Finite Energy Sum Rules in Dense Nuclear Matter

Cristián Villavicencio

CCE & dep. Ciencias Basicas
Universidad del Bio-Bio

CSQCD2026

Barcelona – May 2026

Outline

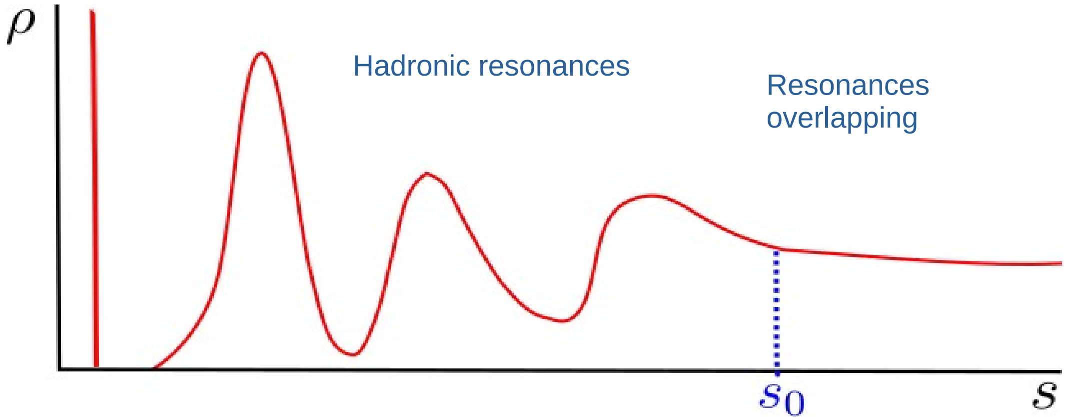
- Introduction to in-medium finite energy sum rules (FESR)
- Charged-pion channel at finite chemical potential
- Nuecleon axial coupling constant at finite density
- QCD vacuum in nuclear lasagna
- Summary and outlook

Introduction to in-medium FESR

Two current correlator $$\Pi_{\mu\nu}(x-y) = i\langle 0|TJ_\mu(x)J_\nu^\dagger(y)|0\rangle$$

Fourier transformation $$\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$$

Spectral function $$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$$



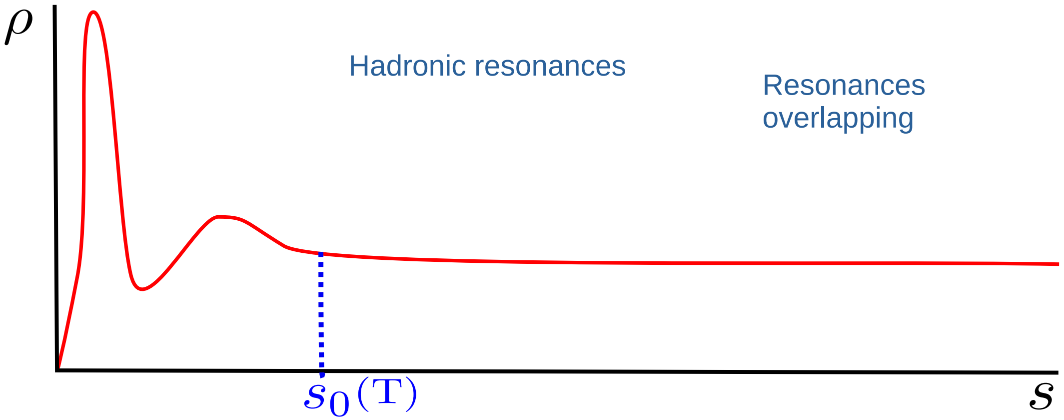
$s_0 \rightarrow$ Hadronic continuum threshold

Introduction to in-medium FESR

Two current correlator $$\Pi_{\mu\nu}(x-y) = i\langle 0|T J_\mu(x) J_\nu^\dagger(y)|0\rangle$$

Fourier transformation $$\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu} q^2 - q_\mu q_\nu)\Pi_T(q^2)$$

Spectral function $$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$$



$s_0 \rightarrow$ Hadronic continuum threshold

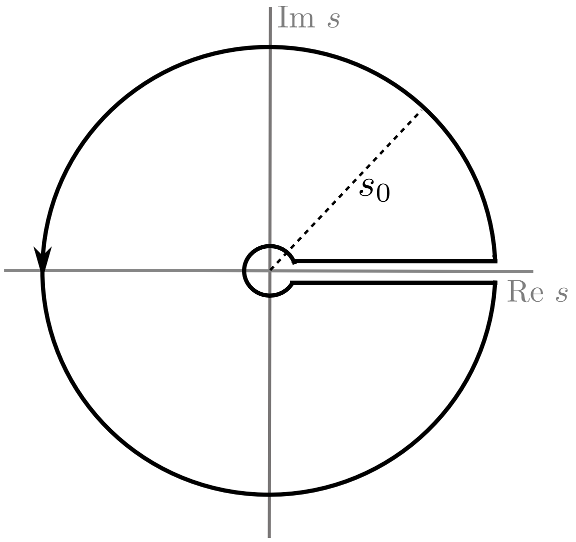
$s_0(T)$ is considered as deconfinement order parameter

In-medium FESR and quark-hadron duality → Cauchy

$$\Pi(\omega, \boldsymbol{p}^2) = \Pi^{\text{even}}(\omega^2, \boldsymbol{p}^2) + \omega \Pi^{\text{odd}}(\omega^2, \boldsymbol{p}^2) \qquad \omega^2 = s$$

$$\int_0^{s_0} \frac{ds}{\pi}\, s^N \, \mathrm{Im}\, \Pi^{\text{had}}(s + i\epsilon, 0) + \oint_{|s|=s_0} \frac{ds}{2\pi i}\, s^N\, \Pi^{\text{QCD}}(s, 0) = \operatorname*{Res}_{s=0} \left[s^N \Pi^{\text{QCD}}(s, \boldsymbol{p}) \right]_{\boldsymbol{p}\to 0}$$

scattering term



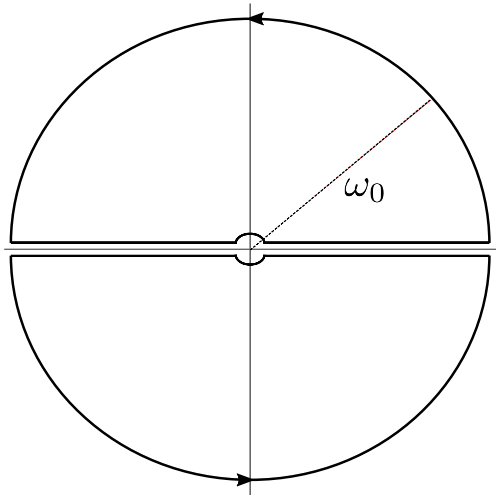
“pac-man” contour

In-medium FESR and quark-hadron duality → Cauchy

Alternative integration path $\omega_0 = \sqrt{s_0}$

$$\int_{-\omega_0}^{\omega_0} \frac{d\omega}{\pi}\, \omega^{n+1} \mathrm{Im}\, \Pi^{\mathrm{had}}(\omega + i\epsilon, 0) \;+\; \oint_{\omega_0} \frac{d\omega}{2\pi i}\, \omega^{n+1} \Pi^{\mathrm{QCD}}(\omega, 0) \;=\; \operatorname*{Res}_{\omega=0} \left[\omega^{n+1} \Pi^{\mathrm{QCD}}(\omega, \boldsymbol{p})\right]_{\boldsymbol{p}\to 0}$$

scattering term



“pokeball” contour

Operator Product Expansion (OPE) → gluon background

$$\Pi^{\mathrm{QCD}}(x,y) = \Pi^{\mathrm{pQCD}}(x-y) + \sum_{n>0} C_n(x-y)\langle\Omega| : O_n(x+y) : |\Omega\rangle$$

Wilson coefficients: short distance (UV) dynamics

Condensates: long distance (IR) information

Medium effects → IR modifications

$$\Pi(p,\Xi) = \Pi_{\mathrm{pQCD}}(p,\nu,\Xi) + \sum_{n>0} C_n(p)\,\langle : \mathcal{O}_n : \rangle(\Xi)$$

$$\Xi = T, \mu, \rho, F_{\mu\nu}, \ldots$$

Operator mixing

Problems with the chiral limit: $\ln(-s/m_q^2)$ terms

Limits $m_u, m_d \rightarrow 0$ do not commute

$$\langle \bar{q}_i q_j \rangle \equiv \langle : \bar{q}_i q_j : \rangle - \int \frac{d^d k}{(2\pi)^d} S_{ji}(k, \nu; G, \Xi)$$

non-normal ordered condensates

$$\Rightarrow \quad \Pi(p, \Xi) = \tilde{\Pi}_{\rm pQCD}(p, \nu, \Xi) + \sum_{n>0} \tilde{C}_n(p, \nu) \, \langle \mathcal{O}_n \rangle(\nu, \Xi)$$

- This replaces $\ln(-s/m_q^2) \rightarrow \ln(-s/\nu^2)$
- pQCD + OPE is renormalizable
- Mixing must be medium-dependent

- Introduction to in-medium finite energy sum rules (FESR)
- **Charged-pion channel at finite chemical potential**
- Nuecleon axial coupling constant at finite density
- QCD vacuum in nuclear lasagna
- Summary and outlook

A. Raya, C.V., Eur. Phys. J. C **85**, 713 (2025)

Correlators *AA, AP, PP*

$$\langle\Omega|T\,A_\mu(x)\,A_\nu^\dagger(0)|\Omega\rangle, \quad \langle\Omega|T\,\partial^\mu A_\mu(x)\,A_\nu^\dagger(0)|\Omega\rangle, \quad \langle\Omega|T\,\partial^\mu A_\mu(x)\,\partial^\nu A_\nu^\dagger(0)|\Omega\rangle$$

Hadronic sector

Axial current $\quad A_0 = -\sqrt{2} f_t \partial_0 \pi \qquad A_i = -\sqrt{2} f_s \partial_i \pi$

Propagator $\quad D_\pi = \dfrac{i}{p_0^2 - v_\pi^2 \boldsymbol{p}^2 - m_\pi^2}$

PCAC $\quad \partial^\mu A_\mu = \sqrt{2} f_t m_\pi^2 \pi \quad \Rightarrow \quad v_\pi = \sqrt{f_s/f_t}$

Correlators *AA, AP, PP*

$$\langle\Omega|T\, A_\mu(x)\, A^\dagger_\nu(0)|\Omega\rangle, \quad \langle\Omega|T\, \partial^\mu A_\mu(x)\, A^\dagger_\nu(0)|\Omega\rangle, \quad \langle\Omega|T\, \partial^\mu A_\mu(x)\, \partial^\nu A^\dagger_\nu(0)|\Omega\rangle$$

QCD sector

Axial current $A_\mu = \bar{d}\,\gamma_\mu\gamma_5\, u$

Quark chemical potential $\mu_q = \frac{\mu_B}{3}$

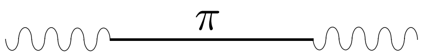
Propagator

$$S_q(p) = \frac{i(\not{p} + m_q)}{p^2 - m_q^2 + i\epsilon p_0(p_0 - \mu)}$$

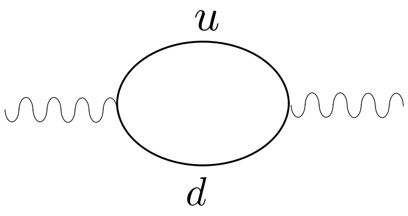
$$= (\not{p} + m_q)\left[\frac{i}{p^2 - m_q^2 + i\epsilon} - 2\pi\theta(p_0(\mu - p_0))\delta(p^2 - m_q^2)\right]$$

Feynman diagrams

Had

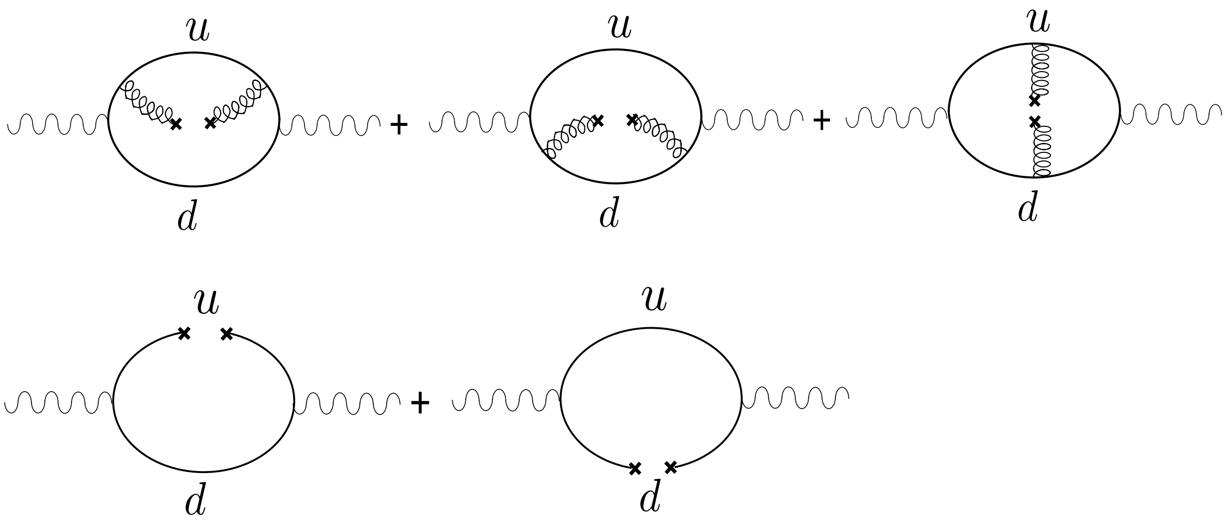


pQCD



+ radiative corrections

OPE O_4



Main results

$\mu_q < \frac{\sqrt{s_0}}{2}$ $\frac{\sqrt{s_0}}{2} \approx 0.4\ \mathrm{GeV}$ (In vacuum)

Considering the scattering term + chemical potential operator mixing

→ No explicit dependence on chemical potential
(Silver Blaze behavior)

$\mu_q > \frac{\sqrt{s_0}}{2}$

Apparently describes Color-Flavor-Locked phase → $SU(3)_{c+L+R}$ models for high μ

$m_\pi^2 \sim m^2$ $v_\pi^2 \sim \frac{1}{3}$ $f_\pi \sim \frac{\mu_q}{2\pi}$ $\langle \bar{q}q \rangle \sim m$

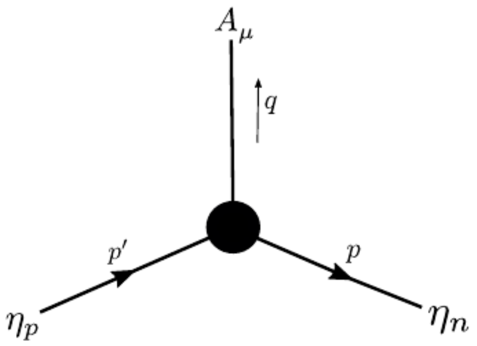
- chromoelectric field vanishes

- Introduction to in-medium finite energy sum rules (FESR)
- Charged-pion channel at finite chemical potential
- **Nuecleon axial coupling constant at finite density**
- QCD vacuum in nuclear lasagna
- Summary and outlook

C.A. Dominguez, M.Loewe, C.V., R.Zamora, Phys. Rev. D **108**, 074024 (2023)

Nucleon-Axial-nucleon correlator → Hadronic sector

$$\Pi_\mu(x,y,z) = -\langle 0|\,\mathcal{T}\,\eta_p(x) A_\mu(y)\,\bar{\eta}_n(z)\,|0\rangle$$



Nucleon interpolating function

$$\eta_N(x) = \lambda_N \psi_N(x)$$

Nucleon field

Current-nucleon coupling

Axial current

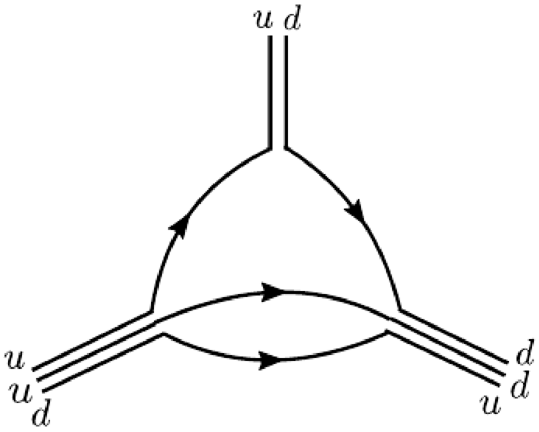
$$A_\mu(y) = \int d^4y'\,\bar{\psi}_p(y') T_\mu(y'-y)\psi_n(y')$$

$$\tilde{T}_\mu(q) = G_A(q^2)\gamma_\mu\gamma_5 + G_P(q^2)\gamma_5\frac{q_\mu}{2m_N} + G_T(q^2)\sigma_{\mu\nu}\gamma_5\frac{q_\nu}{2m_N}$$

$$g_A \equiv G_A(0)$$

Nucleon-Axial-nucleon correlator → QCD sector

$$\Pi_\mu(x,y,z) = -\langle 0|\, \mathcal{T}\, \eta_p(x) A_\mu(y)\, \bar{\eta}_n(z)\, |0\rangle$$



Nucleon interpolating function

$$\eta_p(x) = \epsilon^{abc}\left[u^a(x)^T\, C\gamma^\mu\, u^b(x)\right]\gamma_\mu\gamma_5\, d^c(x),$$

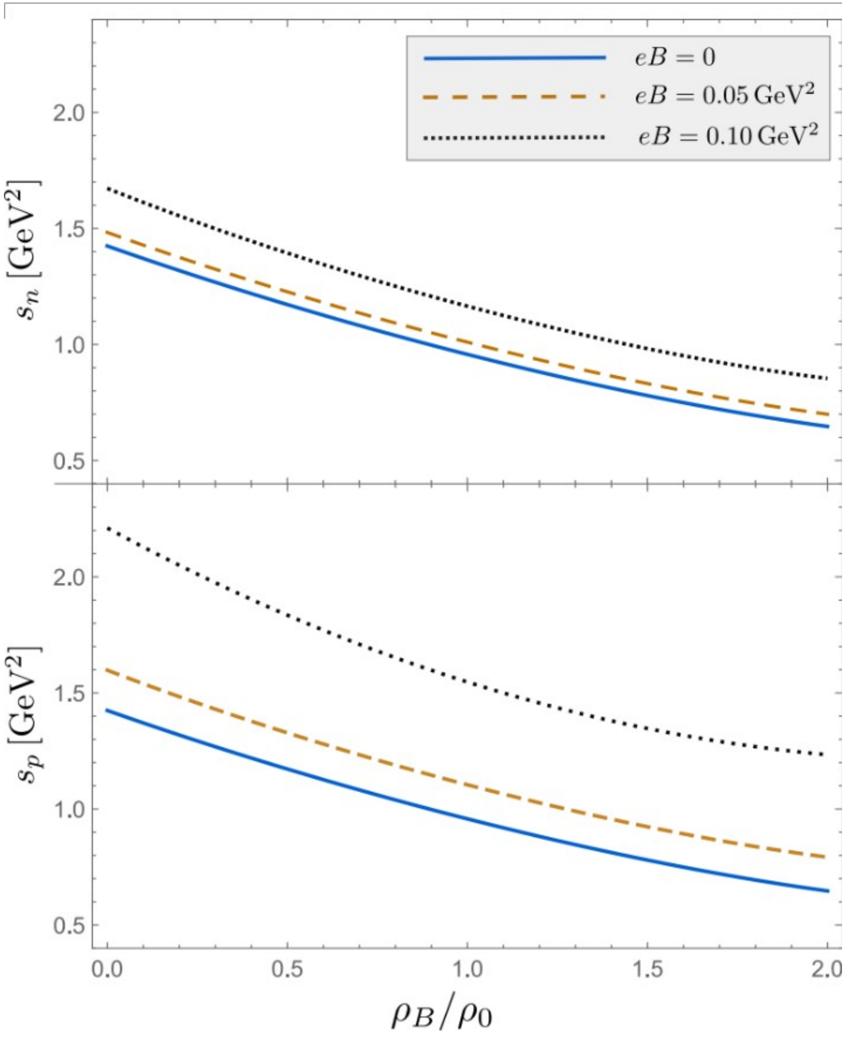
$$\bar{\eta}_n(z) = \epsilon^{abc}\left[\bar{d}^b(z)\,\gamma^\mu C\,\bar{d}^a(z)^T\right]\bar{u}^c(z)\,\gamma_\mu\gamma_5$$

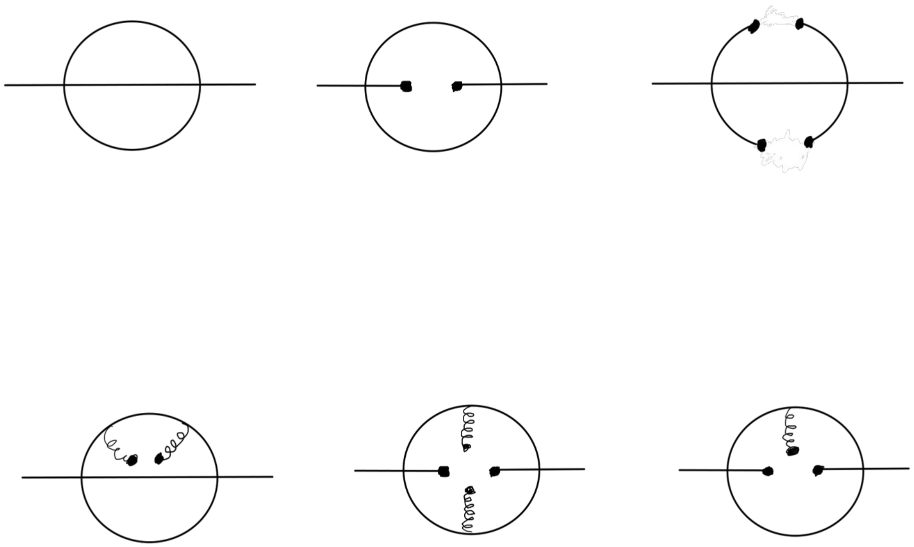
Axial current

$$A_\mu(y) = \bar{d}(y)\,\gamma_\mu\gamma_5\, u(y)$$

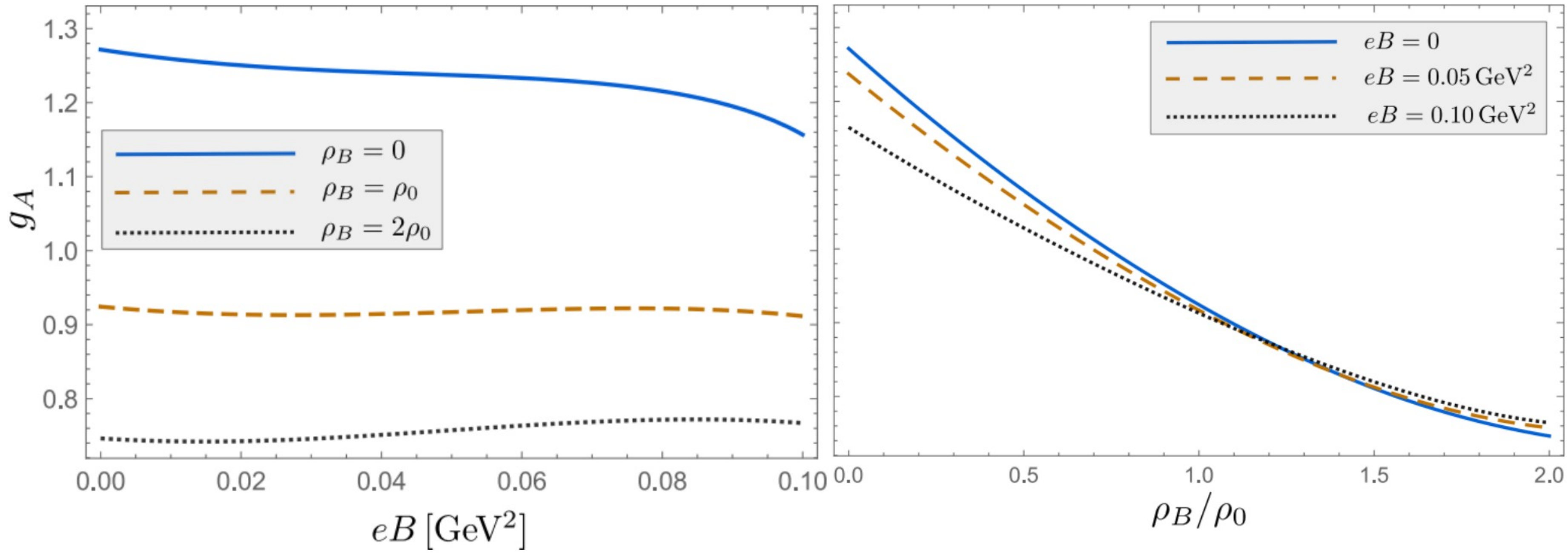
hadronic thresholds and current couplings

- Nucleon-Nucleon correlator





$$g_A = \frac{1}{48\pi^4} \frac{\min(s_n^3, s_p^3)}{\lambda_p \lambda_n}$$



$$g_A^* = g_A(\rho_0) = 0.92$$

(commonly stablished $g_A^* \sim 1$)

- Introduction to in-medium finite energy sum rules (FESR)
- Charged-pion channel at finite chemical potential
- Nuecleon axial coupling constant at finite density
- **QCD vacuum in nuclear lasagna**
- Summary and outlook

F. Canfora, M. Loewe, E. Muñoz, C.V

Skyrme model for layers

Mod.Phys.Lett.A 38 (2023)

$$\mathcal{L} = \frac{f_\pi^2}{4} U^\dagger \partial_\mu U \, U^\dagger \partial^\mu U + \ldots$$

$$U = e^{i\tau_3 F(x)} e^{i\tau_2 H(x)} e^{i\tau_3 G(x)}$$

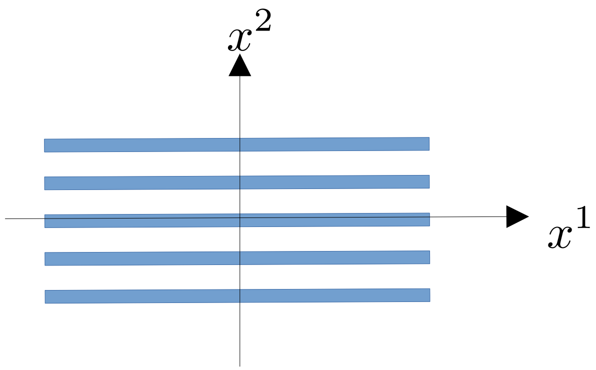
(Euler angles)

Background classical solutions

$$F_0(x) = \frac{1}{2}\left(\frac{2\pi n_1}{L_1}\right) x^1$$

$$H_0(x) = \frac{1}{4}\left(\frac{2\pi}{L_2}\right) x^2$$

$$G_0(x) = 4\left(\frac{2\pi n_3}{L_3}\right)(x^0 - x^3)$$



$$L_3 \gg L_1 \gg L_2$$

The Lagrangian is expanded in quantum fluctuations

$$G(x) = G_0(x^0 - x^3) + \eta(x)/f_\pi$$

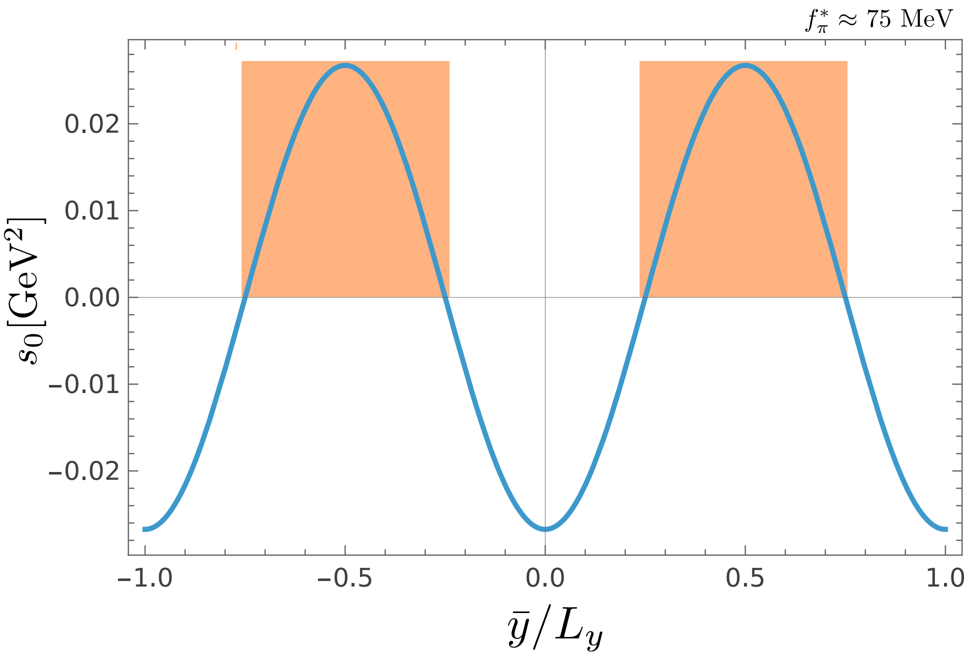
charged axial current

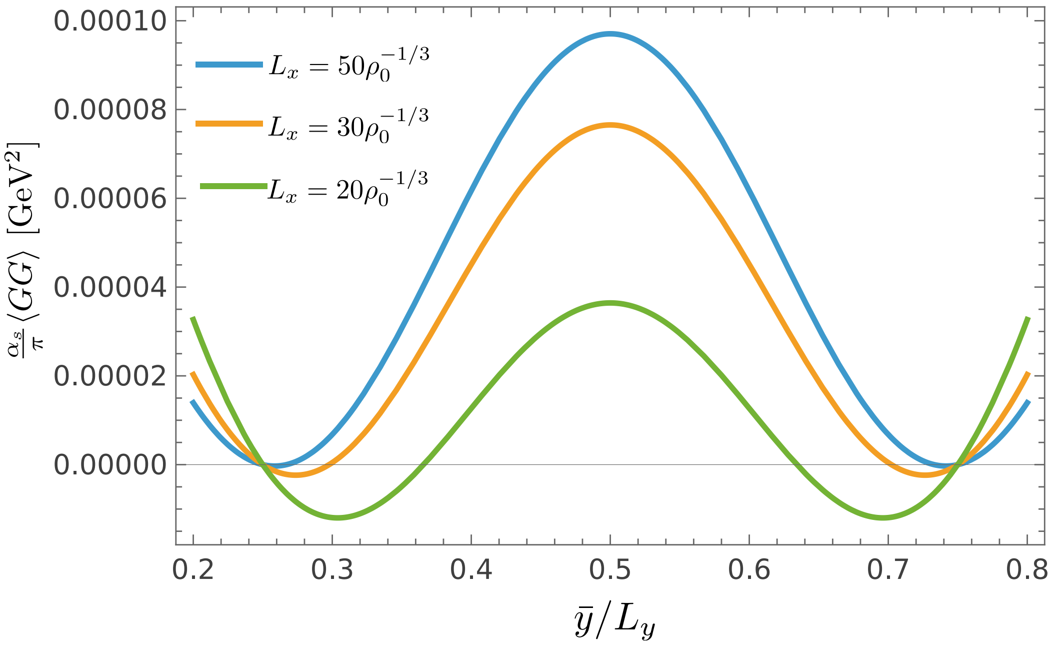
$$A_\mu = f_\pi \left(\frac{1}{2} e^{i2F_0} \sin(2H_0)\partial_\mu \eta + e^{-i2G_0} \left[\partial_\mu H_0 + i\partial_\mu F_0 \sin(2H_0)\right] \eta \right)$$

AA coorelator → non-symmetric and complicated structure

→ “parallel” x^0,x^3 coordinates behaves similar to the vacuum case

Preliminary partial results





Summary

- Interesting tool for exploring high density nuclear and QCD phenomena
- High chemical potential → apparently describes CSC
- Hadronic threshold → diminishes with baryon density
- Nucleon axial coupling constant → agrees with g_A^* ~1
- Nuclear lasagna → at the moment provides consistent results

Outlook

- NN correlators at finite baryon chemical potential
- axial and vector axial coupling improved
- complete analysis of nuclear pasta

Gràcies