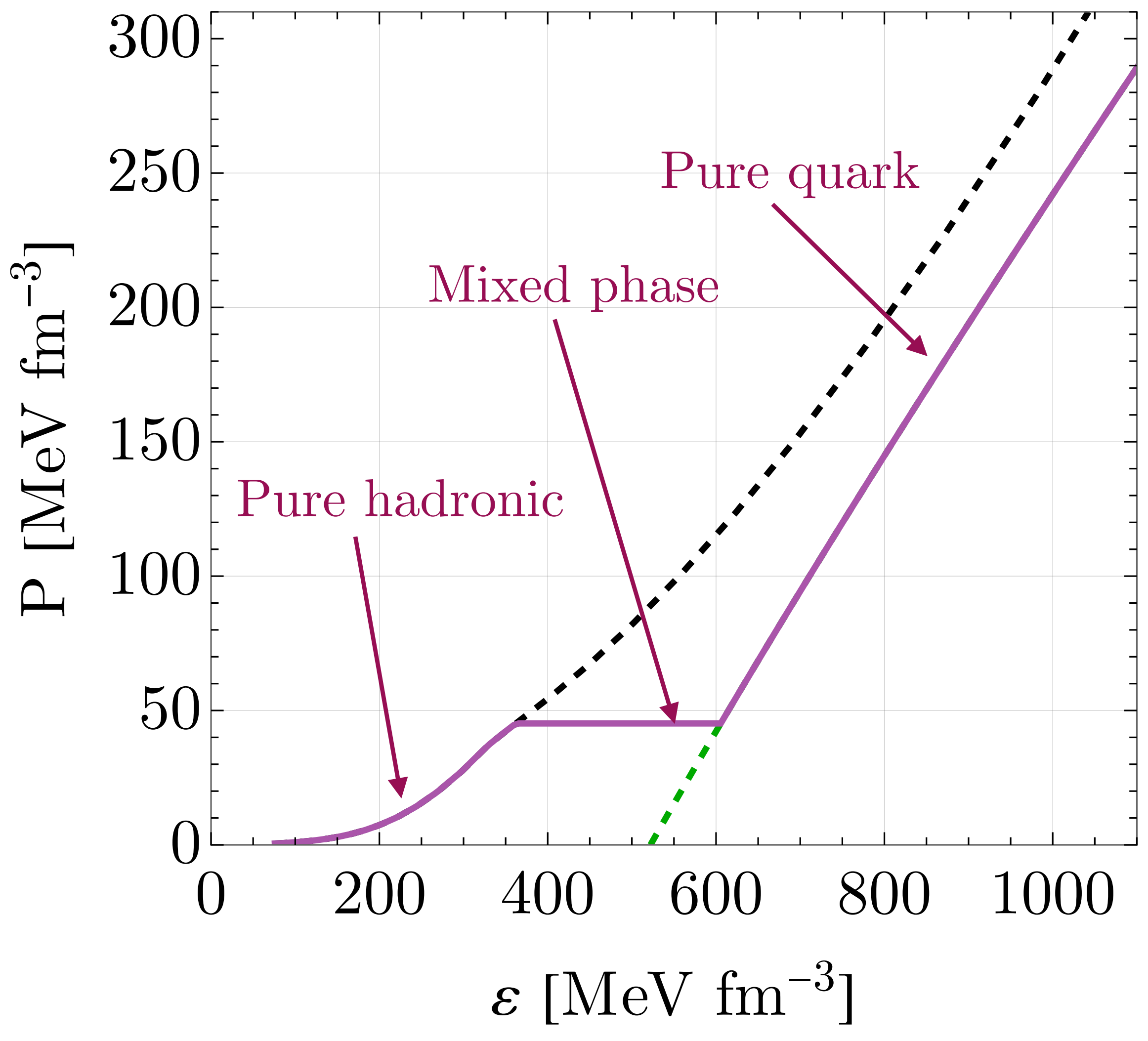
Neutron stars



Hybrid stars

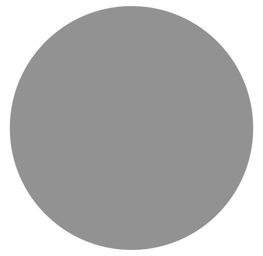
same **stable EOS**



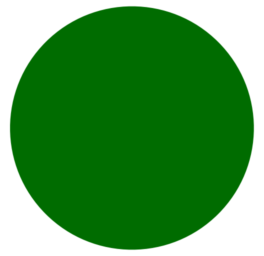


Drago et al. EPJ (2016) **Two-families scenario**

- Absolutely stable SQM (“**Witten**” hypothesis)
- **Quark stars (QSs)** and **Neutron stars (NSs)** coexist



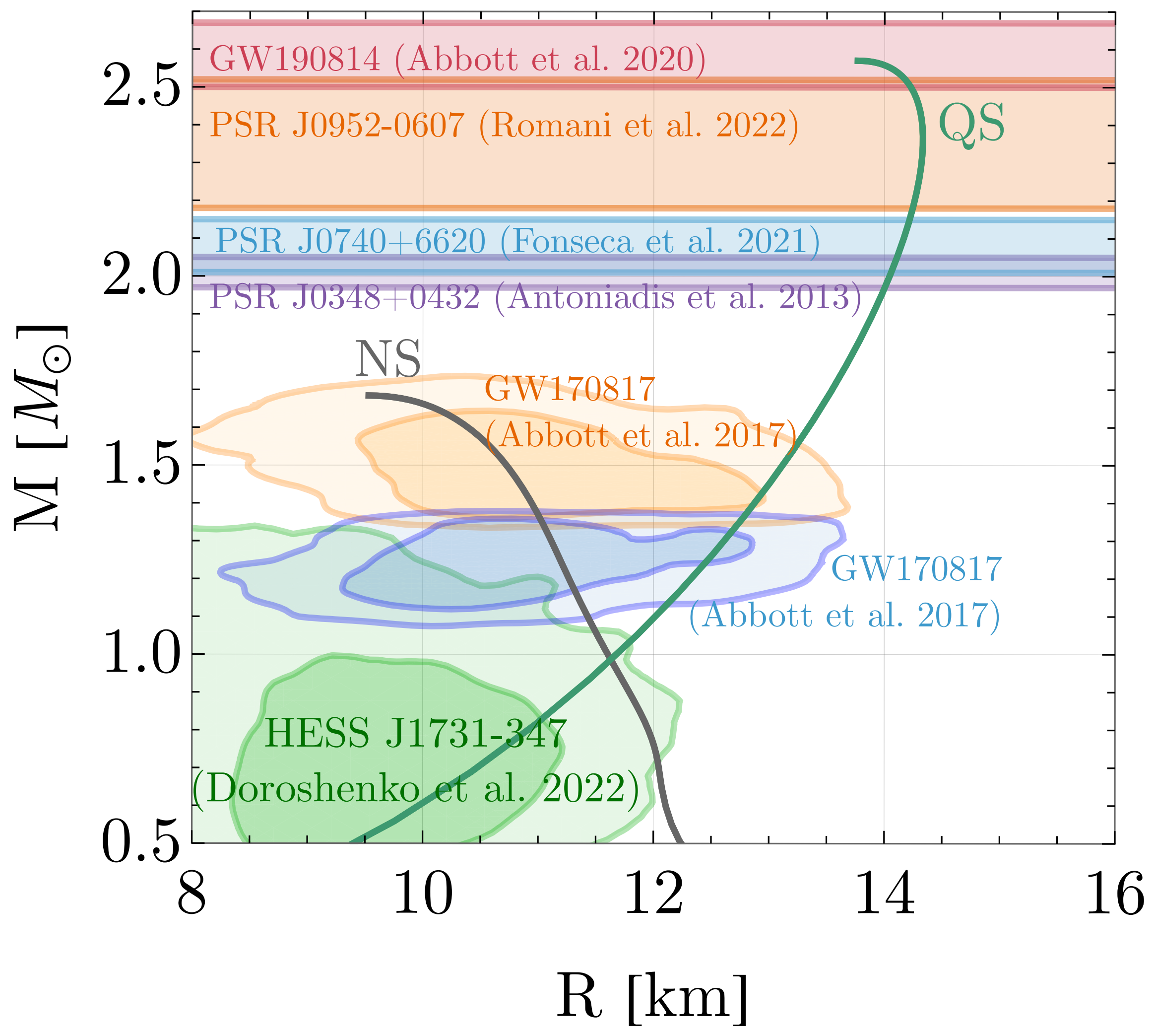
Neutron stars

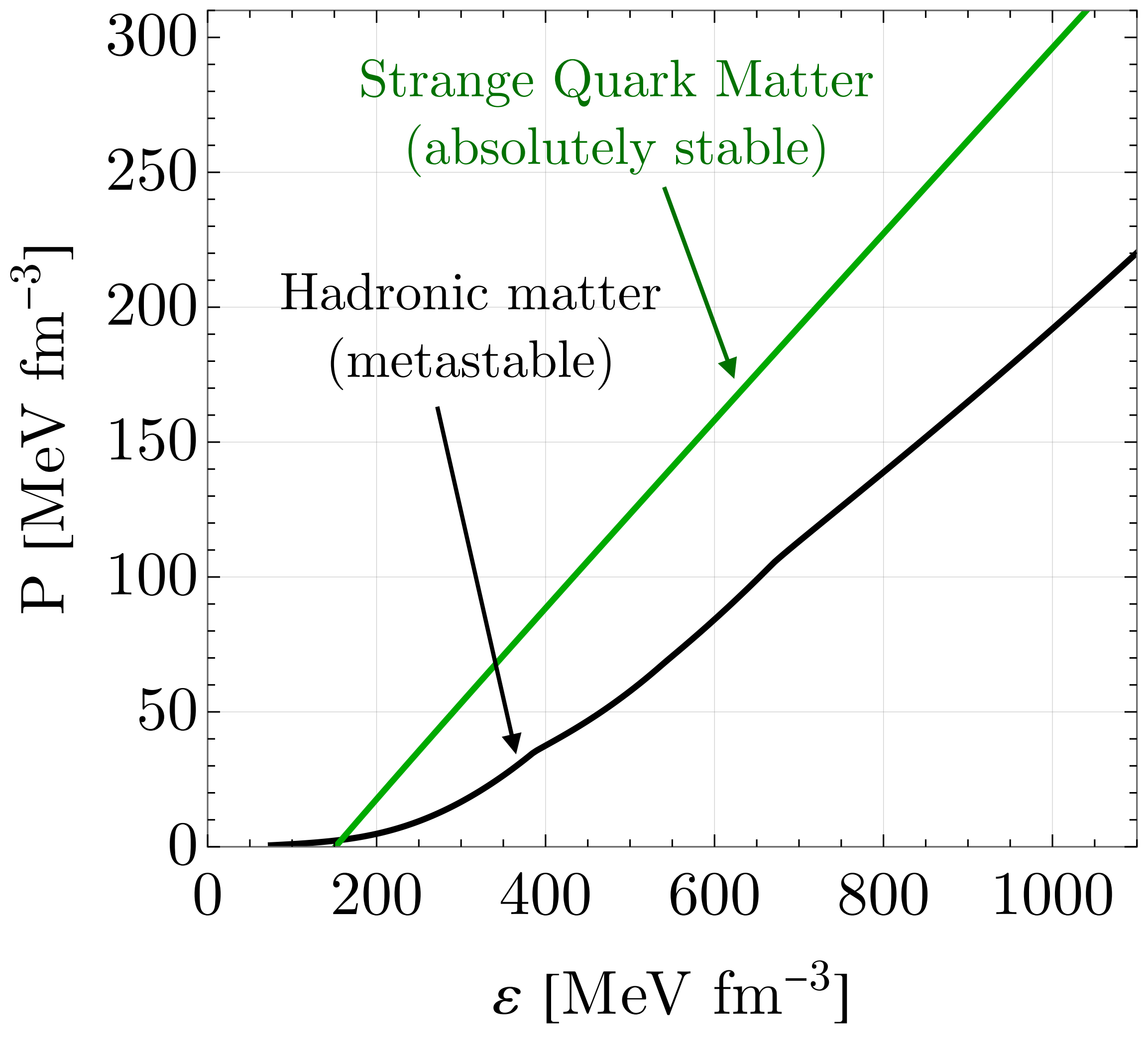


Quark stars

NSs are **metastable**
i.e. described by a
metastable EOS branch

QSs are **stable**
and **self-bound**





Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\big(\theta \mid \{D_i\}, H\big) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\big(\{D_i\}\big)}$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\big(\theta \,|\, \{D_i\}, H\big) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\big(\{D_i\}\big)}$$

Hadron EoS

$$b \in \{n, p, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-\}$$

$$d \in \{\Delta^{++}, \Delta^+, \Delta^0, \Delta^-\}$$

$$g_{\rho N}(n_b) = g_{\rho N}(n_{\rm sat})\exp\Big[-a_\rho\Big(\frac{n_b}{n_{\rm sat}} - 1\Big)\Big]$$

$$\begin{aligned}\mathscr{L}_H = &\sum_b \bar\psi_b\Big[i\gamma_\mu\partial^\mu - m_b + g_{\sigma b}\sigma - g_{\omega b}\gamma_\mu\omega^\mu - g_{\phi b}\gamma_\mu\phi^\mu - g_{\rho b}(n_B)\,\gamma_\mu \boldsymbol{I}_b\cdot\boldsymbol{\rho}^\mu\Big]\psi_b \\ &+\sum_d \bar\psi_{d\nu}\Big[i\gamma_\mu\partial^\mu - m_d + g_{\sigma d}\sigma - g_{\omega d}\gamma_\mu\omega^\mu - g_{\rho d}(n_B)\,\gamma_\mu \boldsymbol{I}_d\cdot\boldsymbol{\rho}^\mu\Big]\psi_d^\nu \\ &+\frac{1}{2}\Big(\partial_\mu\sigma\,\partial^\mu\sigma - m_\sigma^2\sigma^2\Big) - \frac{1}{3}\,b\,m_N(g_{\sigma N}\sigma)^3 - \frac{1}{4}\,c\,(g_{\sigma N}\sigma)^4 \\ &-\frac{1}{4}\,\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}\,m_\omega^2\,\omega_\mu\omega^\mu - \frac{1}{4}\,\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}\,m_\rho^2\,\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu \\ &-\frac{1}{4}\,\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}\,m_\phi^2\,\phi_\mu\phi^\mu\end{aligned}$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Hadron EoS

Quark EoS

$$\Omega_Q(\mu_q) = -\frac{3a_4}{4\pi^2}\mu_q^4 + \frac{3}{4\pi^2}\left(m_s^2 - 4\Delta^2\right)\mu_q^2 + B$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Hadron EoS

$$\theta_H \equiv \{n_{sat}, E_{sat}, K_{sat}, E_{sym}, L_{sym}, m^*/m, U_\Lambda, U_\Sigma, U_\Xi, x_{\sigma\Delta}, x_{\omega\Delta}\}$$

Quark EoS

$$\theta_Q \equiv \{a_4, B, \Delta\}$$

$$\theta = (\theta_H, \theta_Q)$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Drago et al. PRC (2014)

$$0 \lesssim x_{\sigma\Delta} - x_{\omega\Delta} \lesssim 0.2$$

$$V_N - 30\ \text{MeV} \lesssim V_\Delta \lesssim U_N$$

$$\mu_Q(P=0) = \left.\frac{\varepsilon_Q}{n_{B,Q}}\right|_{P=0} < 930\ \text{MeV}$$

$$\mu_Q(P) < \mu_H(P)$$

Category	Label	Info	Likelihood representation
M	PSR J0952−0607	$M = 2.24 \pm 0.17\ \text{M}_\odot$	Gaussian in M
M-R	PSR J0030+0451	$M = 1.44^{+0.15}_{-0.14}\ \text{M}_\odot,\ R = 13.02^{+1.24}_{-1.06}$ km	$q(M,R)$ KDE from posterior samples
M-R	PSR J0740+6620	$M = 2.08 \pm 0.07\ \text{M}_\odot,\ R = 13.7^{+2.6}_{-1.5}$ km	$q(M,R)$ KDE from posterior samples
M-R	PSR J0614−3329	$M = 1.44^{+0.06}_{-0.07}\ \text{M}_\odot,\ R = 10.29^{+1.01}_{-0.86}$ km	$q(M,R)$ KDE from posterior samples
M-R	HESS J1731−347	$M = 0.77^{+0.20}_{-0.17}\ \text{M}_\odot,\ R = 10.4^{+0.86}_{-0.78}$ km	$q(M,R)$ KDE from posterior samples
M-Λ	GW 170817	$\mathcal{M} = 1.186 \pm 0.001\quad \tilde{\Lambda} = 300^{+420}_{-230}$	$q(M_1, M_2, \Lambda_1, \Lambda_2)$ KDE from posterior samples
n_B-E_H^{PNM}	χEFT	PNM band for $E_H^{\text{PNM}}(n_B)$, $n_B \le n_{\text{sat}}$	soft band likelihood from Passarella, Phys.Rev.C 112 (2025)
n_B-P_H^{SNM}	HIC	SNM band for $P_H^{\text{SNM}}(n_B)$, $n_B \le 3\,n_{\text{sat}}$	soft band likelihood from Danielewicz, Science 298 (2002)

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\Big(\theta \mid \{D_i\}, H\Big) = \frac{\mathscr{L}^{(H)}(\theta)\;\pi^{(H)}(\theta)}{Z^{(H)}\Big(\{D_i\}\Big)}$$

Likelihood

$$\mathscr{L}^{(H)}(\theta) = \prod_i \mathscr{L}_i^{(H)}(\theta)$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\big(\theta \mid \{D_i\}, H\big) = \frac{\mathscr{L}^{(H)}(\theta)\, \pi^{(H)}(\theta)}{Z^{(H)}\big(\{D_i\}\big)}$$

Likelihood

$$\mathscr{L}^{(H)}(\theta) = \prod_i \mathscr{L}_i^{(H)}(\theta)$$

$$\mathscr{L}_i^{(1\mathrm{F})}(\theta) \propto \int_{M_{\mathrm{NS,min}}^{(1\mathrm{F})}(\theta)}^{M_{\mathrm{NS,max}}^{(1\mathrm{F})}(\theta)} dM\, q_i\big[M, R_{\mathrm{NS}}^{\mathrm{TOV}}(M;\theta)\big]$$

$$\mathscr{L}_i^{(2\mathrm{F})}(\theta) \propto \sum_{f\in\{\mathrm{NS,QS}\}} \int_{M_{f,\mathrm{min}}^{(2\mathrm{F})}(\theta)}^{M_{f,\mathrm{max}}^{(2\mathrm{F})}(\theta)} dM\, q_i\Big[M, R_f^{\mathrm{TOV}}(M;\theta)\Big]\, \eta_f^{(2\mathrm{F})}(M;\theta) \equiv \mathscr{L}_{\mathrm{NS},i}^{(2\mathrm{F})}(\theta) + \mathscr{L}_{\mathrm{QS},i}^{(2\mathrm{F})}(\theta)$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\, \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Likelihood

$$\mathscr{L}^{(H)}(\theta) = \prod_i \mathscr{L}_i^{(H)}(\theta)$$

$$\mathscr{L}_i^{(1F)}(\theta) \propto \int_{M_{\rm NS,min}^{(1F)}(\theta)}^{M_{\rm NS,max}^{(1F)}(\theta)} dM\, q_i\left[M, R_{\rm NS}^{\rm TOV}(M;\theta)\right]$$

$$\mathscr{L}_i^{(2F)}(\theta) \propto \sum_{f\in\{\rm NS,QS\}} \int_{M_{f,\rm min}^{(2F)}(\theta)}^{M_{f,\rm max}^{(2F)}(\theta)} dM\, q_i\left[M, R_f^{\rm TOV}(M;\theta)\right]\, \eta_f^{(2F)}(M;\theta) \equiv \mathscr{L}_{{\rm NS},i}^{(2F)}(\theta) + \mathscr{L}_{{\rm QS},i}^{(2F)}(\theta)$$

$1.17 M_\odot$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\Big(\theta \mid \{D_i\}, H\Big) = \frac{\mathscr{L}^{(H)}(\theta)\, \pi^{(H)}(\theta)}{Z^{(H)}\Big(\{D_i\}\Big)}$$

Likelihood

$$\mathscr{L}^{(H)}(\theta) = \prod_i \mathscr{L}_i^{(H)}(\theta)$$

$$\mathscr{L}_i^{(1F)}(\theta) \propto \int_{M_{\mathrm{NS,min}}^{(1F)}(\theta)}^{M_{\mathrm{NS,max}}^{(1F)}(\theta)} dM\, q_i\Big[M, R_{\mathrm{NS}}^{\mathrm{TOV}}(M;\theta)\Big]$$

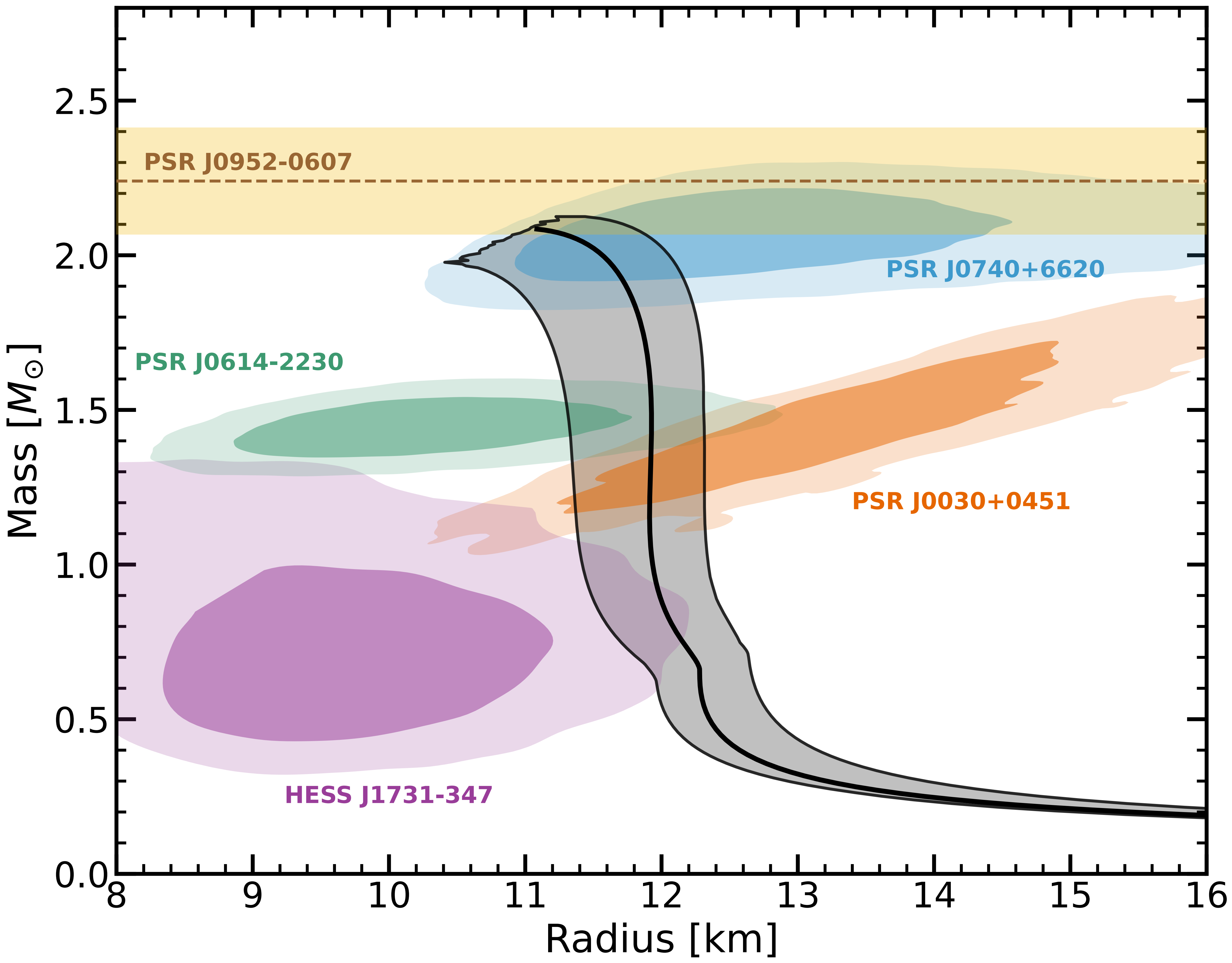
$$M_{conv}(\theta) = M(n_{S,c} = n_{sat})$$

$$\mathscr{L}_i^{(2F)}(\theta) \propto \sum_{f \in \{\mathrm{NS,QS}\}} \int_{M_{f,\min}^{(2F)}(\theta)}^{M_{f,\max}^{(2F)}(\theta)} dM\, q_i\Big[M, R_f^{\mathrm{TOV}}(M;\theta)\Big]\, \eta_f^{(2F)}(M;\theta) \equiv \mathscr{L}_{\mathrm{NS},i}^{(2F)}(\theta) + \mathscr{L}_{\mathrm{QS},i}^{(2F)}(\theta)$$

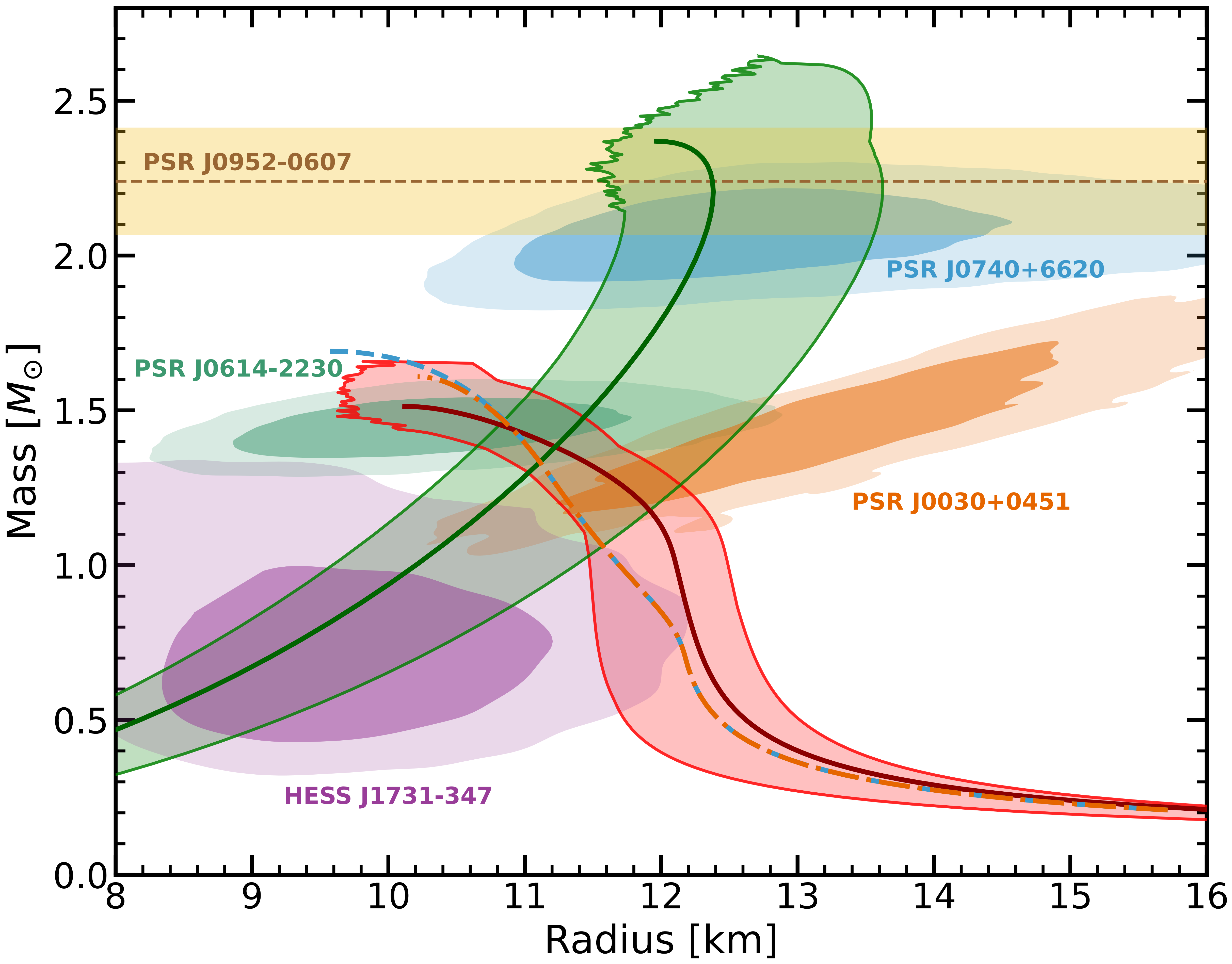
$$1.17 M_\odot$$

Testing NS–QS coexistence via Bayesian Inference

One-Family scenario

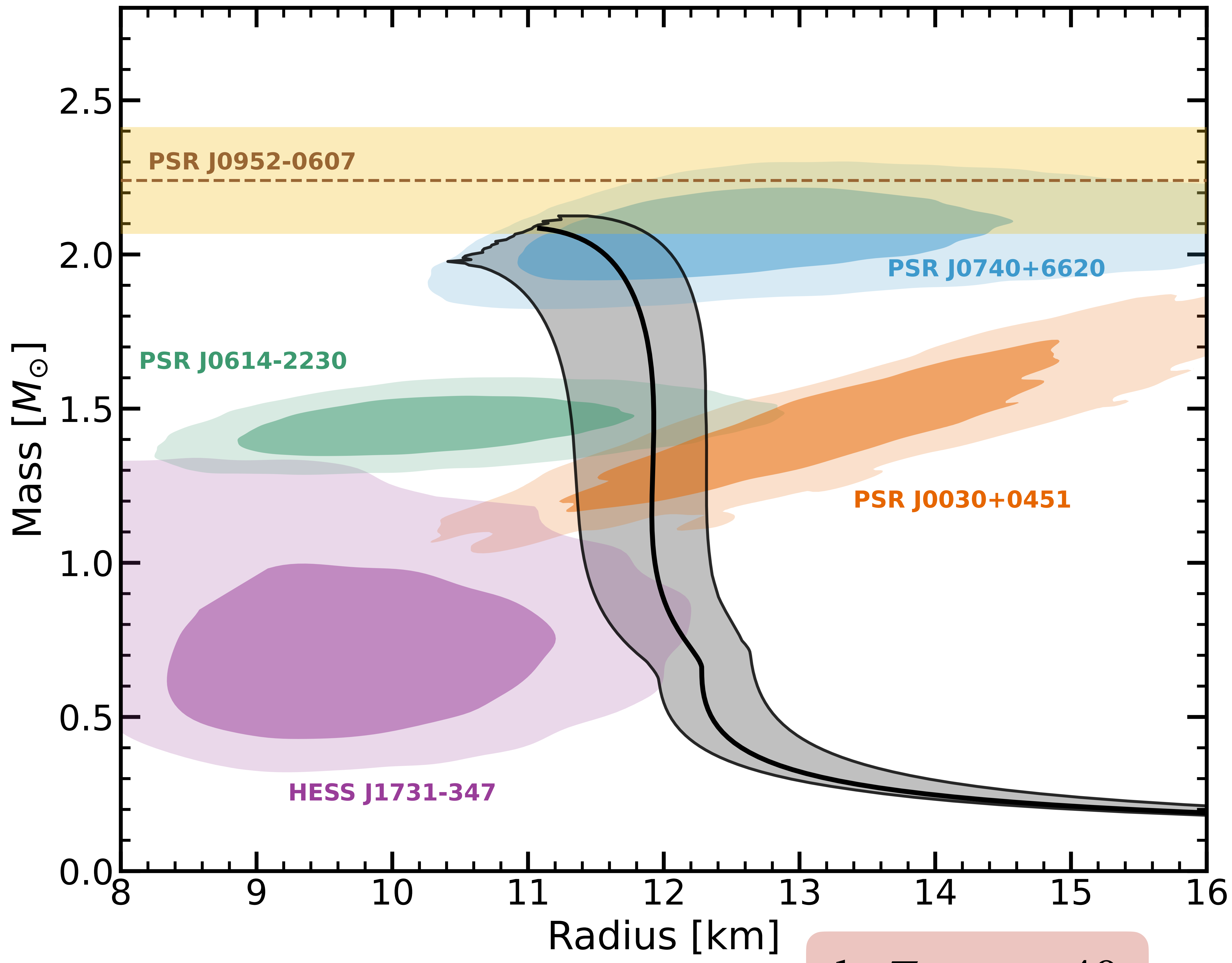


Two-Family scenario



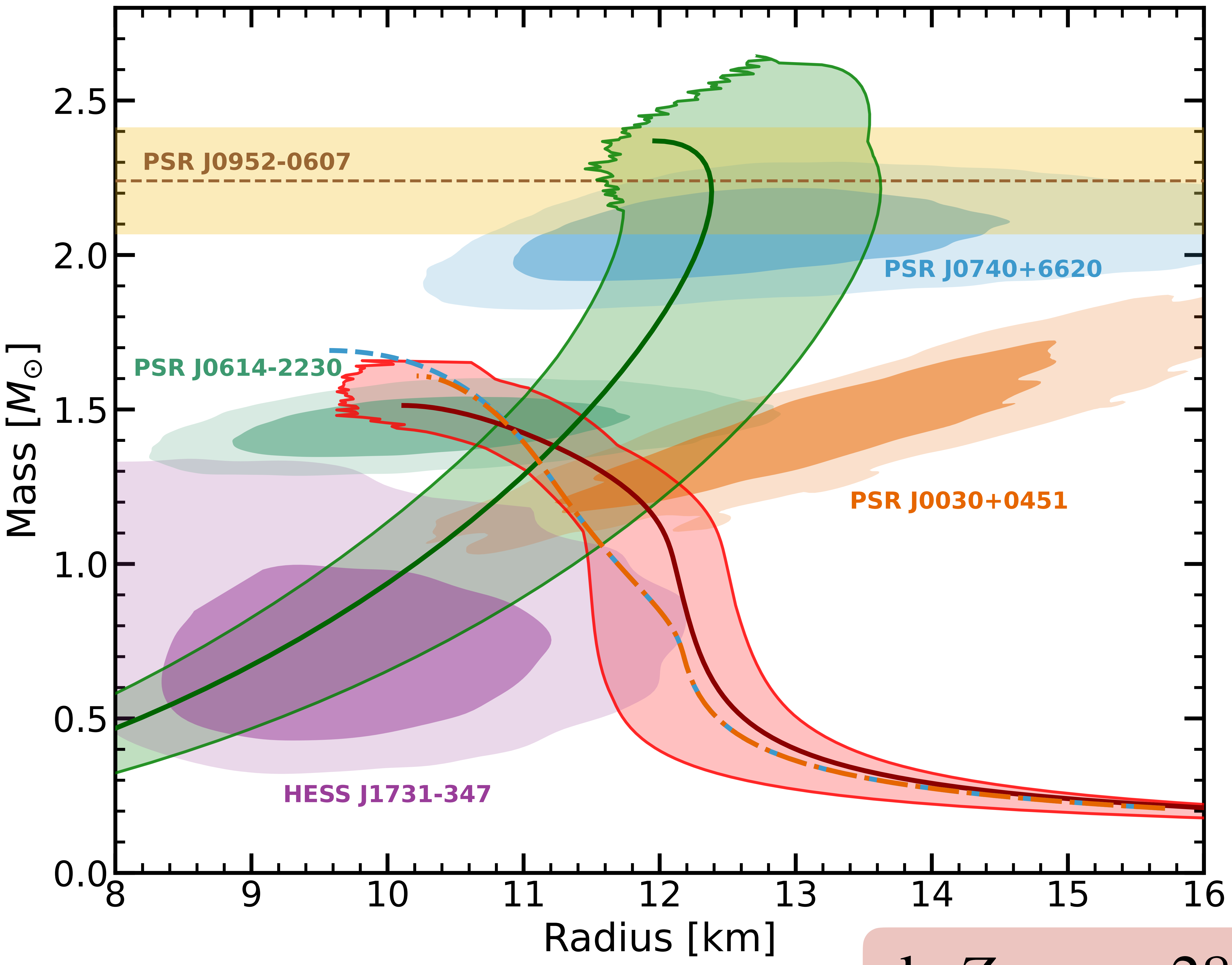
Testing NS–QS coexistence via Bayesian Inference

One-Family scenario



$\ln Z_{1F} \approx -40$

Two-Family scenario

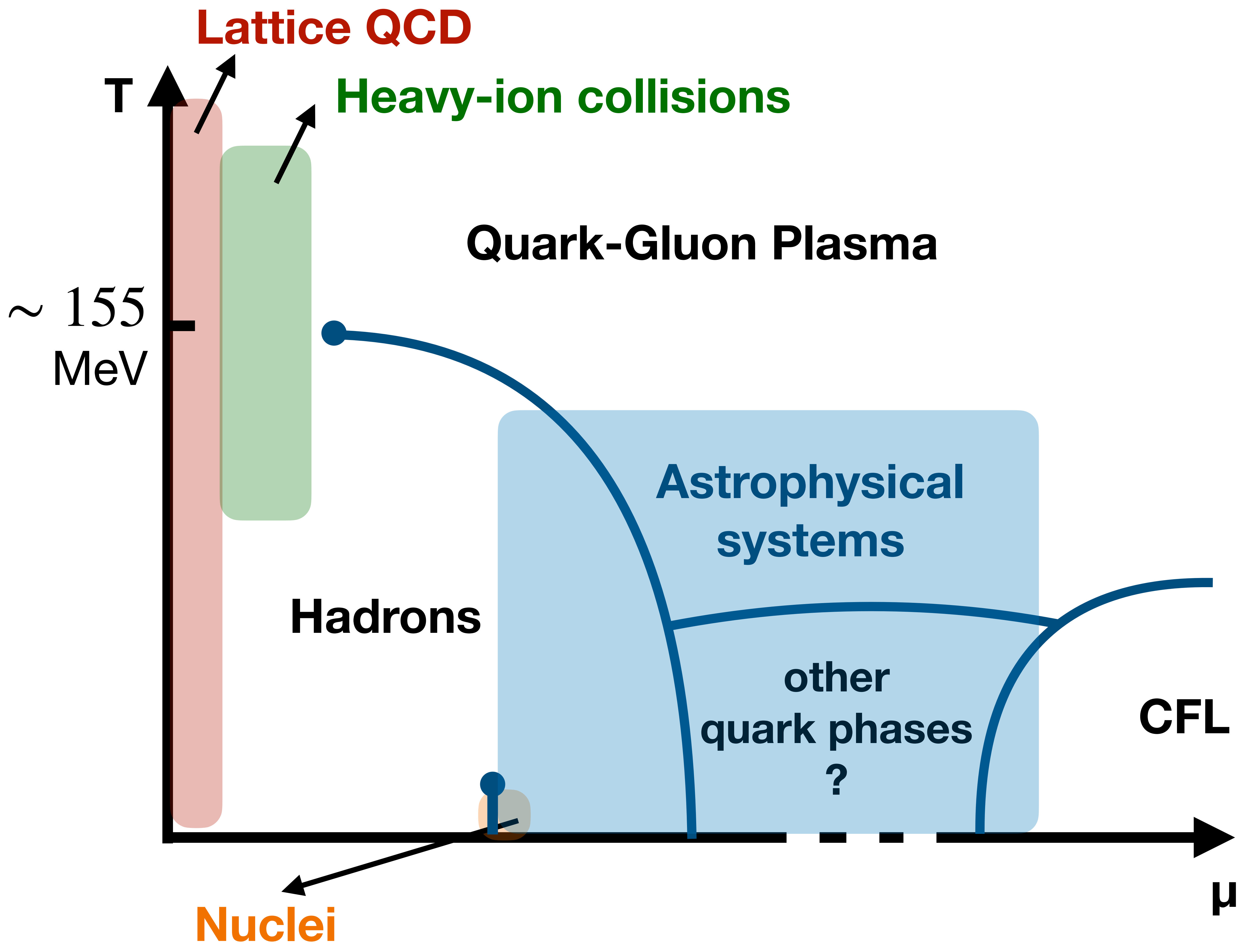


$\ln Z_{2F} \approx -28$

Table of Contents

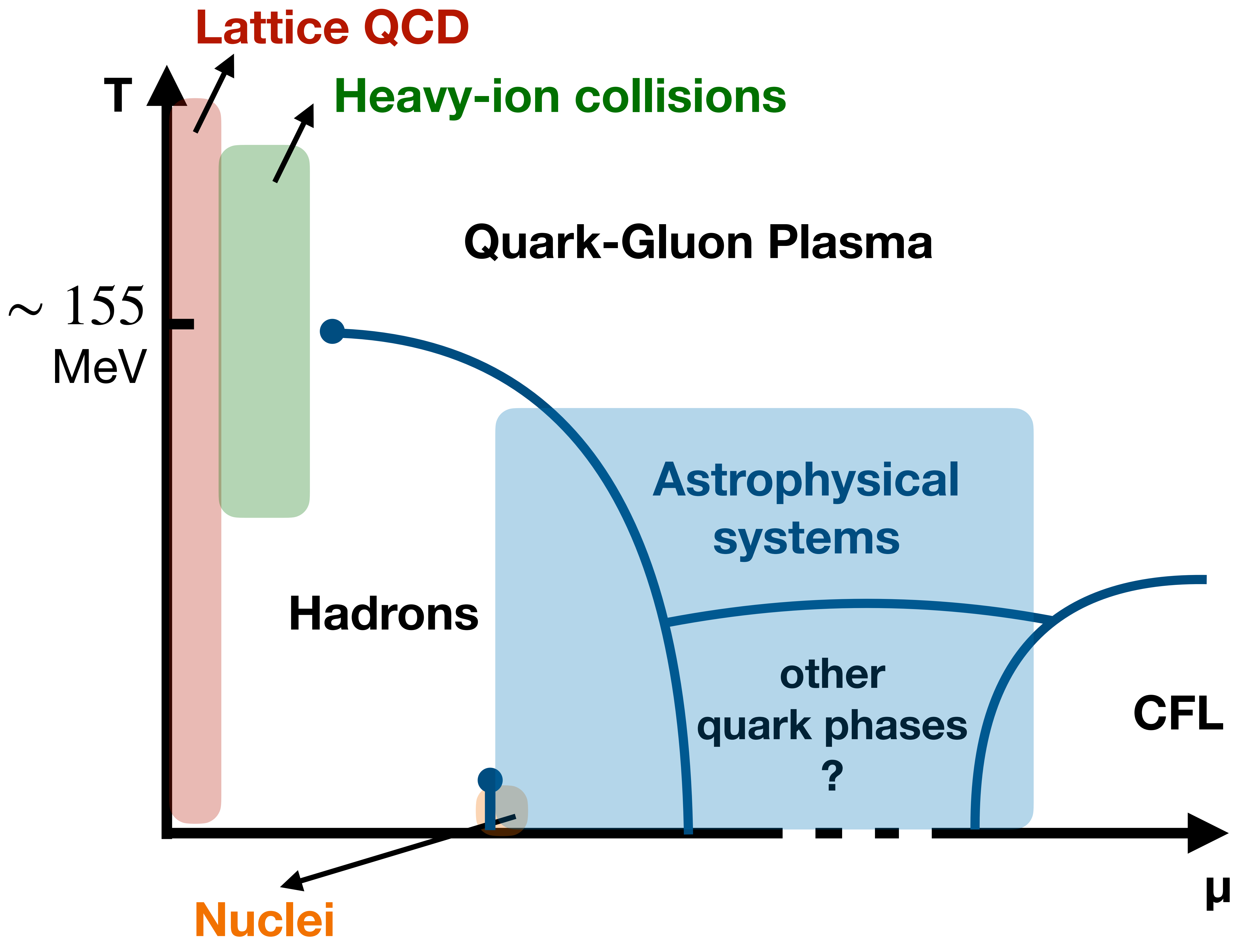
Modeling finite-temperature EoSs

QCD phase diagram



QCD phase diagram

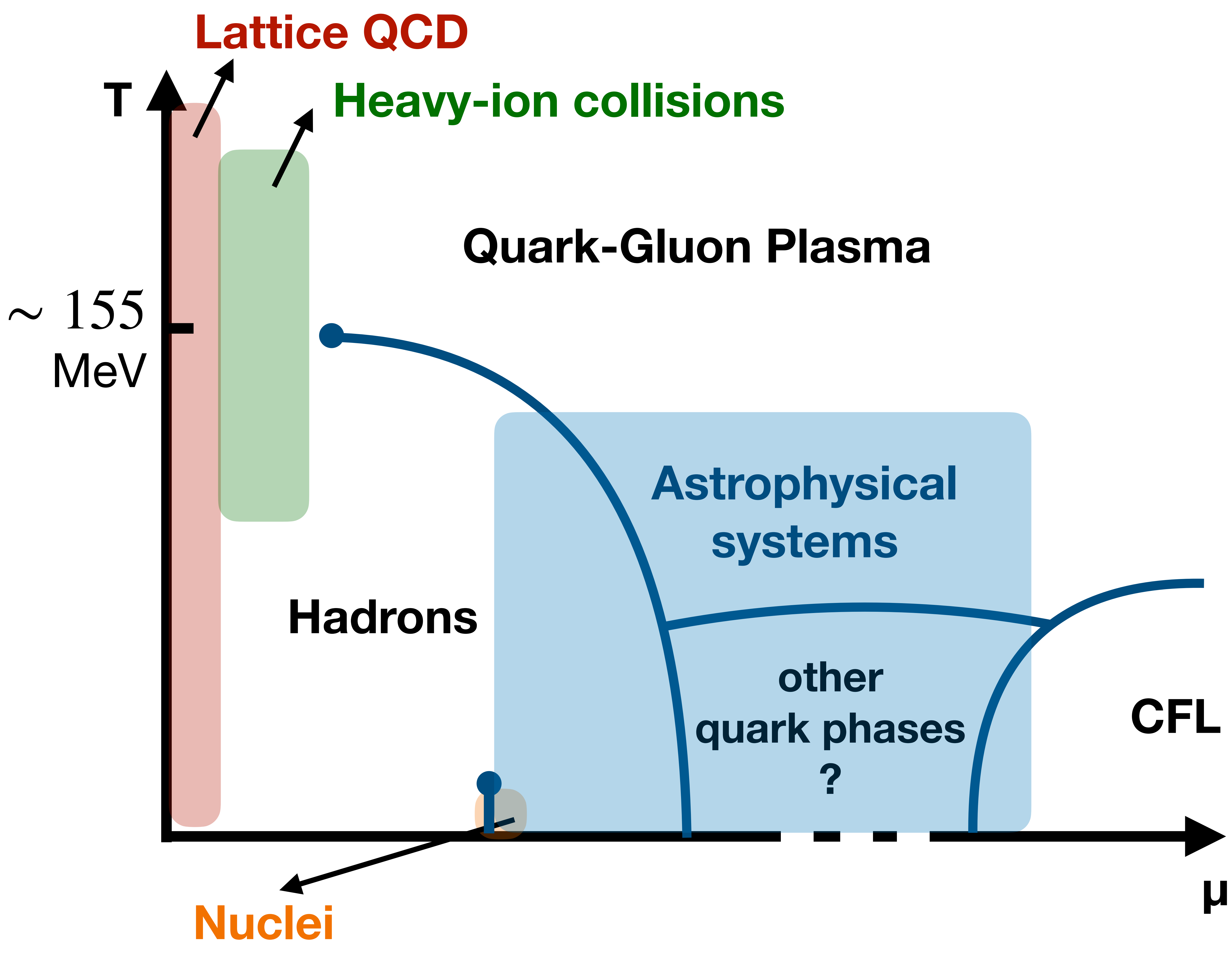
- Regimes of low temperatures and high densities: neutron stars and compact stars



QCD phase diagram

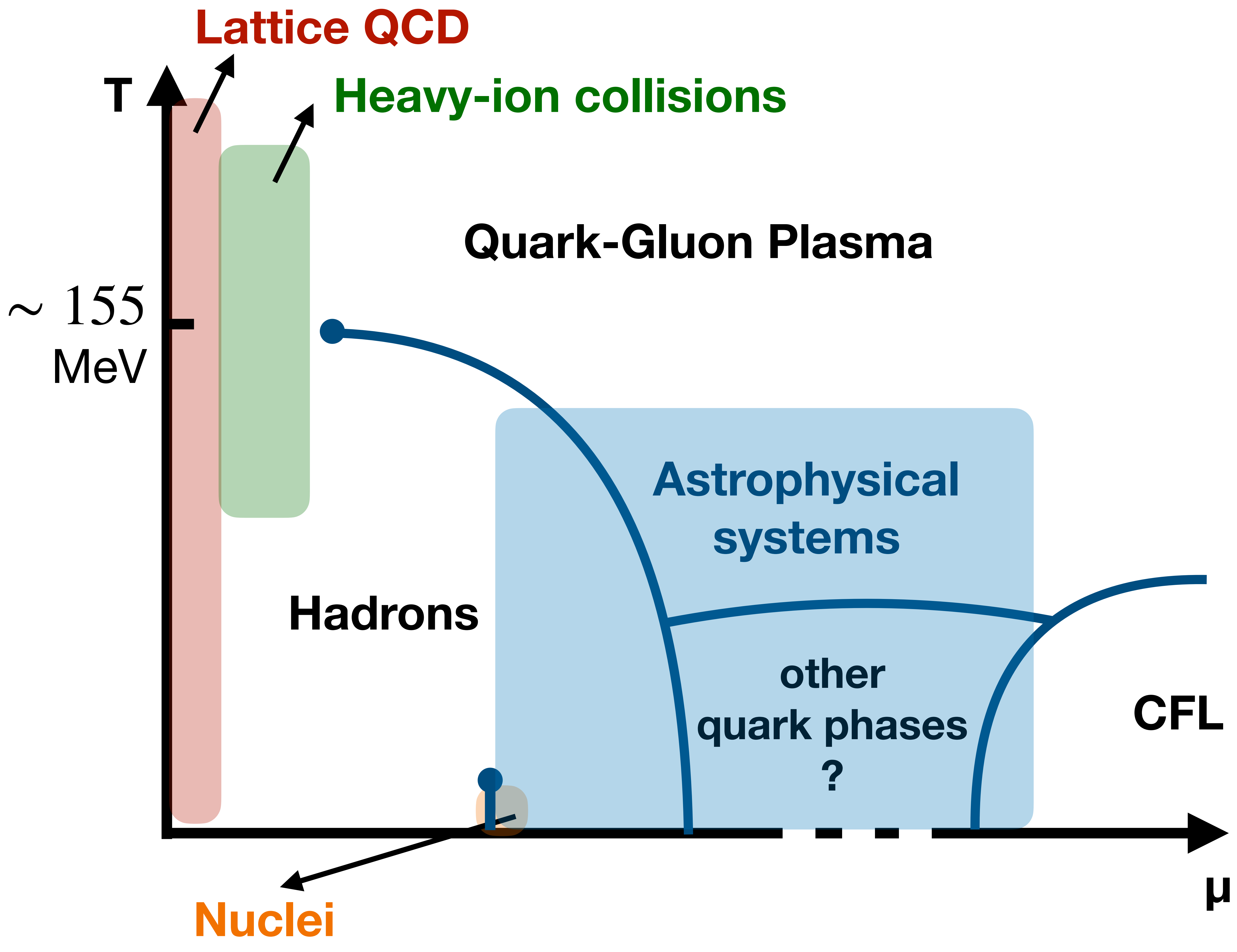
- Regimes of low temperatures and high densities: neutron stars and compact stars

- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions



QCD phase diagram

- Regimes of low temperatures and high densities: neutron stars and compact stars
- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions
- Only a few EOS are studied in both regimes simultaneously and with several model-dependent parameters: a complete description is still challenging.



Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU}(3)_c$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln \frac{\phi}{\phi_0} \right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln \chi \right).$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $SU(3)_c$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln\frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln\chi\right).$$

The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle\mathscr{V}(\chi)\rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2}m_\chi^{*2}\left\langle\Delta_\chi^2\right\rangle .$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln\frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln\chi\right).$$

The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle\mathscr{V}(\chi)\rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2}m_\chi^{*2}\langle\Delta_\chi^2\rangle\,.$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln \frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln \chi\right).$$

The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle \mathscr{V}(\chi) \rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle .$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.
- Inclusion **perturbative gluon-gluon interaction** for $T \gg T_c$

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln \frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln \chi\right).$$

The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle \mathscr{V}(\chi) \rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle .$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.
- Inclusion **perturbative gluon-gluon interaction** for $T \gg T_c$

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln \frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln \chi\right).$$

The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle \mathscr{V}(\chi) \rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle .$$

Infrared cut-off $K(\chi) = \dfrac{A}{1 - \chi}$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)
P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2026)

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.
- Inclusion **perturbative gluon-gluon interaction** for $T \gg T_c$
- Investigation of the impact of **thermal fluctuations** on the dilation field

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.
- Inclusion **perturbative gluon-gluon interaction** for $T \gg T_c$
- Investigation of the impact of **thermal fluctuations** on the dilation field

It is possible to divide the glueball into two distinct components: the mean field $\bar{\chi}$ and the fluctuating part Δ_χ, with $\langle \Delta_\chi \rangle = 0$.

$$\chi = \bar{\chi} + \Delta_\chi$$

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

- **Gluon condensate** dynamics dominated below T_c.
- Condensate evaporates into **quasi-free gluons** at higher temperatures.
- Inclusion **perturbative gluon-gluon interaction** for $T \gg T_c$
- Investigation of the impact of **thermal fluctuations** on the dilation field

It is possible to divide the glueball into two distinct components: the mean field $\bar{\chi}$ and the fluctuating part Δ_χ, with $\langle \Delta_\chi \rangle = 0$.

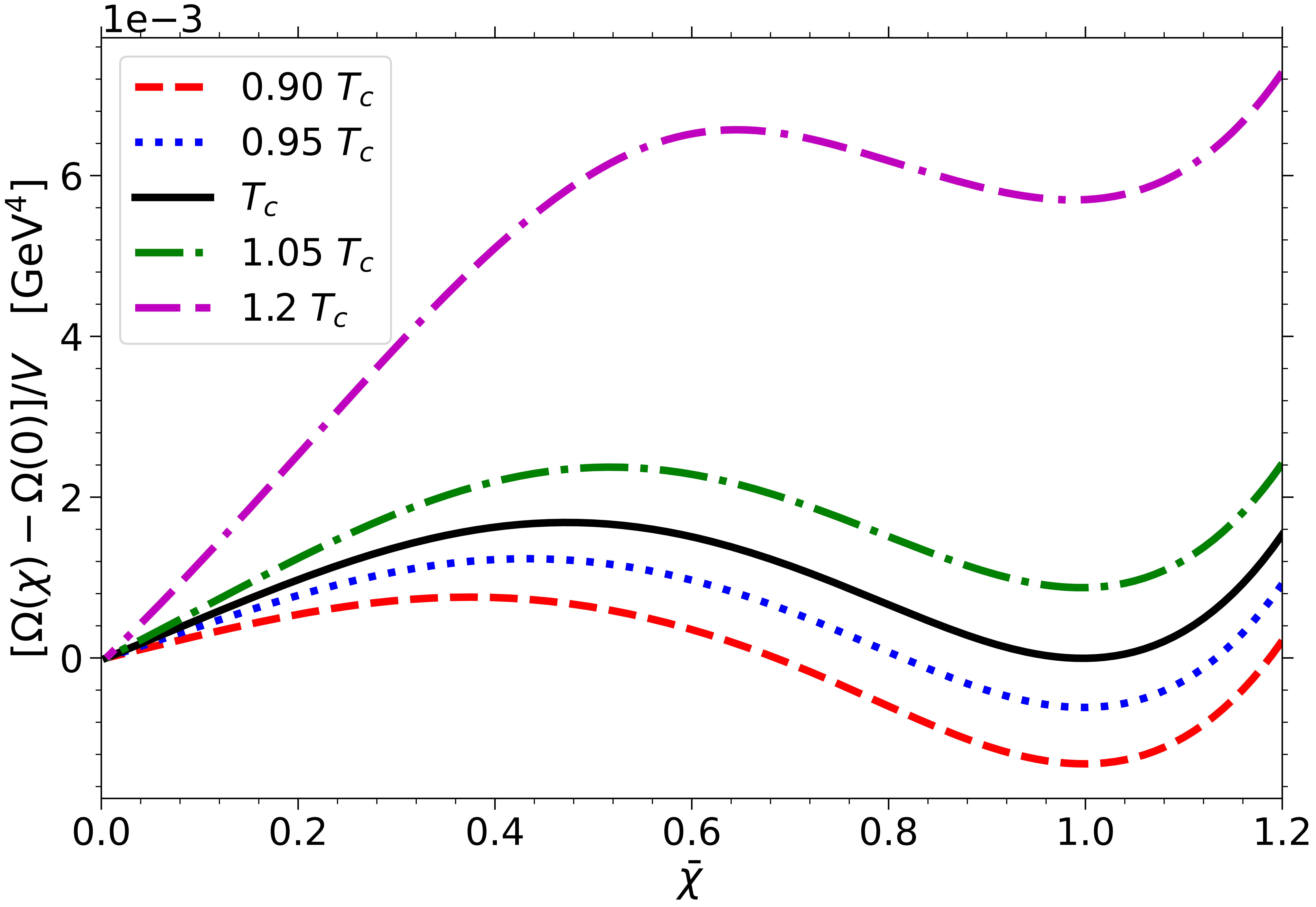
$$\chi = \bar{\chi} + \Delta_\chi$$

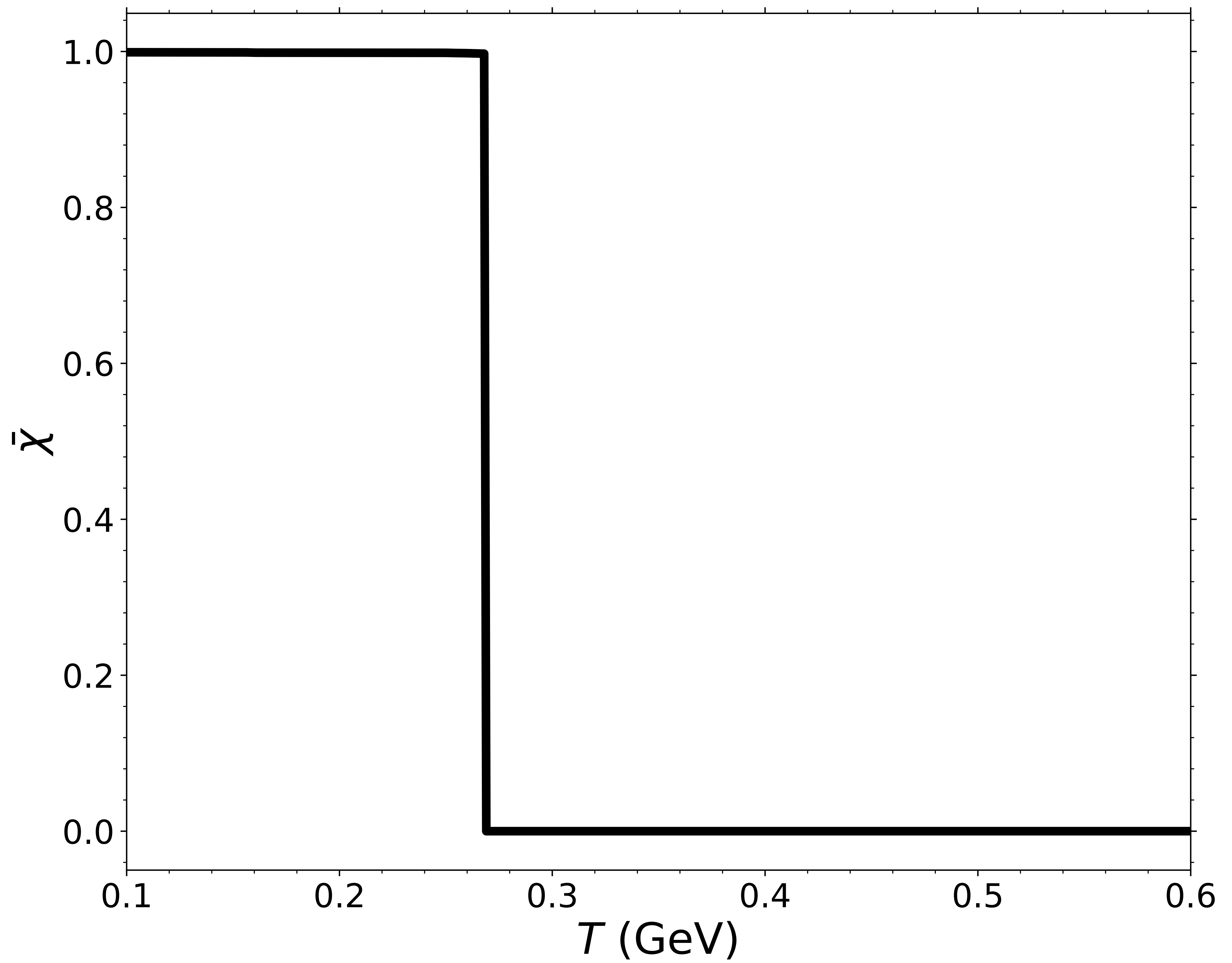
$$\langle \Delta_\chi^2 \rangle = \frac{1}{2\pi^2 \phi_0^2} \int_0^\infty \frac{k^2}{\epsilon_\chi} \frac{1}{e^{\beta \epsilon_\chi} - 1} dk$$

$$P_\chi(z) = \sqrt{\frac{1}{2\pi \langle \Delta_\chi^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\chi^2 \rangle} \right)$$

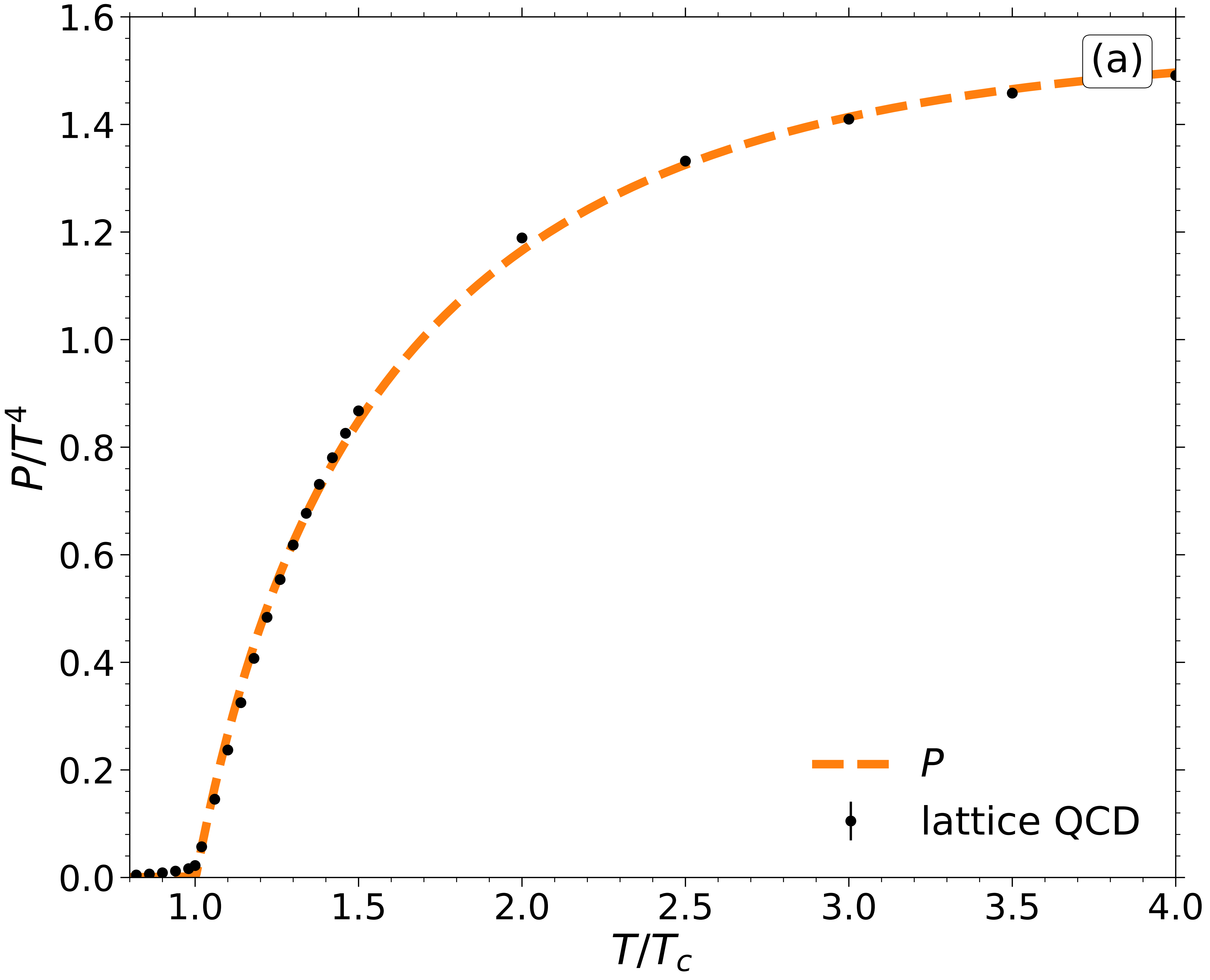
$$\langle f(\chi) \rangle = \int_{-\infty}^{\infty} P_\chi(z) f(\bar{\chi} + z)\, dz$$

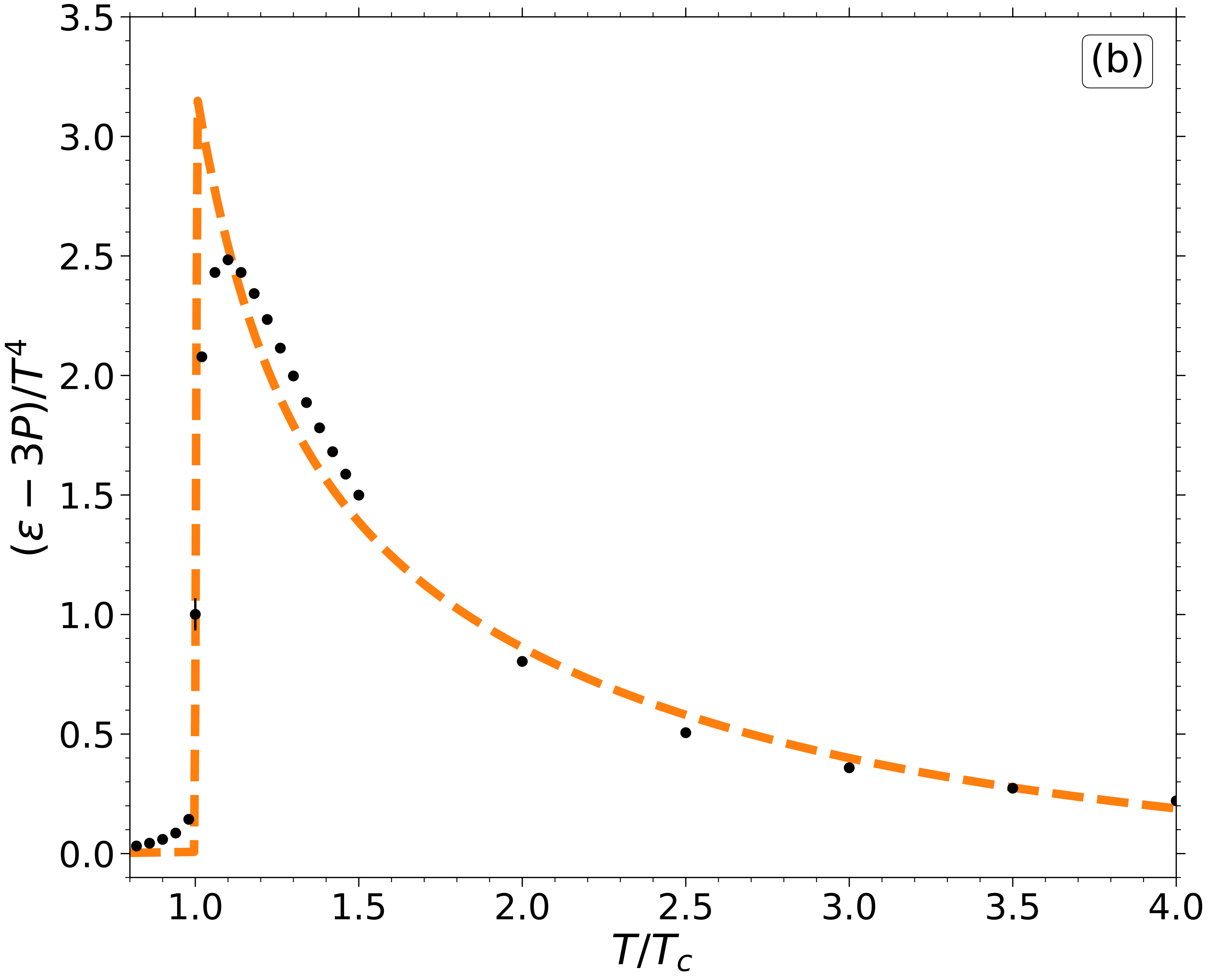
Pure gauge $\mathrm{SU(3)_c}$ sector





Pure gauge $\mathrm{SU}(3)_c$ sector





Pure gauge $\mathrm{SU}(3)_c$ sector

Pure gauge $\mathrm{SU(3)_c}$ sector

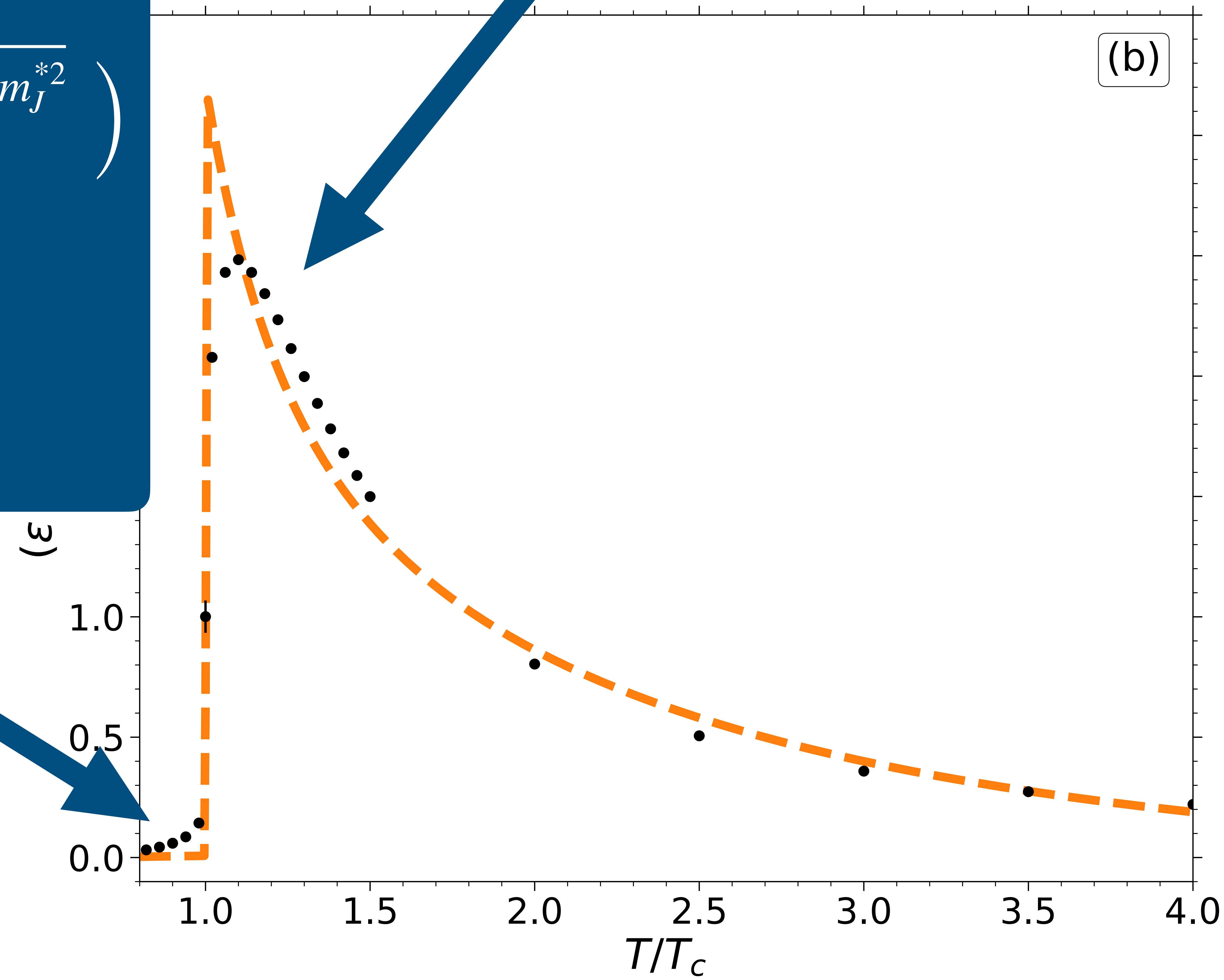
$$P_{\mathrm{dil}_J} = -(2J+1)T\int \frac{dk^3}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{k^2+m_J^{*2}}}\right)$$

$$\frac{\tilde{\Omega}(\chi,T)}{V} = \frac{\Omega(\chi,T)}{V} - \sum_J^N P_{\mathrm{dil}_J}$$

Pure gauge $\mathrm{SU}(3)_c$ sector

$$P_{\mathrm{dil}_J} = -(2J+1)T\int \frac{dk^3}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{k^2+m_J^{*2}}}\right)$$

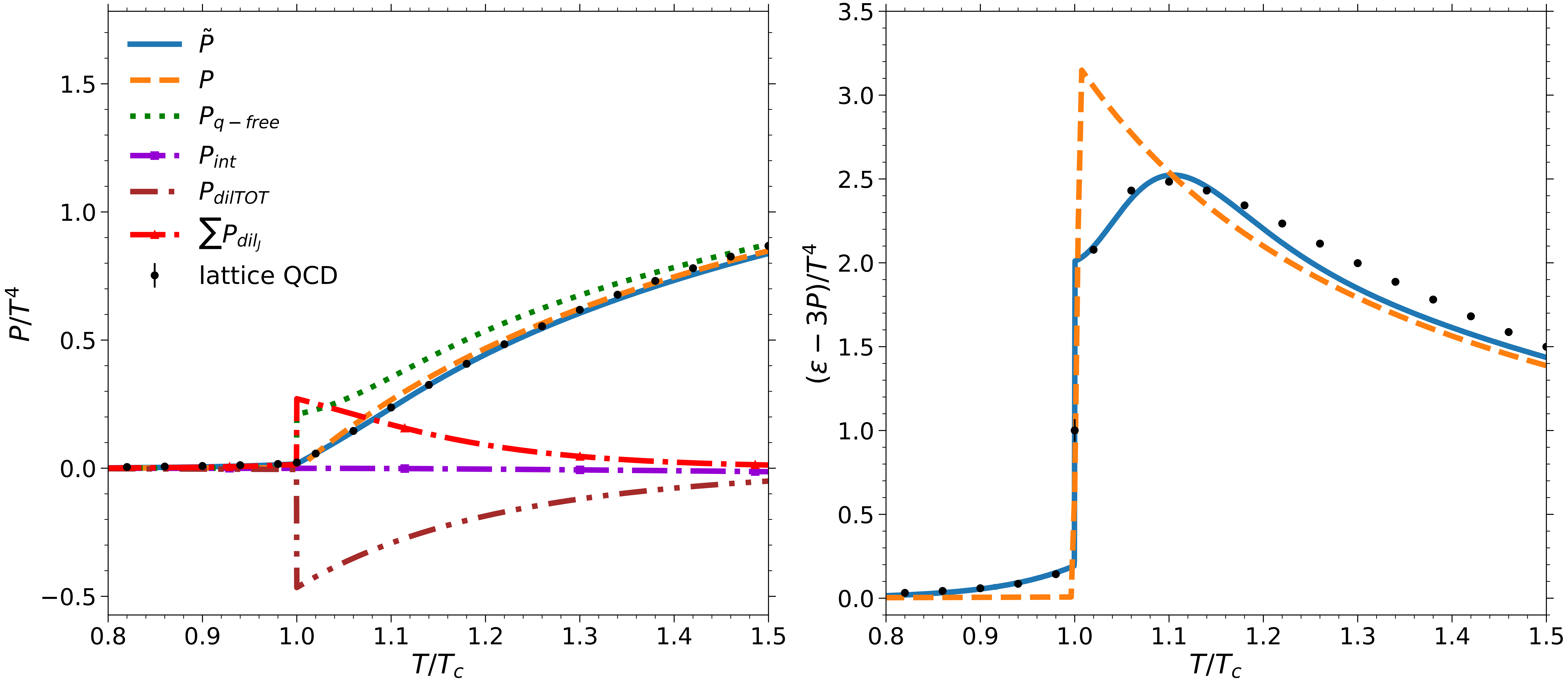
$$\frac{\tilde{\Omega}(\chi, T)}{V} = \frac{\Omega(\chi, T)}{V} - \sum_{J}^{N} P_{\mathrm{dil}_J}$$



Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1-\chi^2} = \frac{A}{1-\bar{\chi}^2 - \langle \Delta_\chi^2 \rangle \cdot F(T)}$$

Pure gauge $\mathrm{SU}(3)_c$ sector



Pure gauge $\mathrm{SU}(3)_c$ sector

Table of Contents

Modeling finite-temperature EoSs

- Introduction
- Testing NS–QS coexistence via Bayesian Inference
- Modeling finite-temperature EoS:
  - Pure Gauge $SU(3)_c$ sector
  - **Meson and baryon sector**
- Summary
- Outlooks

Meson and baryon sector at finite chemical potential

We consider the **meson** and **baryon** sector at **finite chemical potential** through the introduction of an effective Lagrangian that incorporates broken scale in addition to explicit broken chiral symmetry

Meson and baryon sector at finite chemical potential

We consider the **meson** and **baryon** sector at **finite chemical potential** through the introduction of an effective Lagrangian that incorporates broken scale in addition to explicit broken chiral symmetry

$$\tilde{\mathscr{L}} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{4}\boldsymbol{B}_{\mu\nu}\cdot\boldsymbol{B}^{\mu\nu} + \frac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu + \frac{1}{2}G_{b\phi}\phi^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu$$

$$+\left[(G_4)^2\omega_\mu\omega^\mu\right]^2 - \tilde{\mathscr{V}} + \bar{N}\left[\gamma^\mu\left(i\partial_\mu - g_\omega\omega_\mu - \frac{1}{2}g_\rho\boldsymbol{b}_\mu\cdot\boldsymbol{\tau}\right) - g\sqrt{\sigma^2+\pi^2}\right]N$$

$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2}B_0\delta\chi^4\ln\frac{\sigma^2+\pi^2}{\sigma_0^2} + \frac{1}{2}B_0\delta\chi^2\left[\frac{\sigma^2+\pi^2}{\sigma_0} - \frac{\chi^2}{2}\right] - \frac{1}{4}\epsilon_1'\chi^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) - \chi^2\right] - \frac{3}{4}\epsilon_1'.$$

Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$
$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

Meson and baryon sector at finite chemical potential

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\, P(z_i, \langle \Delta_i^2 \rangle)\, A(\mathbf{\Phi}_i + z_i)$$

$$\mathbf{\Phi} = (\phi, \sigma, \pi_0, \pi_+, \pi_-)$$

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\left(\bar{\chi}^2 + \langle \Delta_\chi^2 \rangle\right)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\left(\bar{\chi}^2 + \langle \Delta_\chi^2 \rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3 \ln\chi - 2B_0\,\delta\chi^3 \ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$

$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle \Delta\omega_\mu \Delta\omega^\mu \rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle \Delta b_\mu \Delta b^\mu \rangle\right) = 0$$

Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\left(\bar\chi^2 + \langle\Delta_\chi^2\rangle\right)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\left(\bar\chi^2 + \langle\Delta_\chi^2\rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$

$$-2\,\epsilon_1'\,\bar\chi\,\frac{\bar\sigma}{\sigma_0} - m_\omega^2\,\bar\chi\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar\chi\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\,P(z_i,\langle\Delta_i^2\rangle)\,A(\mathbf{\Phi}_i + z_i)$$

$$\mathbf{\Phi} = (\phi,\sigma,\pi_0,\pi_+,\pi_-)$$

$$\langle\Delta_i^2\rangle = \frac{n_i}{2\pi^2}\int_0^\infty \frac{k^2}{\epsilon_i^*}\,\frac{1}{e^{\beta(\epsilon_i^*-\mu_i^*)}-1}\,dk$$

$$\epsilon_i^* = \sqrt{k^2 + m_i^{*2}}$$

$$n_i = \begin{cases} 1 & \text{for } \phi,\ \sigma, \pi_0,\ \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu,\ b_{0\mu},\ b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$

$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\,P(z_i, \langle\Delta_i^2\rangle)\,A(\mathbf{\Phi}_i + z_i)$$

$$\mathbf{\Phi} = (\phi, \sigma, \pi_0, \pi_+, \pi_-)$$

$$\langle\Delta_i^2\rangle = \frac{n_i}{2\pi^2}\int_0^\infty \frac{k^2}{\epsilon_i^*}\,\frac{1}{e^{\beta(\epsilon_i^*-\mu_i^*)}-1}\,dk$$

$$\epsilon_i^* = \sqrt{k^2 + m_i^{*2}}$$

$$n_i = \begin{cases} 1 & \text{for } \phi,\ \sigma, \pi_0,\ \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu,\ b_{0\mu},\ b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

$$m_\pi = m_\pi(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\sigma = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\chi = m_\chi(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\rho = m_\rho(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_N = m_N(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$

Meson and baryon sector at finite chemical potential

Meson and baryon sector: phase diagram

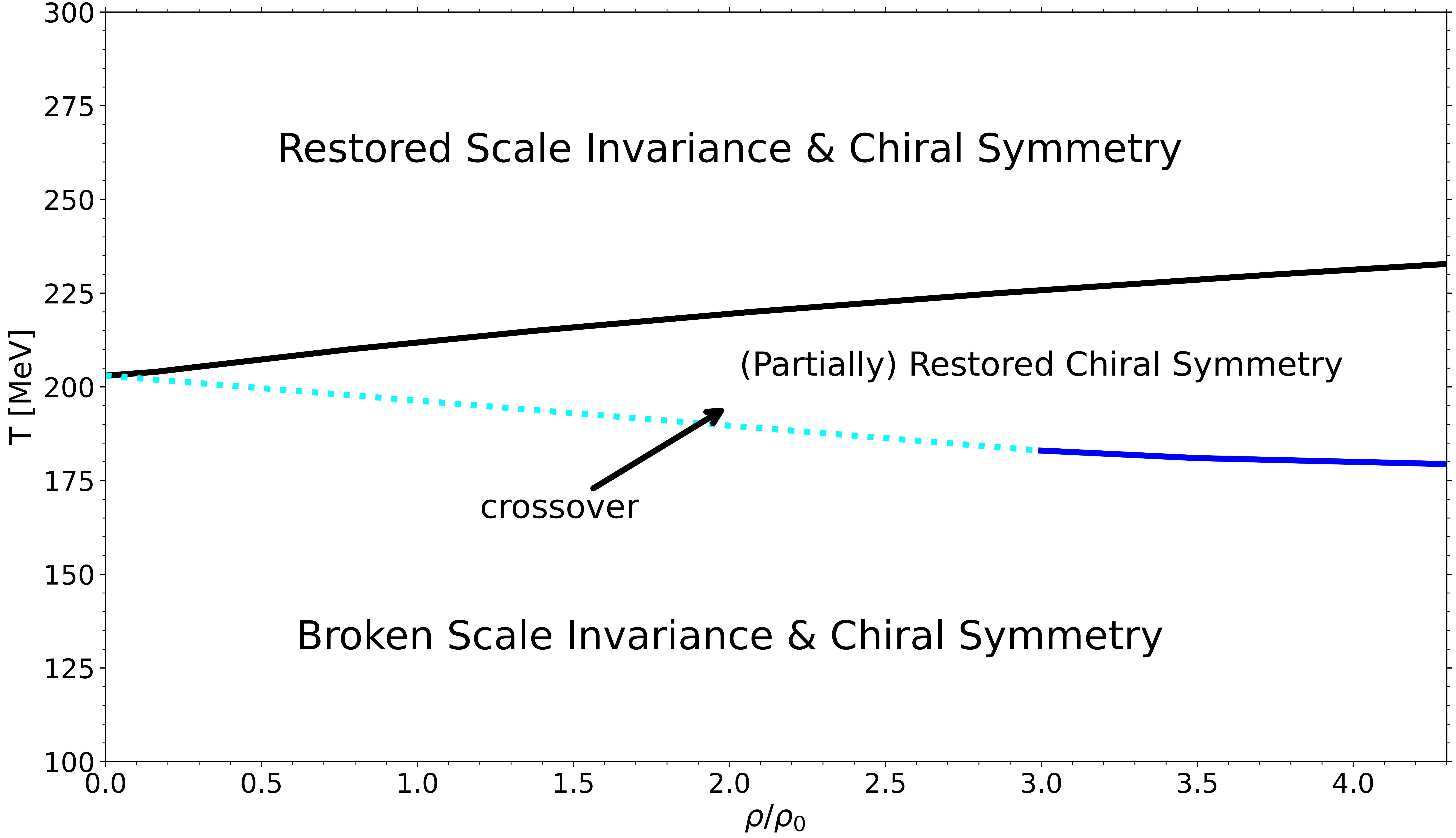


Table of Contents
Summary

Summary

- Development of a numerical pipeline for **RMF predictions** of the dense-matter EOS and **Bayesian analysis** for the **two-family scenario**

- Phenomenological approach **reproduces main EOS lattice QCD** results for pure gauge $SU(3)_c$ thermodynamics, consistently showing a **first-order phase transition**, **solving** the **stability problems** of the complex potential.

- A model beyond mean field has been developed, including **mesonic** and **baryonic sectors**, enabling the exploration of a **wide range** of **temperatures** and **densities**, including **thermodynamic fluctuations**.

Table of Contents
Outlooks

Outlooks

Outlooks

- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon

Outlooks

- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon
- **QCD phase diagram** in a large range of temperature and densities

Outlooks

- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon
- **QCD phase diagram** in a large range of temperature and densities
- High-energy astrophysical phenomena:
  - **neutron star** structure (**two family scenario** through Bayesian analysis)

Outlooks

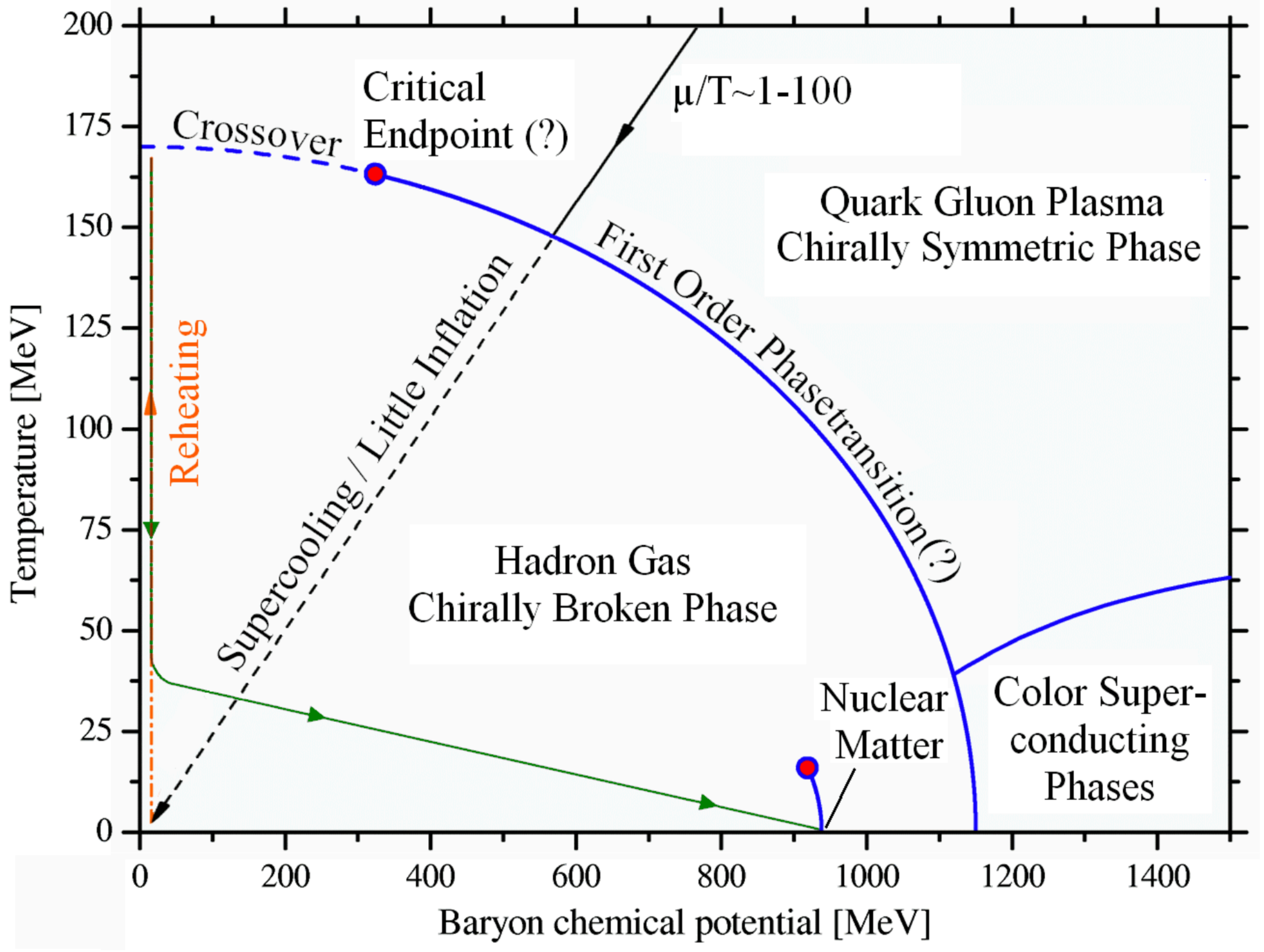
- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon
- **QCD phase diagram** in a large range of temperature and densities
- High-energy astrophysical phenomena:
  - **neutron star** structure (**two family scenario** through Bayesian analysis)
  - Using future observations in the **two family scenario**

Outlooks

- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon
- **QCD phase diagram** in a large range of temperature and densities
- High-energy astrophysical phenomena:
  - **neutron star** structure (**two family scenario** through Bayesian analysis)
  - Using future observations in the **two family scenario**
  - Testing the coexistence of **hybrid stars** and **strange quark stars** with Bayesian inference

Outlooks

- Take into account new degrees of freedom:
  - $SU(3)_f$ **quarks**
  - hadronic phase with **strange** meson and baryon
- **QCD phase diagram** in a large range of temperature and densitie
- High-energy astrophysical phenomena:
  - **neutron star** structure (**two family scenario** through Bayesian analysis)
  - Using future observations in the **two family scenario**
  - Testing the coexistence of **hybrid stars** and **strange quark stars** with Bayesian inference
  - **cosmological trajectories (little inflation scenario)** in the QCD epoch of the early universe



J. Schaffner-Bielich et al., Phys.Rev.D 85, 103506 (2012)

Thank you

Backup slides

Two family scenario

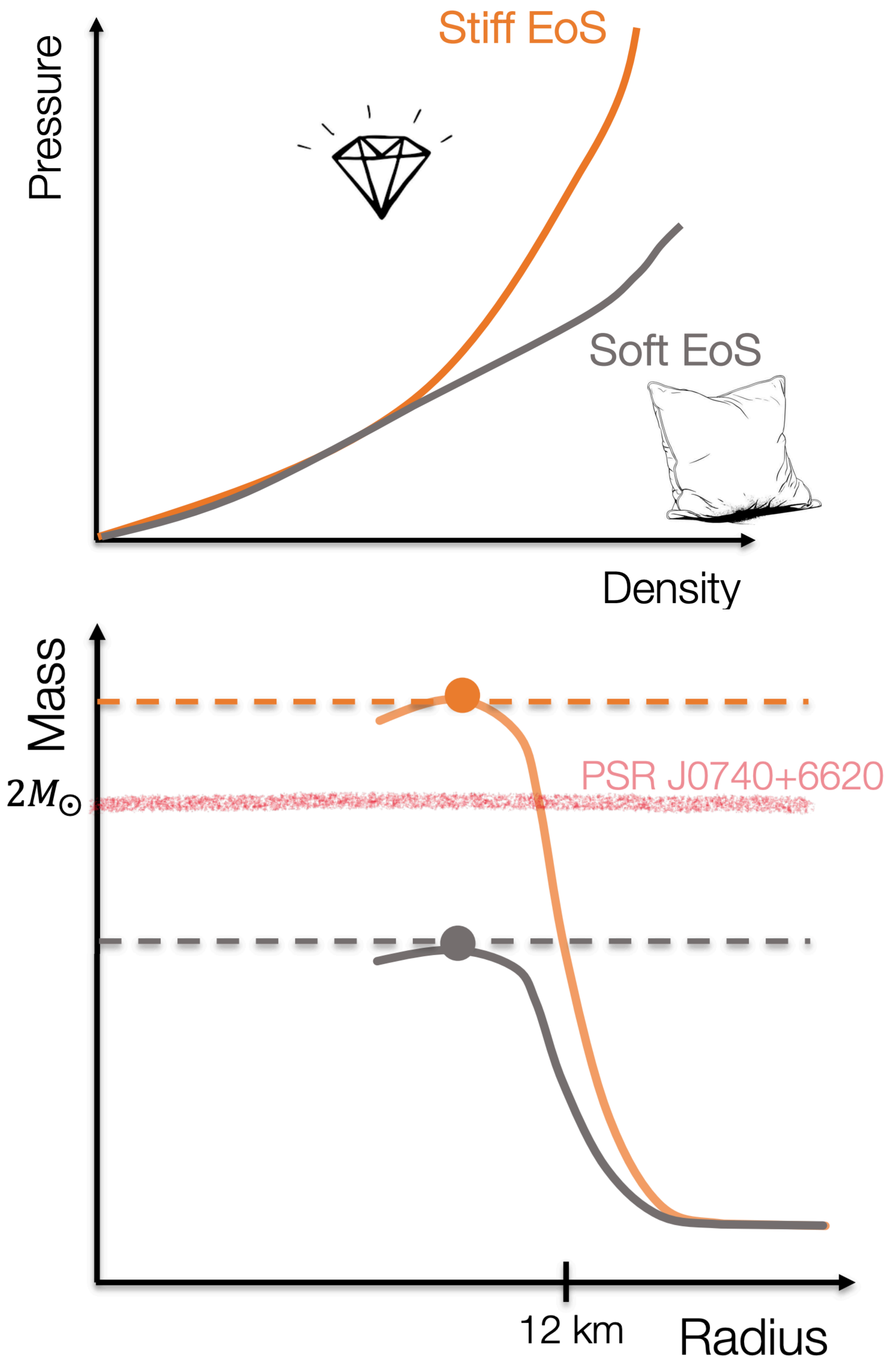
Two family scenario

- **Hyperon and Delta Puzzle**:

  expected appearance of hyperons ($> 2-3\ n_{\rm sat}$) and Δ-resonance significantly softens the EoS *[Lavagno et al. Phys.Rev.C (2014)]*.

- Observational Conflict:

  soft EoSs fail to support observed $\sim 2M_\odot$ NS, which mandate a stiff EoS.



Two family scenario

- **Hyperon and Delta Puzzle**:

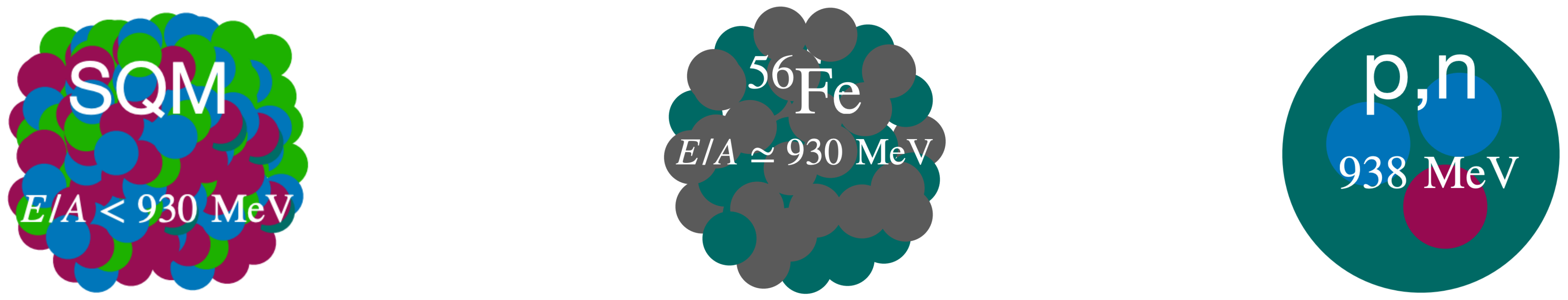
  expected appearance of hyperons ($> 2 - 3\ n_{sat}$) and Δ-resonance significantly softens the EoS *[Lavagno et al. Phys.Rev.C (2014)]*.

- Observational Conflict:

  soft EoSs fail to support observed $\sim 2M_{\odot}$ NS, which mandate a stiff EoS.

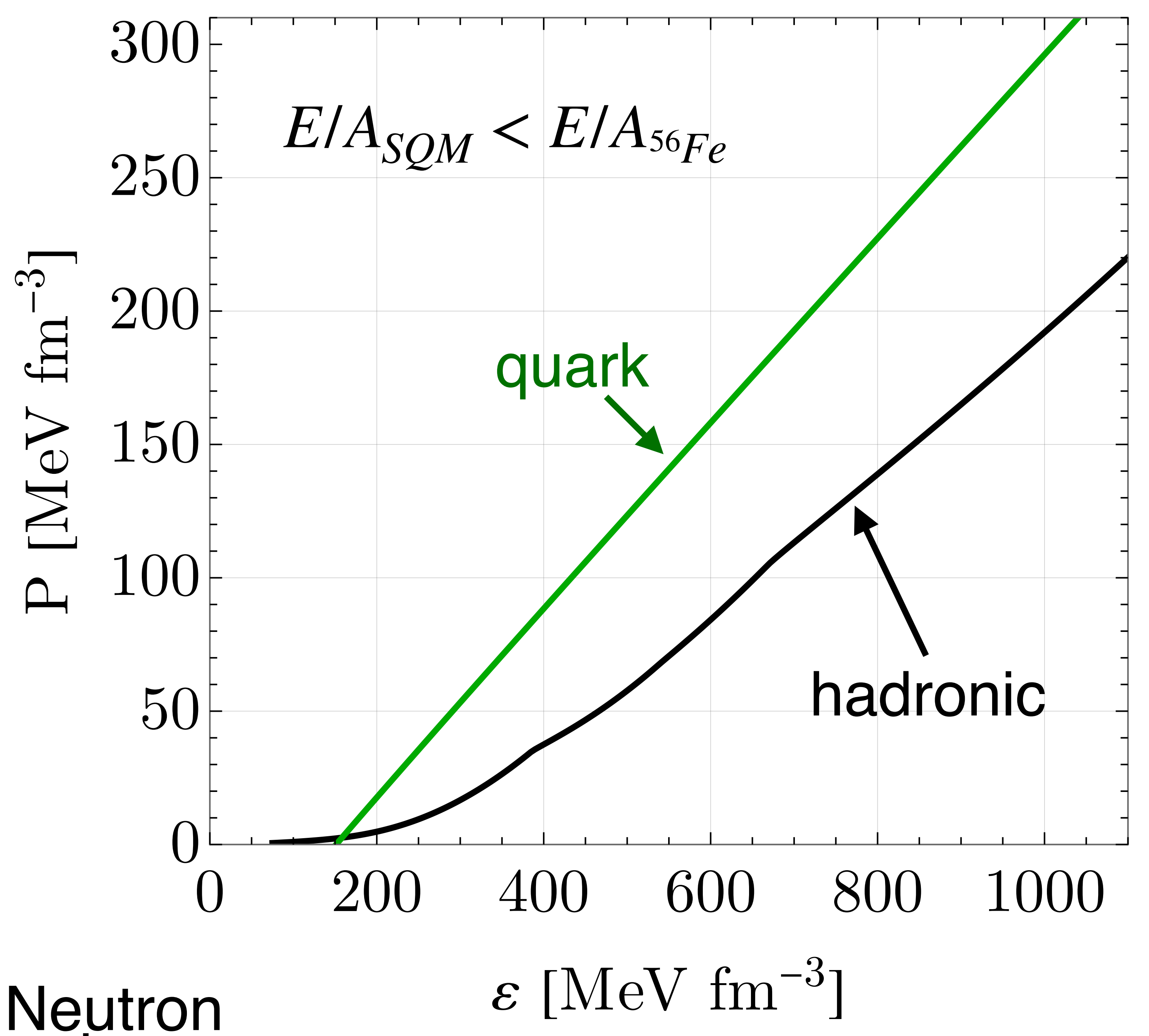
The **Two-Family Scenario**

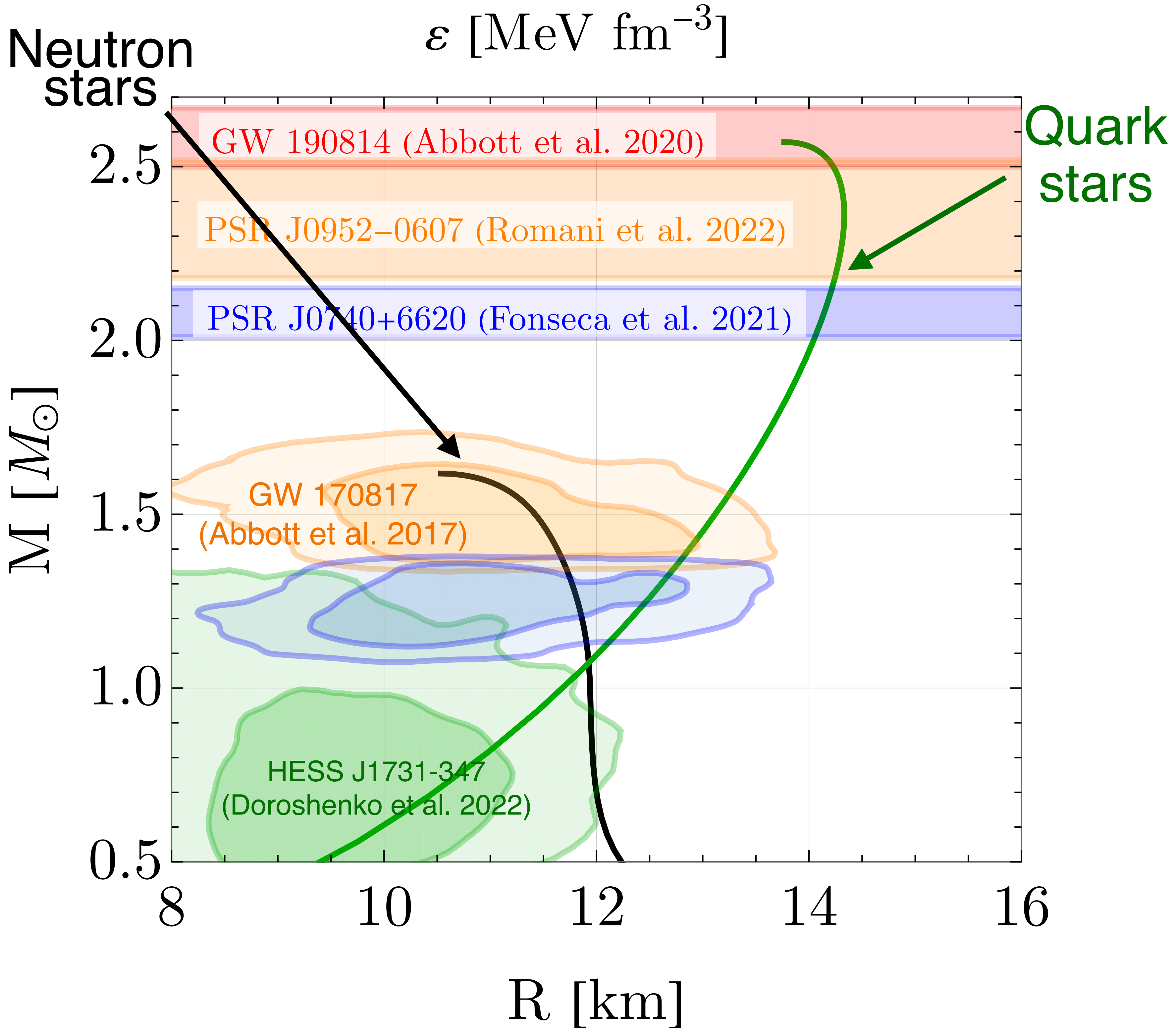
- Based on Bodmer-Witten hypothesis



- Mechanism: a strong, **first-order phase transition** in the core.
- Creates two disconnected, stable Mass-Radius branches:
  - **Hadronic:** Soft branch, $M_{max} < 2M_{\odot}$.
  - **Quark:** Stiff branch, $M_{max} \geq 2M_{\odot}$.
- Once reached deconfinement conditions, **HS** converts to **QS**

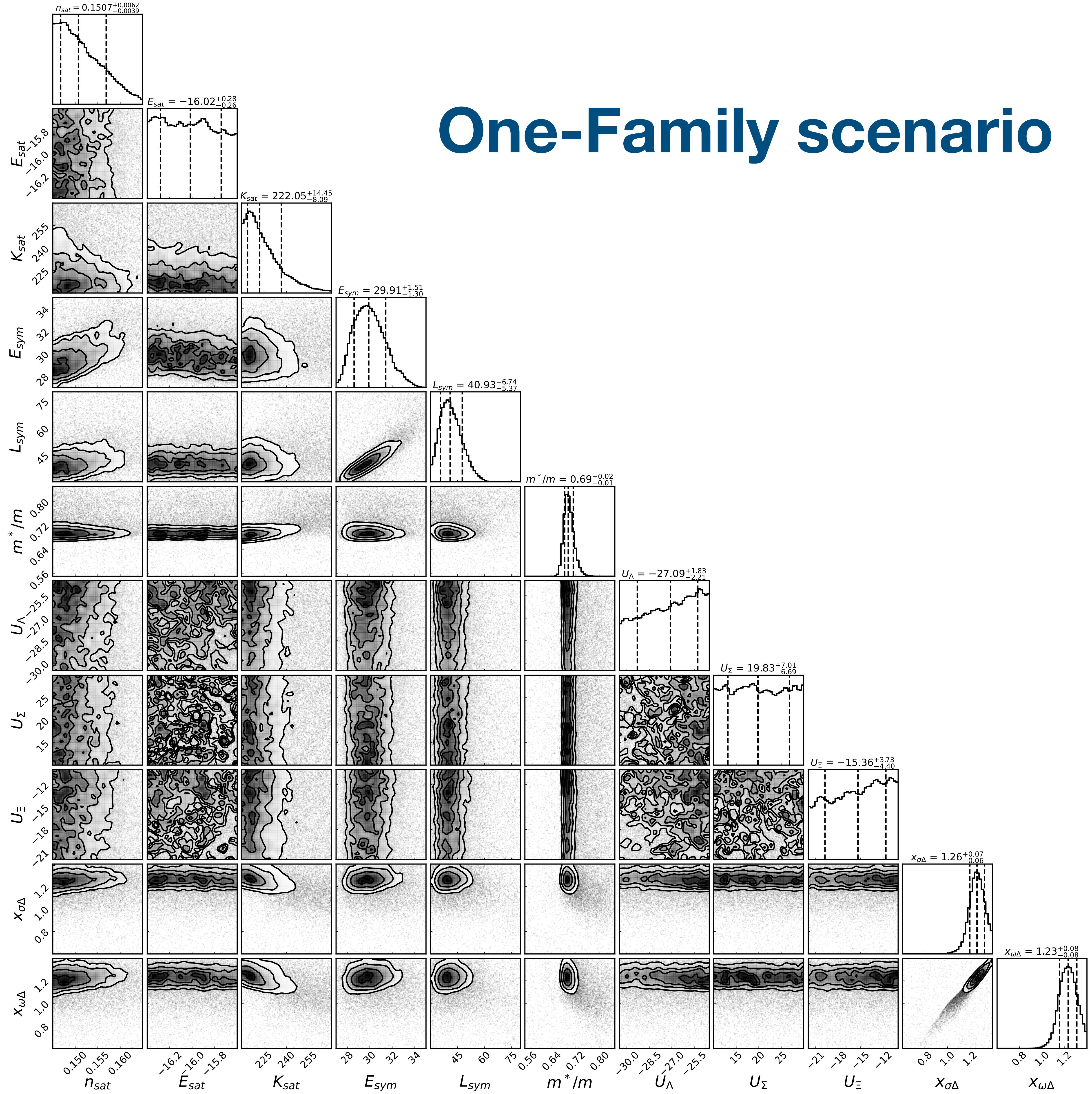
[Astrophys.J. 974 (2024); Universe 11 (2025) 258; JHEAp 50 (2026)]



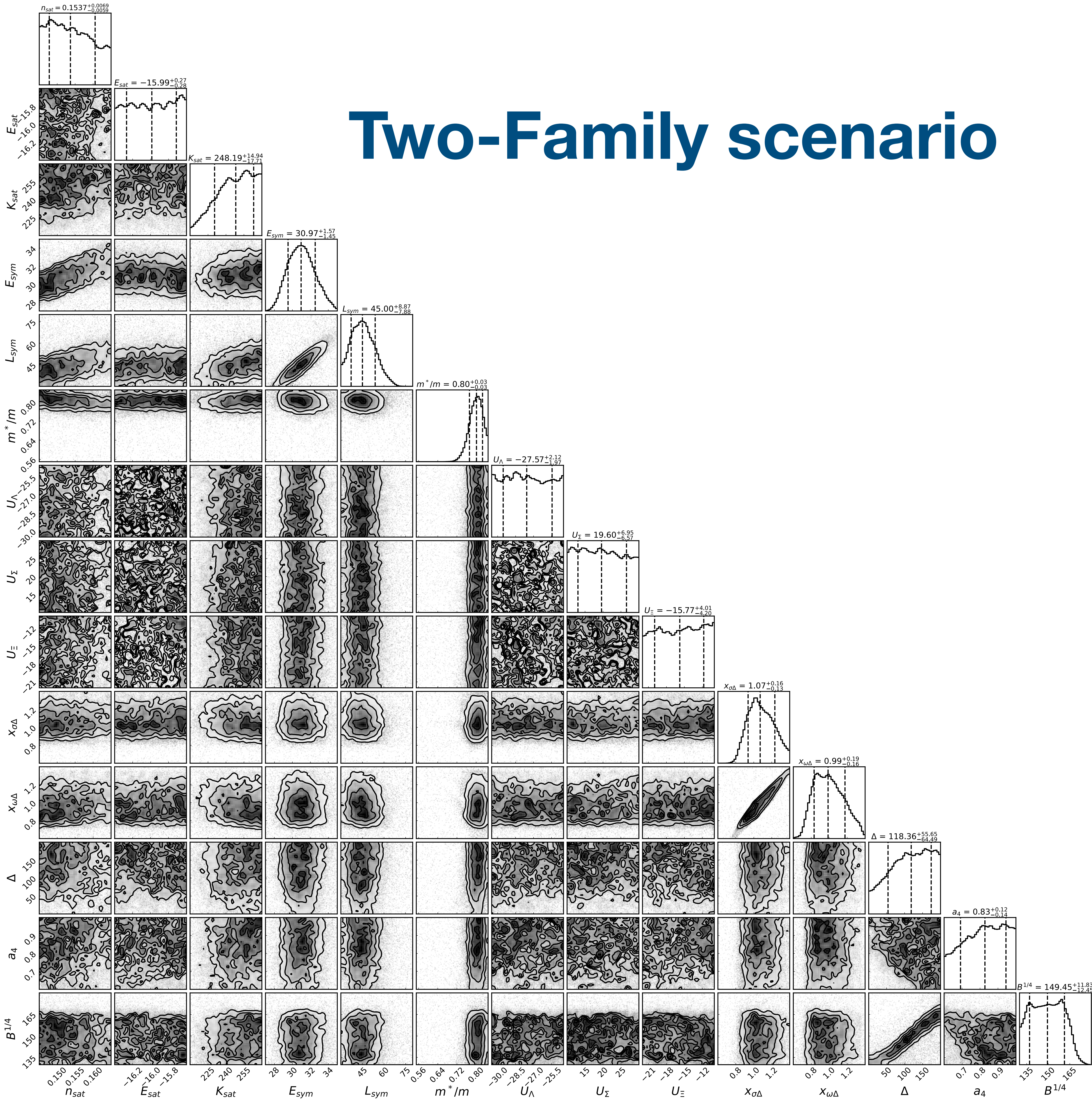


RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario



Two-Family scenario



RMF predictions for the dense-matter EOS and application to NSs

$$\mathscr{L}^{(2F)}_{\mathrm{GW170817}}(\theta) \propto \int_{M^{(2F)}_{\mathrm{NS,min}}(\theta)}^{M^{(2F)}_{\mathrm{NS,max}}(\theta)} dM_1 \int_{M^{(2F)}_{\mathrm{NS,min}}(\theta)}^{M^{(2F)}_{\mathrm{NS,max}}(\theta)} dM_2\, q_{\mathrm{GW170817}}\left[M_1, M_2, \Lambda^{\mathrm{TOV}}_{\mathrm{NS}}(M_1;\theta), \Lambda^{\mathrm{TOV}}_{\mathrm{NS}}(M_2;\theta)\right]\ \eta^{(2F)}_{\mathrm{NS\text{-}NS}}(M_1, M_2;\theta)$$

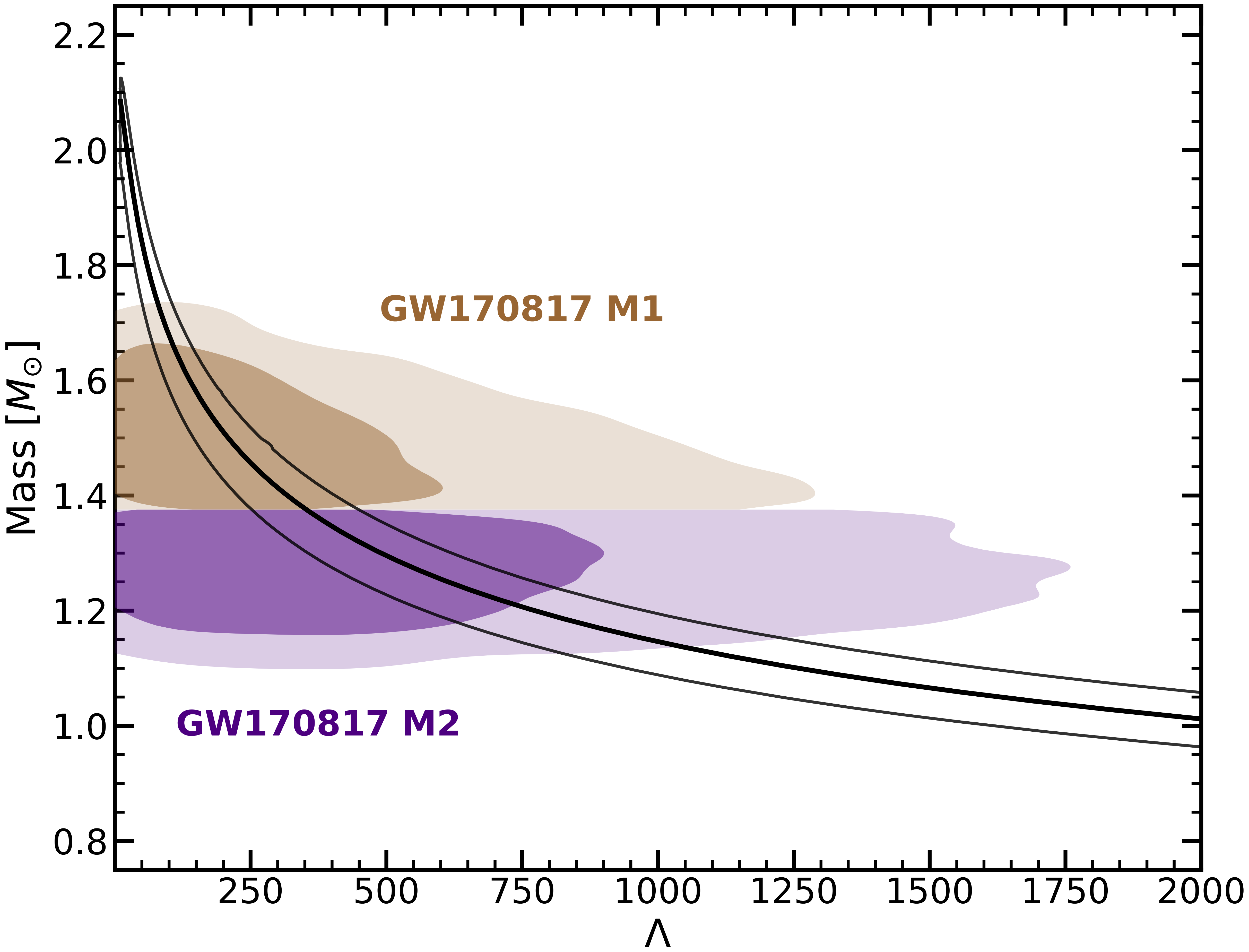
$$+ \int_{M^{(2F)}_{\mathrm{NS,min}}(\theta)}^{M^{(2F)}_{\mathrm{NS,max}}(\theta)} dM_1 \int_{M^{(2F)}_{\mathrm{QS,min}}(\theta)}^{M^{(2F)}_{\mathrm{QS,max}}(\theta)} dM_2\, q_{\mathrm{GW170817}}\left[M_1, M_2, \Lambda^{\mathrm{TOV}}_{\mathrm{NS}}(M_1;\theta), \Lambda^{\mathrm{TOV}}_{\mathrm{QS}}(M_2;\theta)\right]\ \eta^{(2F)}_{\mathrm{NS\text{-}QS}}(M_1, M_2;\theta)$$

$$+ \int_{M^{(2F)}_{\mathrm{QS,min}}(\theta)}^{M^{(2F)}_{\mathrm{QS,max}}(\theta)} dM_1 \int_{M^{(2F)}_{\mathrm{NS,min}}(\theta)}^{M^{(2F)}_{\mathrm{NS,max}}(\theta)} dM_2\, q_{\mathrm{GW170817}}\left[M_1, M_2, \Lambda^{\mathrm{TOV}}_{\mathrm{QS}}(M_1;\theta), \Lambda^{\mathrm{TOV}}_{\mathrm{NS}}(M_2;\theta)\right]\ \eta^{(2F)}_{\mathrm{QS\text{-}NS}}(M_1, M_2;\theta)$$

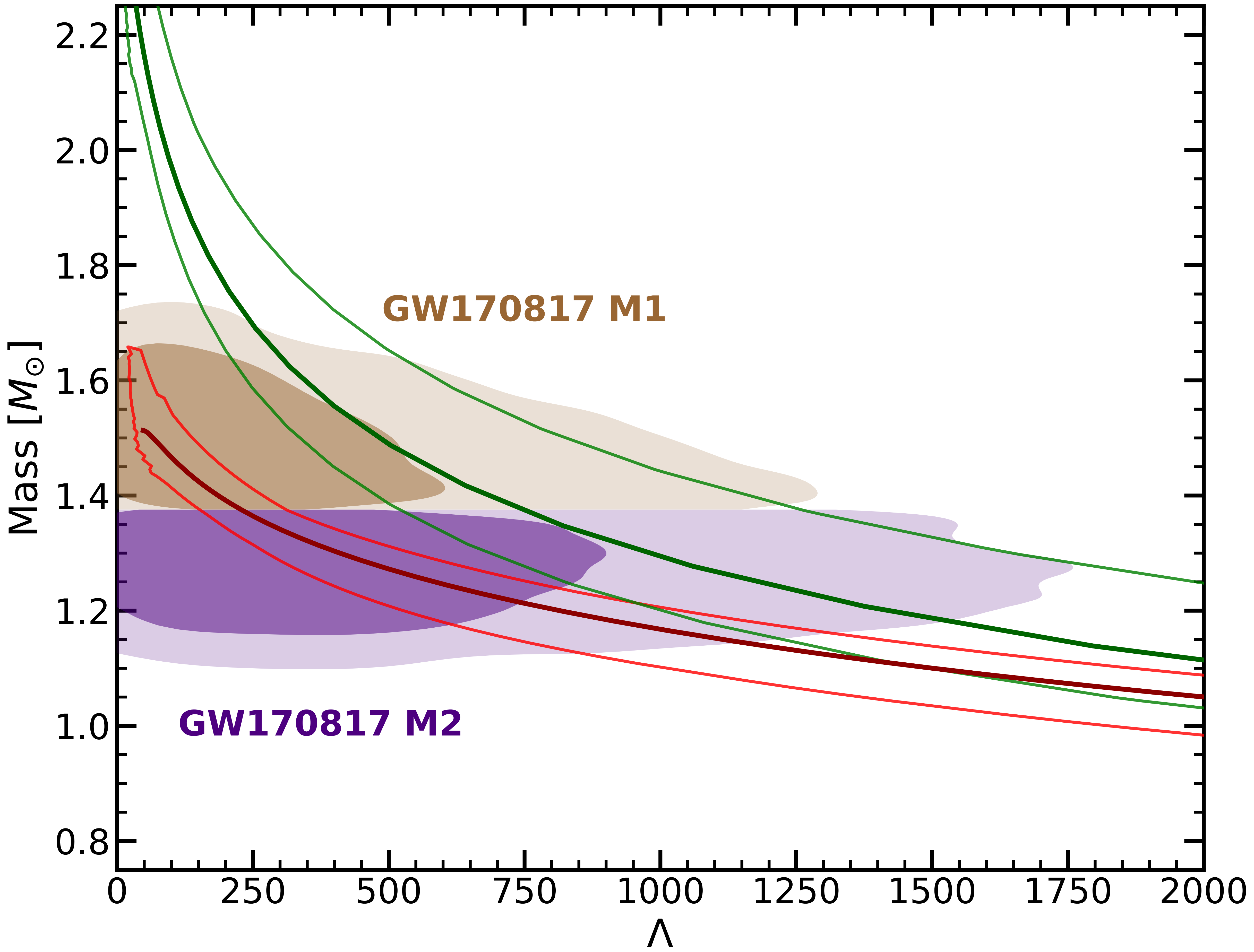
$$+ \int_{M^{(2F)}_{\mathrm{QS,min}}(\theta)}^{M^{(2F)}_{\mathrm{QS,max}}(\theta)} dM_1 \int_{M^{(2F)}_{\mathrm{QS,min}}(\theta)}^{M^{(2F)}_{\mathrm{QS,max}}(\theta)} dM_2\, q_{\mathrm{GW170817}}\left[M_1, M_2, \Lambda^{\mathrm{TOV}}_{\mathrm{QS}}(M_1;\theta), \Lambda^{\mathrm{TOV}}_{\mathrm{QS}}(M_2;\theta)\right]\ \eta^{(2F)}_{\mathrm{QS\text{-}QS}}(M_1, M_2;\theta)$$

RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario

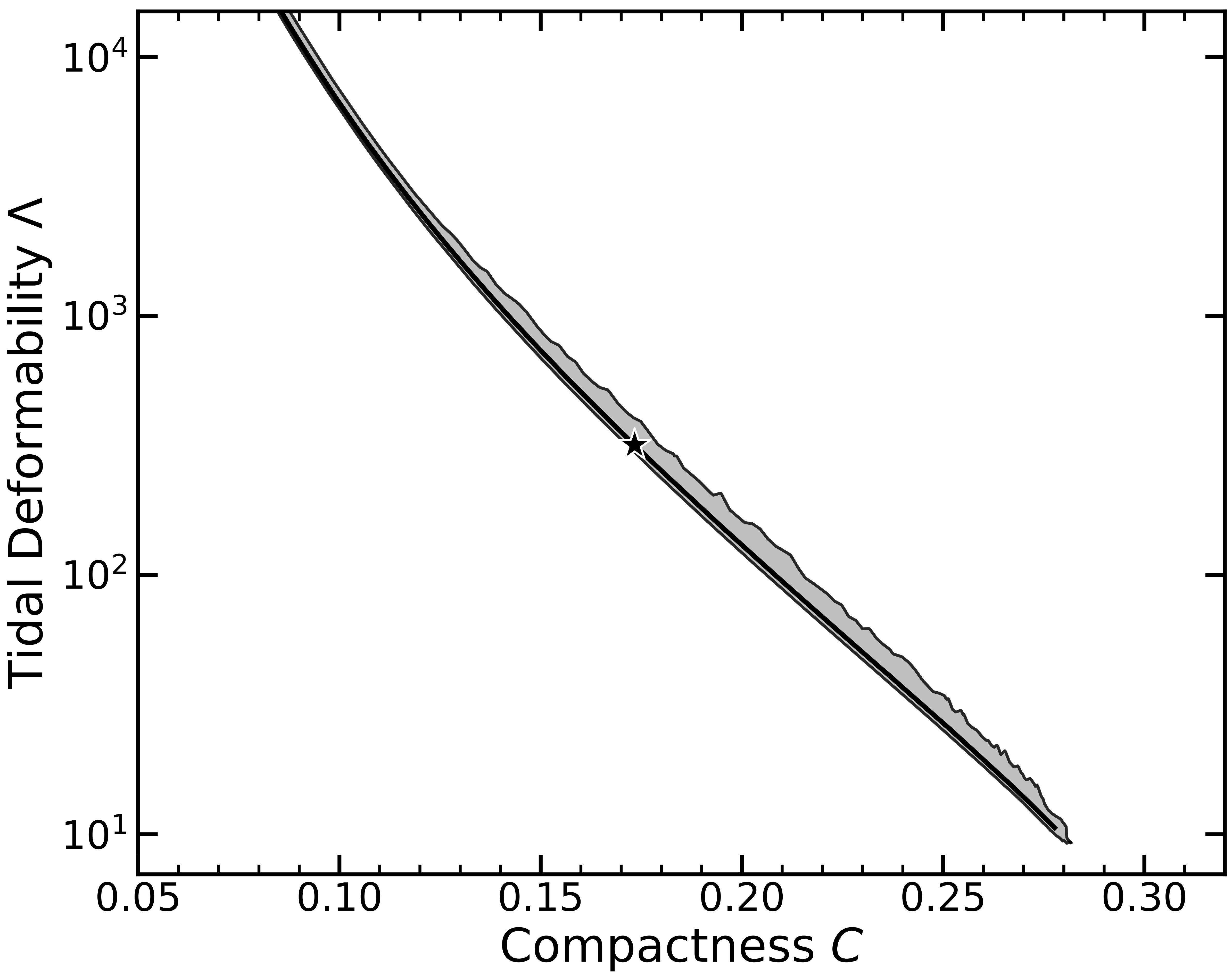


Two-Family scenario

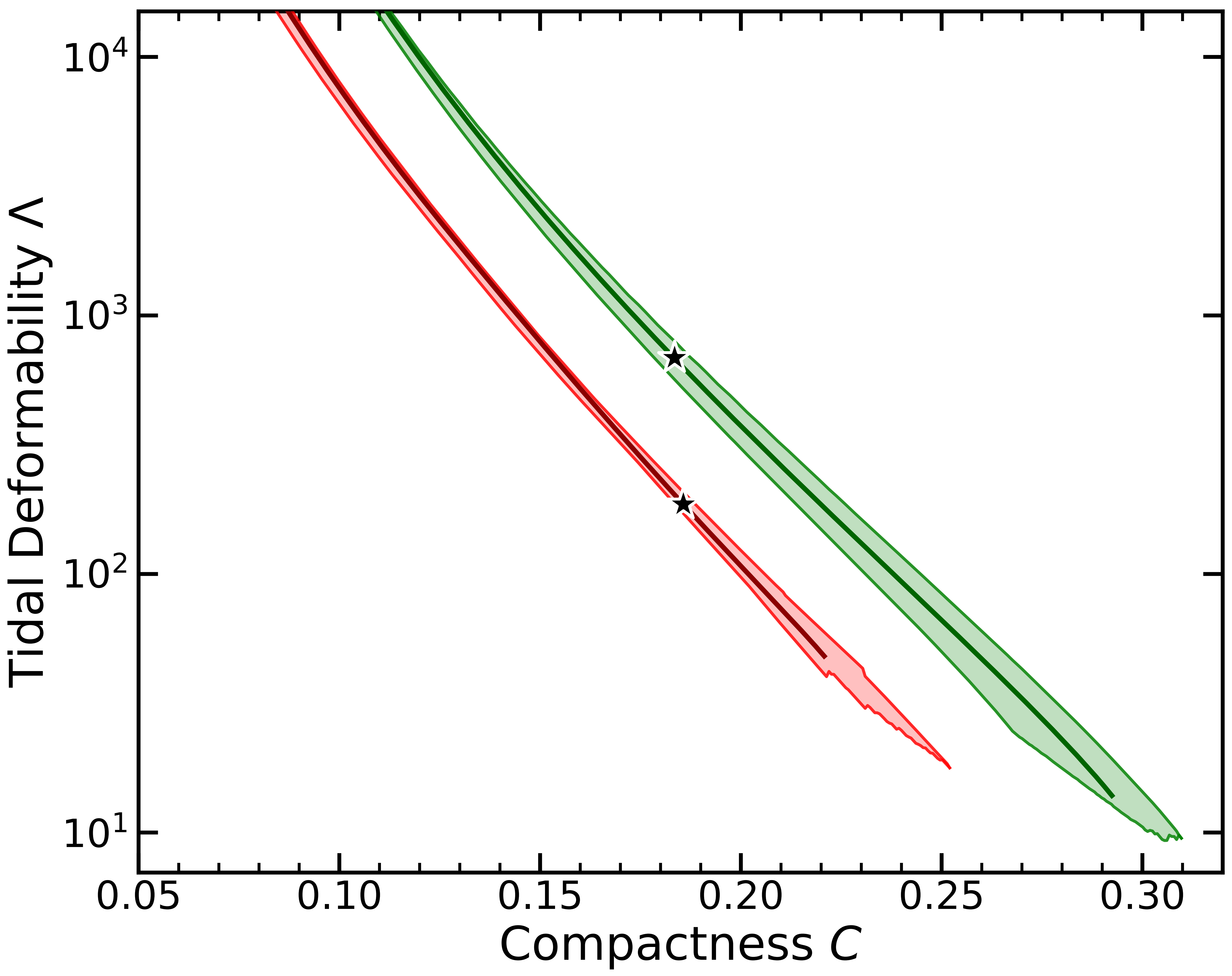


RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario

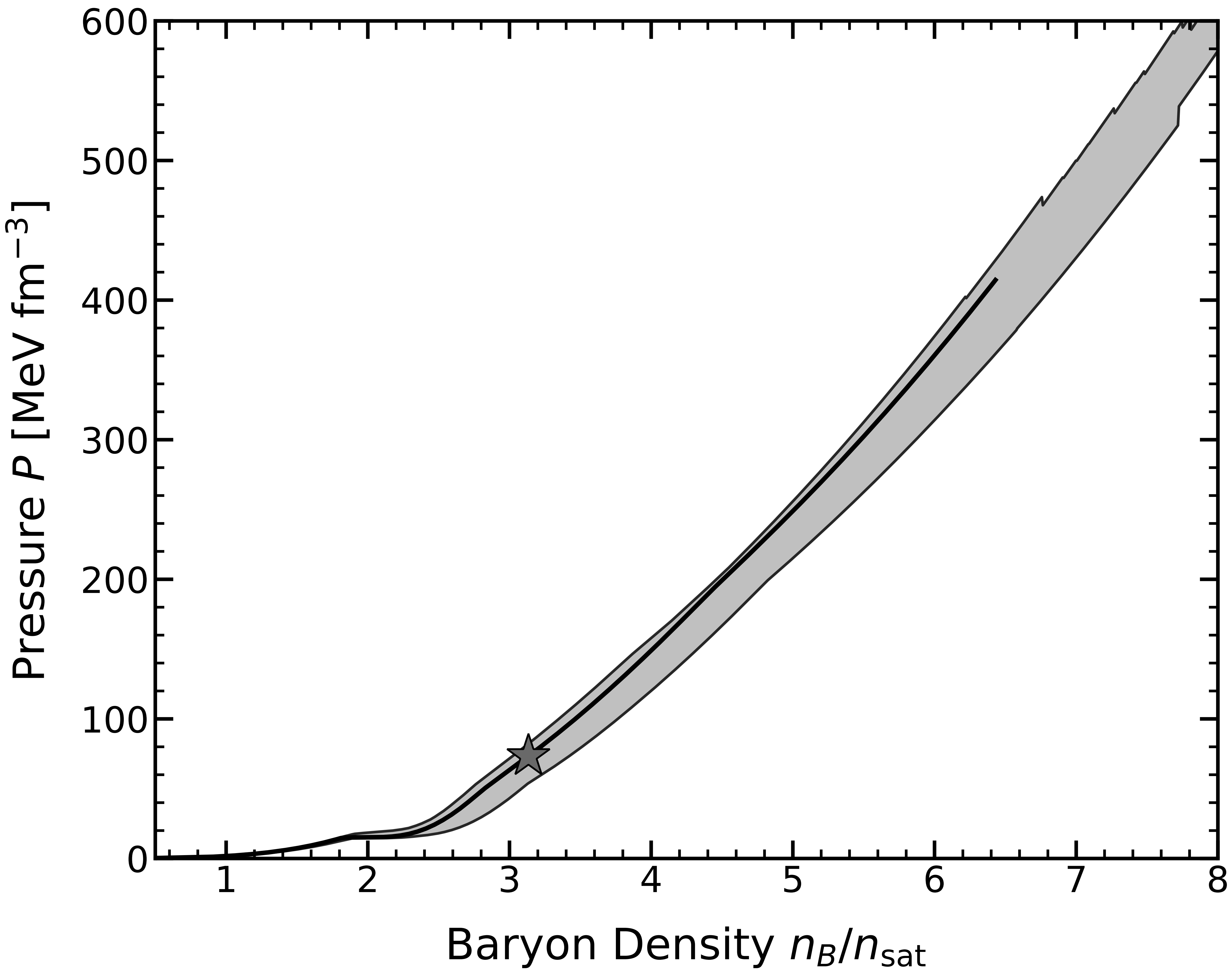


Two-Family scenario

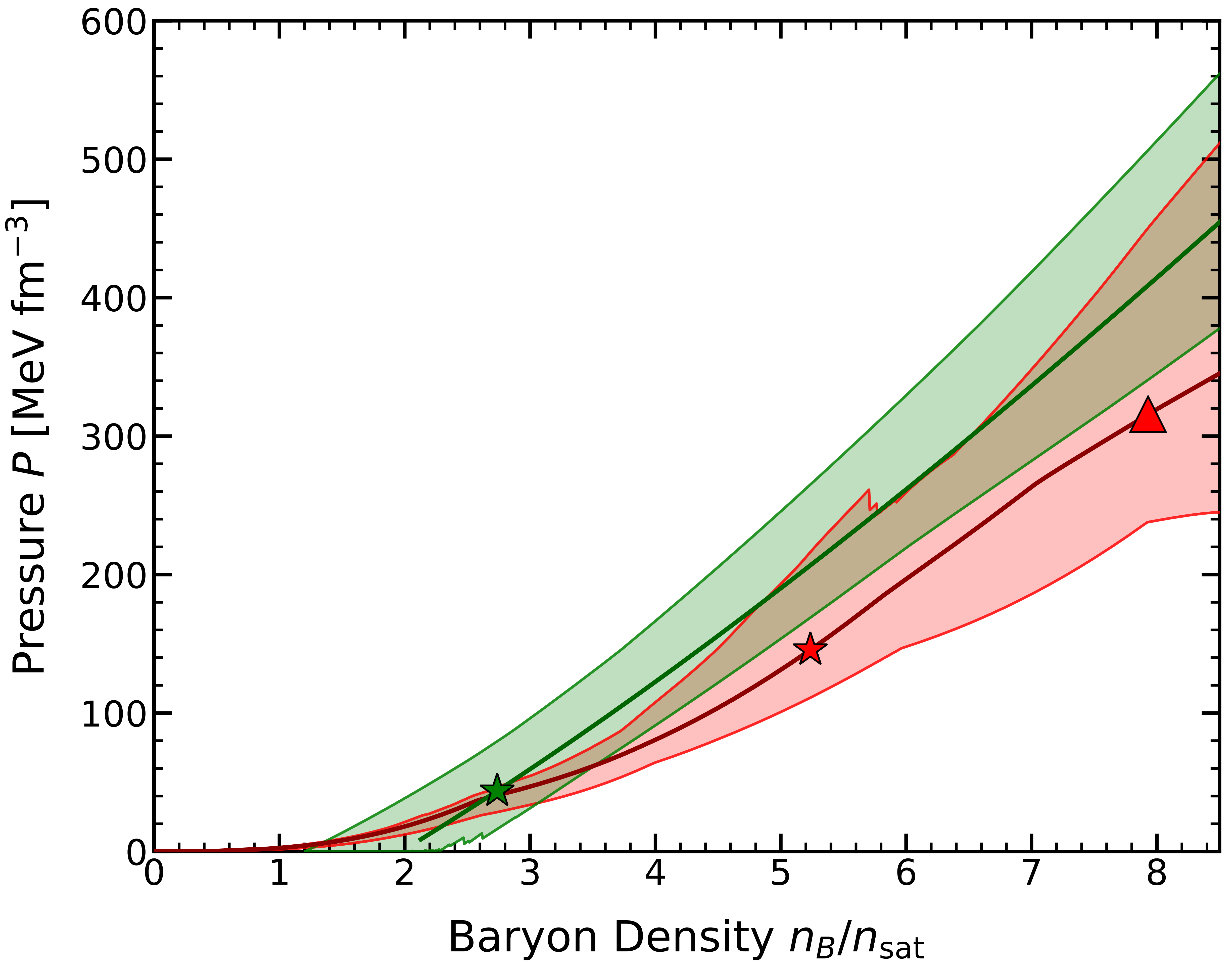


RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario

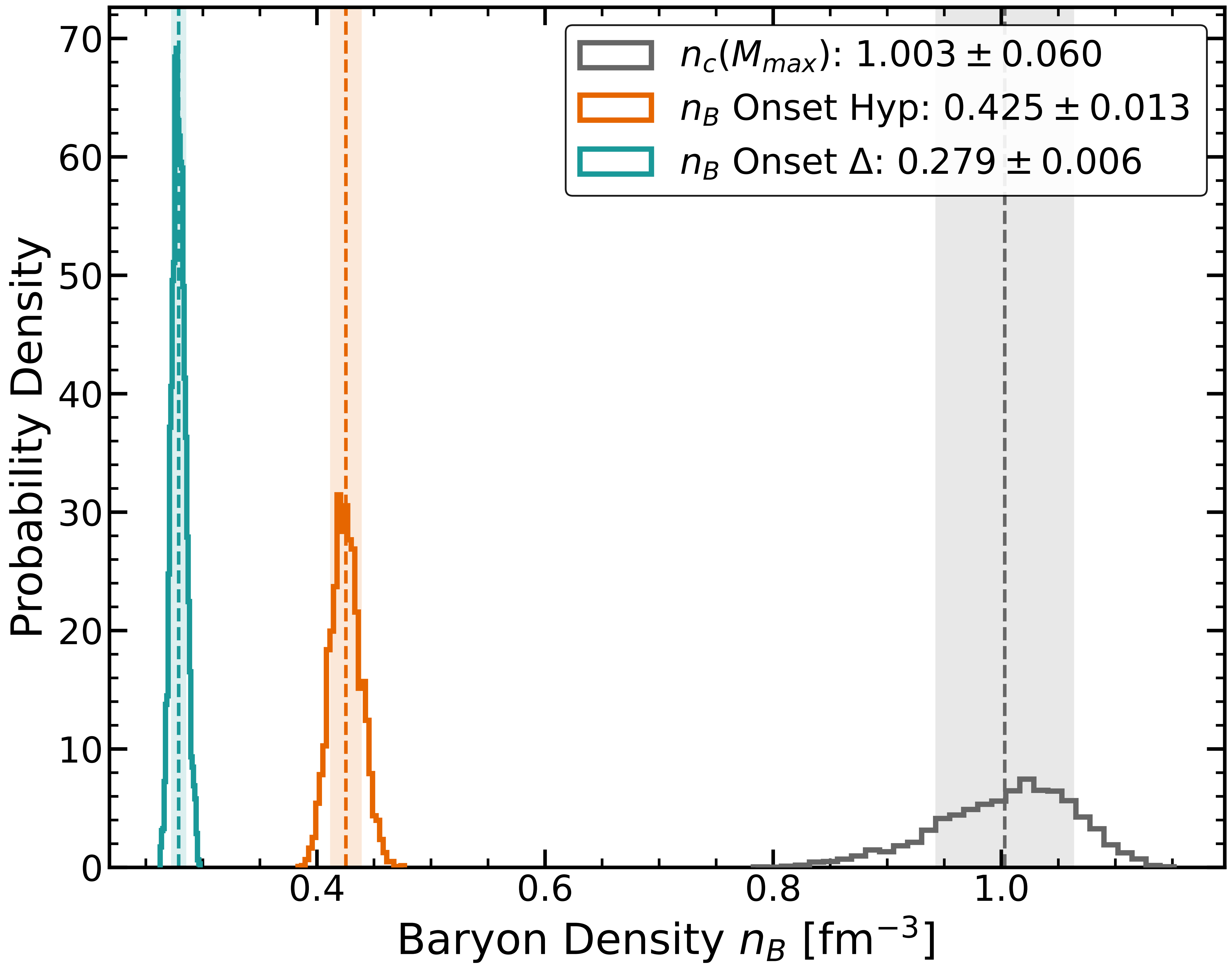


Two-Family scenario

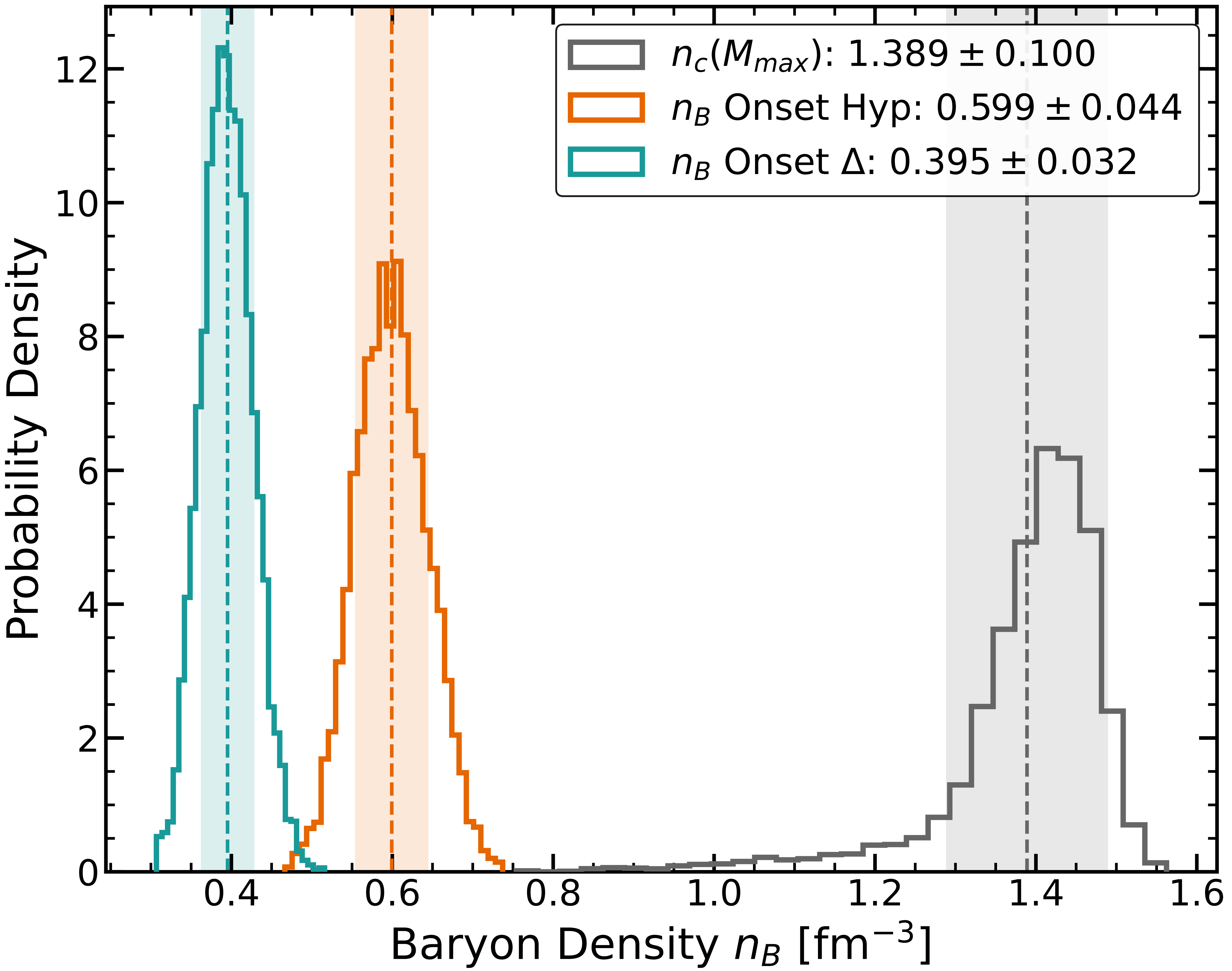


RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario

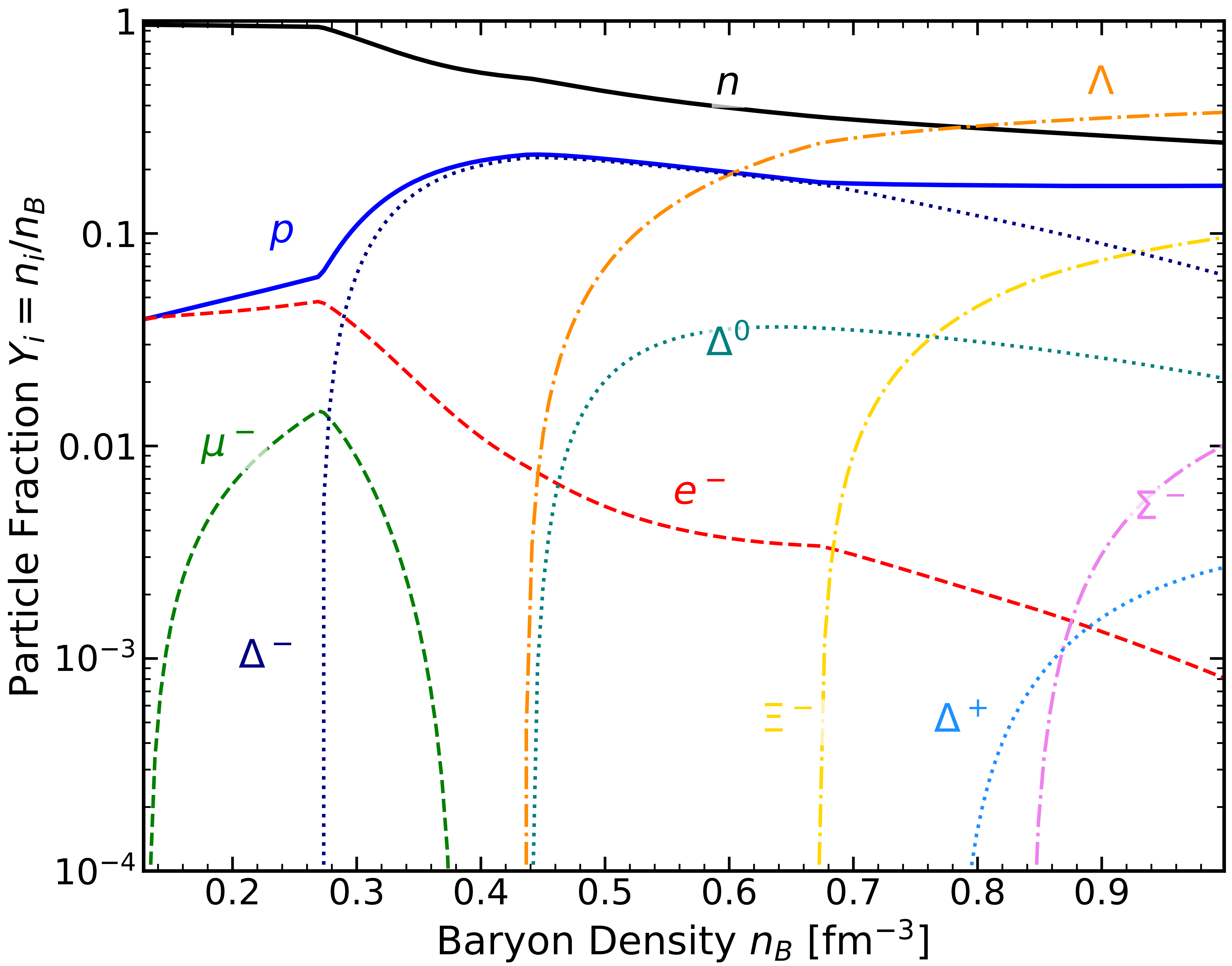


Two-Family scenario

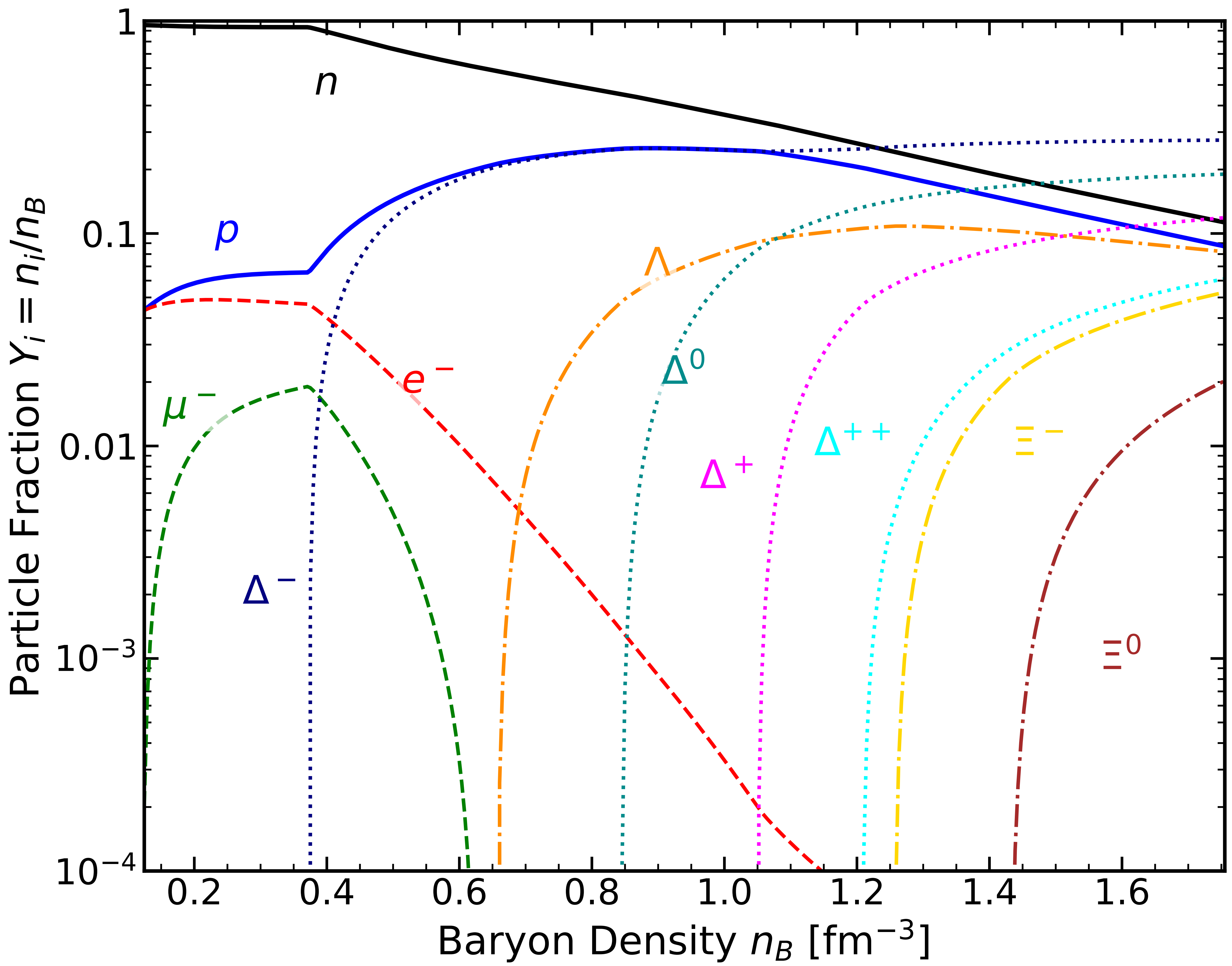


RMF predictions for the dense-matter EOS and application to NSs

One-Family scenario



Two-Family scenario



Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

$$0 \lesssim x_{\sigma\Delta} - x_{\omega\Delta} \lesssim 0.2$$

$$V_N - 30\ \mathrm{MeV} \lesssim \mathrm{V}_\Delta \lesssim \mathrm{U}_\mathrm{N}$$

$$\mu_Q(P=0) = \left.\frac{\varepsilon_Q}{n_{B,Q}}\right|_{P=0} < 930\ \mathrm{MeV}$$

$$\mu_Q(P) < \mu_H(P)$$

Category	Label	Info	Likelihood representation
M	PSR J0952−0607	$M = 2.24 \pm 0.17\ \mathrm{M}_\odot$	Gaussian in M
M-R	PSR J0030+0451	$M = 1.44^{+0.15}_{-0.14}\ \mathrm{M}_\odot,\ R = 13.02^{+1.24}_{-1.06}$ km	$q(M,R)$ KDE from posterior samples
M-R	PSR J0740+6620	$M = 2.08 \pm 0.07\ \mathrm{M}_\odot,\ R = 13.7^{+2.6}_{-1.5}$ km	$q(M,R)$ KDE from posterior samples
M-R	PSR J0614−3329	$M = 1.44^{+0.06}_{-0.07}\ \mathrm{M}_\odot,\ R = 10.29^{+1.01}_{-0.86}$ km	$q(M,R)$ KDE from posterior samples
M-R	HESS J1731−347	$M = 0.77^{+0.20}_{-0.17}\ \mathrm{M}_\odot,\ R = 10.4^{+0.86}_{-0.78}$ km	$q(M,R)$ KDE from posterior samples
M-Λ	GW 170817	$\mathcal{M} = 1.186 \pm 0.001\quad \tilde{\Lambda} = 300^{+420}_{-230}$	$q(M_1, M_2, \Lambda_1, \Lambda_2)$ KDE from posterior samples
n_B-E_H^{PNM}	χEFT	PNM band for $E_H^{\mathrm{PNM}}(n_B)$, $n_B \leq n_{\mathrm{sat}}$	soft band likelihood from Passarella, Phys.Rev.C 112 (2025)
n_B-P_H^{SNM}	HIC	SNM band for $P_H^{\mathrm{SNM}}(n_B)$, $n_B \leq 3\, n_{\mathrm{sat}}$	soft band likelihood from Danielewicz, Science 298 (2002)

Testing NS–QS coexistence via Bayesian Inference

Parameter	Prior [min, max]	Marg. Posterior 1F (Median $\pm 1\sigma$)	Max. Likelihood 1F	Marg. Posterior 2F (Median $\pm 1\sigma$)	Max. Likelihood 2F
$n_{\rm sat}$ (fm^{-3})	$[0.145, 0.165]$	$0.1506^{+0.0061}_{-0.0039}$	0.1499	$0.1540^{+0.0065}_{-0.0058}$	0.1562
$E_{\rm sat}$ (MeV)	$[-16.4, -15.6]$	$-16.01^{+0.27}_{-0.27}$	-16.24	$-15.97^{+0.25}_{-0.28}$	-15.89
$K_{\rm sat}$ (MeV)	$[210.0, 270.0]$	$222.52^{+15.64}_{-8.49}$	214.22	$246.21^{+15.92}_{-17.92}$	254.84
$E_{\rm sym}$ (MeV)	$[27.0, 35.0]$	$29.85^{+1.49}_{-1.32}$	28.89	$31.01^{+1.52}_{-1.40}$	31.54
$L_{\rm sym}$ (MeV)	$[30.0, 80.0]$	$40.57^{+6.58}_{-5.37}$	35.69	$44.96^{+8.43}_{-7.77}$	48.13
m^*/m	$[0.55, 0.85]$	$0.70^{+0.02}_{-0.01}$	0.68	$0.81^{+0.02}_{-0.03}$	0.81
U_Λ (MeV)	$[-30.5, -24.5]$	$-27.12^{+1.83}_{-2.15}$	-26.18	$-27.48^{+2.04}_{-2.05}$	-24.66
U_Σ (MeV)	$[10.0, 30.0]$	$20.15^{+6.87}_{-6.76}$	13.89	$19.85^{+7.01}_{-6.68}$	13.12
U_Ξ (MeV)	$[-22.0, -10.0]$	$-15.59^{+3.86}_{-4.20}$	-11.00	$-15.95^{+4.12}_{-4.02}$	-11.49
$x_{\sigma\Delta}$	$[0.6, 1.4]$	$1.26^{+0.07}_{-0.06}$	1.27	$1.10^{+0.16}_{-0.15}$	0.85
$x_{\omega\Delta}$	$[0.6, 1.4]$	$1.23^{+0.08}_{-0.07}$	1.23	$1.02^{+0.19}_{-0.18}$	0.75
Δ (MeV)	$[0.0, 200.0]$	—	—	$113.52^{+58.00}_{-67.32}$	108.88
a_4	$[0.6, 1.0]$	—	—	$0.83^{+0.11}_{-0.13}$	0.61
$B^{1/4}$ (MeV)	$[130.0, 180.0]$	—	—	$147.52^{+12.00}_{-11.54}$	148.98

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\, \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Likelihood

$$\mathscr{L}^{(H)}(\theta) = \prod_i \mathscr{L}_i^{(H)}(\theta)$$

- Y. Suwa et al, Mon. Not. Roy. Astron. Soc. 481, 3305 (2018).
- B. Muller et al., Phys. Rev. Lett.134, 071403 (2025)

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\big(\theta \mid \{D_i\}, H\big) = \frac{\mathcal{L}^{(H)}(\theta)\, \pi^{(H)}(\theta)}{Z^{(H)}\big(\{D_i\}\big)}$$

Likelihood

$$\mathcal{L}^{(H)}(\theta) = \prod_i \mathcal{L}_i^{(H)}(\theta)$$

$$\mathcal{L}_i^{(1\mathrm{F})}(\theta) \propto \int_{M_{\mathrm{NS,min}}^{(1\mathrm{F})}(\theta)}^{M_{\mathrm{NS,max}}^{(1\mathrm{F})}(\theta)} dM\, q_i\big[M, R_{\mathrm{NS}}^{\mathrm{TOV}}(M;\theta)\big]$$

$$\mathcal{L}_i^{(2\mathrm{F})}(\theta) \propto \sum_{f\in\{\mathrm{NS,QS}\}} \int_{M_{f,\mathrm{min}}^{(2\mathrm{F})}(\theta)}^{M_{f,\mathrm{max}}^{(2\mathrm{F})}(\theta)} dM\, q_i\Big[M, R_f^{\mathrm{TOV}}(M;\theta)\Big]\ \eta_f^{(2\mathrm{F})}(M;\theta) \equiv \mathcal{L}_{\mathrm{NS},i}^{(2\mathrm{F})}(\theta) + \mathcal{L}_{\mathrm{QS},i}^{(2\mathrm{F})}(\theta)$$

- Y. Suwa et al, Mon. Not. Roy. Astron. Soc. 481, 3305 (2018).
- B. Muller et al., Phys. Rev. Lett.134, 071403 (2025)

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\Big(\theta \mid \{D_i\}, H\Big) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\Big(\{D_i\}\Big)}$$

Bayesian evidence

$$Z^{(H)}\Big(\{D_i\}\Big) = \int d\theta\ \mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)$$

Testing NS–QS coexistence via Bayesian Inference

Bayes' Theorem

$$P\left(\theta \mid \{D_i\}, H\right) = \frac{\mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)}{Z^{(H)}\left(\{D_i\}\right)}$$

Bayesian evidence

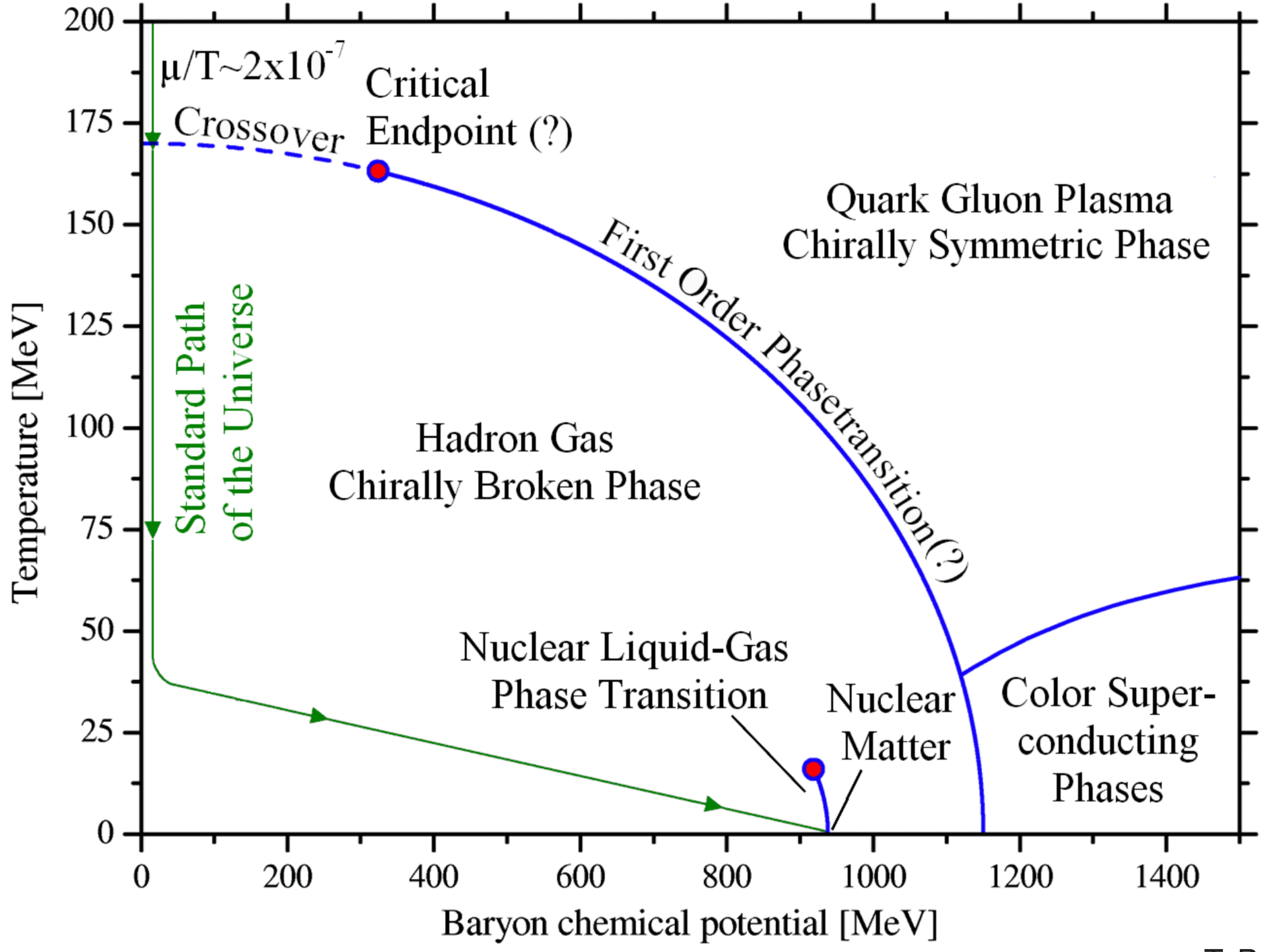
$$Z^{(H)}\left(\{D_i\}\right) = \int d\theta\ \mathscr{L}^{(H)}(\theta)\ \pi^{(H)}(\theta)$$

$$Z_{1\mathrm{F}} \approx -40$$

$$Z_{2\mathrm{F}} \approx -28$$

$$\mathscr{B}_{2\mathrm{F},1\mathrm{F}} \approx 10$$

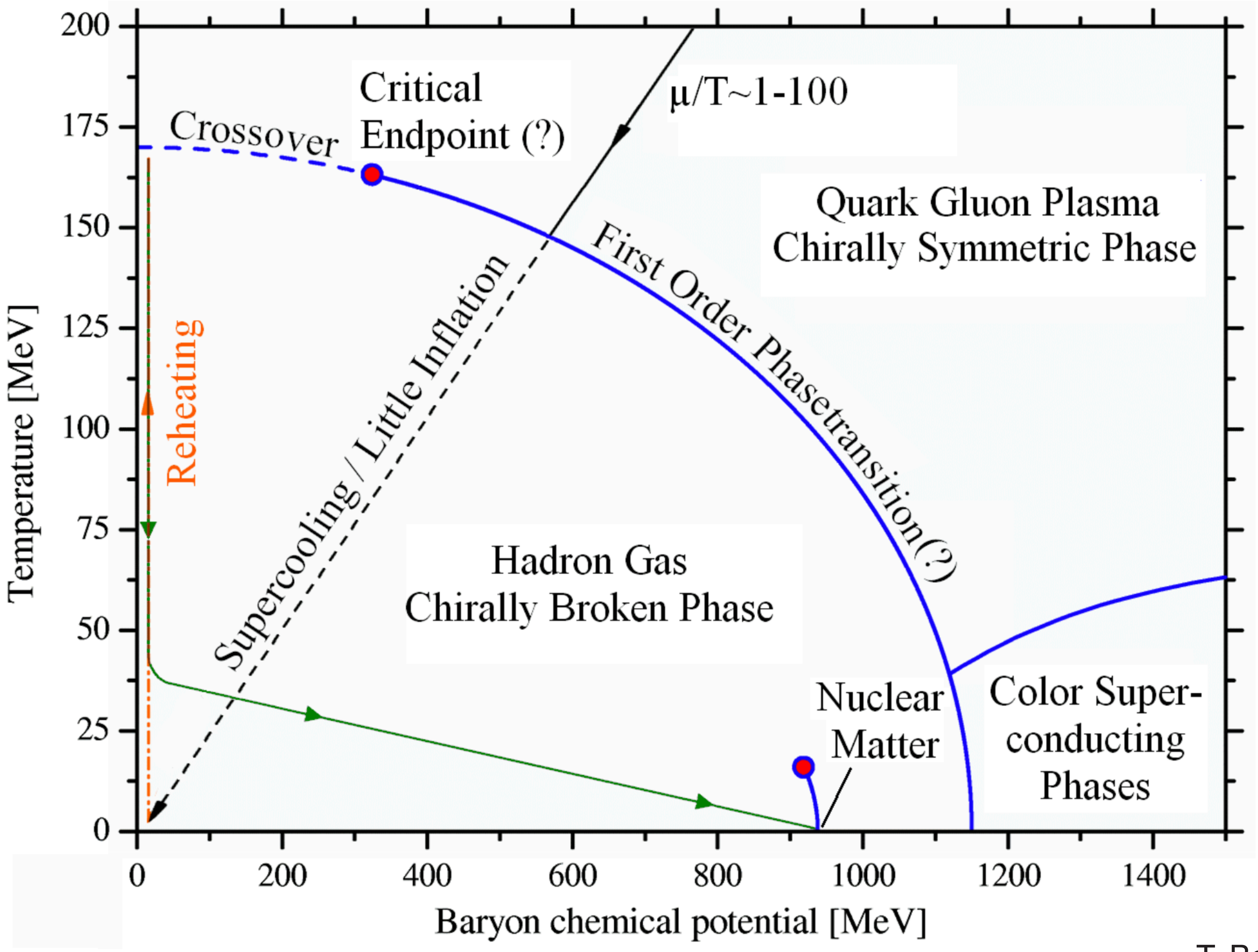
Little inflation scenario



- Negligible baryon density: $\mu_B \ll T$
- QCD phase transition: **crossover**

T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Little inflation scenario



- Non vanishing μ_B: $\mu_B/T \sim O(1)$
- $\Omega(T, V, \mu_B)$ has two minima
- System stucks in **false vacuum**

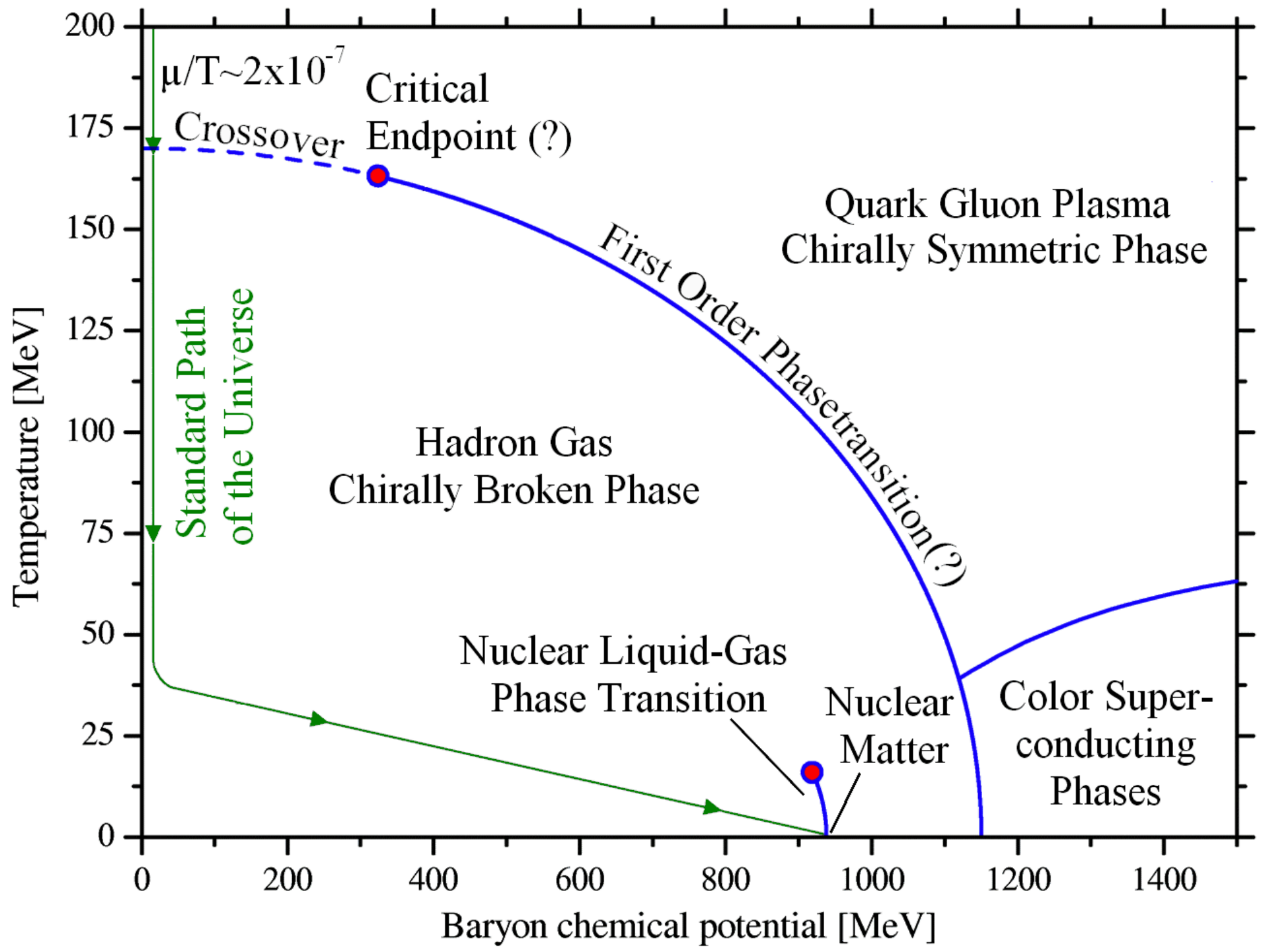
$\Rightarrow$ **LITTLE INFLATION**

- **First order phase transition**: release of false vacuum energy causes **reheating**

T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory



T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Equation of State for: cosmic trajectories

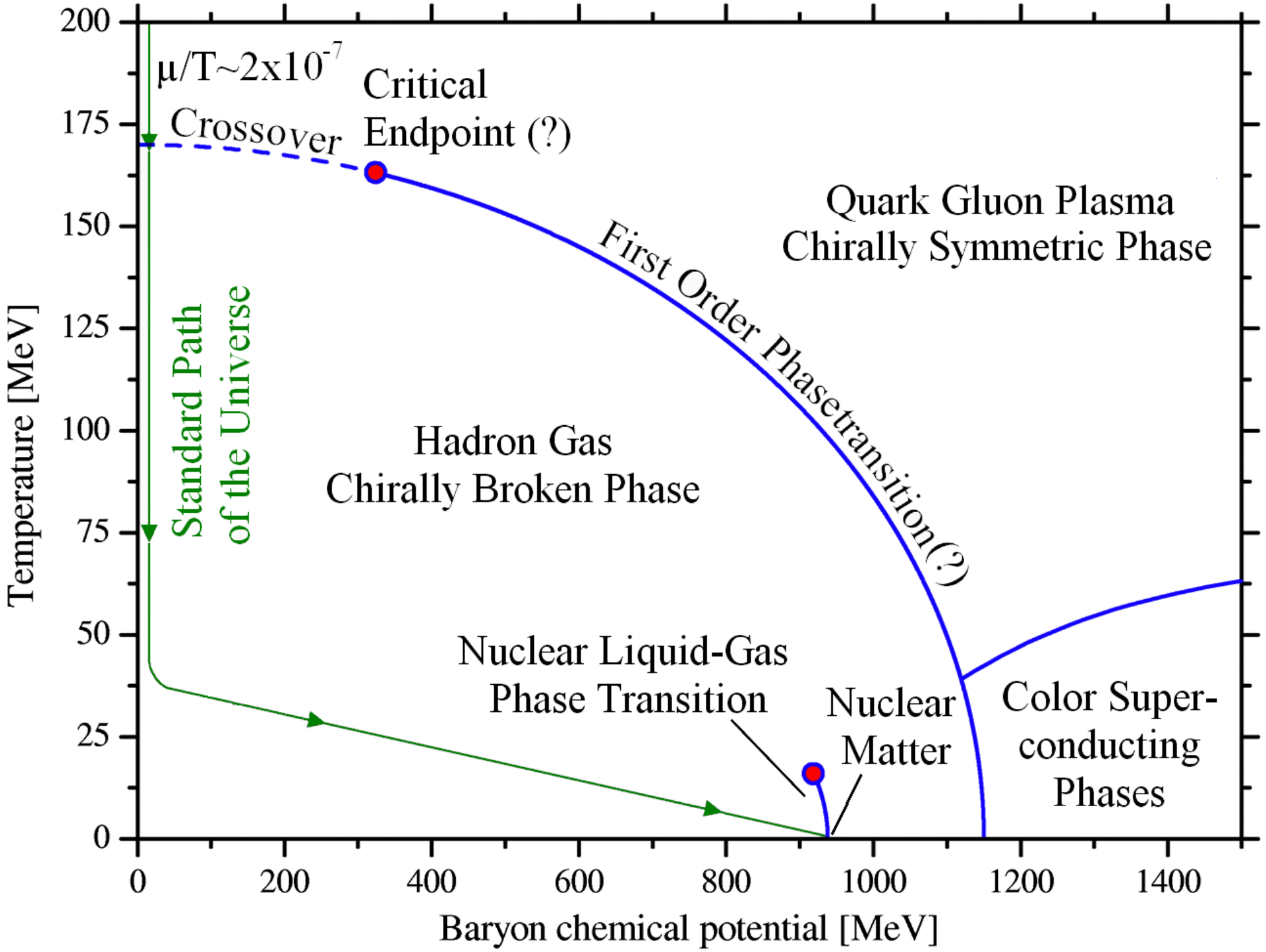
When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Conservation laws

1. Lepton number: $l_\alpha s = n_\alpha + n_{\nu_\alpha}, \alpha = e, \mu, \tau$

2. Baryon number: $bs = \sum_i B_i n_i$

3. Electric charge: $qs = \sum_i Q_i n_i$



T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Conservation laws

l_α free input parameters

1. Lepton number: $l_\alpha s = n_\alpha + n_{\nu_\alpha}\,,\ \alpha = e, \mu, \tau$

$b = 8.6 \times 10^{-11}$

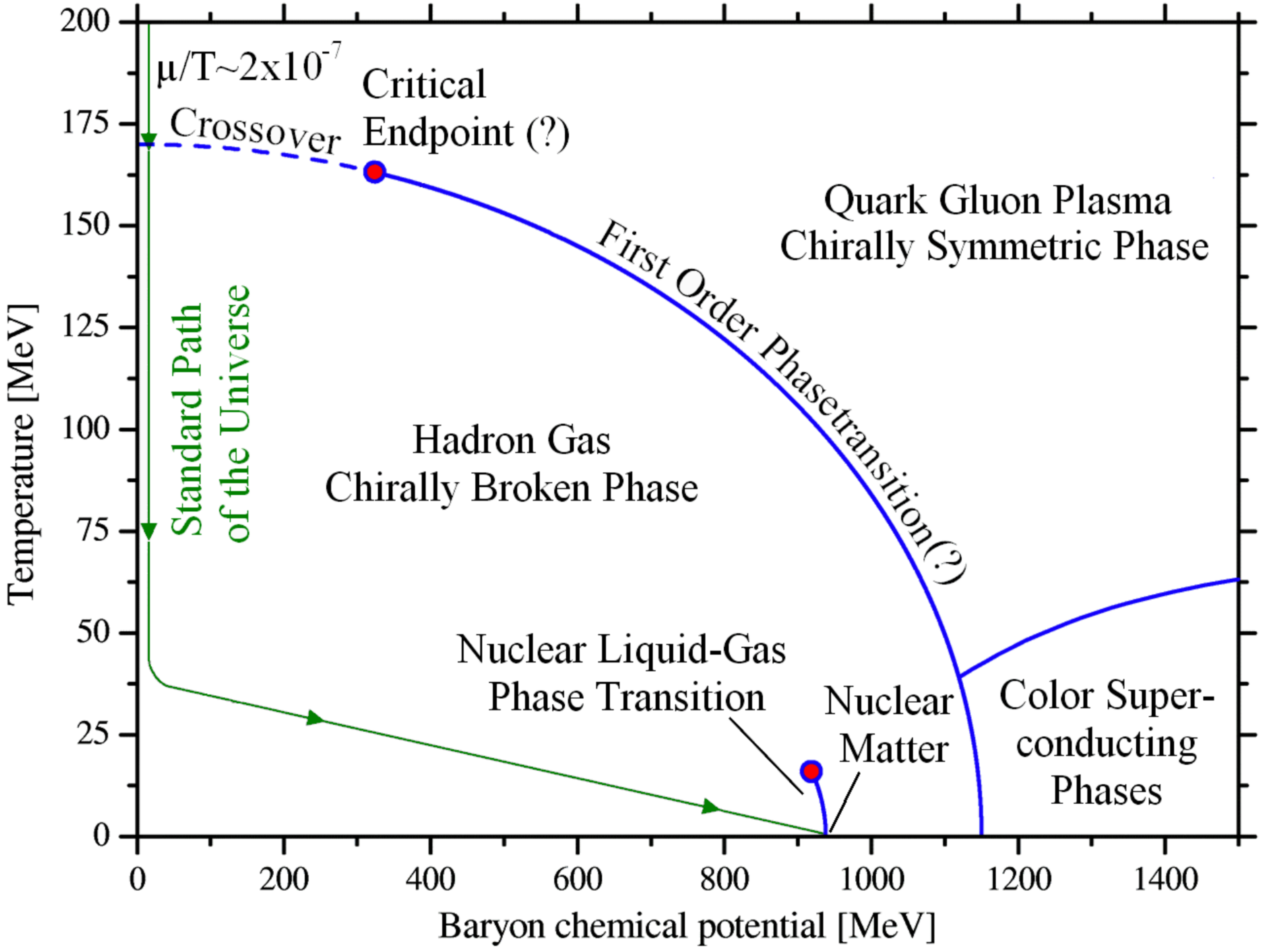
2. Baryon number: $bs = \sum_i B_i n_i$

$q = 0$ (charge neutrality)

3. Electric charge: $qs = \sum_i Q_i n_i$

5 conservation laws
→5 equations
→5 chemical potentials

$\mu_{L_\alpha}, \mu_B, \mu_Q$



T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Conservation laws

l_α free input parameters

1. Lepton number: $l_\alpha s = n_\alpha + n_{\nu_\alpha}$, $\alpha = e, \mu, \tau$

$b = 8.6 \times 10^{-11}$

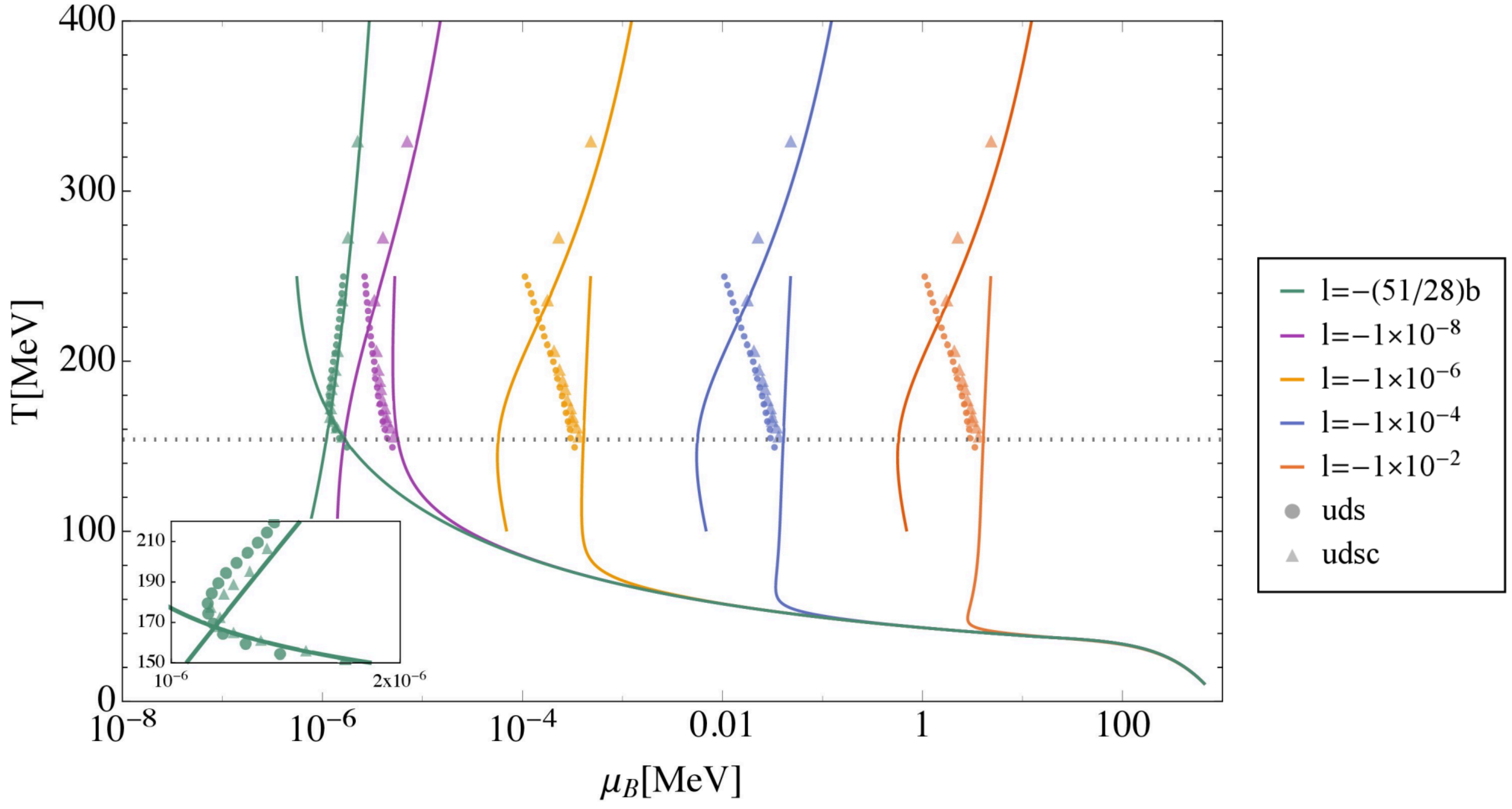
2. Baryon number: $bs = \sum_i B_i n_i$

$q = 0$ (charge neutrality)

3. Electric charge: $qs = \sum_i Q_i n_i$

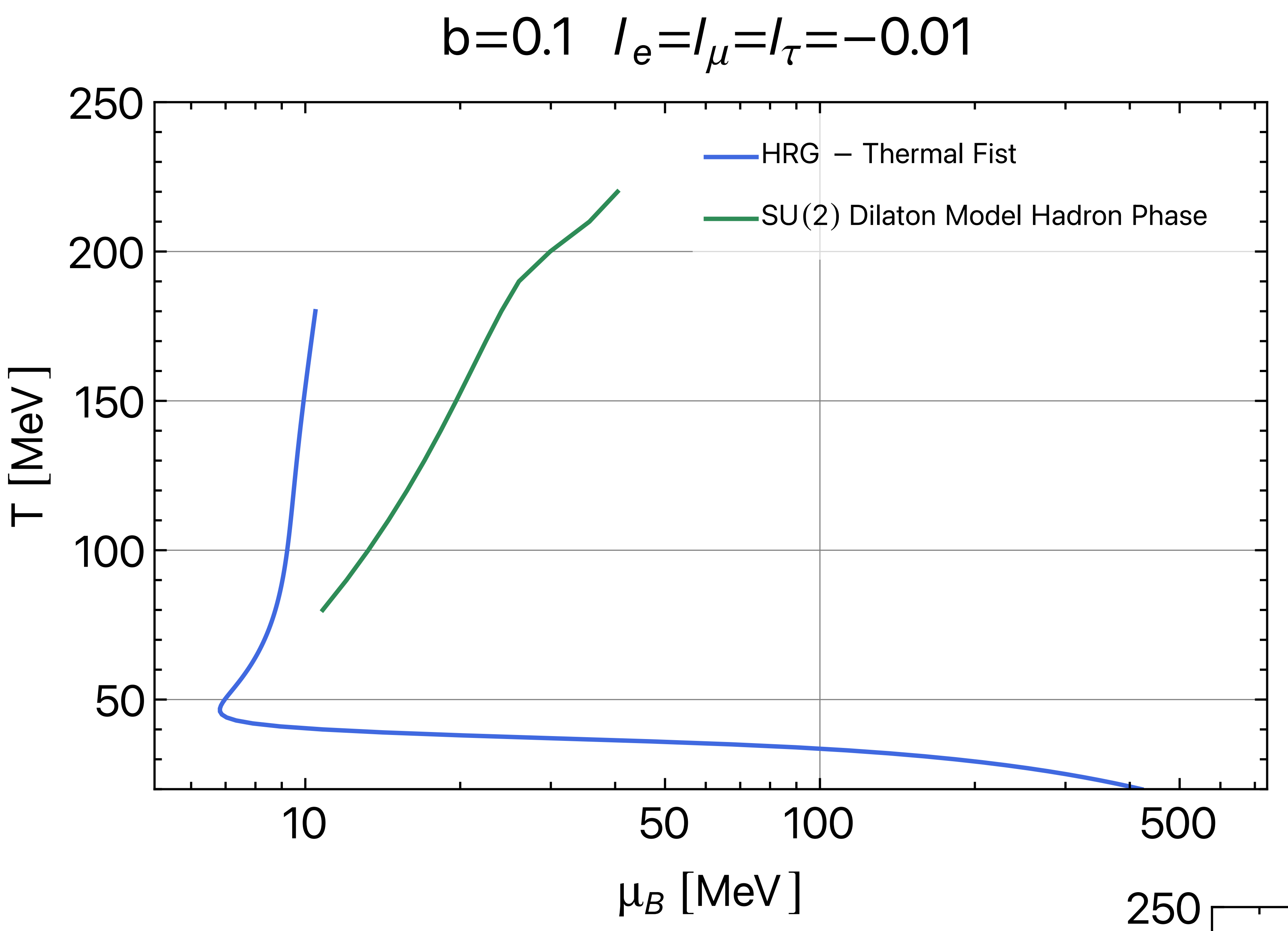
5 conservation laws
→5 equations
→5 chemical potentials

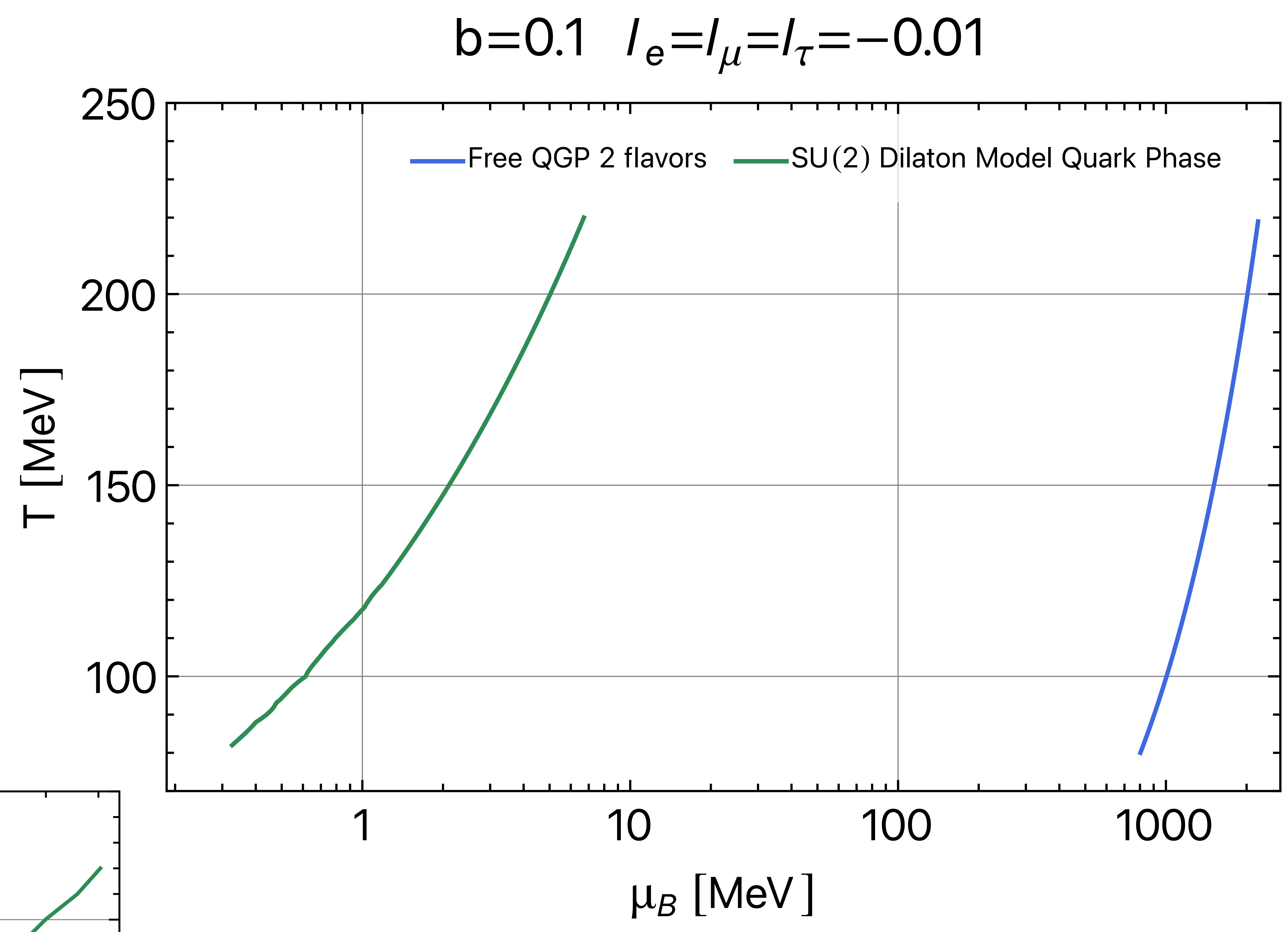
$\mu_{L_\alpha}, \mu_B, \mu_Q$

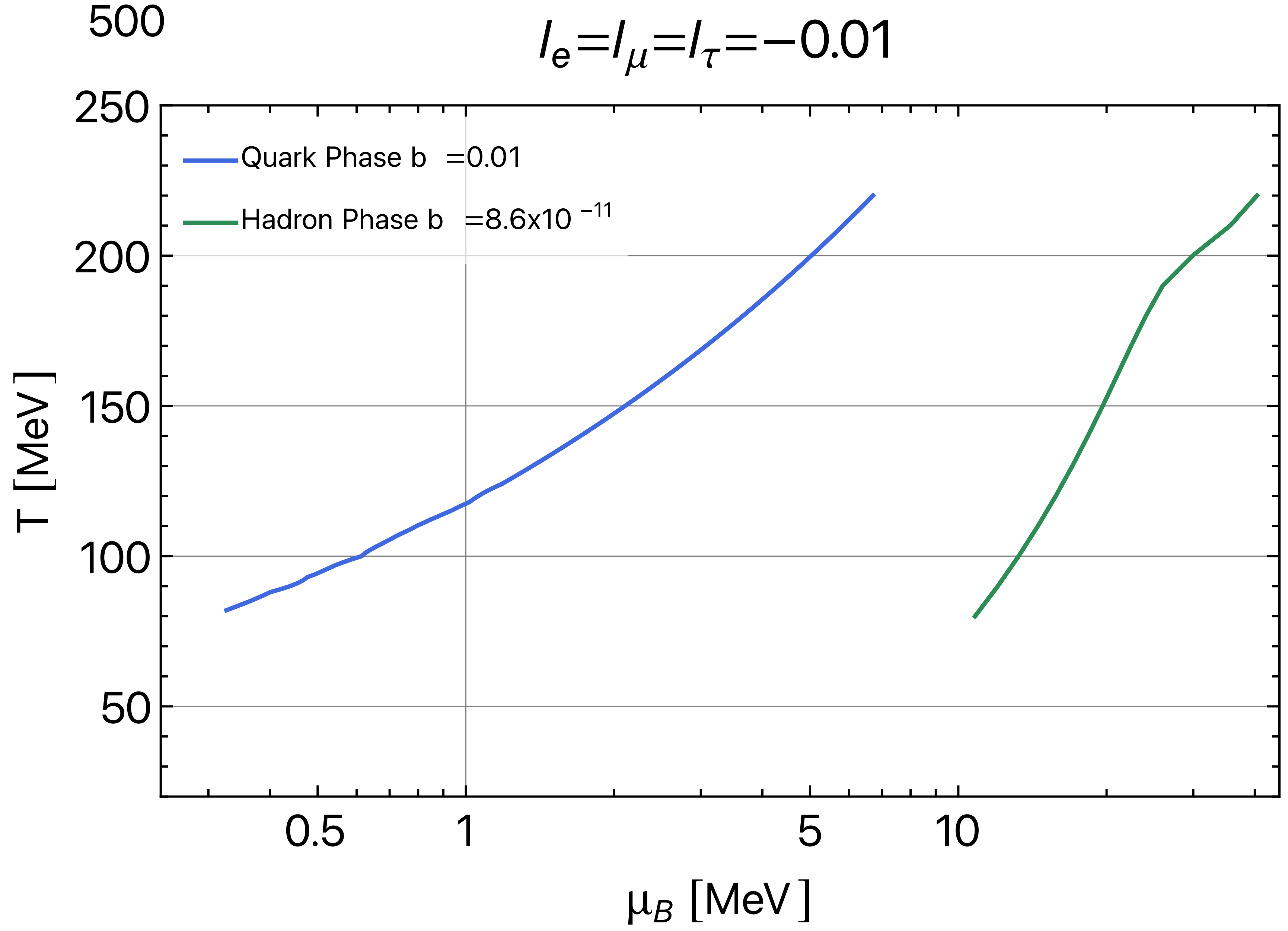


Mandy M. Wygas et al., PRL 121, 201302 (2018)

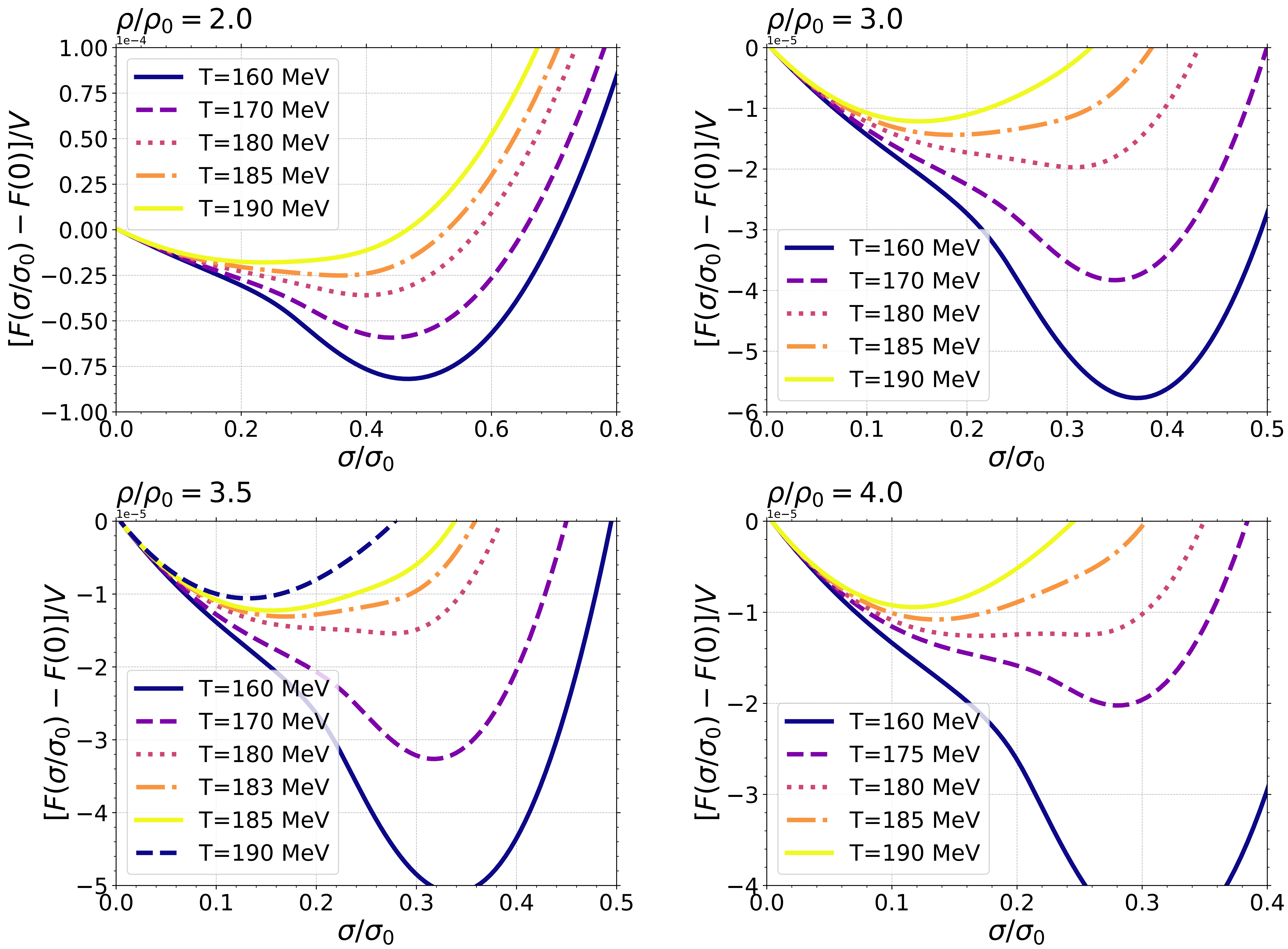
Equation of State for: cosmic trajectories







Meson and baryon sector at finite chemical potential



Little inflation scenario

T = 180 MeV, ρ = 4.00 ρ_0

Numerical details

4 field equations:

$\chi, \sigma, \omega, \rho$

6 mass equations:

$$m_\chi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\sigma = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\pi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\omega = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\rho = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_N = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

2 conservation equation:

Total baryonic charge

Total electric charge

$$\begin{cases} \vdots \\ \langle \Delta_i^2 \rangle = \dfrac{n_i}{2\pi^2} \displaystyle\int_0^\infty dk \, \frac{k^2}{e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle)} \frac{1}{\exp\left[\beta\left(e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle) - \mu_i^*\right)\right] - 1} \\ \vdots \end{cases}$$

$$e_i^* = \sqrt{k^2 + m_i^{*2}}$$

Symmetries of the model

Scale anomaly

A theory is invariant under scale transformations if its action remains constant when the fields are transformed as follows under the scale operator $U(\lambda)$:

$$\phi(x) \rightarrow U^{+}\phi(x)U(\lambda) = \lambda^{\Delta}\phi(\lambda x)$$

where Δ is the scale dimension of ϕ. The action remains unchanged *only if* no dimensional parameter is present: scale invariance is broken due to quantum corrections due to the appearance of a dimensional parameter Λ.

Chiral symmetry

Chiral symmetry is spontaneously broken in the QCD vacuum. The vacuum is non-trivial and not invariant under the chiral transformation $SU(N_f)_L \times SU(N_f)_R$, even in the chiral limit, so that the order parameter $< \bar{q}^i_L q^j_R >$ acquires a non-zero expectation value. By increasing temperature and/or density, the quark condensate melts, restoring chiral symmetry.

Pure gauge $SU(3)_c$ sector

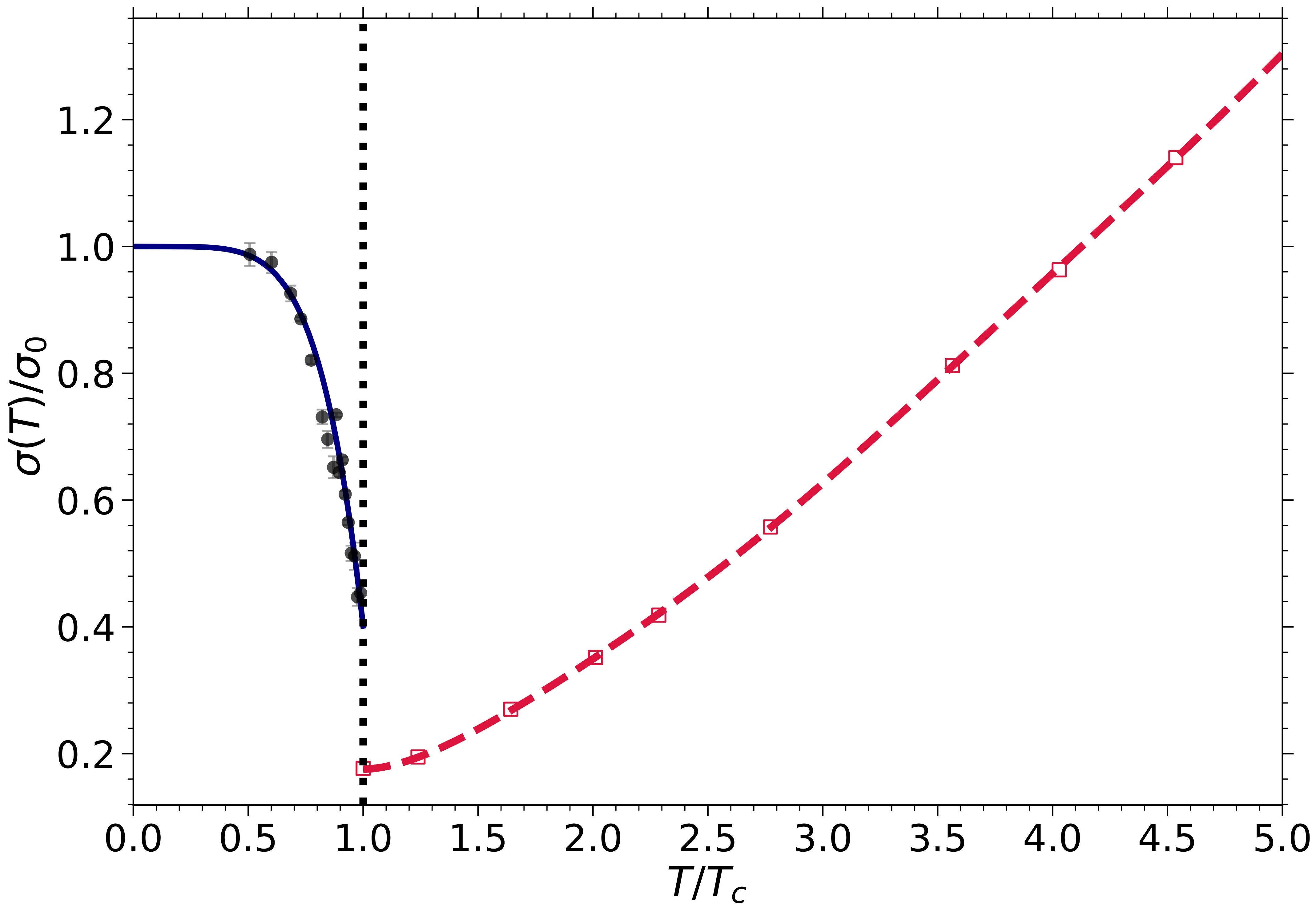
To improve accuracy with latticeQCD data:

- Inclusion **excited glueballs**:

$$M^2(J,T) = \frac{1}{\alpha'}(J-\alpha_0)\sigma(T) \simeq 4(J-0.58)\sigma(T)$$

$$P_{\text{dil}_J} = -(2J+1)T\int \frac{dk^3}{(2\pi)^3}\ln\left(1-e^{-\beta\sqrt{k^2+m_J^{*2}}}\right)$$

$$\frac{\tilde{\Omega}(\chi,T)}{V} = \frac{\Omega(\chi,T)}{V} - \sum_J^N P_{\text{dil}_J}$$



- N. Cardoso and P. Bicudo, Phys. Rev. D 85, 077501 (2012).
- G. Boyd et a., Nuclear Physics B 469, 419 (1996)

Pure gauge $\mathrm{SU(3)_c}$ sector

To improve accuracy with latticeQCD data:

- Inclusion excited glueballs;
- Modification **infrared cut-off**:

$$I = \frac{s}{T^3} - \frac{4P}{T^4} \Rightarrow s/T^3 \gg 4P/T^4$$

Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1-\chi^2} = \frac{A}{1-\bar{\chi}^2 - \langle \Delta^2 \rangle \cdot F(T)}$$

Form factor

$$F(T) = \frac{\Theta(T-T_c)}{1+\left(\frac{T-T_c}{\gamma}\right)^2}$$

Meson and baryon sector at finite chemical potential

The **thermodynamical potential** is

$$\frac{\tilde{\Omega}}{V} = \langle \tilde{\mathscr{V}} \rangle - \frac{1}{2} m_\omega^2 \chi^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \chi^2 b_0^2 - \frac{G_4}{4} \omega_0^4 - \frac{1}{2} m_\sigma^{*2} \langle \Delta_\sigma^2 \rangle - \frac{1}{2} m_\pi^{*2} \langle \boldsymbol{\pi}^2 \rangle - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle$$

$$+ \frac{T}{2\pi^2} \int dk\, k^2 \left[\ln(1 - e^{-\beta \epsilon_\sigma}) + 3 \ln(1 - e^{-\beta \epsilon_\omega}) \right] + \frac{T}{2\pi^2} \int dk\, k^2 \left[\sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\pi^* - \mu_\pi^{*a})}) + 3 \sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\rho^* - \mu_\rho^{*a})}) \right]$$

$$- \frac{T}{\pi^2} \sum_{i=p,n} \int dk_i\, k_i^2 \left[\ln\left(1 + e^{-\beta(E_i^* - \mu_i^*)}\right) + \ln\left(1 + e^{-\beta(E_i^* + \mu_i^*)}\right) \right]$$

$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2} B_0 \delta \chi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} + \frac{1}{2} B_0 \delta \chi^2 \left[\frac{\sigma^2 + \pi^2}{\sigma_0} - \frac{\chi^2}{2} \right] - \frac{1}{4} \epsilon_1' \chi^2 \left[\frac{4\sigma}{\sigma_0} - 2 \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] - \frac{3}{4} \epsilon_1' .$$

Meson and baryon sector: thermal fluctuations

$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i^*} \frac{1}{e^{\beta(\epsilon_i^* - \mu_i^*)} - 1} dk$$

$$n_i = \begin{cases} 1 & \text{for } \phi, \sigma, \pi_0, \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu, b_{0\mu}, b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

$$P_\sigma(z) = \sqrt{\frac{1}{2\pi\langle \Delta_\sigma^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\sigma^2 \rangle} \right),$$

$$P_{\boldsymbol{\pi}}(y) = \sqrt{\frac{2}{\pi}} \left(\frac{3}{\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right)^{(3/2)} \exp\left(-\frac{3y^2}{2\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right),$$

$$\langle O(\bar{\sigma} + \Delta_\sigma, \boldsymbol{\pi}^2) \rangle = \int_{-\infty}^{\infty} dz P_\sigma(z) \int_0^\infty dy y^2 P_{\boldsymbol{\pi}}(y) O(\bar{\sigma} + z, y^2) \, .$$

The thermodynamical potential in the mean field approximation

$$\frac{\Omega(\chi,T)}{V} = \langle \mathscr{V}(\chi) \rangle - P_{\text{dil}}(\chi,T) - P_{\text{q-free}}(\chi,T) - P_{\text{int}}(\chi,T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \left\langle \Delta_\chi^2 \right\rangle$$

Where

$$P_{\text{dil}}(\chi,T) = \frac{1}{6\pi} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_\chi^{*2}}} \frac{1}{e^{\beta\sqrt{k^2+m_\chi^{*2}}} - 1}$$

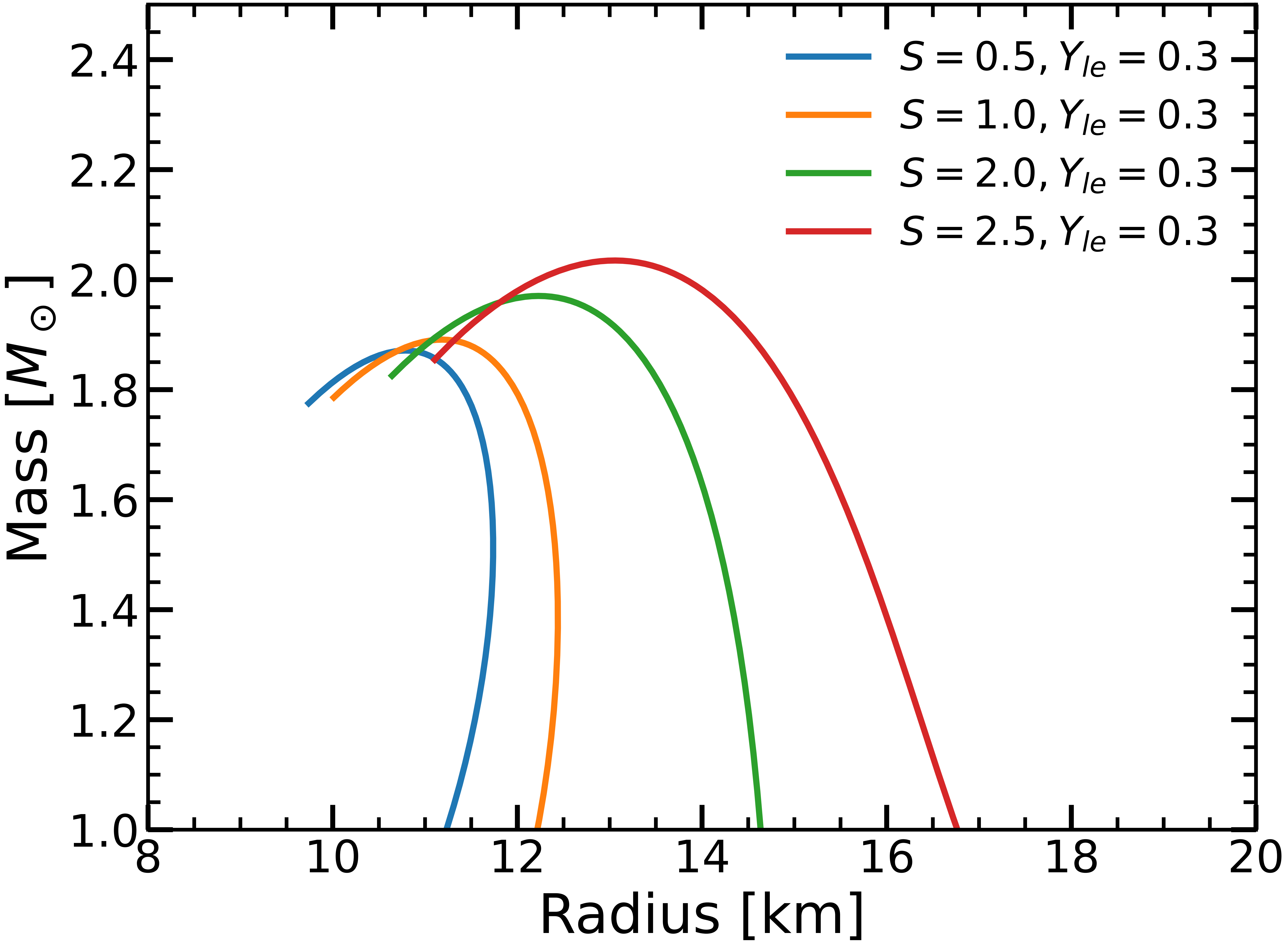
$$P_{\text{q-free}}(\chi,T) = -2(N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-k/T}\right) \Theta\left(k - K(\chi)\right)$$

$$K(\chi) = \frac{A}{(1-\chi)}$$

$$g^2(T) = \frac{48\pi^2}{11 N_c \ln\left(\frac{T^2 + S^2}{\Lambda^2}\right)}$$

$$P_{\text{int}}(\chi,T) = g^2 N_c (N_c^2 - 1) \left\{ (-3) \left[\int \frac{d^3k}{(2\pi)^3} \frac{1}{k} N_B\left(\frac{k}{T}\right) \Theta\left(k - K(\chi)\right) \right]^2 + \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{1}{k_1 k_2} N_B\left(\frac{k_1}{T}\right) N_B\left(\frac{k_2}{T}\right) \right.$$
$$\left. \times \Theta\left(k_1 - K(\chi)\right) \Theta\left(k_2 - K(\chi)\right) \times \left[\frac{9}{4} \Theta\left(|\boldsymbol{k}_1 + \boldsymbol{k}_2| - K(\chi)\right) - \frac{1}{4} \Theta\left(|\boldsymbol{k}_1 - \boldsymbol{k}_2| - K(\chi)\right) \right] \right\}$$

PNS with dilaton model



Pure gauge $\mathrm{SU}(3)_c$ sector