Compact Stars with Renormalization Group Improved Cold and Dense Quark Matter

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Compact Stars in the QCD phase diagram,
Barcelona, May 18-22, 2026

Based on arXiv:2408.16674 (L. Fernandez, JLK),
arXiv:2501.14935 (JLK, M.B Pinto, C. Providencia, T. Restrepo),
and preliminary results from work in progress (L. Fernandez, JLK, M. Pinto, C. Providencia, C. Ratti, T. Restrepo (arXiv:2606.***)

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Renormalization Group Optimized Perturbation (RGOPT)

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Application to hybrid stars

Context, motivations

Slow convergence of thermal (medium) perturbative expansion:

- Hard sector $p \sim T, \mu$: follows standard (fixed order) perturbative expansion
- Soft sector $p \sim g_s T, g_s \mu$: resummation gives non-analytic terms in expansion:

  $\to \sqrt{\alpha_S}$ at $T \neq 0$: not good convergence.

  $\to \ln^k(\alpha_S)$ at $T = 0, \mu \neq 0$: much better convergence. Yet sizable residual renormalization scale uncertainties in pQCD, mainly from the hard sector, below $\mu_B \lesssim 2$ GeV ($n \lesssim 30 - 40 n_s$): relevant range for compact stars
- EoS impacted by pQCD even down to lower μ_B region (see Komoltsev today's talk)
- Crucial to calculate higher orders in the weak coupling expansion: potentially drastically reduced uncertainties: full N^3LO in sight! Gorda et al 2021; 2023 (see A. Vuorinen today's review)
- Complementary path: resummation /going beyond perturbative expansion

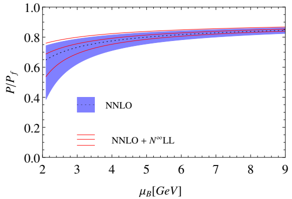
  In particular for QCD: HTLpt (Andersen, Braaten, Strickland '99 + many) $\simeq$ resummation within Hard Thermal Loop (HTL) Effective Theory (Braaten, Pisarski '92)
- Our approach ($T, \mu \neq 0$) "Renormalization Group optimized perturbation": resummation operates via "RG-dressed" soft masses, obtained from RG Equation

Context, motivations

- More precisely for $\mu_B \neq 0$, $T = 0$: importantly $P^{N^2LO}_{pQCD}$ ($m_q = 0$, or $m_s \neq 0$) is perturbatively RG-invariant $\rightarrow$ residual scale dependence is *N3LO* $\mathcal{O}(\alpha_s^3)$.

  Yet sizable scale dependence (below $\mu_B \lesssim 2$ GeV): originates dominantly from hard sector: $P_{N^2LO} = P_f \left(1 - (2/\pi)\alpha_s - \alpha_s^2[0.87 + 0.30 \ln \alpha_s + 0.91 \ln(\Lambda/\mu)]\right)$

  -In soft sector, using RG methods in HTL, leading logarithms $\alpha_s^{k+2} \ln^k(\alpha_s)$ were resummed to all orders, significantly reducing scale dependence (Fernandez, JLK 2021)



  $\leftarrow$ bands: $\mu \leq \Lambda \leq 4\mu$

  But only partly balances the large scale dependence in the hard sector.

- This N^2LO hard sector scale dependence worsens somehow for $m_s \neq 0$ (Kurkela, Romatschke, Vuorinen 2010)
- RG resummation may also improve the hard sector, complementarily to higher order calculations

Renormalization Group Optimized Perturbation (RGOPT)

Generic prescription (both for $T, \mu \neq 0$ or in vacuum):
"add and subtract" an auxiliary (unphysical) mass, treating $m\,\delta$ as interaction:

$\mathcal{L}_{QCD}(m, \alpha_s) \to \mathcal{L}_{QCD}(m(1-\delta)^a, \delta\,\alpha_s)$
• $T, \mu \neq 0$: expansion about quasi-particle mass $m \sim g_s\,T$ (or $m \sim g_s\,\mu$)

• Start from *renormalized* (massive) LO, NLO,...$P(m, \alpha_s, T, \mu)$ at $\mathcal{O}(\alpha_s^k)$:

1) first step: (re)introduce perturbatively RG-invariant (RGI) vacuum contributions:
$P_{pQCD} \to P_{pQCD} + P_{vac}^{RGI}$, $\quad (P_{vac} \propto m^4$: often neglected since medium-independent)
crucial for RG properties: massive theories get vacuum energy anomalous dimension!

2) $m_q \to m\,(1-\delta)^a$, $\alpha_s \to \delta\,\alpha_s$, expand in δ to LO, NLO, ...$\mathcal{O}(\delta^k)$; take $\delta \to 1$ afterwards
$\rightsquigarrow P^{(k)}(m, \alpha_s, T, \mu; \delta = 1)$ has remnant m-dependence, fixed by further prescription.

3) $a = \dfrac{\gamma_0}{b_0}$ uniquely guarantees (LO) RGI of modified perturbation

4) Fix trial mass m from RG [JLK, A. Neveu '2013, '2015; JLK, M.B. Pinto '2015]

$$\mathrm{RG}\left[P^{(k)}(m, \alpha_s, T, \mu; \delta = 1)\right] = 0 \qquad \text{with } \mathrm{RG} \equiv \Lambda\frac{\partial}{\partial\Lambda} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} - \gamma_m(\alpha_s)\,m\frac{\partial}{\partial m}$$

$\to$ self-consistent "mass gap" solution $\tilde{m}_{RG}(\alpha_s, T, \mu) \neq 0 \to P^{(k)}(\tilde{m}_{RG}(\alpha_s, T, \mu), \alpha_s, T, \mu)$

RGOPT main features

Similar to "screened perturbation" (SPT) $\phi^4(T \neq 0)$ (Parwani '92, Karsch et al '97), or for hot QCD, to HTLpt (Andersen, Braaten, Strickland '99, Andersen et al '2002-2014)

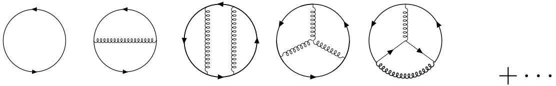
But those works take $m \to m\,(1-\delta)^a$ with $a = 1$, and fix m by "stationarity":
$\partial_m P^{(k)}(m, \alpha_s, \ldots) = 0$,   or more simply taking $m = m_{Debye}$

RGOPT main differences:

- $m \to m\,(1-\delta)^{\frac{\gamma_0}{2b_0}}$ goes beyond "add and subtract" trick:
  at increasing orders, multi-solutions $\tilde{m}(g_S, T, \mu, \ldots)$ may appear, while $\tilde{m}_{RG}$ unique at a given order (RG consistency removes spurious solutions).
- trial mass m primarily fixed by RG equation (rather than stationarity).
- Depart from strict (agnostic) pQCD, but basically rooted in pQCD and parameter-free determination: final (physical) result $P(\tilde{m}, g_S, \Lambda; T, \mu) \rightsquigarrow P(\Lambda^{QCD}_{\overline{MS}}, T, \mu)$
- Caveat: non-linear mass gap equations for $m(g_s, T, \mu)$: may give (beyond LO)
  unphysical (non-real) $\tilde{m}$ solutions (common issue for SPT, HTLpt, RGOPT)
  artifacts of renormalization scheme $\rightsquigarrow$ possibly cured by scheme change
  (or alternatively by perturbatively expanded $\tilde{m}_{RG}$ solutions, always real)

Digression: previous RGOPT results (in vacuum and for $T \neq 0$)

In vacuum: previous estimates of chiral symmetry breaking order parameter $\langle \bar{q} q \rangle$:



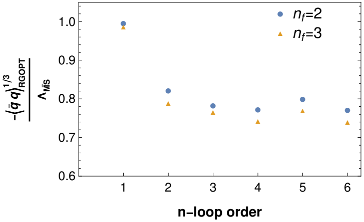
pQCD known to 5-loop order (for arbitrary m_q)! (Baikov, Chetyrkin 2018; Maier, Marquard, 2020)

RGOPT $\to \langle \bar{q} q \rangle^{1/3}_{m_q=0}(2\,\mathrm{GeV}) \simeq -(0.78 \pm 0.02)\Lambda^{\mathrm{QCD}}_{\overline{\mathrm{MS}}}$ (JLK, A.Neveu, '15, '20)

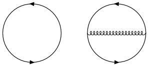
Parameter free determination

•Compares well with latest most precise lattice value (Aoki et al, FLAG coll., 2020)

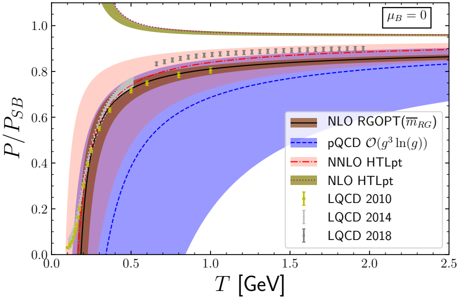
•Good convergence properties exhibited to higher orders: already realistic at NLO:



$T \neq 0$ QCD: NLO RGOPT vs pQCD, HTLpt and Lattice



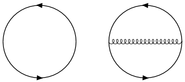
$+ \cdots$ (gluon sector)



High T Pressure scale dependence ($\pi T \leq \Lambda \leq 4\pi T$) [JLK, M.B. Pinto, T. Restrepo '2021]

- Drastic improvement of residual scale dependence w.r.t. pQCD and HTLpt
- Very good agreement with lattice data (2010) in intermediate region $0.25 < T < 1$ GeV
- But fail to reproduce pseudo-critical T_c region (same for HTLpt)

Cold Quark Matter ($T = 0, \mu \neq 0$): massive quark case

Consider first only LO + NLO: 

$$\frac{P_q^{PT}}{N_f} \equiv P_{matter}(\mu, m) + P_{vacuum}(m) = \Theta(\mu - m) \left\{ \frac{N_c}{12\pi^2} \left[\mu p_F \left(\mu^2 - \frac{5}{2} m^2 \right) + \frac{3}{2} m^4 \ln(\frac{\mu + p_F}{m}) \right] \right.$$
$$\left. -2 \frac{g_s^2}{(2\pi)^4} \left[3 \left[m^2 \ln(\frac{\mu + p_F}{m}) - \mu p_F \right]^2 - 2 p_F^4 \right] \right\}$$
$$-\frac{m^4}{8\pi^2} \left(\frac{3}{4} - \ln \frac{m}{\Lambda} \right) - 2 g_s^2 \frac{m^4}{(2\pi)^4} \left(\ln^2(\frac{m}{\Lambda}) - \frac{4}{3} \ln(\frac{m}{\Lambda}) + \frac{3}{4} \right)$$

$g_s^2 = 4\pi\alpha_s$, generic quark mass $m \neq 0$, $p_F = \sqrt{\mu^2 - m^2}$

First step: RG invariance adds a zero-point energy: $P_q^{PT} \to P_q^{PT} - \mathcal{E}_0(\mathrm{m}, \mathrm{g}, \mathrm{T} = 0)$:

$\mathcal{E}_0(m, g_s^2) = m^4 \left(\frac{s_0}{g_s^2} + s_1 + s_2 g_s^2 + \cdots \right)$ (connected to *vacuum energy anomalous dimension*)

$s_0 = -\frac{1}{(4\pi)^2 (b_0 - 2\gamma_0)}$, $s_1 = \cdots$ entirely determined from (massive) RG coefficients

Simple illustration: one-loop RGOPT quark matter pressure

$$P^{RGOPT}(m \to m(1-\delta)^{\frac{\gamma_0}{2b_0}}, g_s^2 \to \delta g_s^2)|_{\mathcal{O}(\delta^0),\delta\to 1} =$$

$$N_f N_c \left\{ \frac{m^4}{(4\pi)^2\, b_0\, g_s^2} + \frac{1}{12\pi^2}\left[\mu p_F\left(\mu^2 - \frac{5}{2}m^2\right) + \frac{3}{2}m^4\left(L_\mu - \frac{3}{4}\right)\right]\right\}$$

$$L_\mu \equiv Log\frac{\mu}{\Lambda} + Log[1 + \frac{p_F}{\mu}]$$

Dressed mass solution $\tilde{m}(g_s,\mu)$ at leading order:

$$\tilde{m}^2 = \mu^2\left(\frac{\sqrt{1+4C(\tilde{m},\mu,g_s)}-1}{2C(\tilde{m},\mu,g_s)}\right) \;,\quad C(m,\mu,g) = \left(\frac{1}{2b_0 g_s^2} - \frac{1}{2} + L_\mu\right)^2$$

(JLK, M.B Pinto, T. Restrepo '19)

Implicit equation in $m \to$ all-order $\tilde{m}(g_s,\mu)$ resummation

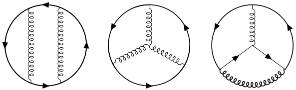
•Perturbatively expanded, it has form of a screening mass $\tilde{m} \sim \# g_s\, \mu$

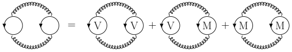
•Exhibits (one-loop exact) RG invariance (generic RGOPT feature):

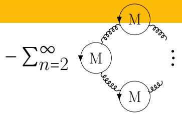
since one-loop running obeys $g_s^{-2}(\Lambda) = g_s^{-2}(\Lambda_0) + 2b_0 \ln\frac{\Lambda}{\Lambda_0}$

at NNLO: hard and soft (ring sum) enter with full m_q-dependence

(Kurkela, Romatschke, Vuorinen 2010)







$$P^{hard}_{NNLO} \sim \frac{g_s^4}{(4\pi)^4}\frac{2\mu^4}{\pi^2}\left\{ f_{analytic}(m,p_F) + C_A\left(-\frac{11}{3}\ln\frac{\hat{m}}{2} - \frac{71}{9} + G_1(\hat{m})\right) + C_F\left(\frac{17}{4} + G_2(\hat{m})\right) + N_f\left(\frac{2}{3}\ln\frac{\hat{m}}{2} + \frac{11}{9} + G_3(\hat{m})\right) + G_4(\hat{m})\right\}$$

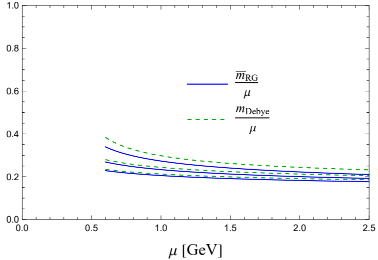
$\hat{m} = m/\mu$, $G_k(\hat{m})$ numerical functions obtained from fitting massive master integrals

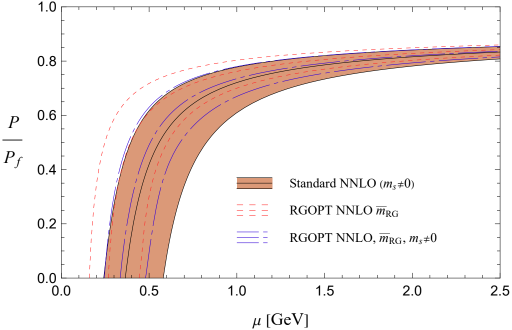
- Includes also NNLO massive vacuum contributions

- RGOPT ($m_s \neq 0$): $m_{u,d} \to m\,(1-\delta)^{\frac{\gamma_0}{2b_0}}$, $m_s \to m_s + m\,(1-\delta)^{\frac{\gamma_0}{2b_0}}$, $\alpha_s \to \delta\,\alpha_s$
  (i.e. physical m_s unaffected)

- Common RG-dressed mass m for u, d, s

NNLO RGOPT pressure for cold quark matter [JLK, L. Fernandez,2408.16674]





Left: NNLO $\tilde{m}_{RG}$ vs m_{Debye}, $\mu \leq \Lambda \leq 4\mu$. Right: NNLO RGOPT vs pQCD $P(\mu)$, $\mu \leq \Lambda \leq 4\mu$

Sizably reduced residual scale dependence, although more moderate compared to RGOPT for hot QCD.
May be traced to: -strong m_q depence at NNLO (and $m_q \sim \# \alpha_s^{1/2} \mu_B \gg m_s$)
-already well-behaving $T = 0, \mu \neq 0$ standard pQCD.

Applications to compact stars: comparison with pQCD

From NNLO RGOPT pressure expression, we consider as usual

- chemical equilibrium and charge neutrality:

$$\mu_u = \mu_d - \mu_e \equiv \mu, \quad \mu_s = \mu_d,$$

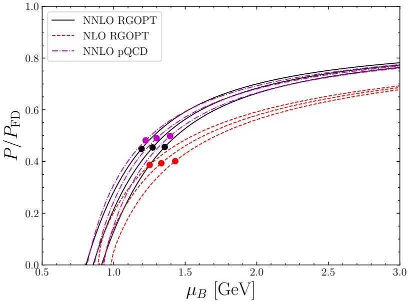
$$\frac{2}{3}\rho_{\rm up} - \frac{1}{3}\rho_{\rm down} - \frac{1}{3}\rho_{\rm strange} - \rho_{\rm electron} = 0,$$

  with $\rho_i = \frac{\partial P_{\rm RGOPT}}{\partial \mu_i}|_\Lambda$

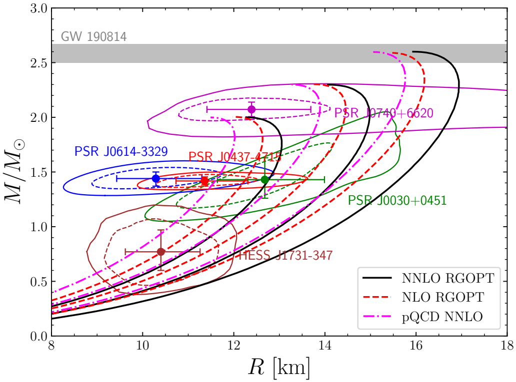
- Thermodynamically consistent pressure and density: (non-trivial construction in our case, since $\tilde{m}_{RG}(\alpha_S, \Lambda)$ entails extra scale dependence Λ)
- We provide thermodynamically consistent P, ρ as pocket formulas obtained from appropriate fits.
  (NB other relevant input: NLO running $\alpha_S(\Lambda)$ and strange quark mass $m_s(\Lambda)$)
- We solve TOV $M(R)$ relations for such pure quar star Equation of states
- We compare NLO, N2LO RGOPT with N2LO pQCD, varying renormalization scale $X \simeq 3\Lambda/\mu_B$ to match representative NS data for compact star candidates (NICER, GW) (in particular some with maximal masses)

(strange) quark stars: RGOPT versus pQCD

NB previous NLO RGOPT for quark stars: JLK, M.B. Pinto, C. Providencia, T. Restrepo, 2501.14935



$P(\mu_B)$ (β-equilibrated matter),
$X \simeq 3\Lambda/\mu_B$ matching $M_{\rm max} = 2, 2.3$ and $2.6 M_\odot$
Caution: X values different for RGOPT and pQCD!



$M(R)$ of quark stars for the different aproximations compared to NS data

(strange) quark stars: RGOPT versus pQCD

	X $\simeq 3\Lambda/\mu_B$	$M_{\rm max}$ $(M_\odot)$	$R_{\rm max}$ (km)	$R_{1.4}$ (km)	$R_{0.77}$ (km)	$\rho_B^{c,\rm max}$ (ρ_0)	$\mu_B^{c,\rm max}$ (GeV)	$\rho_B^{\rm surf,max}$ (ρ_0)	$\mu_B^{\rm surf,max}$ GeV	$\alpha_s^{\rm surf}$
NNLO RGOPT	3.08	2.00	12.3	12.7	11.0	4.98	1.357	0.90	0.915	0.44
	3.58	2.30	14.1	14.2	12.1	4.08	1.272	0.74	0.856	0.41
	4.10	2.60	15.9	15.5	13.2	3.36	1.194	0.61	0.806	0.39
NLO RGOPT	3.63	2.00	12.0	12.2	10.4	4.95	1.430	1.03	0.969	0.38
	3.97	2.30	13.7	13.5	11.4	3.98	1.337	0.86	0.930	0.37
	4.30	2.59	15.5	14.6	12.7	3.21	1.251	0.73	0.890	0.36
NNLO pQCD	2.95	2.00	11.2	11.6	9.85	5.48	1.394	1.31	0.923	0.45
	3.26	2.30	13.3	12.9	11.0	4.40	1.300	1.04	0.863	0.44
	3.56	2.60	15.1	14.2	12.0	3.62	1.223	0.84	0.813	0.43

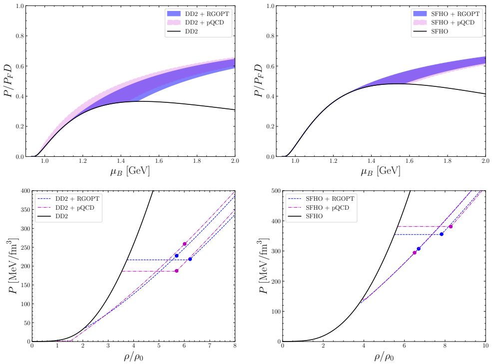
QS properties at NLO, NNLO RGOPT, NNLO pQCD for $N_f = 2+1$, $X \simeq 3\Lambda/\mu_B$ matching $M_{\rm max} = 2,\ 2.3$ and $2.6 M_\odot$.

- reduced RGOPT X-scale dependence $\rightsquigarrow$ larger ΔX spanned w.r.t. pQCD to match stars.
- α_s (slightly) smaller at surface: somewhat "more perturbative".
- for NNLO RGOPT, $\mu_B^{surface}$ compatible with Bodmer-Witten hypothesis (strange QM), pQCD too.

Hybrid stars: RGOPT versus pQCD hybrid EoS

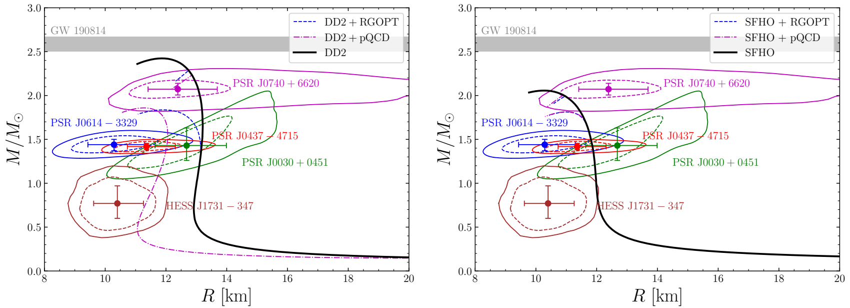
To build an hybrid EOS, accounting for hadronic phase uncertainties, we select two representative soft and stiff EoS respectively, both based on relativistic mean field (RMF)

- SFHo (Steiner, Hempel, Fischer 2013) as soft EOS;
- DD2 (Typel, Röpke, Klähn, Blaschke, Wolter 2009) as reasonably stiff EoS.
- Then doing a standard Maxwell construction for first order phase transition (FOPT)



Bands: X-scale range that can produce a quark core

Hybrid stars: RGOPT versus pQCD hybrid EoS (preliminary results!)



$M(R)$ for NNLO RGOPT, pQCD and DD2 (left)/SFHO (right) hadronic EOS

Higher X-values produces lower maximum star masses.

Depending on hadronic EoS prescription, and X scale values, hybrid stars show very different fate (stable hybrid branch, or immediately unstable)

NB an illustration in our model of some behavior explained in Prof. Alford review at this conference.

Hybrid stars: RGOPT versus pQCD hybrid EoS (preliminary results!)

	X $\simeq 3\Lambda/\mu_B$	$M_{\rm max}$ ($M_\odot$)	$M_{\rm core}/M_{\rm max}$	$R_{\rm max}$ (km)	$R_{\rm core}$ (km)	$\rho_B^{c,\rm max}$ (ρ_0)	$R_{1.4}$ (km)	$\rho_B^{c,1.4}$ (ρ_0)	μ_B^t (GeV)	α_s^t	$\rho_B^{t,\rm had}$ (ρ_0)
DD2 +	2.00	2.28	3×10^{-4}	12.7	0.509	6.23	13.2	2.19	1.484	0.426	3.74
RGOPT	2.59	1.84	0.591	11.9	8.12	5.70	13.1	2.33	1.101	0.475	2.06
DD2 +	2.16	2.23	2×10^{-4}	12.8	0.474	5.70	13.2	2.19	1.432	0.413	3.53
pQCD	2.80	1.86	0.979	11.3	10.6	6.01	12.0	2.67	0.962	0.460	0.76
SFHO +	2.30	1.99	1×10^{-4}	11.0	0.355	7.08	11.9	3.20	1.567	0.372	5.51
RGOPT	2.98	1.81	0.282	11.0	5.41	6.70	11.9	3.20	1.263	0.362	3.79
SFHO +	2.24	2.00	$< 10^{-4}$	10.9	0.0239	7.16	11.8	3.23	1.596	0.374	5.66
pQCD	2.86	1.81	0.232	11.0	5.25	6.50	11.8	3.23	1.283	0.368	3.91

Hybrid star properties for the different models. X scale values are those that reproduce a finite quark core.

•Here X_{max} such that it can allow the matching of the two EoS.

•Some observed differences between pQCD and RGOPT for μ_B^{trans}, α_s^{trans}, but also influenced by hadronic EoS part.

Summary

- RGOPT prescription departs from strict (agnostic) pQCD, but basically rooted in pQCD and parameter-free determination
- At NNLO it sizably reduces the residual scale dependence w.r.t. pQCD, both for symmetric quark matter or beta-equilibrated, thermodynamically consistent pressure
- Better scale dependence translates into visible differences between RGOPT and pQCD when matching NS data (RGOPT appears more stable)
- Preliminary applications to hybrid star EoS: interesting qualitative differences with pQCD (to be finalized)
- Future prospects: extend RGOPT to full QCD thermodynamics