Dual Approach to Quarkyonic Matter at Finite Temperature

Dyana C. Duarte

Compact Stars in the QCD phase diagram – CSQCD 2026
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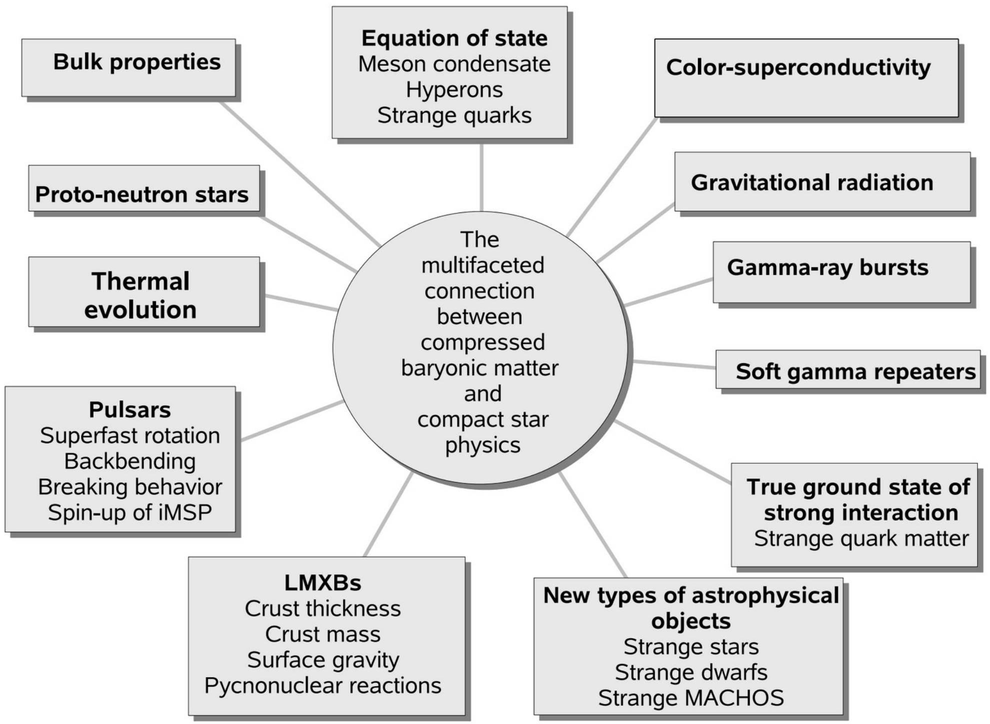
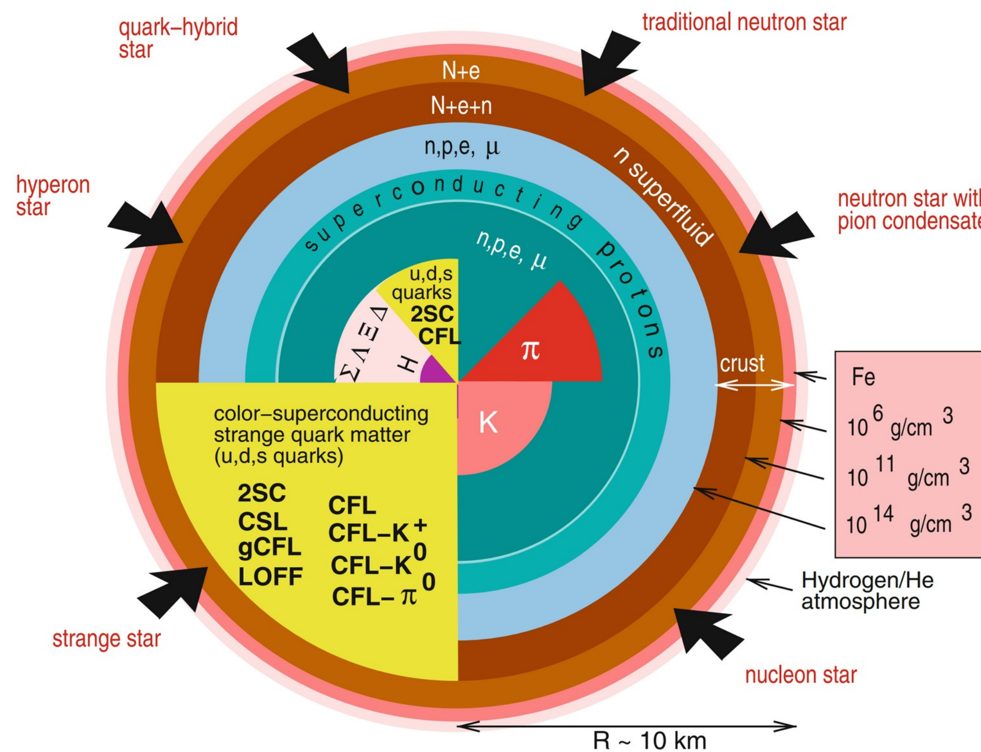
Universidade Federal de Santa Maria
SEDES SAPIENTIÆ
1960

Neutron stars as laboratories of dense QCD

Weber *et. al*, Prog.Part.Nucl.Phys.59, 94(2007)

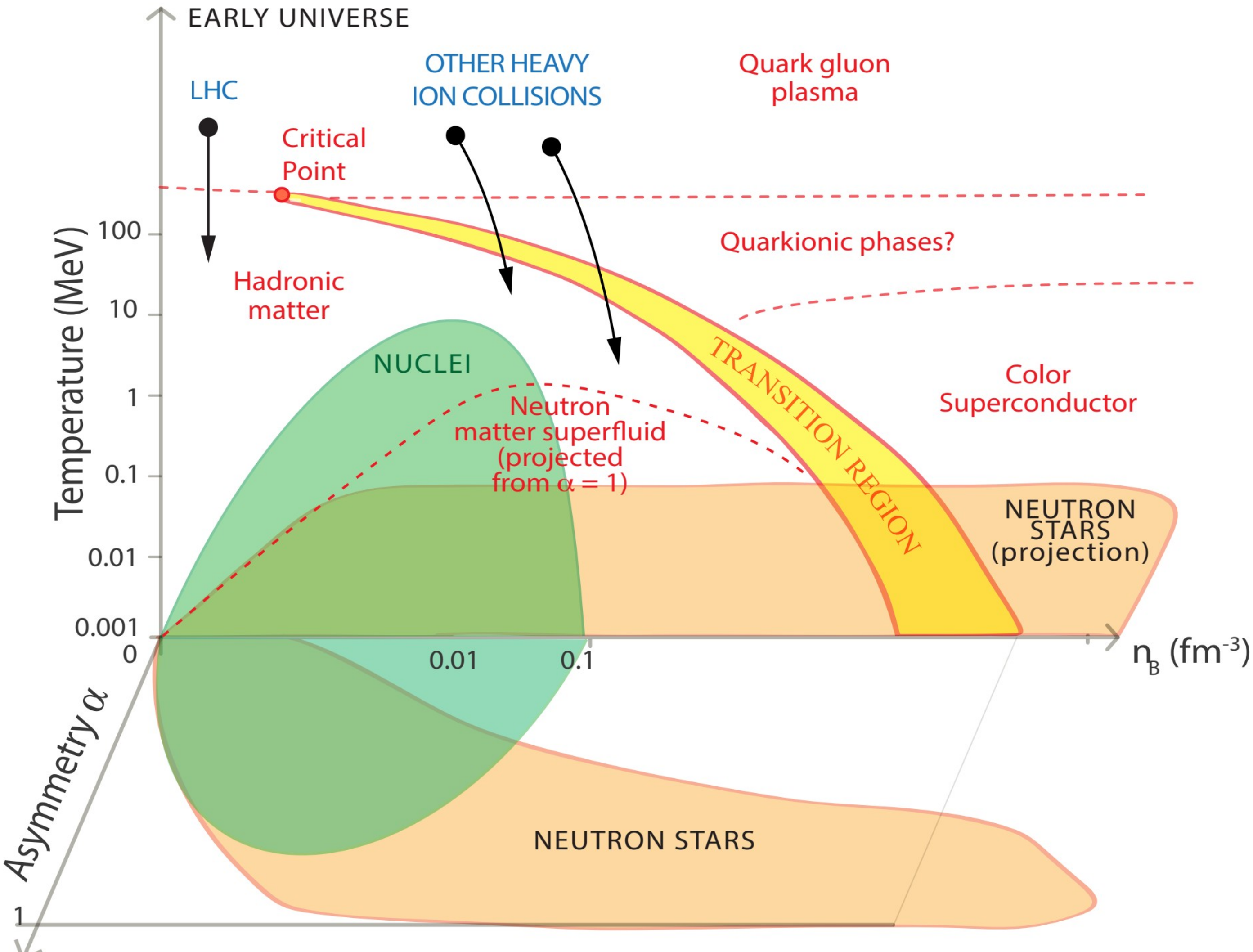


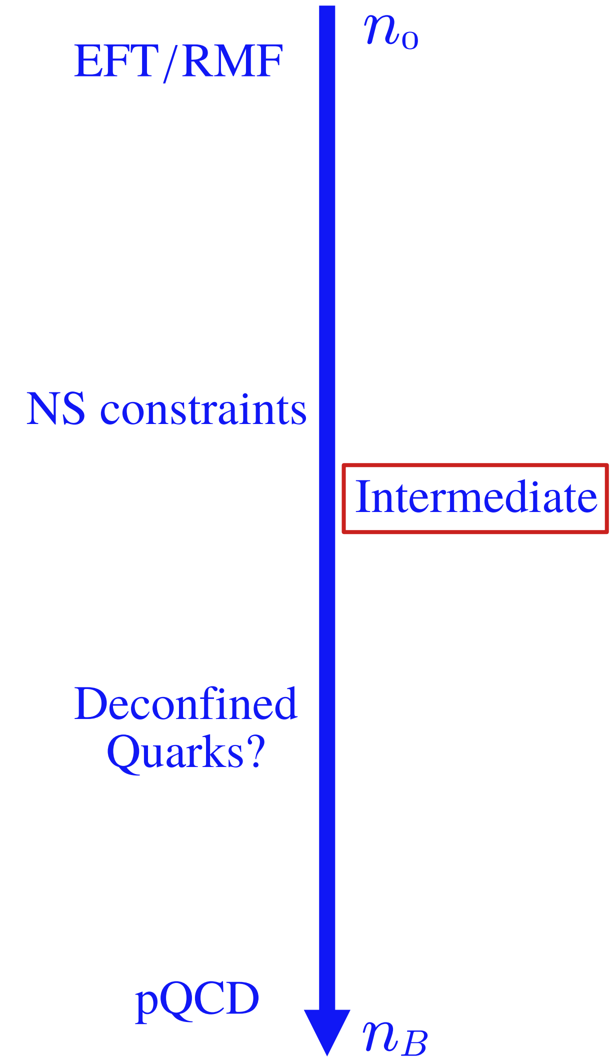
Weber, Prog.Part.Nucl.Phys, 54, 193(2005).



The challenge of describing intermediate-density QCD

Watts *et.al*, Proc. Science, arXiv:1501.00042 (2014)





Quarkyonic Matter

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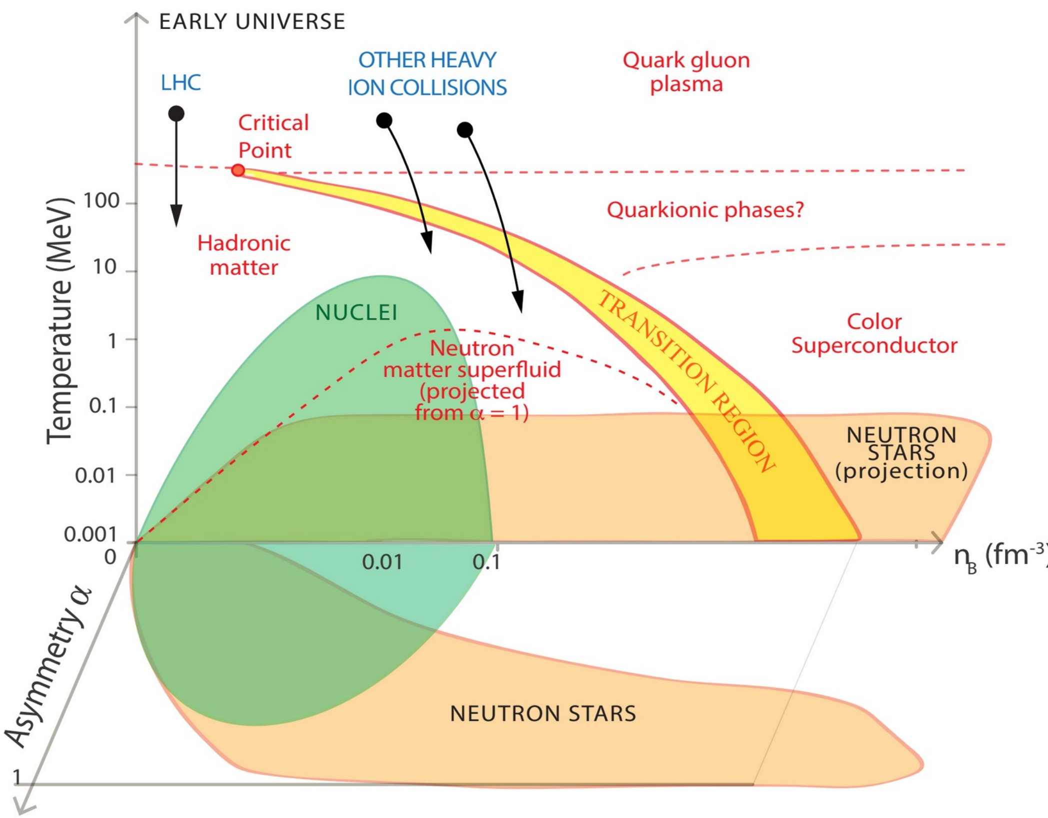
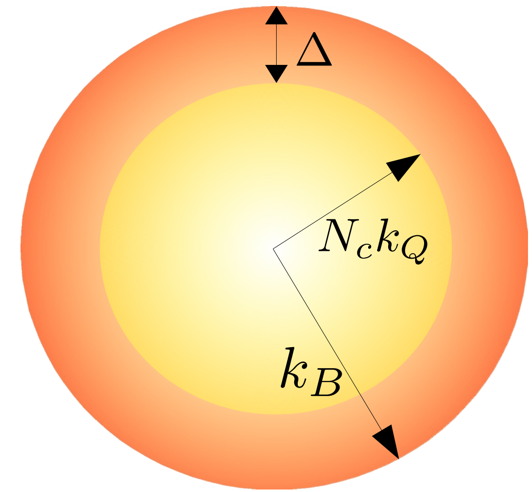




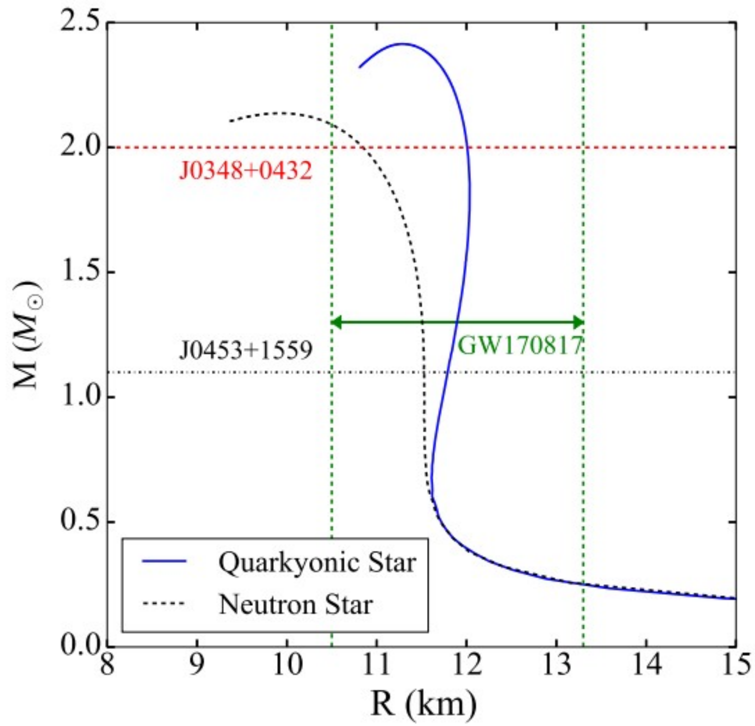
Pressure when $\mu_Q > M_q$ and $n_B \neq 0$:

$$\mathcal{O}(N_c^0) \to \mathcal{O}(N_c)$$

(hadron) $\to$ (quarkyonic)

Weakly interacting quark system?
Baryonic system?

Constraints to the EoS of NS: phenomenologic models



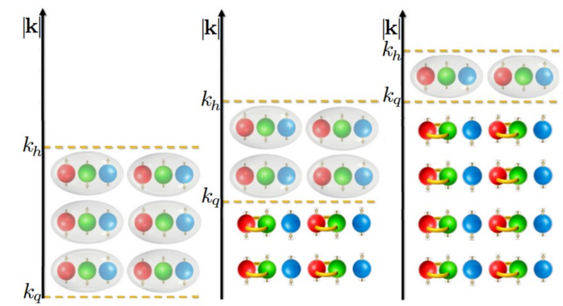
Model for the dependence of Δ with the baryon density:

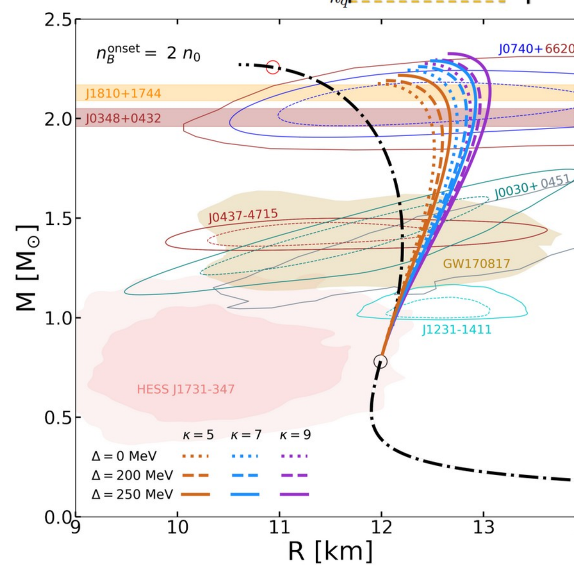
$$\Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

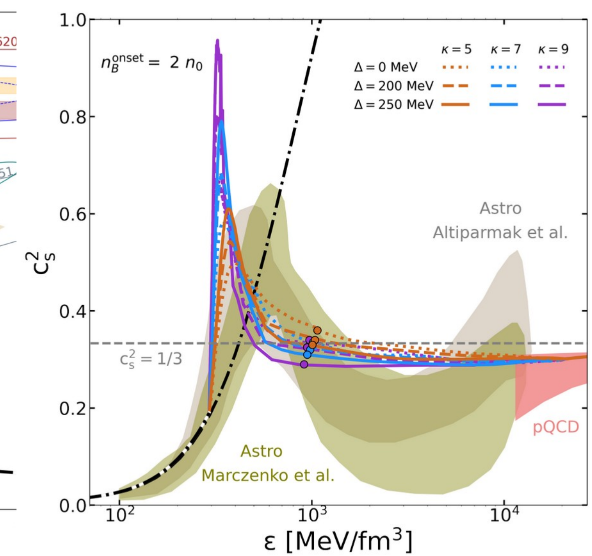
McLerran and Reddy,
PRL 122,122701 (2019)

Including color superconductivity

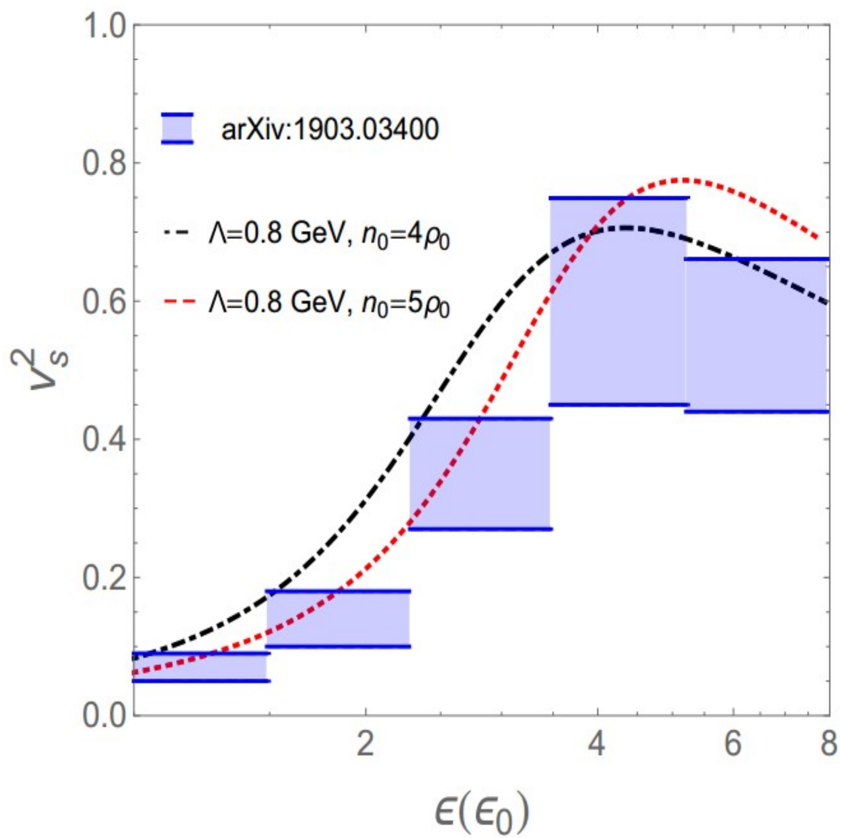
Gärtlein, Ivanytskyi, Sagun, Lopes,
PRD 113, 034017 (2026)







Constraints to the EoS of NS: dynamical models

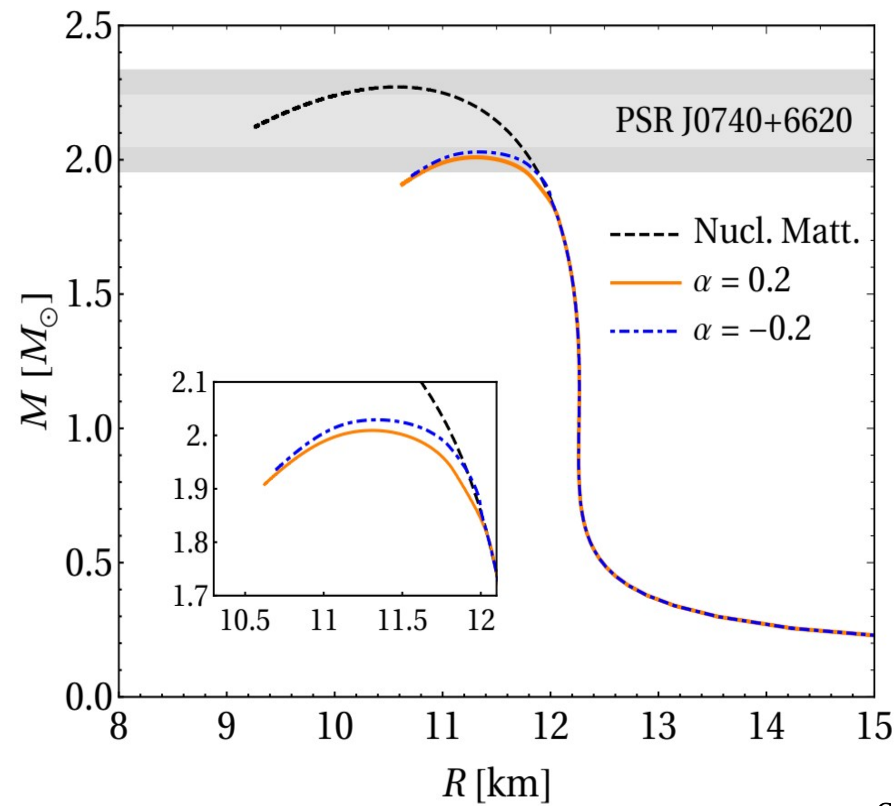


Sound velocity obtained from an equation of state extracted from neutron stars properties using deep neural network.

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Three flavor excluded volume model, shell appearing dynamically with the increase of baryon density.

DD, Jeong, Hernandez-Ortiz, PRC 102 065202 (2020)



Quarkyonic EFT

Field theory of quarkyonic matter: nucleonic and quark degrees of freedom in the same description: How to deal with??

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- Nucleons are composed by quarks: since inside the Fermi sea the lower momentum states are occupied by quarks, the nucleons cannot propagate in the region of momentum $k_N < N_c k_Q$.

- In addition to quarkyonic matter, such EFT should reduce to a theory of nucleons and mesons at low densities, and evolve to quarks at high density and temperature.

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- Nucleons are composed by quarks: since inside the Fermi sea the lower momentum states are occupied by quarks, the nucleons cannot propagate in the region of momentum $k_N < N_c k_Q$.
- In addition to quarkyonic matter, such EFT should reduce to a theory of nucleons and mesons at low densities, and evolve to quarks at high density and temperature.

Solution: Inclusion of a negative metric ghost field, that fill precisely the same state as the quarks, and cancel away the degrees of freedom of unconstrained nucleons.

Quarkyonic EFT

Nucleons can be thought of as an ensemble of quarks: If there is a quark Fermi sea with a chemical potential μ_Q in the low momentum states, the quarks composing the nucleon cannot occupy the same states.

$$\rho^G = \frac{1}{1+e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1+e^{\beta(N_C E_Q + \mu_G)}}$$

Density of the states in which nucleons cannot exist: Region of quark Fermi sea up to $\mu_G = N_c \mu_Q$.

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Density of quarks in a noninteracting gas of quarks and nucleons;

$$\rho^N_{\mathrm{const.}} = \rho^n = \frac{1}{1+e^{\beta(E_N - \mu_N)}} - \frac{1}{1+e^{\beta(E_N - \mu_G)}}$$

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Density of nucleons constrained to not propagate in the quark Fermi sea:

Nucleons are placed in a shell of momenta above the momenta of quark Fermi sea, where they are not Pauli blocked!

Quarkyonic EFT

$$\mathcal{L}_{\rm eff} = \overline{N}\left(i\partial\!\!\!/ - M_N + \gamma^0\mu_N\right)N + \overline{G}\left(i\partial\!\!\!/ - M_N + \gamma^0\mu_G\right)G$$
$$+\,\overline{Q}\left(i\partial\!\!\!/ - M_Q + \gamma^0\mu_Q\right)Q$$

<u>Ghosts:</u> Nucleon fields with M_N and $\mu_G = N_c\mu_Q$, with the same Lorentz structure of nucleonic field, satisfying anti-periodic boundary conditions in immaginary time. It is, however, a commuting field!

But what does it mean? Let $S(\mu,M)$ be the propagator for a given field.

$$Z_{\rm Grassman} = \det{}^{-1} S(\mu, M)$$
$$Z_{\text{c-number}} = \det S(\mu, M)$$

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$$\Rightarrow$$

$$\Omega = -gT \int \frac{d^3k}{(2\pi)^3} \left\{ \ln\left[1 + e^{-\beta(E_N - \mu_N)}\right] - \ln\left[1 + e^{-\beta(E_N - \mu_G)}\right] + N_c \ln\left[1 + e^{-\beta(E_Q - \mu_Q)}\right] \right\}$$

$$\mathcal{E}_{\text{quarkyonic}} = \underbrace{\mathcal{E}_N - \mathcal{E}_G}_{\text{Energy density of nucleons}} + \mathcal{E}_Q$$

DD, K.S. Jeong, S. Hernandez-Ortiz,L.McLerran, PRD 104, L091901(2021)

Quarkyonic EFT: MF theory

$$n_v = \int_0^{k_N} dk \frac{k^2}{2\pi^2}$$

- Independent parameters: n_N, n_Q related as $n_B = n_N - n_G + n_Q$

$$\begin{aligned} S &= \int d^4x \Big\{ \overline{\psi}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_N^* + M_N - g_\sigma \sigma\right)\psi \\ &+ \overline{G}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_G^* + M_N - g_\sigma \sigma\right) G \\ &+ \frac{M_\sigma^2}{2}\sigma^2 + \frac{M_v^2}{2} V^2 \Big\} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \varepsilon &= \varepsilon_{\rm kin}^N(\mu_N) - \left(1 - \frac{1}{N_c^3}\right)\varepsilon_{\rm kin}^N(N_c \mu_Q) \\ &+ \frac{g_v^2}{2M_v^2}\Big[n_v(\mu_N) - n_v(N_c\mu_Q)\Big]^2 \\ &- \frac{g_\sigma^2}{2M_\sigma^2}\left[n_s(\mu_N) - \left(1 - \frac{1}{N_c^3}\right) n_s(N_c\mu_Q)\right]^2 \end{aligned}$$

DD, K.S. Jeong, S. Hernandez-Ortiz,L.McLerran, PRC 107, 065201.

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$$n_v = \int_0^{k_N} dk \frac{k^2}{2\pi^2}$$

$$\mu_G = N_c(\mu_Q - \Delta M) + g_v V_0$$

$$\begin{aligned} S &= \int d^4x \Big\{ \overline{\psi} \left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_N^* + M_N - g_\sigma \sigma \right) \\ &\quad + \overline{G} \left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_G^* + M_N - g_\sigma \sigma \right) G \\ &\quad + \frac{M_\sigma^2}{2}\sigma^2 + \frac{M_v^2}{2} V^2 \Big\} \end{aligned}$$

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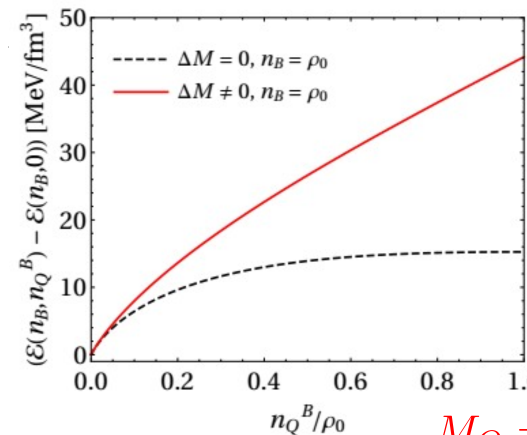
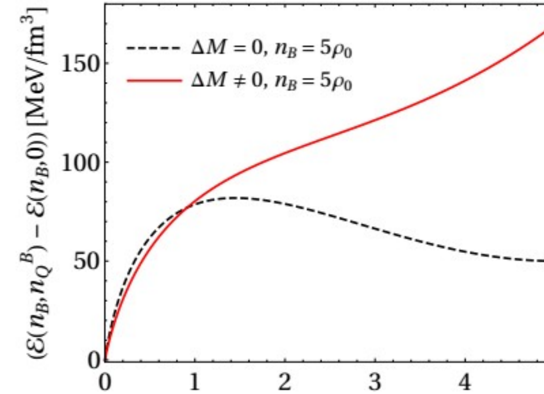
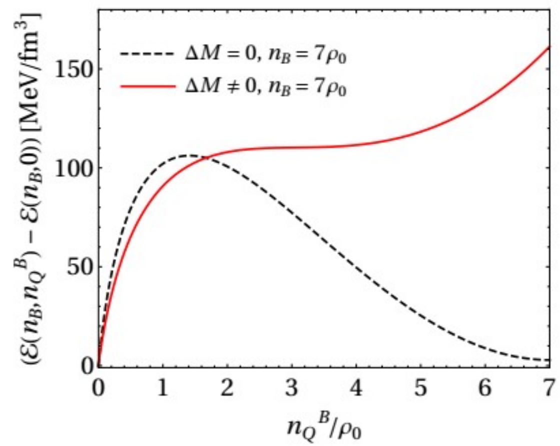
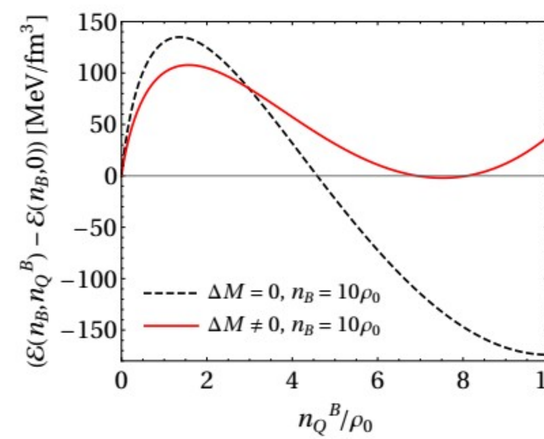




$$M_Q = M_N/N_c + \Delta M$$

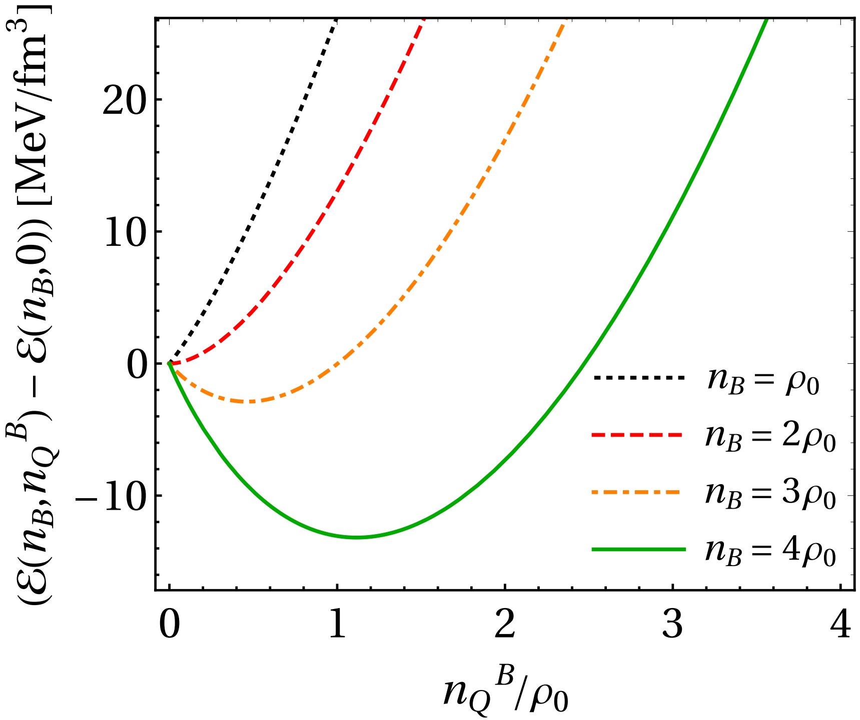




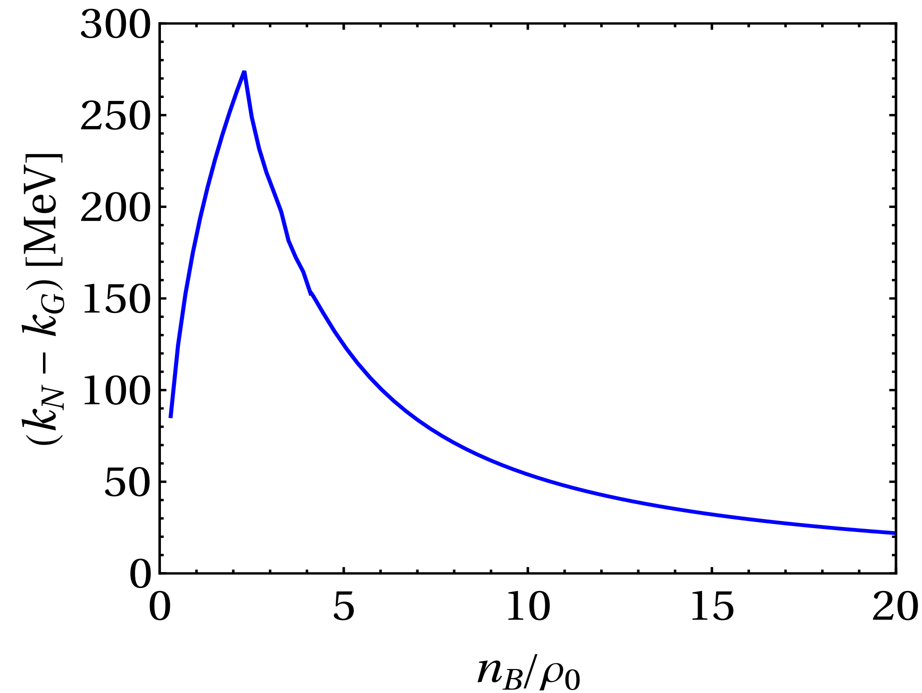
DD, K.S. Jeong, S. Hernandez-Ortiz, L.McLerran, PRC 107, 065201.

Quarkyonic EFT at $T = 0$

Interactions are important!! Phenomenological relation: $\mu_Q = M_Q + \frac{n_B^Q}{\Lambda^2}$



Continuous transition at $T = 0$.



Shell thickness

DD, K.S. Jeong, S. Hernandez-Ortiz,L.McLerran, PRC 107, 065201 (2023).

The Entropy Paradox in Quarkyonic Matter

- Quarkyonic matter: specific shell structure in momentum space.
- Finite T: f = -ε + Ts = - p + $\mu_B n_B$ must be extremized instead of energy dentsity.
- Low-momentum region: baryon distribution function is suppressed: $f_B = \dfrac{1}{N_c^3}$

$$s = -\frac{\gamma}{2\pi^2}\int_0^\infty dk\,k^2\Big[f_D^+\ln f_D^+ + (1-f_D^+)\ln(1-f_D^+) + f_D^-\ln f_D^- + (1-f_D^-)\ln(1-f_D^-)\Big], \qquad f_D^\pm = \frac{1}{1+e^{\beta(E\pm\mu)}}$$

- Applying this formula directly to the $1/N_c^3$ distribution yields a non-zero entropy at zero temperature: Violation of the third law of Thermodynamics!
- The "under-occupation" is incorrectly understood as thermal disorder by standard formulas.

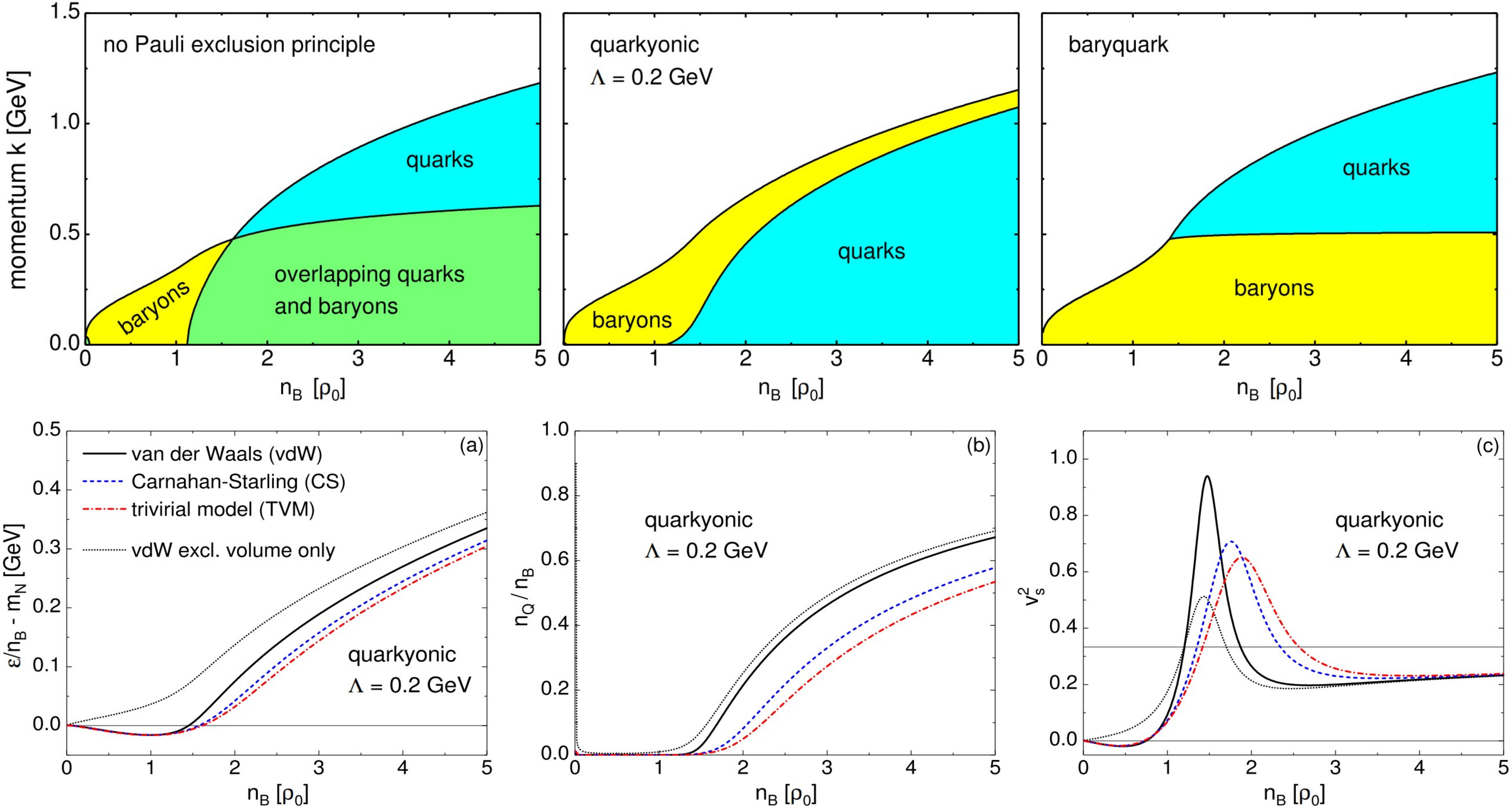
The Entropy Paradox in Quarkyonic Matter

- Microscopic Origin of the Constraint: The suppression is not due to “missing” particles, but to a restricted Fock space!
- The Pauli exclusion principle acting on constituent quarks reduces the number of physically available baryon states.
- Statistical framework redefinition: factorized baryon distribution $f_B(k) \to g(k) f_D(k)$.
- $g(k)$: Density of states – accounts for the reduced phase-space volume available due to quark saturation.
- $f_D(k)$: Thermal occupation – represents the actual thermal excitations.
- Thermodynamic Consistency is recovered when the entropy is redefined as being weighted by $g(k)$:

$$
\begin{aligned}
s &= -\frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2 g(k) \Big[f_D^+ \ln f_D^+ + (1-f_D^+)\ln(1-f_D^+) \\
&\quad + f_D^- \ln f_D^- + (1-f_D^-)\ln(1-f_D^-) \Big].
\end{aligned}
$$

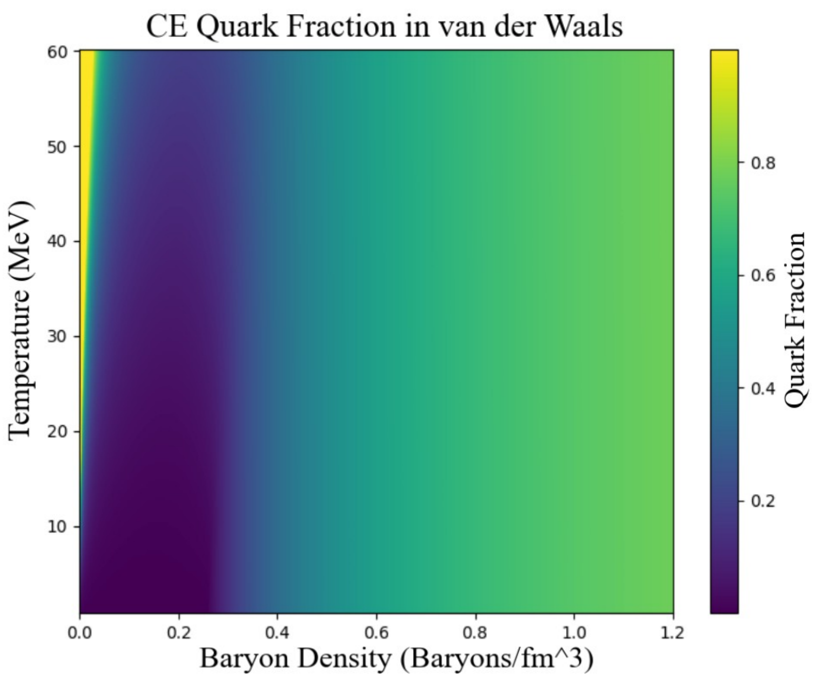
- At $T = 0$: all available states are fully occupied, leading to a perfectly ordered system with zero entropy!

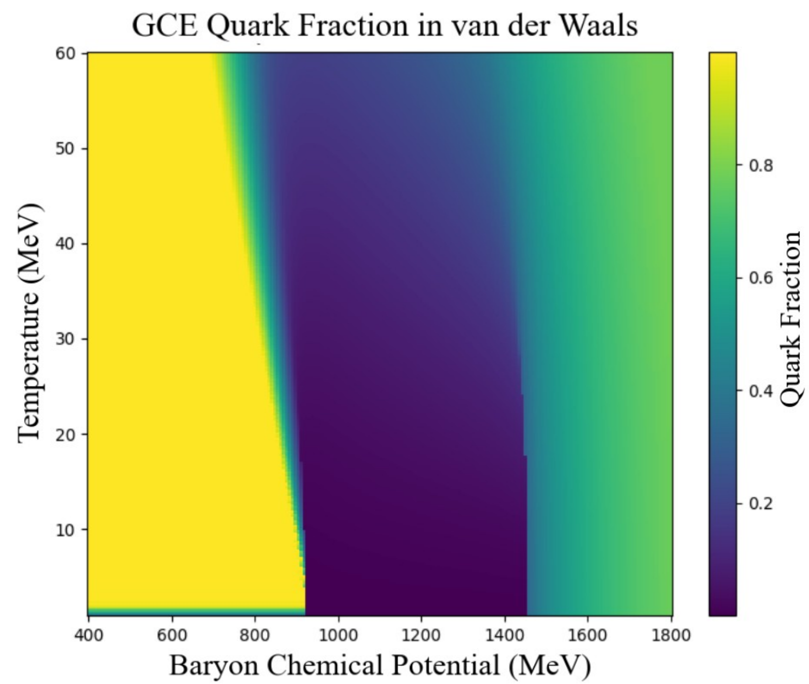
Quarkyonic matter with quantum van der Waals theory



Poberezhnyuk, Stoecker, Vovchenko, Phys.Rev.C 108, 045202 (2023)

Model Predictions at Finite Temperature





- At finite temperatures we see an unphysical transition to quark matter at low densities
- Entropic contributions of quarks must be suppressed to recover low-density confinement

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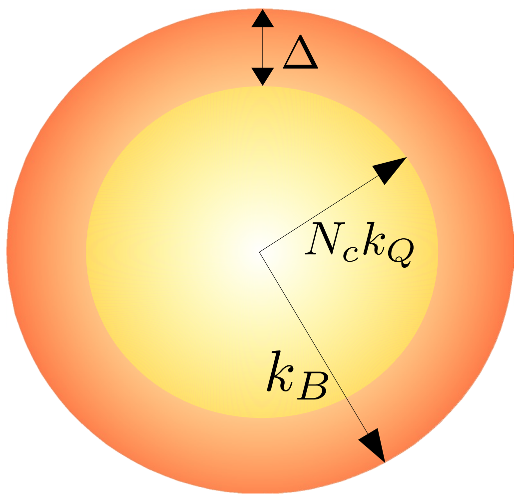
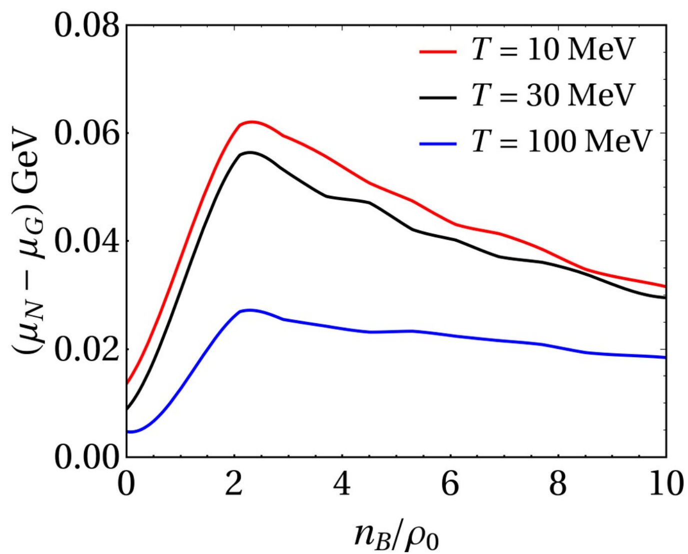
Natural Sciences and Mathematics

T. Moss presentation for progress evaluation @Huston U. (private communication).

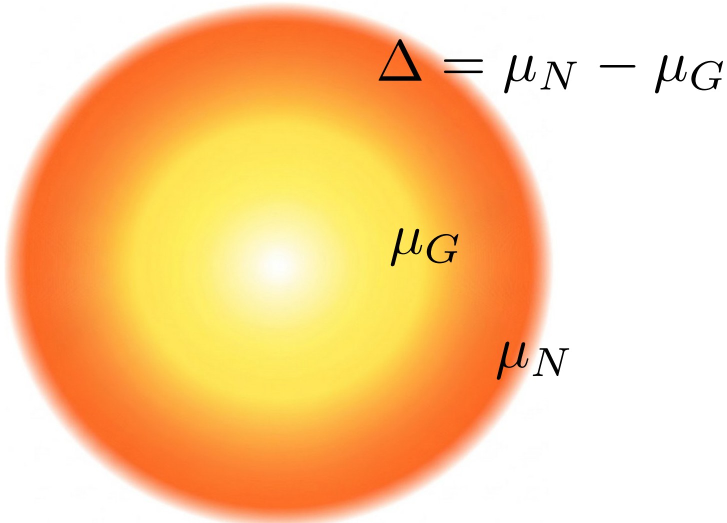
Why EFT+ghost is naturally suited for finite temperature.

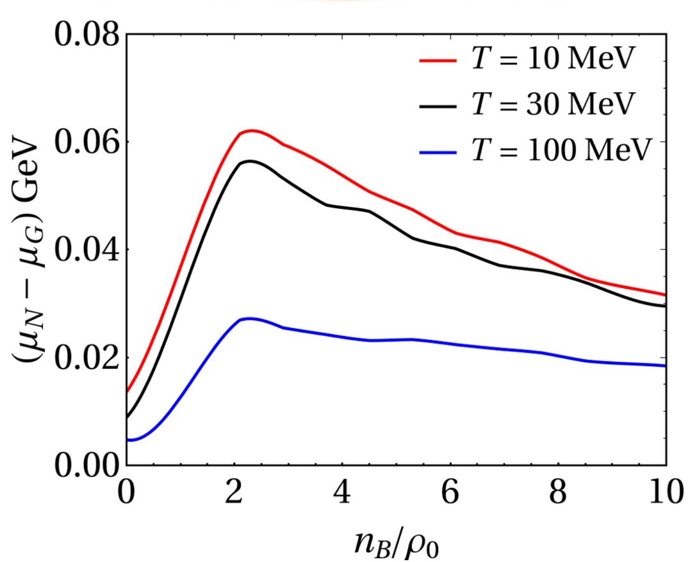




- Thermal excitations must preserve the constrained phase space.
- In the ghost formulation, the restricted baryonic sector is implemented directly at the level of the partition function.
- The cancellation mechanism naturally suppresses unphysical low-momentum baryonic states.
- Thermal effects can then be incorporated consistently without modifying the underlying shell structure.

Why EFT+ghost is naturally suited for finite temperature.





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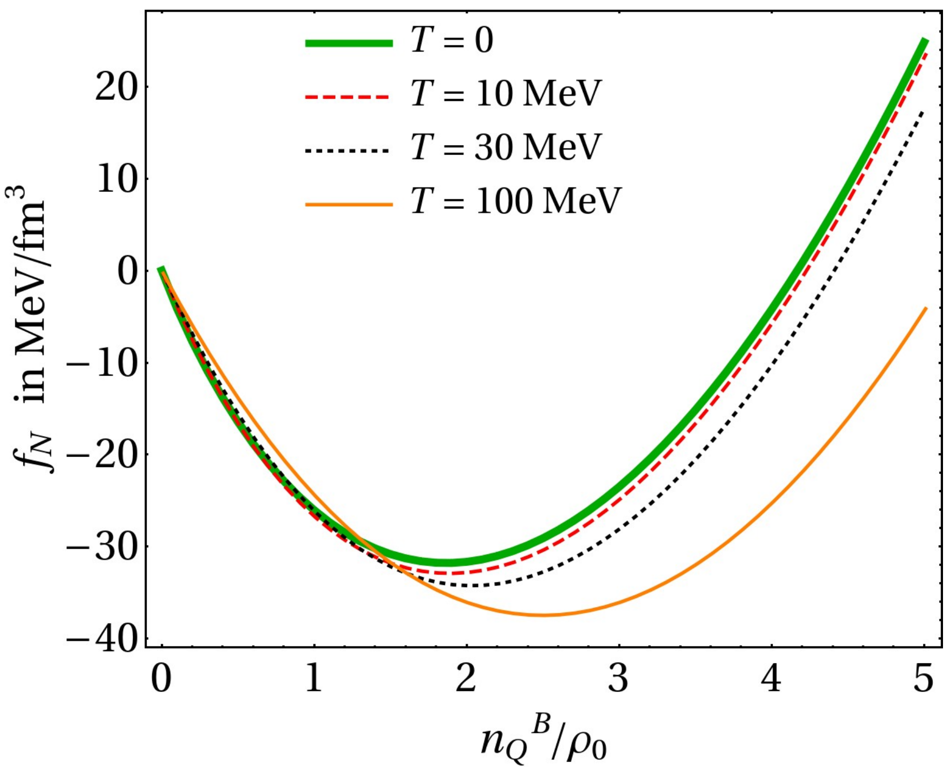
What should we expect at finite temperature?

- Thermal broadening of the shell
- Enhanced quark population
- Smooth crossover behavior
- Competition between confinement and thermal excitations

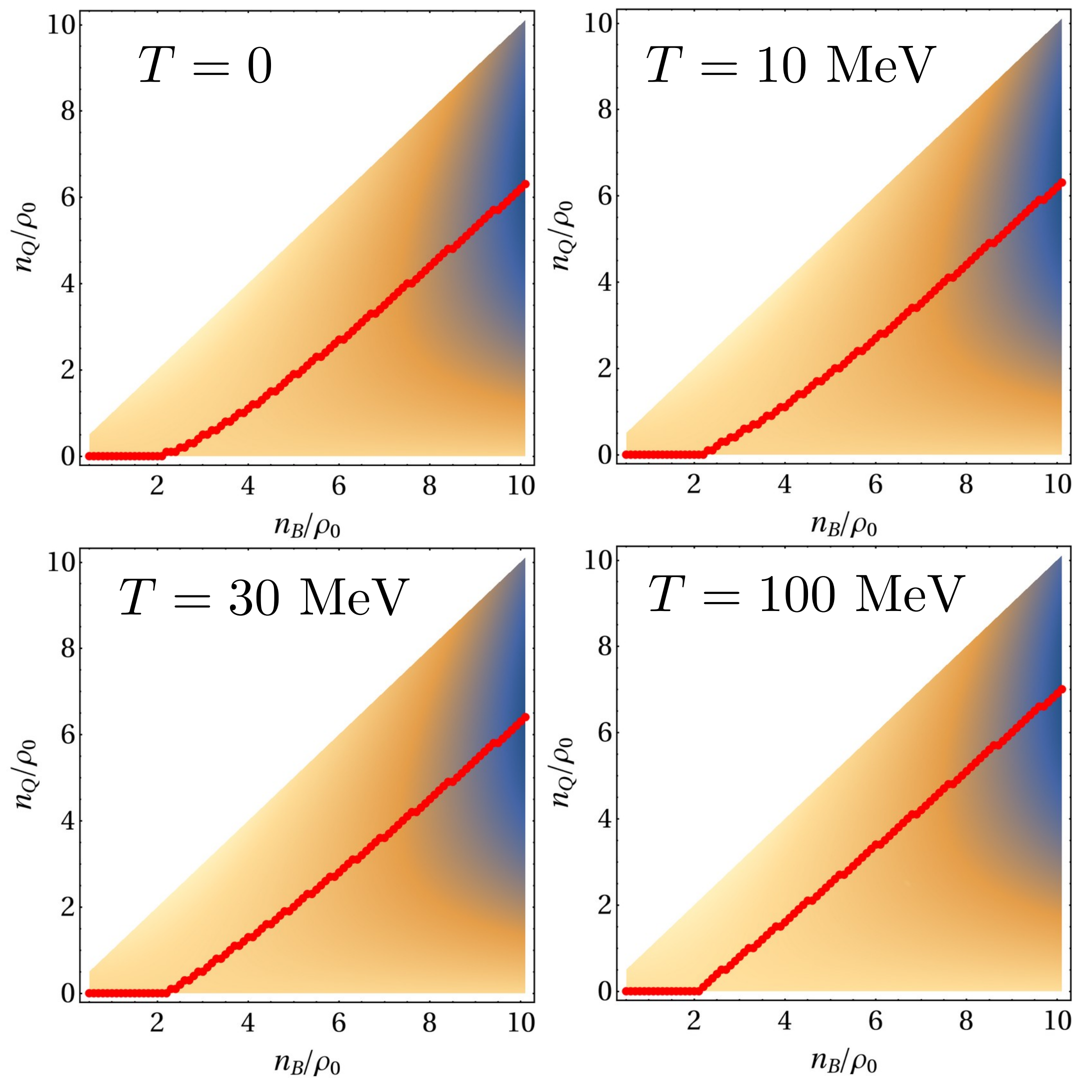
$$\mu_Q^B = N_c \left[\left(M_Q + n_Q^B \frac{a}{M_Q^2} + a\frac{T^2}{M_Q} \right) \Theta(r - n_Q^B) + \left(\kappa (n_Q^B)^{\frac{1}{3}} - \frac{\pi^2}{12} \frac{T^2}{\kappa (n_Q^B)^{1/3}} \right) \Theta(n_Q^B - r) \right]$$

DD, S. Hernandez-Ortiz, W. Tavares, in progress

Preliminar Results



$$f_N = f(n_B, n_Q, T) - f(n_B, 0, T)$$



Final Remarks

- Quarkyonic matter provides a natural framework for intermediate-density QCD
- Current GW data do not exclude quarkyonic scenarios. Therefore, extending quarkyonic matter to finite temperature is astrophysically relevant.
- Challenges at finite T, due to constrained phase space.
- EFT with ghosts solve the hadron-quark duality and the 3rd law of Thermodynamics.
- Finite T effects favor the quark population, but have small effects on the onset of quark Fermi sea.

Thanks for your attention!

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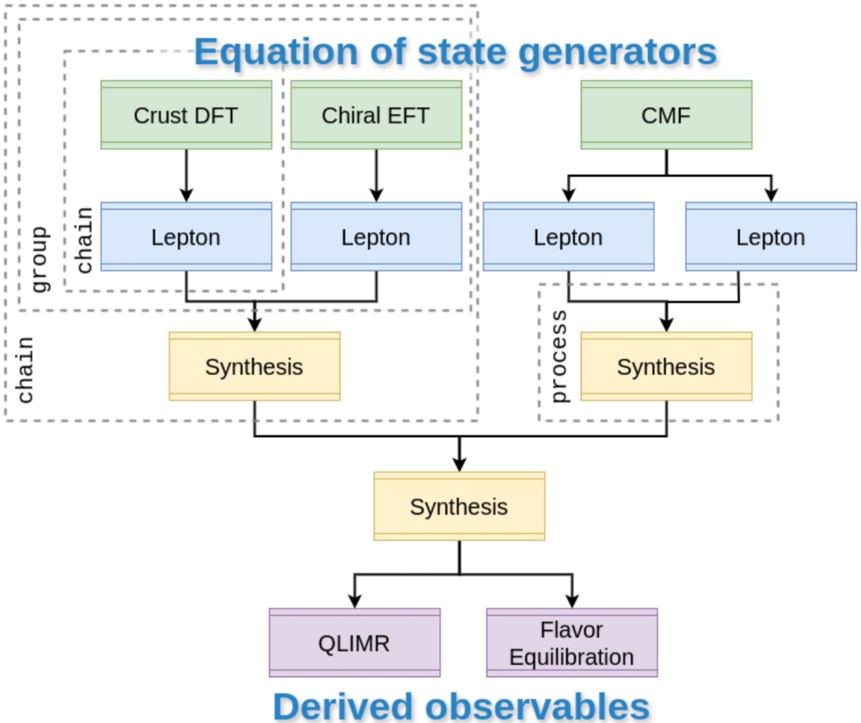
Mateus Reinke talk on Monday

muses // Calculation Engine



What is MUSES?

MUSES is an **interdisciplinary team** of physics experts in lattice QCD, nuclear physics, gravitational wave astrophysics, relativistic hydrodynamics, and research software engineers. The goal of the collaboration is to help **find answers to questions that bridge nuclear physics, heavy-ion physics, and gravitational phenomena** such as:

- What exists within the core of a neutron star?
- What temperatures are reached when two neutron stars collide?
- What can nuclear experiments with heavy-ion collisions teach us about the strongest force in nature and neutron stars?

What are MUSES modules?

Modules are the atomic processing units of a MUSES workflow. There are several types of modules, including those that calculate equations of state (EoS) and those that derive observable quantities from the EoS.

The equation of state modules include:

- **Chiral EFT** – `chiral_eft (v1.0.1)`
- **Chiral Mean Field** – `cmf_solver (v1.0.3)`
- **Crust DFT** – `crust_dft (v1.0.3)`
- **Ising 2D T'-Expansion Scheme (Ising-2DTExS)** – `eos_ising_texs_2d (v1.0.2)`
- **4D Taylor-expanded lattice (BQS)** – `eos_taylor_4d (v1.0.2)`
- **4D lattice T-Expansion Scheme (4D-TExS)** – `eos_texs_4d (v1.0.1)`
- **Holographic** – `holographic_eos (v1.3.0)`
- **Lepton** – `lepton (v1.0.3)`
- **NJL** – `njl (v1.0.0)`

The observables modules include:

- **Flavor Equilibration** – `flavor_equilibration (v1.0.0)`
- **QLIMR** – `qlimr (v1.0.2)`
- **Synthesis** – `synthesis (v1.0.4)`

Visit this page to view a more detailed list of registered modules.

Quarkyonic module soon!

Backup

Quarkyonic EFT

- Independent parameters: nucleon density $n_{_N}$ and quark baryon density $n_Q^B = n_Q / N_c$

- At fixed total baryon density...

$$n_B = n_N - n_G(n_Q^B) + n_Q^B$$

$$dn_B = dn_N + \left(1 - \frac{dn_G(n_Q^B)}{dn_Q^B}\right) dn_Q^B = 0$$

... the configuration that minimized the energy density is....

$$d\varepsilon = \frac{\partial \varepsilon}{\partial n_N} dn_N - \frac{\partial \varepsilon}{\partial n_G(n_Q^B)} dn_G(n_Q^B) + \frac{\partial \varepsilon}{\partial n_Q^B} dn_Q^B = 0$$

... and this leads to the relation

$$\mu_B = \frac{d\varepsilon}{dn_B} = \mu_N$$

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$\mu_Q^B \simeq N_c \mu_Q$
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Dynamically generated shell of nucleons: Excluded Volume Model

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2} \int_{k_F}^{k_F+\Delta} dk\, k^2$$

$$\varepsilon = \frac{N_f}{\pi^2} \left(1 - \frac{n_N}{n_0}\right) \int_{k_F}^{k_F+\Delta} dk\, k^2 \sqrt{k^2 + M^2} + \varepsilon_Q$$

Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\, k^2 = \dfrac{N_f}{3\pi^2} k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\, k^2 \sqrt{k^2 + m^2} \end{cases}$$

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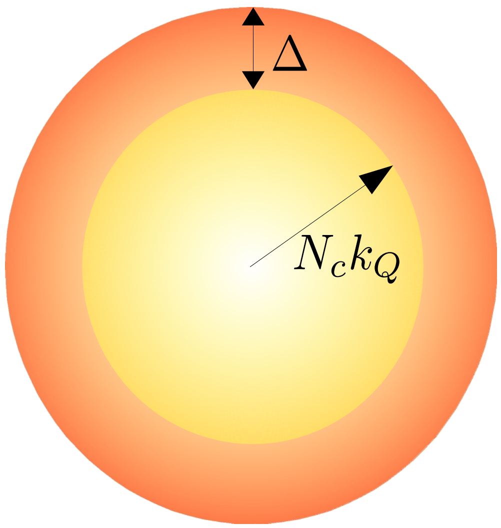

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

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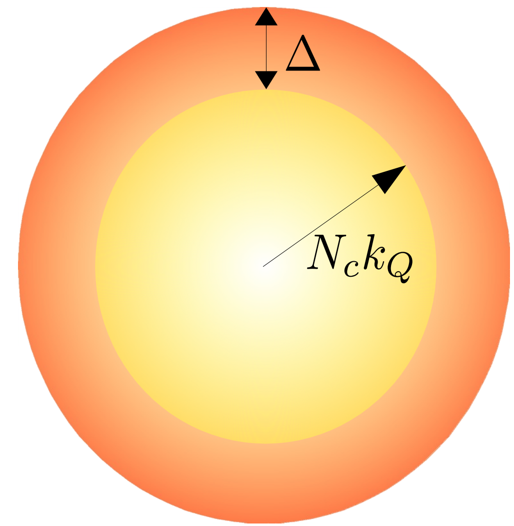

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At small k_Q quark density increase very fast to generate acceptable sound velocity ➡ Modification in the low density Fermi distribution in a way that does not affect its behavior for large Fermi momenta:

$$1 \to \frac{\sqrt{k_Q^2+\Lambda^2}}{k_Q}$$

$$n_Q = \frac{N_f}{3\pi^2}\left[\left(k_Q^2+\Lambda^2\right)^{3/2} - \Lambda^3\right]$$

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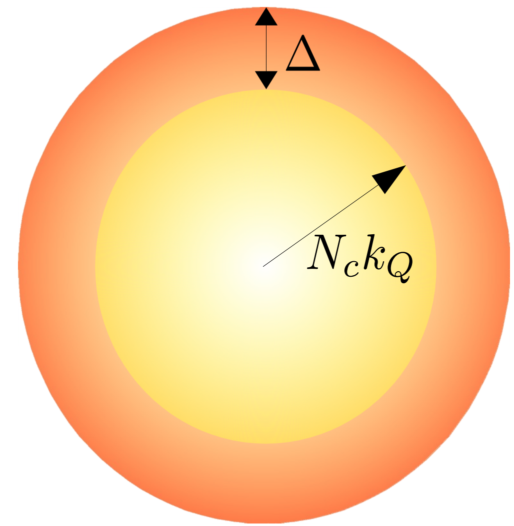

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At the minimum of the energy density: $\mu_N = N_c\,\mu_Q$

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Quarkyonic EFT: Excluded volume model

- By assuming $\mu_G = N_c \mu_Q$ one may derive the excluded volume model and verify the shell-like distribution on nucleons in the ideal gas limit.

$$\Omega = -p = \varepsilon_n - \mu_N n_n - (\mu_n - \mu_G) n_G + \varepsilon_{\tilde{Q}} - \mu_{\tilde{Q}} n_{\tilde{Q}}$$

$$\begin{aligned} \varepsilon_n &= \varepsilon_N - \varepsilon_G \\ &= 4 \int_{k_G}^{k_N} dk \frac{k^2}{2\pi^2} \sqrt{k^2 + M_N^2} \end{aligned}$$

$$n_n = n_N - n_G = 4 \int_{k_G}^{k_N} dk \frac{k^2}{2\pi^2}$$

$$k_{N,G} = \sqrt{\mu_{N,G}^2 - M_N^2}$$

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$$\begin{aligned}\varepsilon_n^{\rm ex} &= \varepsilon_N^{\rm ex} - \varepsilon_G^{\rm ex} \\ &= 4 \int_{k_G^{\rm ex}}^{k_N^{\rm ex}} dk \frac{k^2}{2\pi^2} \sqrt{k^2 + M_N^2}\end{aligned}$$

$$n_n^{\rm ex} = \frac{n_N - n_G}{1 - \frac{n_N - n_G}{n_0}} = 4 \int_{k_G^{\rm ex}}^{k_N^{\rm ex}} dk \frac{k^2}{2\pi^2}$$

$$k_{N,G}^{\rm ex} = \sqrt{(\mu_{N,G}^{\rm ex})^2 - M_N^2}$$

Quarkyonic EFT: Excluded volume model

- The other quantities in the reduced volume are related to the ones of the total system from:

$$\varepsilon^{\mathrm{ex}} = \frac{\varepsilon}{1 - \frac{n}{n_0}}$$

$$\mu^{\mathrm{ex}} = \frac{\partial \varepsilon^{\mathrm{ex}}}{\partial n^{\mathrm{ex}}}$$

- At fixed total baryon density (and zero temperature) the system configuration is obtained by extremizing the energy density, and the chemical potentials are related as $\mu_B = \mu_{N.}$.