Peaks of the Speed of Sound in Dense QCD Matter

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Compact stars in the QCD phase diagram

Barcelona, May 2026

Outline

- Motivation
- Medium Separation Scheme (MSS)
- How do medium effects can influence the phase diagram of **QCD**?
- Effective models of **QCD** under extreme conditions X LQCD
- Conclusions and perspectives

Why Study the Speed of Sound?

The squared speed of sound is defined as

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

where:

- $P =$ pressure
- $\epsilon =$ energy density

It measures the response of the medium to compression.

In dense QCD matter, c_s^2 is one of the most important probes of:

- phase transitions,
- equation of state stiffness,
- microscopic degrees of freedom,
- neutron-star structure,

Physical Interpretation

Interpretation

The quantity c_s^2 determines how efficiently pressure reacts to changes in energy density.

- Large c_s^2:
  - stiff equation of state,
  - matter resists compression.
- Small c_s^2:
  - soft equation of state,
  - matter compresses easily.

Typical limits:

$$\begin{aligned} c_s^2 &\approx 0 \quad \text{soft matter / phase transition region,} \\ c_s^2 &\to \frac{1}{3} \quad \text{ultrarelativistic conformal limit.} \end{aligned}$$

Conformal Limit violation: First evidence

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Editors' Suggestion

Sound Velocity Bound and Neutron Stars

Paulo Bedaque and Andrew W. Steiner
Phys. Rev. Lett. **114**, 031103 – Published January 2015

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If the bound on the speed of sound is actually violated – as it is strongly suggested by our results– the speed of sound, as a function of the energy density, has a peculiar shape. It raises from small values, reaches a maximum with $v_s^2 > 1/3$, lowers to a local minimum with $v_s^2 < 1/3$ and then raises again approaching $v_s^2 = 1/3$ from below at high densities. We find remarkable that such a conclusion can be derived from well established facts.

Conformal Limit violation: First evidence

PHYSICAL REVIEW LETTERS

Sound Velocity Bound and Neutron Stars

Paulo Bedaque and Andrew W. Steiner
Phys. Rev. Lett. **114**, 031103 – Published 21 January 2015

Using nuclear EFT at low density and the observed existence of $2M_\odot$ neutron stars, Bedaque and Steiner showed that any realistic equation of state must violate the conformal bound $c_s^2 \leq 1/3$ at intermediate densities.

GW constraints

GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

GW190425

- $R_{M=1.4M_\odot} \leq 15$ km and M_{max} $(2-3)M_\odot$

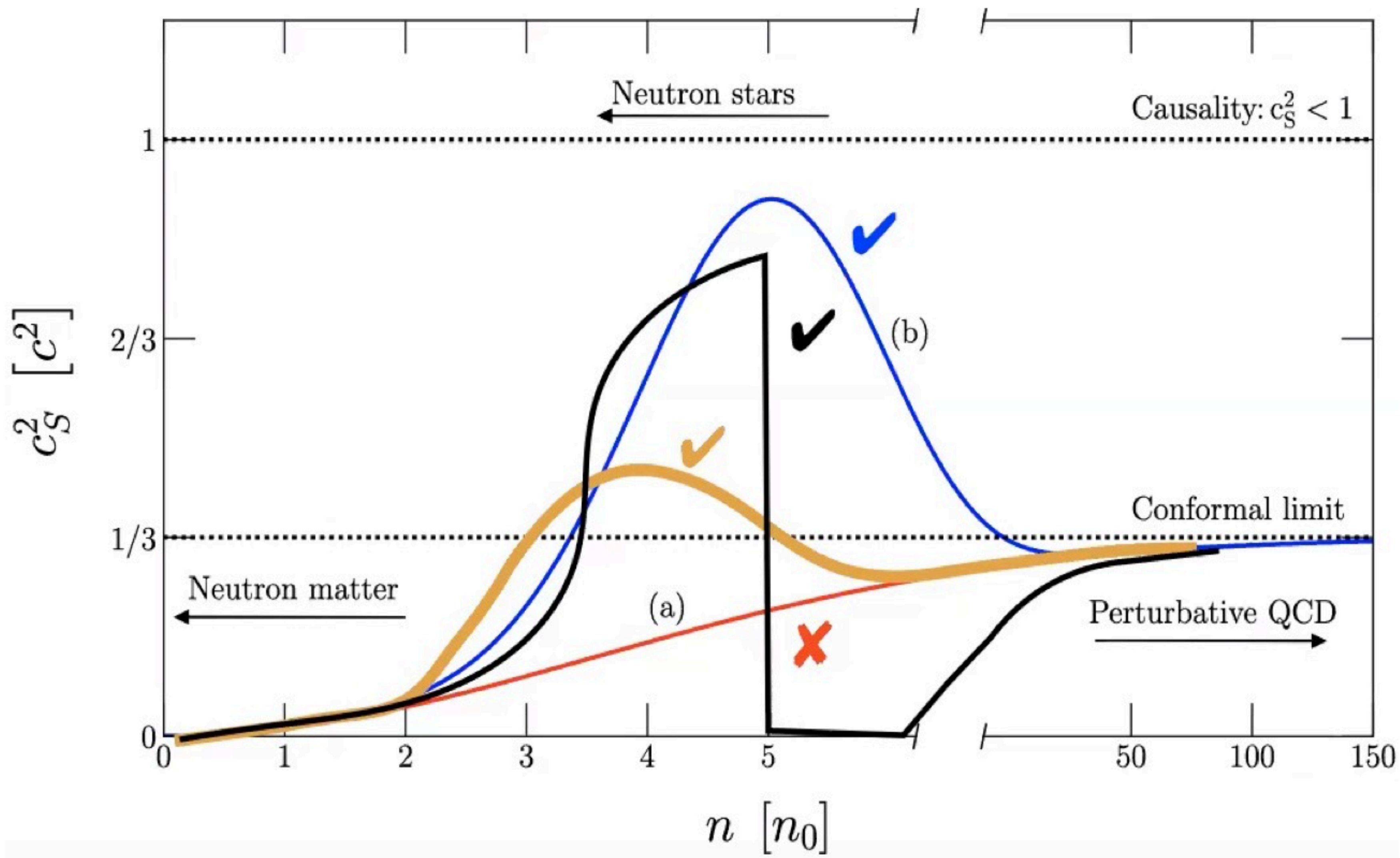


Fig. from S. Reddy presentation @BNL (2021)

GW constraints

GW170817

- $R_{M=1.4M_{\odot}} \leq 13.5$ km and $M_{max} > 2M_{\odot}$

Breakdown of the models

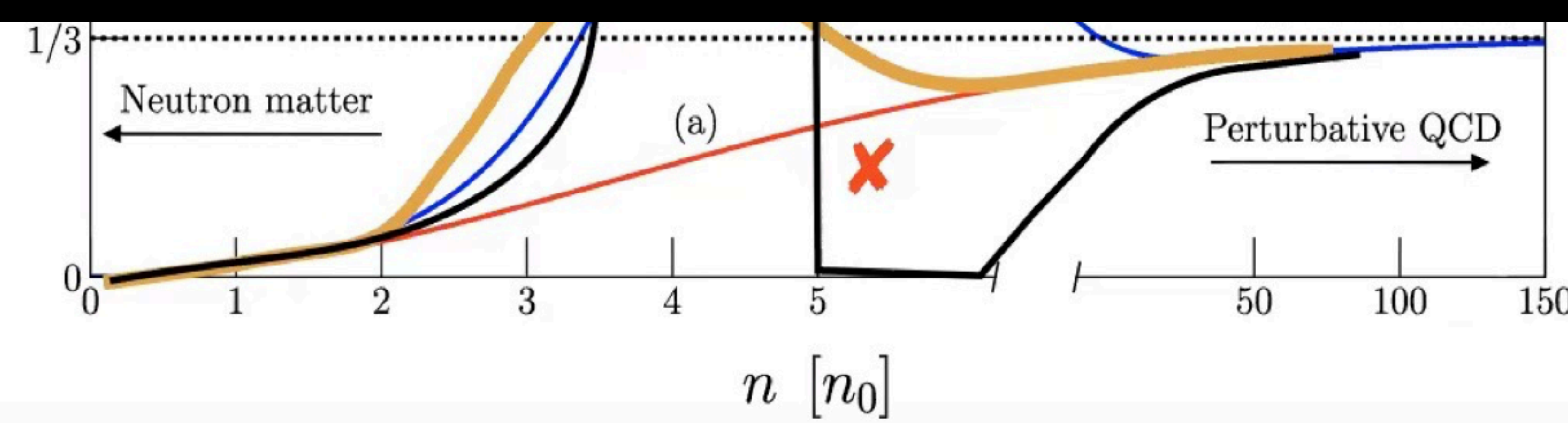
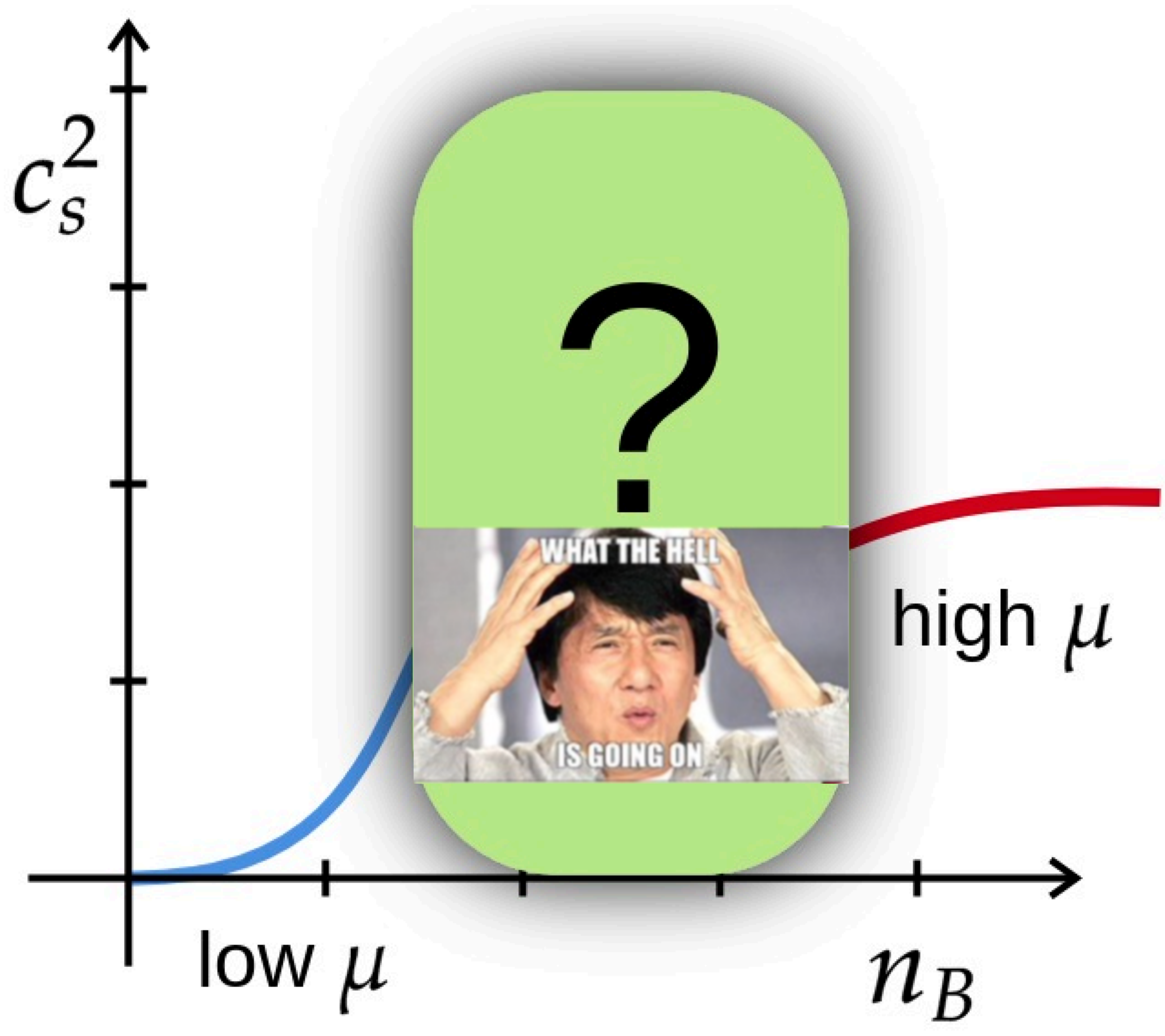


Fig. from S. Reddy presentation @BNL (2021)

To make progress



Insights from LQCD

QCD under extreme conditions

- Finite temperature T
- Finite magnetic field eB
- Chiral chemical potential μ_5
- Two color QC2D
- Finite isospin chemical potential μ_I

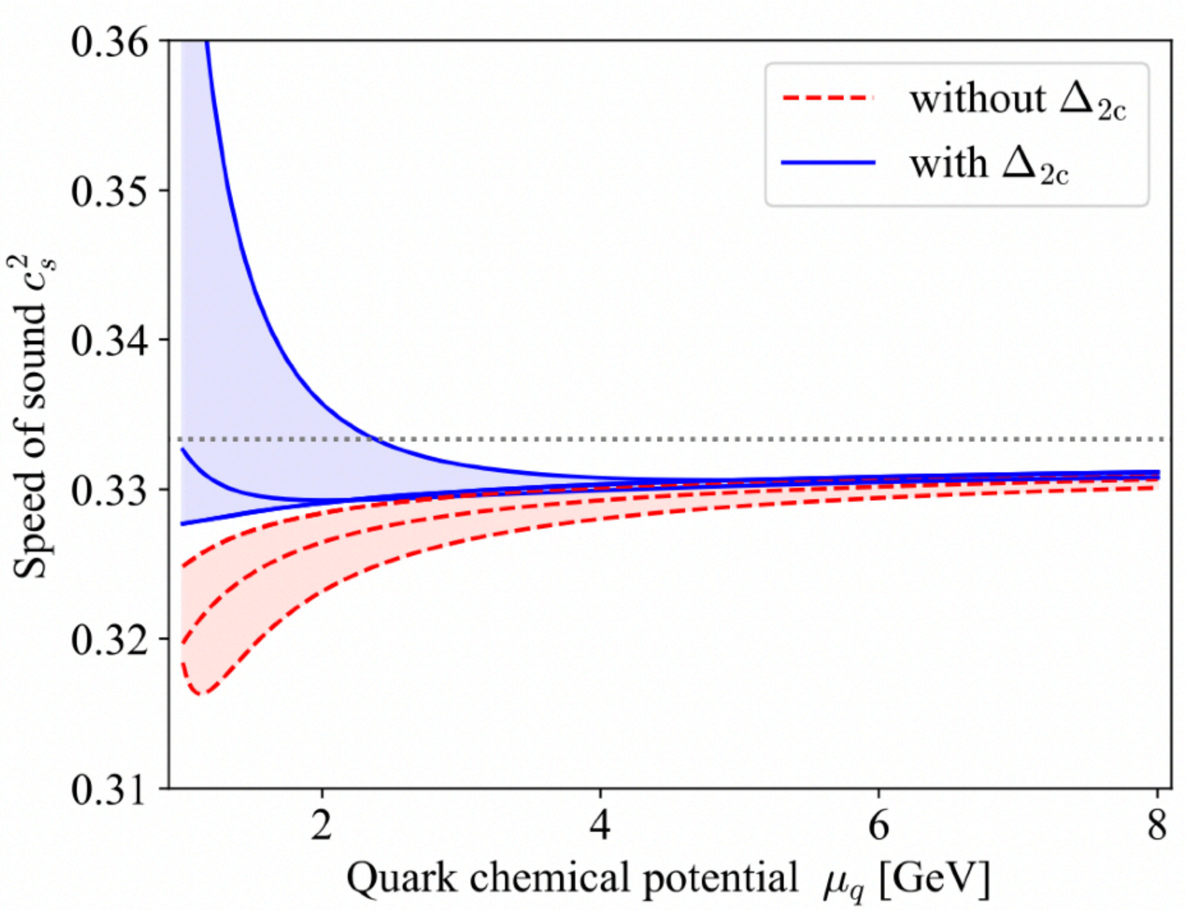
QCD under extreme conditions

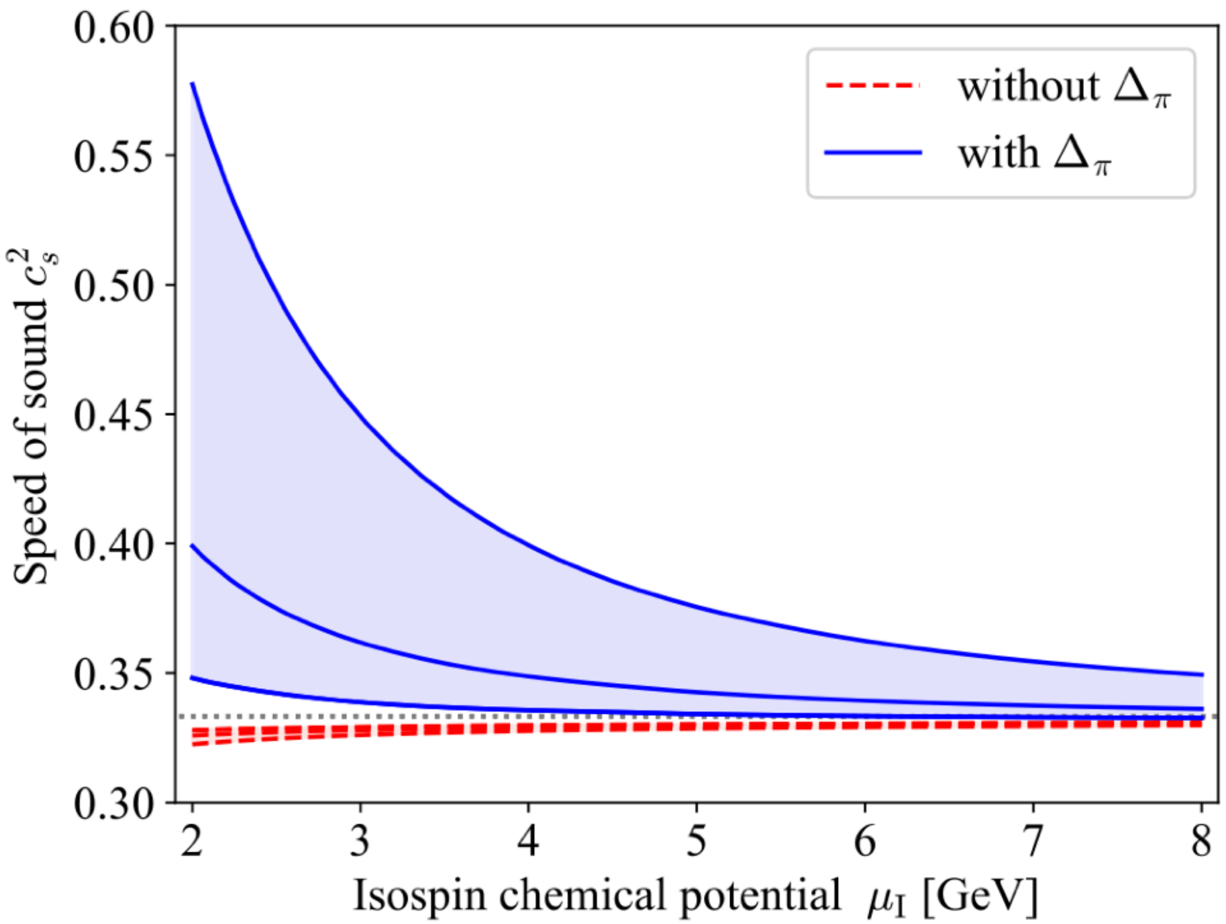
- Finite temperature T
- Finite magnetic field eB
- Chiral chemical potential μ_5
- Two color QC2D
- Finite isospin chemical potential μ_I

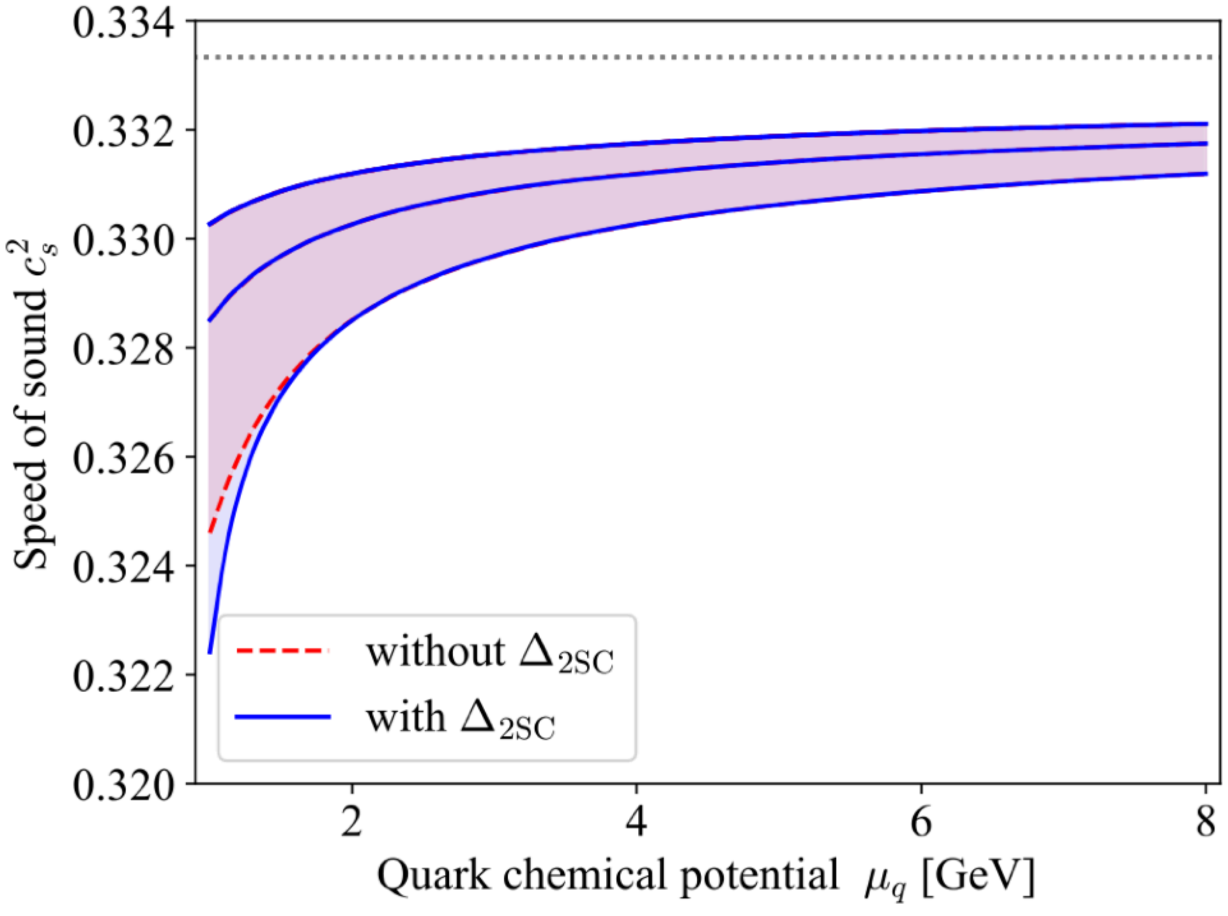
NO
Sign Problem!

LQCD can be used as a benchmark platform for comparing different effective models used in the literature.

Conformal Limit







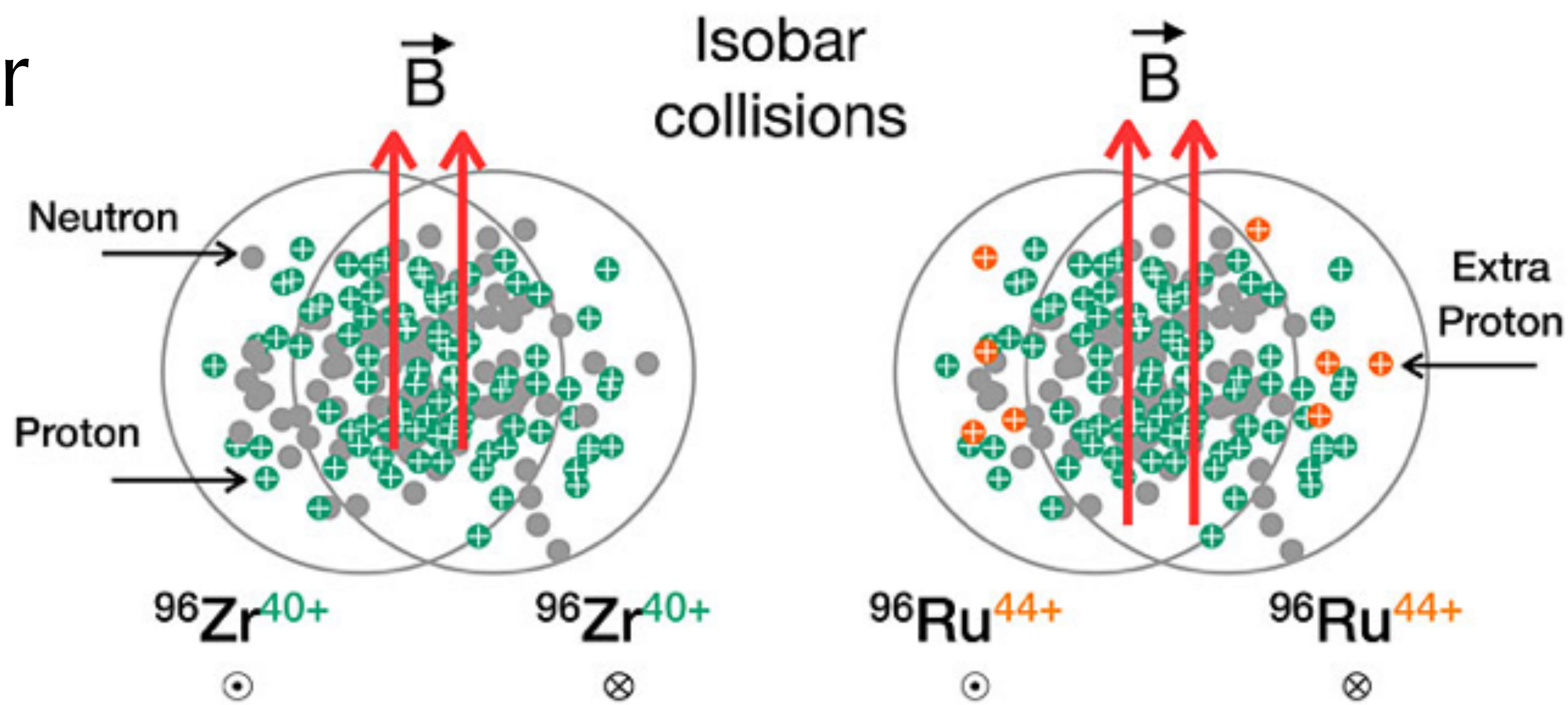
K. Fukushima and S. Minato, Phys. Rev. D **111**, 094006 (2025)

Isospin-asymmetric QCD

Motivation

Why relevant?

- In RHIC isobar program



- Excess of neutrons over protons

- Neutron star interiors

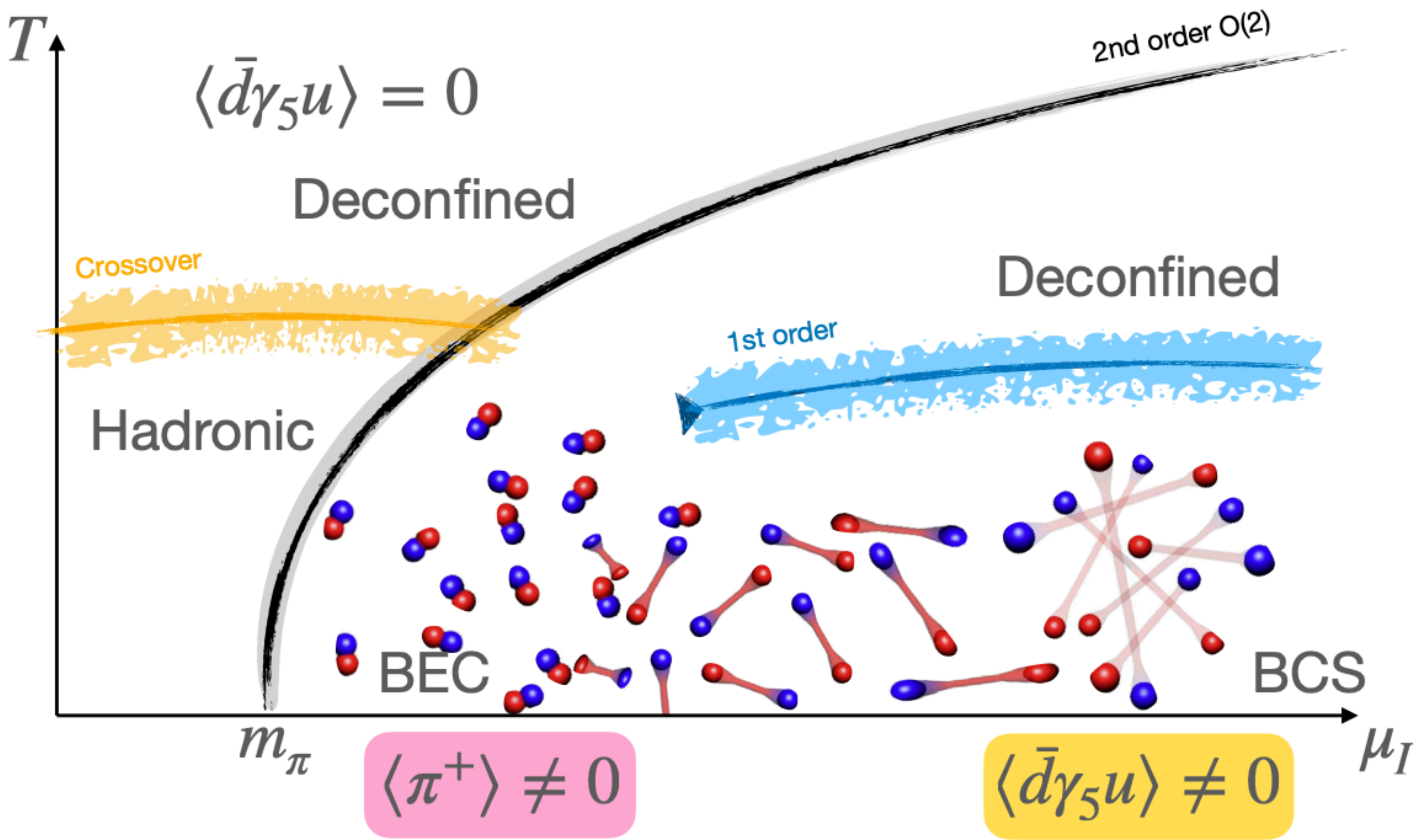
$$n_I = n_u - n_d$$

Brookhaven Lab, RHIC isobar collisions — "Search for Chiral Magnetic Effect", 31 Aug 2021

QCD at $\mu_I \neq 0$ & $\mu_B = 0$

Conjectured phase diagram

- Son & Stephanov PRL 2001

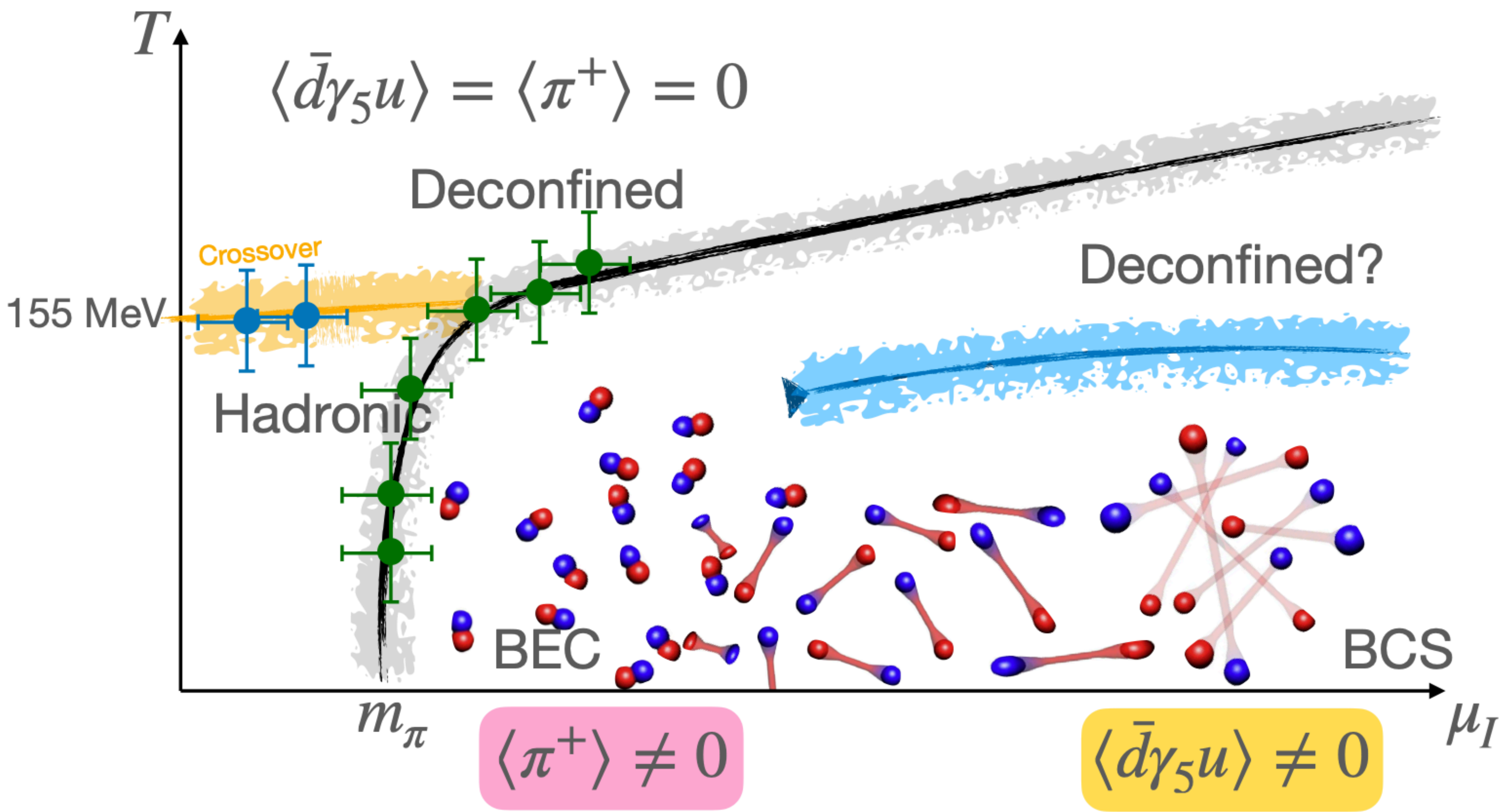


QCD at $\mu_I \neq 0$ & $\mu_B = 0$

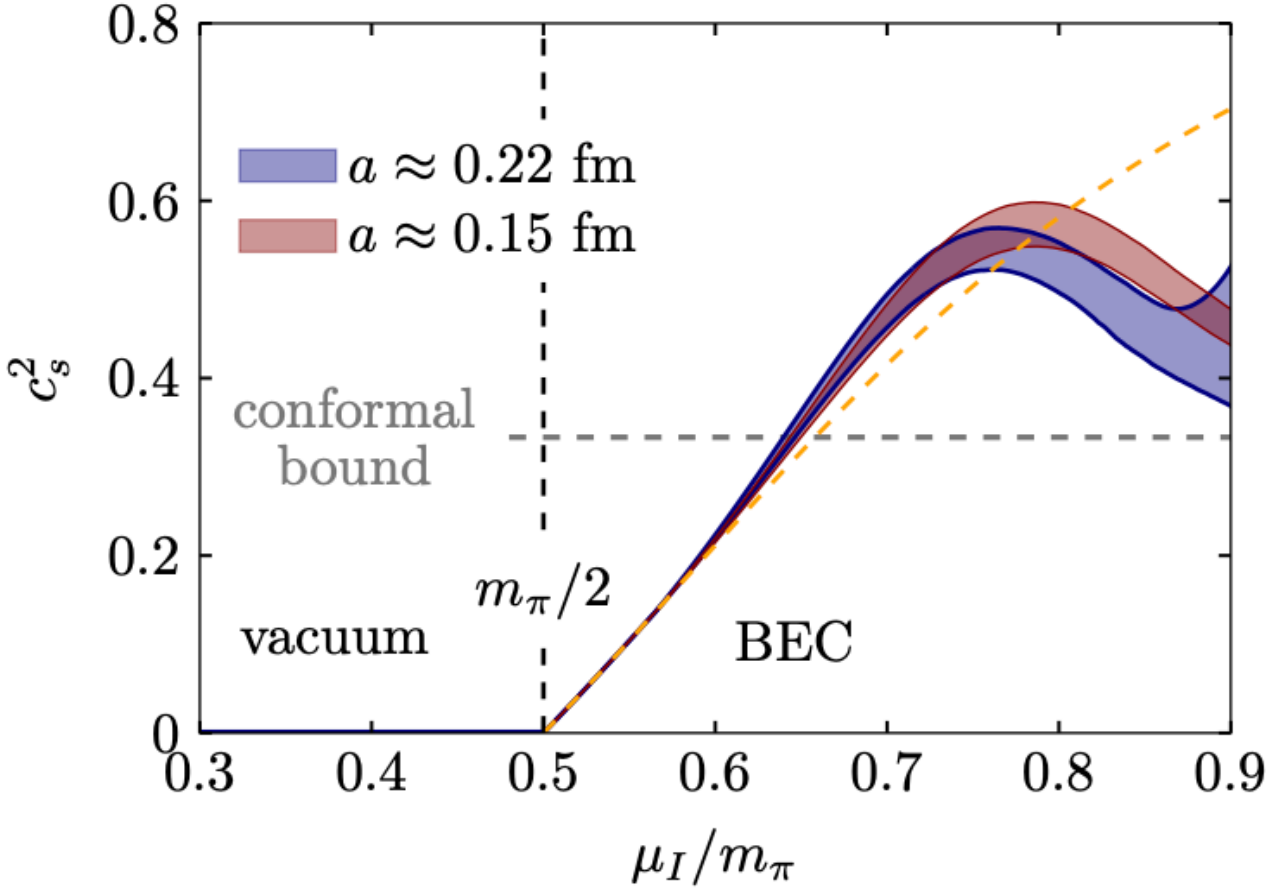
Conjectured phase diagram

- Status in 2023 - much recent work from Brandt, Cuteri & Endrodi

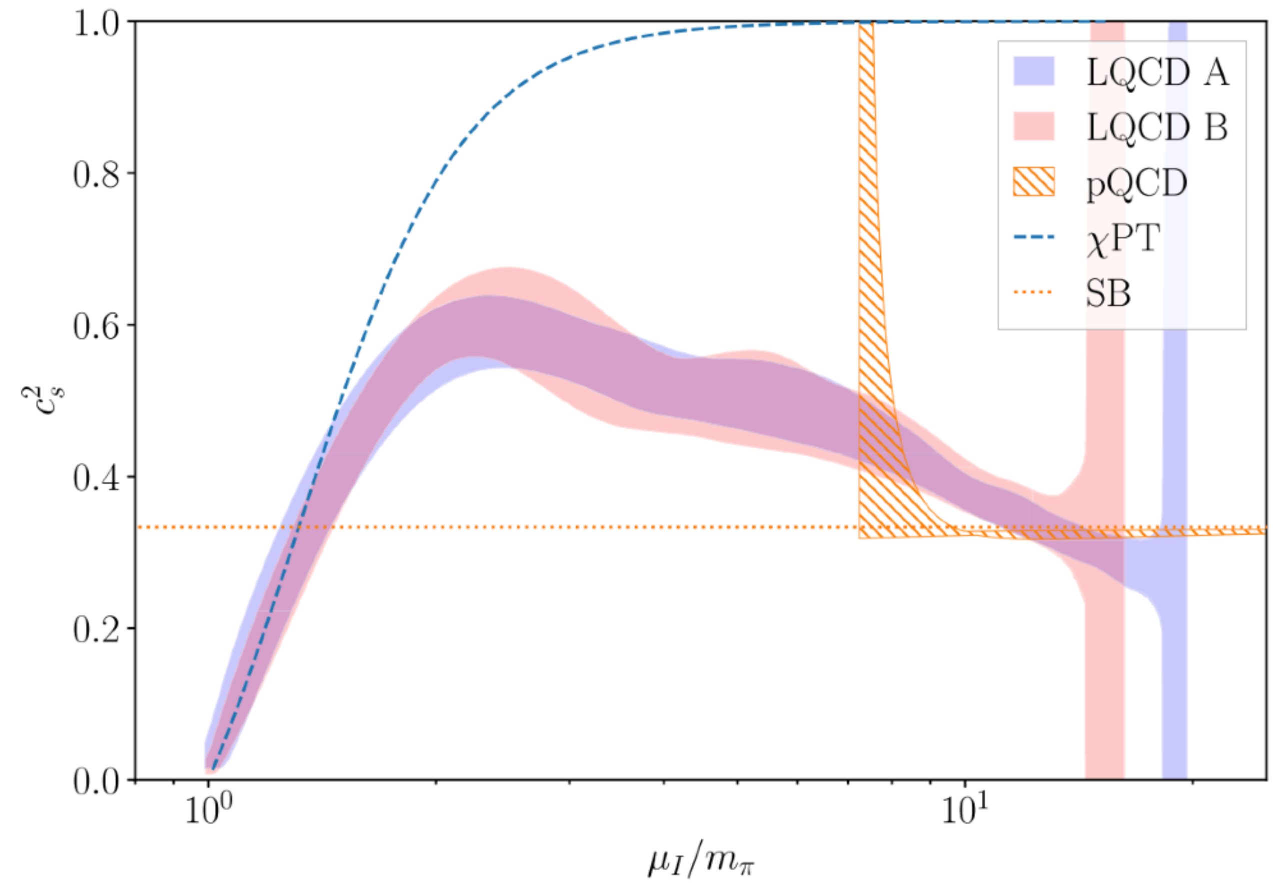
JHEP 07 (2023) 055 • e-Print: 2212.14016 [hep-lat]



Recent LQCD results: $T = 0,\ \mu_B = 0$ and $\mu_I \neq 0$



B.B. Brandt, F. Cuteri, G. Endrodi, JHEP 07 (2023) 055

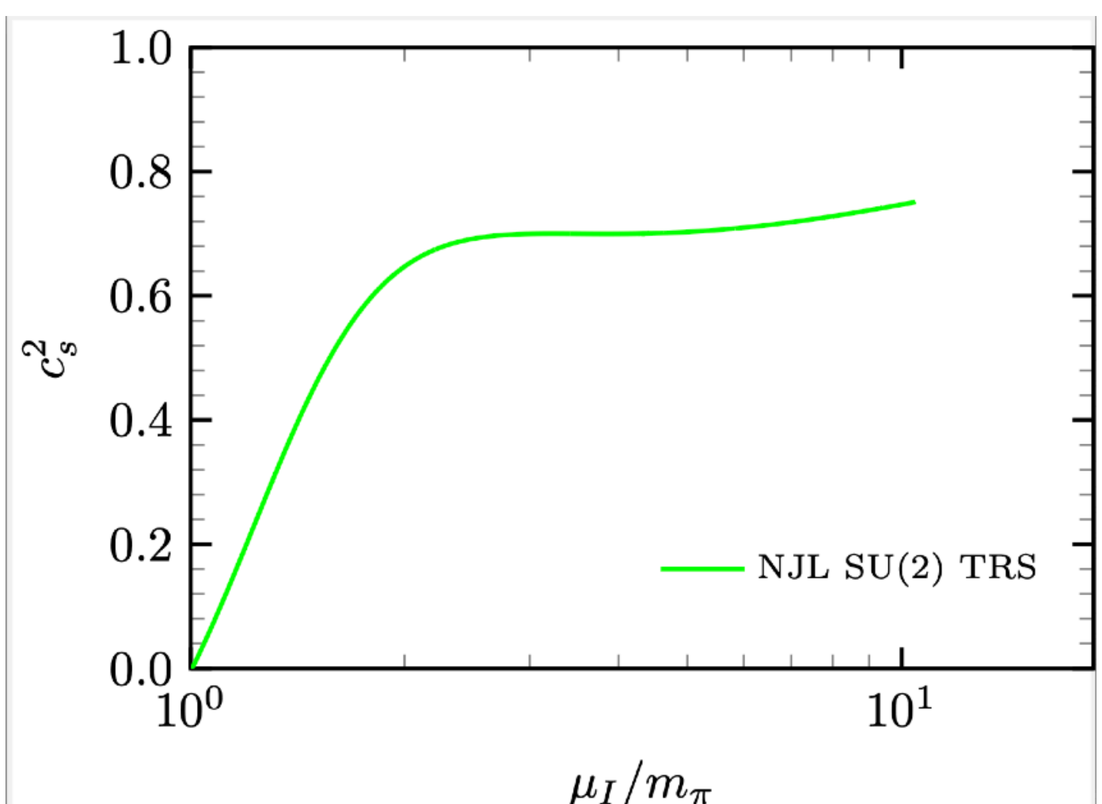


PHYSICAL REVIEW D **108,** 114506 (2023)

Lattice quantum chromodynamics at large isospin density

Ryan Abbott,[1,2,*] William Detmold,[1,2] Fernando Romero-López,[1,2] Zohreh Davoudi,[3,4] Marc Illa,[5] Assumpta Parreño,[6] Robert J. Perry,[6] Phiala E. Shanahan,[1,2] and Michael L. Wagman[7]

(NPLQCD Collaboration)



PHYSICAL REVIEW LETTERS **134,** 011903 (2025)

Editors' Suggestion Featured in Physics

QCD Constraints on Isospin-Dense Matter and the Nuclear Equation of State

Ryan Abbott,[1,2] William Detmold,[1,2] Marc Illa,[3] Assumpta Parreño,[4] Robert J. Perry,[4] Fernando Romero-López,[1,2] Phiala E. Shanahan,[1,2] and Michael L. Wagman[5]

(NPLQCD Collaboration)

SU(2) NJL model

$T = 0, \mu_B = 0$ and $\mu_I \neq 0$

In the mean-field approximation, the thermodynamic potential is given by

$$\Omega_{\rm NJL} = \frac{\sigma^2 + \Delta^2}{4G} - 2N_c \int_\Lambda \frac{d^3k}{(2\pi)^3} \left(E_k^+ + E_k^-\right)$$

Needs regularization!

where $E_k^\pm = \sqrt{\left(E_k \pm \frac{\mu_I}{2}\right)^2 + \Delta^2}$, $E_k = \sqrt{k^2 + M^2}$

From these equations we obtain

$$\sigma = 4GN_cM\ I_\sigma$$
$$\Delta = 4GN_c\Delta\ I_\Delta$$

with the definitions

$$I_\sigma = \sum_{s=\pm 1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} \frac{E_k + s\mu_I}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$
$$I_\Delta = \sum_{s=\pm 1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$

TRS scheme

Medium Separation Scheme

Consider an energy relation ω depending on chemical potentials μ and function(s) of a condensate $\langle\cdots\rangle$. ➡ $\omega(k) = \sqrt{[f(\vec{k}) \pm \mu]^2 + \langle\cdots\rangle}$

Chiral chemical potential

$\mu \rightarrow \mu_5$

$\langle\cdots\rangle \rightarrow M_q \ (\sim \langle\bar{q}q\rangle)$

$$\omega(k) = \sqrt{(|\vec{k}| \pm \mu_5)^2 + M_q^2}$$

M_q : constituent (quark) mass

Isospin chemical potential

$\mu \rightarrow \mu_I$

$\langle\cdots\rangle \rightarrow \Delta_\pi \ (\sim \langle\bar{q}i\gamma_5\tau_\pm q\rangle)$

$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_I\right)^2 + \Delta_\pi^2}$$

Δ_π : pion condensate

Quark chemical potential

$\mu \rightarrow \mu_q$

$\langle\cdots\rangle \rightarrow \Delta_c \ (\sim \langle qq\rangle)$

$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_q\right)^2 + \Delta_c^2}$$

Δ_c : diquark condensate

M_q	$\mu_5, \ \mu_I, \ \mu_q$	Δ_π	Δ_c	$\langle\cdots\rangle$
constituent (quark) mass	(chiral, isospin, quark chemical potentials)	(pion condensate)	(diquark condensate)	(condensate(s))

Medium Separation Scheme

$$I_\Delta = \frac{1}{\pi}\sum_{j=\pm 1}\int_{-\infty}^{+\infty} dx \int_\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{x^2+(E_k+j\mu_I)^2+\Delta^2}$$

Using the identity

$$\frac{1}{x^2+(E_k+j\mu_I)^2+\Delta^2} = \frac{1}{x^2+k^2+M_0^2} + \frac{M_0^2-\Delta^2-\mu_I^2-M^2-2j\mu_I E_k}{(x^2+k^2+M_0^2)\left[x^2+(E_k+j\mu_I)^2+\Delta^2\right]}$$

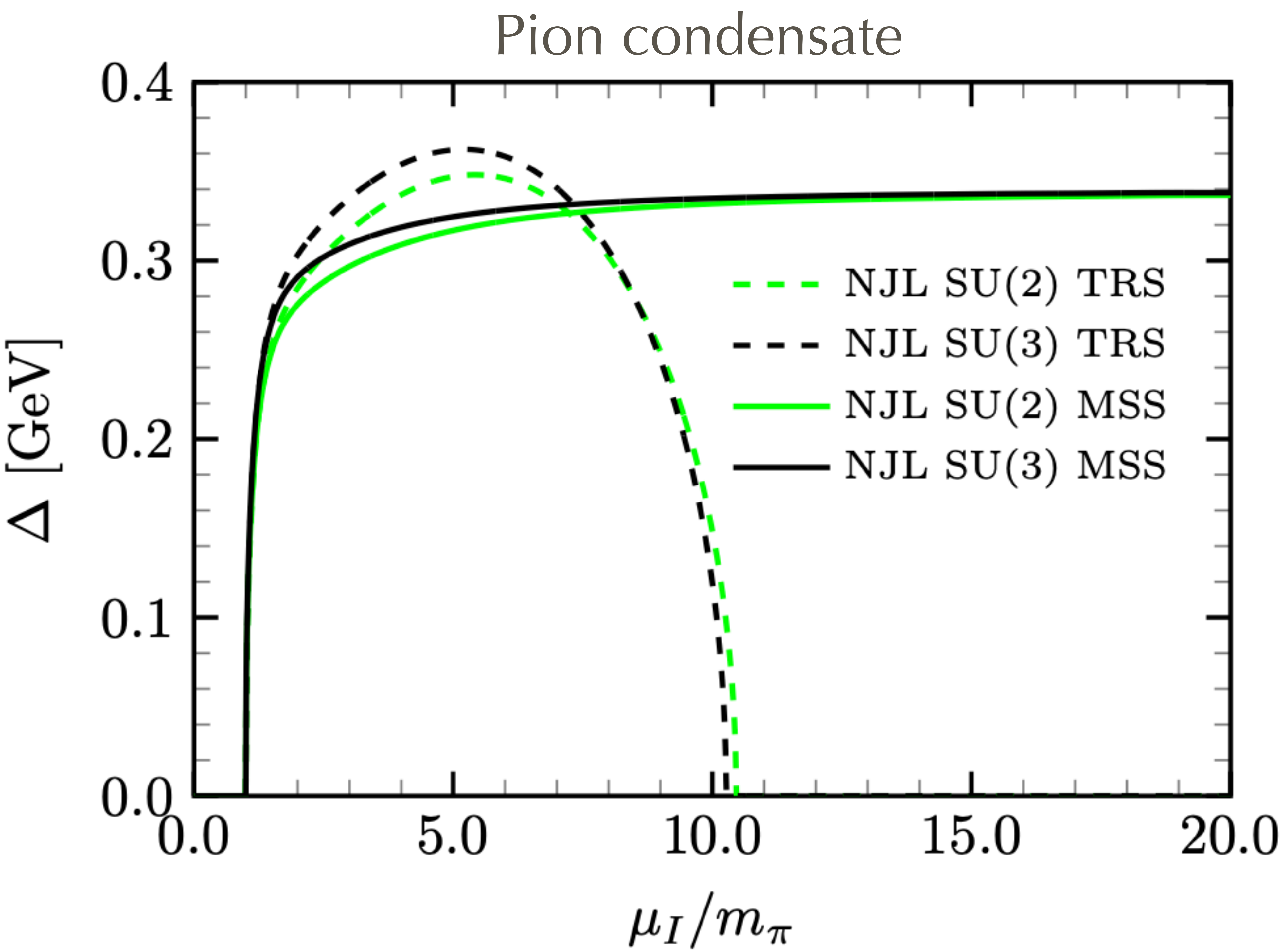
M_0 is the vacuum quark mass!

$$I_\Delta = \frac{1}{2}\underbrace{I_{\text{quad}}(M_0)}_{\text{Divergent}} - (\Delta^2 - M_0^2 - 2\mu_I^2 + M^2)\underbrace{I_{\log}(M_0)}_{\text{Divergent}} + \underbrace{I_{\text{finite}}}_{\text{No Regulator}}$$

$$I_{\text{quad}}(M_0) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{p^2+M_0^2}}$$

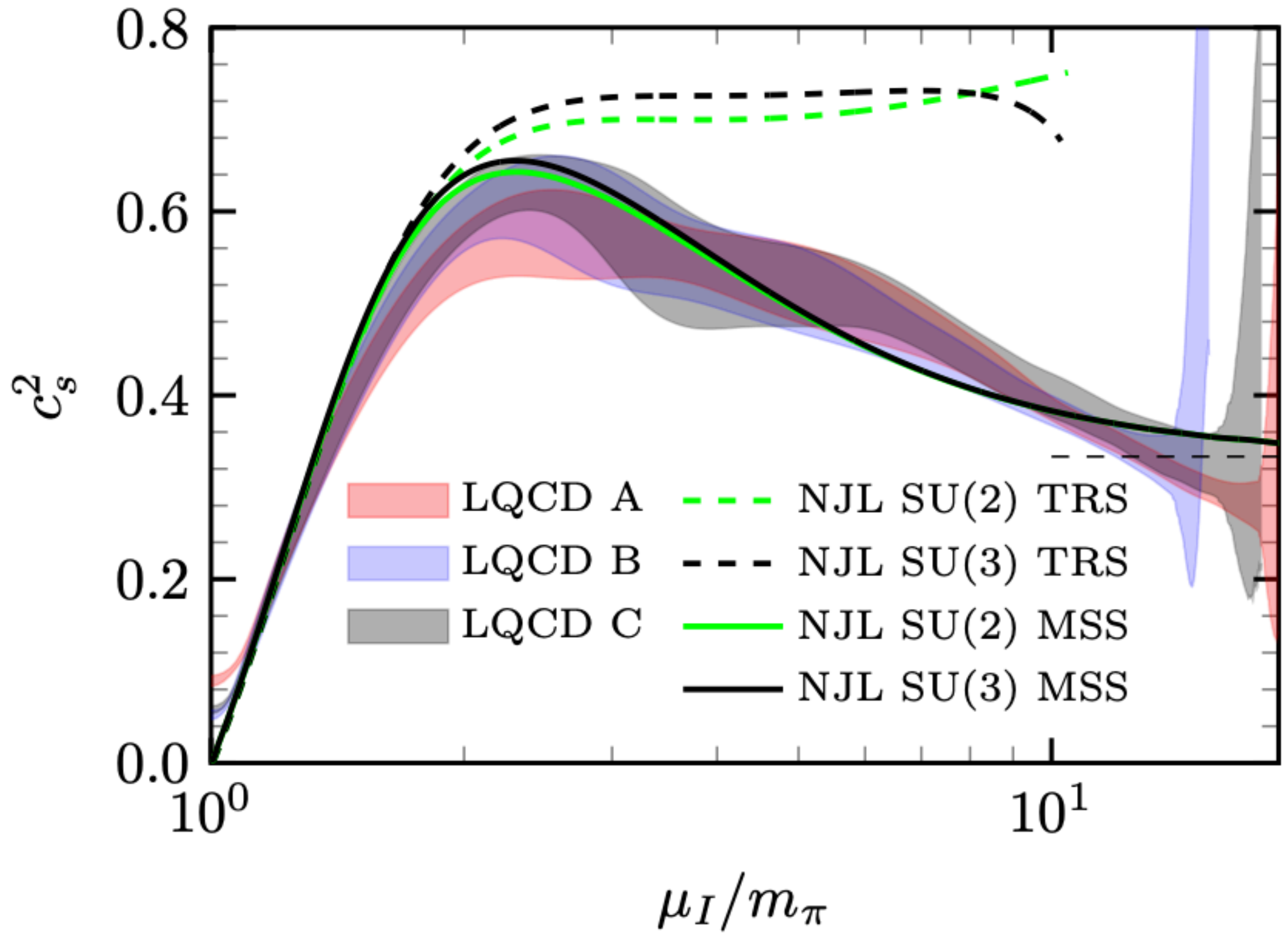
$$I_{\log}(M_0) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+M_0^2)^{\frac{3}{2}}}$$

T=0 NJL + isospin asymmetry



RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, Phys. Rev. D **112**, L091903 (2025)

T=0 Sound velocity



$$\epsilon = -P + \mu_I n_I$$
$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

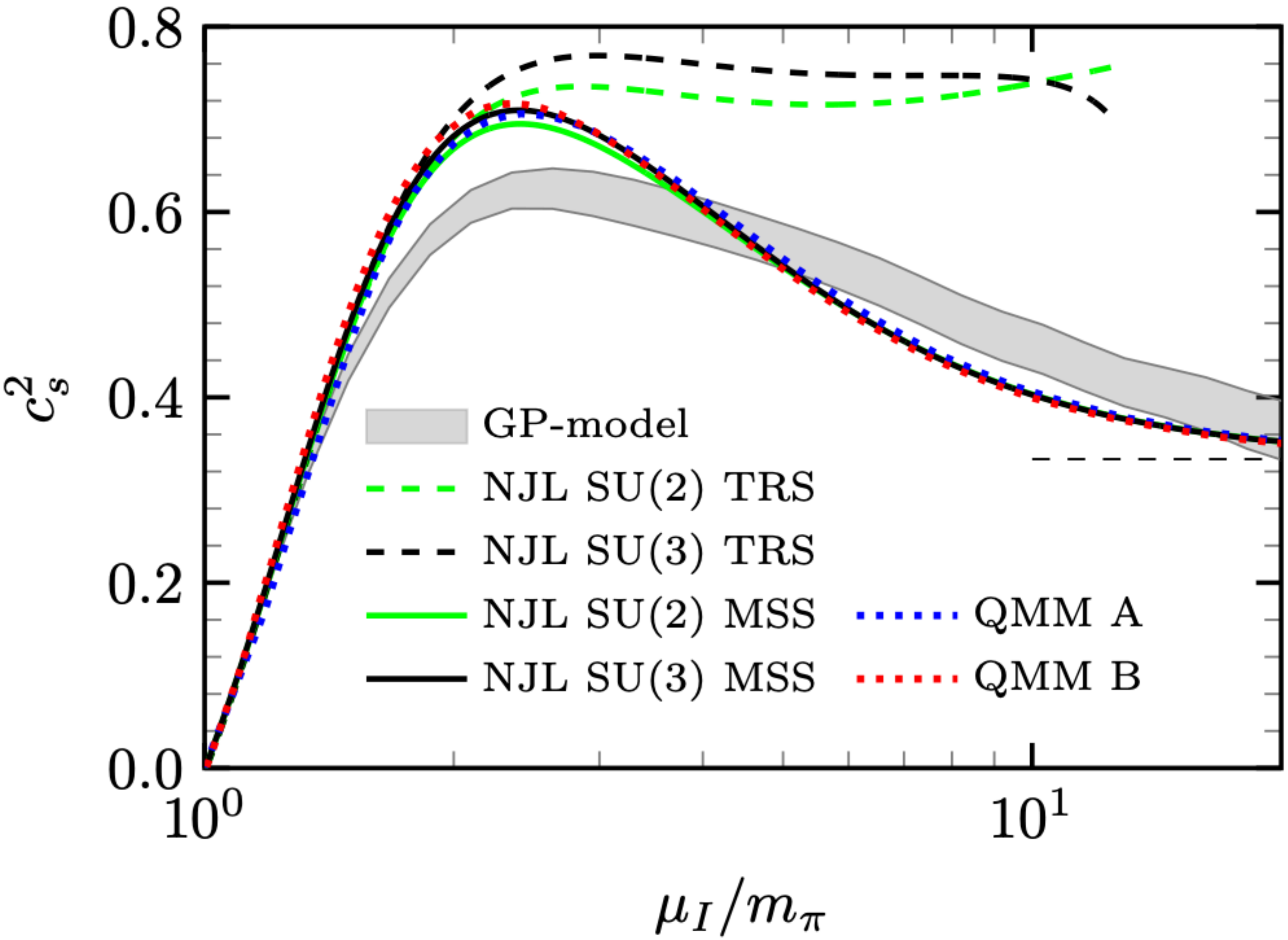
This data does not control for finite lattice spacing effects, and there are also uncontrolled systematics at large and small μ_I

Updated bounds based on the continuum limit lattice QCD results at the physical quark masses, χPT, and perturbative QCD through the GP-model.

RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, Phys. Rev. D **112**, L091903 (2025)

R. Abbott et al. [NPLQCD], Phys. Rev. Lett. **134**, no.1, 1 (2025)

Speed of sound peak in isospin QCD: a natural prediction of the Medium Separation Scheme



No adjust of NJL parameters, Just disentangle medium contributions from the vacuum!

RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, Phys. Rev. D **112**, L091903 (2025)

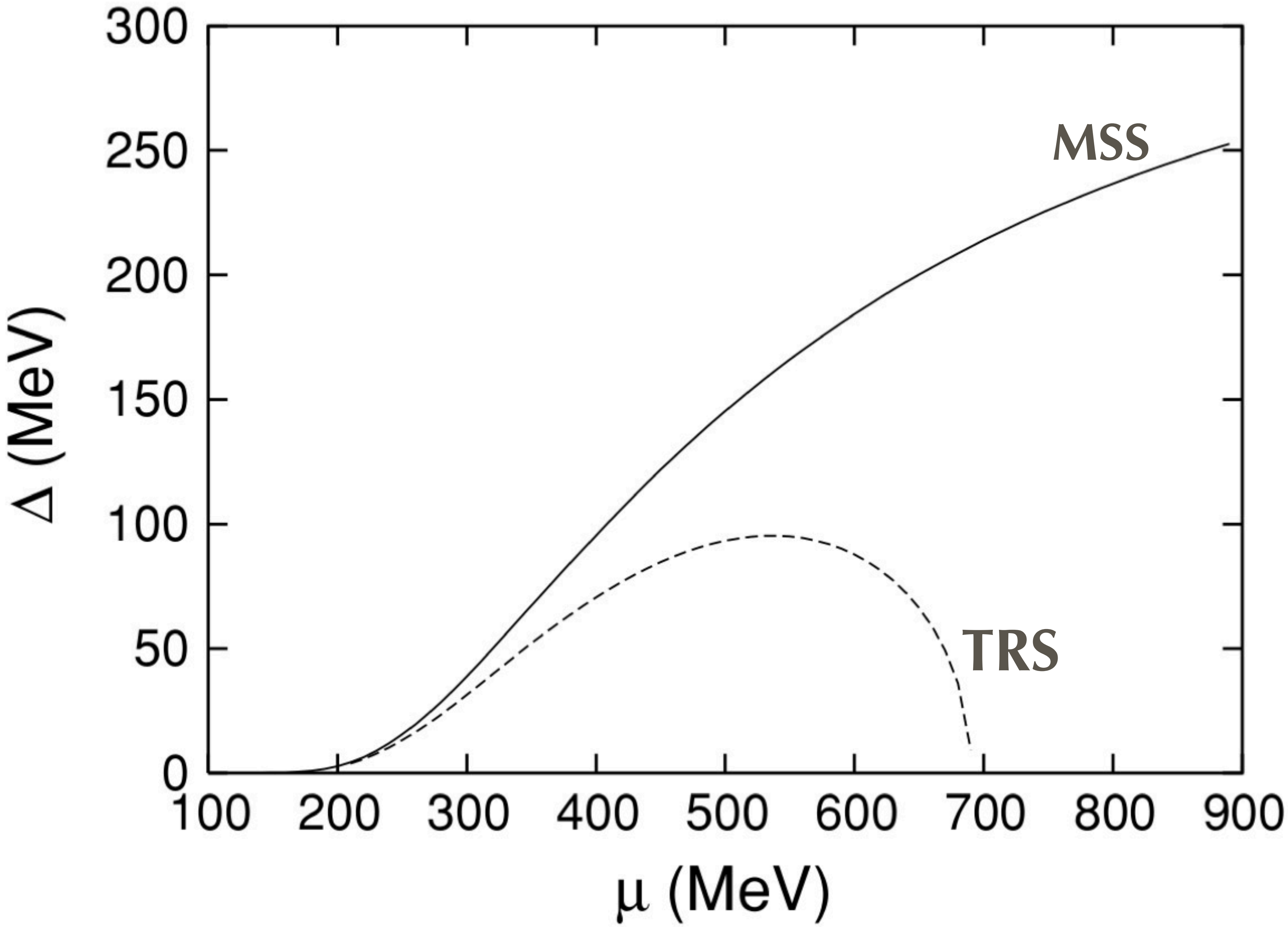
R. Abbott et al. [NPLQCD], Phys. Rev. Lett. **134**, no.1, 1 (2025)

(A) J. O. Andersen and M. P. Nødtvedt, Phys. Rev. D **113**, 014026 (2026)

(B) B. B. Brandt, V. Chelnokov, G. Endrodi, G. Marko, D. Scheid and L. von Smekal, Phys. Rev. D 112, 054038 (2025)

2SC

NJL+MSS



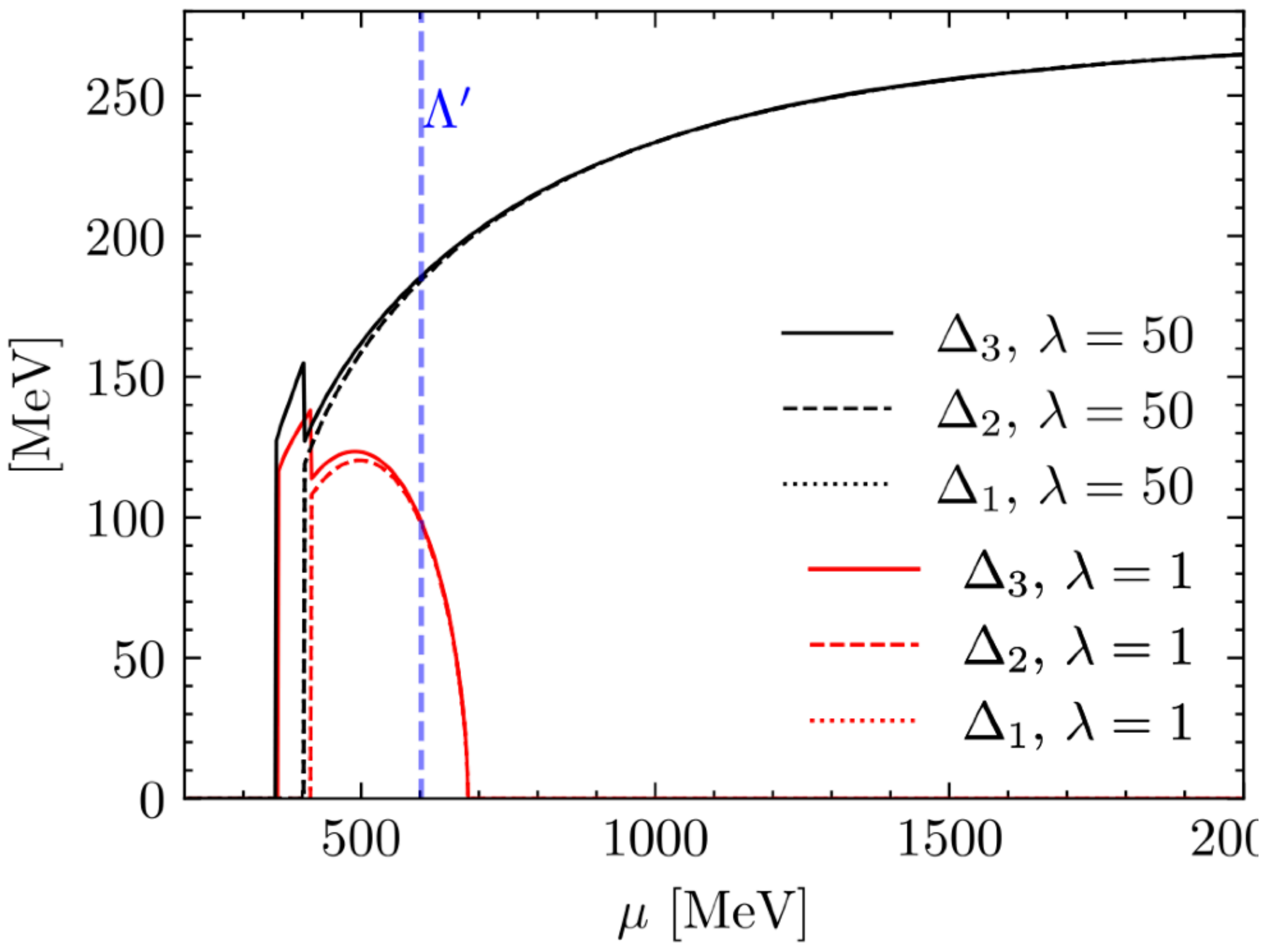
RLSF, G. Dallabona, O.A. Battistel and G. Krein, PRC 73, 018201 (2006)

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

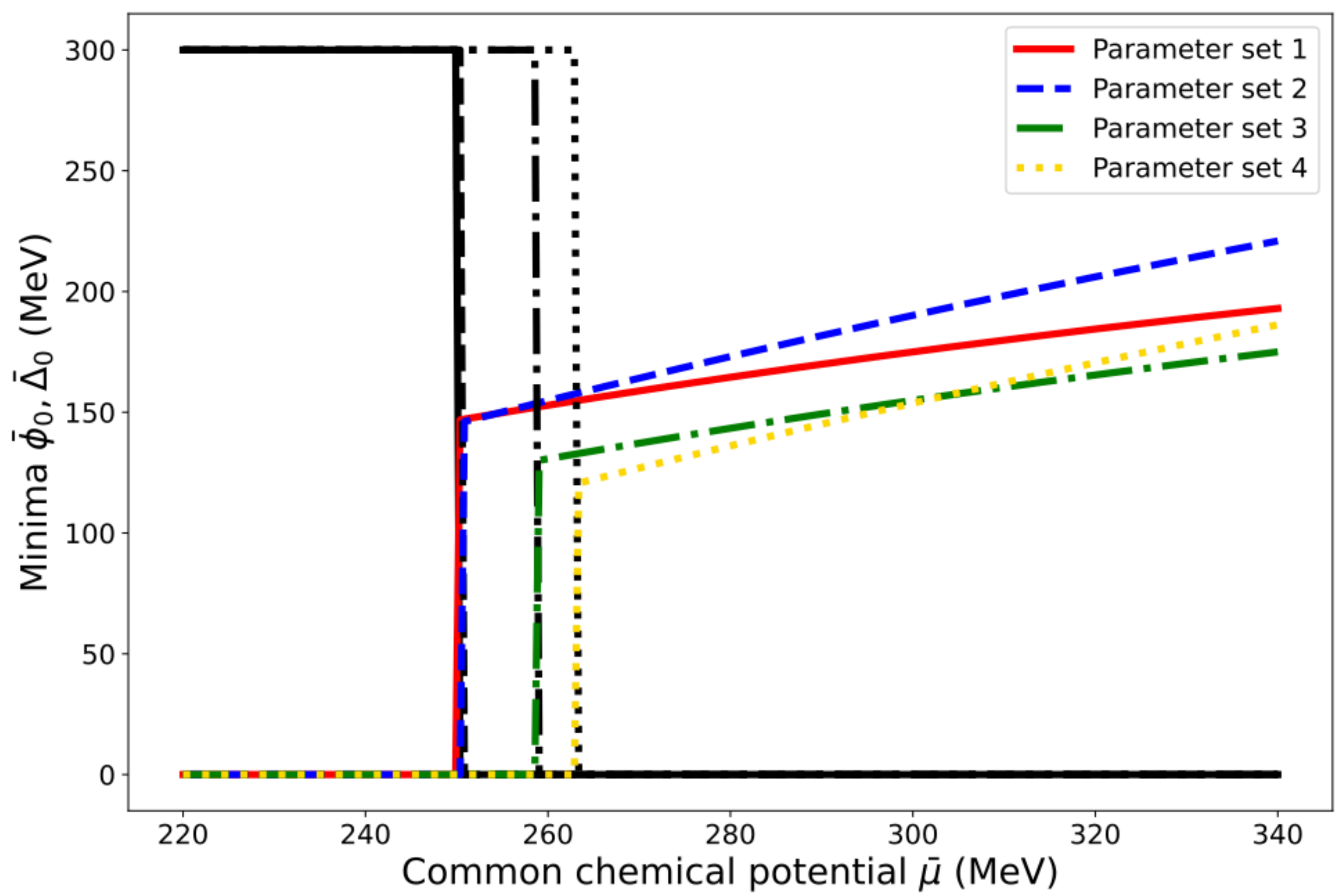
D.T. Son, Phys. Rev. D 59, 094019 (1999)

2SC CSC

RGNJL and QMM



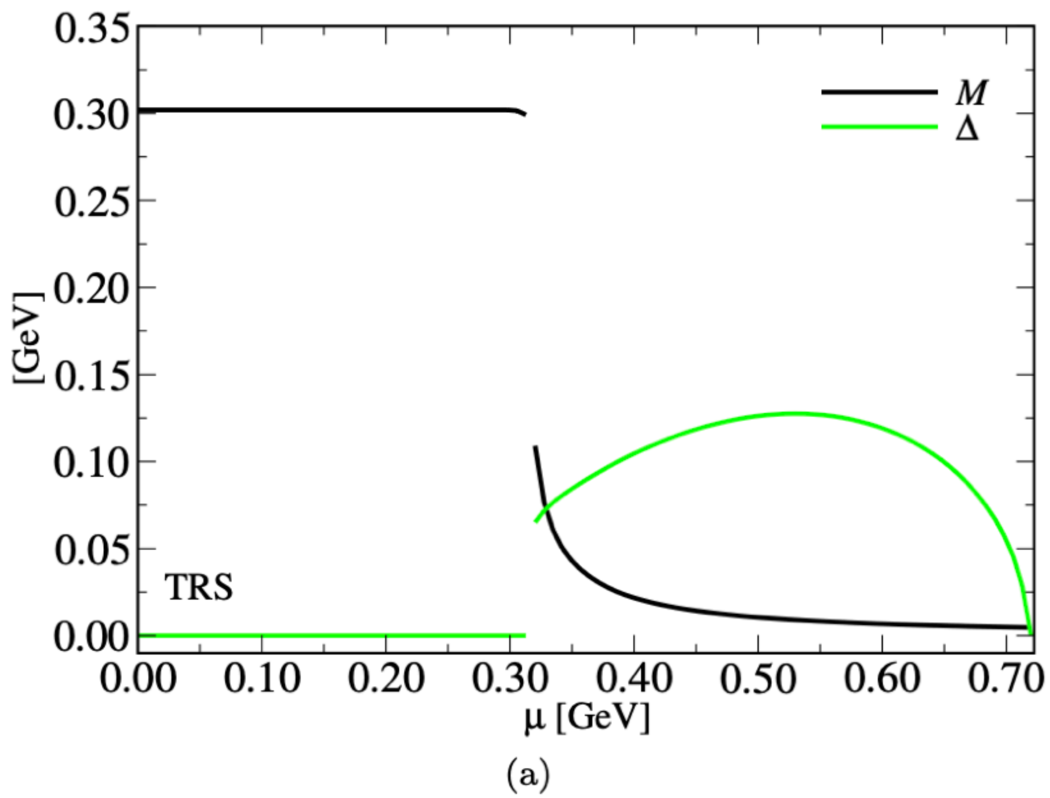
H. Gholami, M. Hofmann and M. Buballa, PRD 111, 014006 (2025)

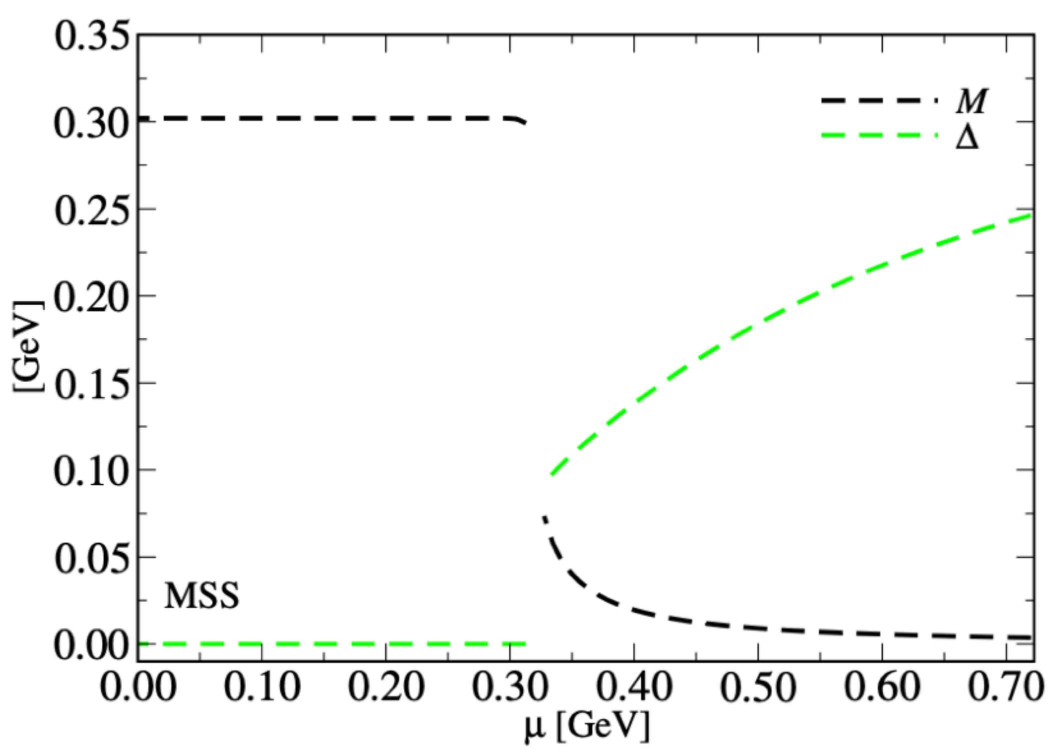


J.O. Andersen and M. P. Nødtvedt, PRD 111, 034031 (2025)

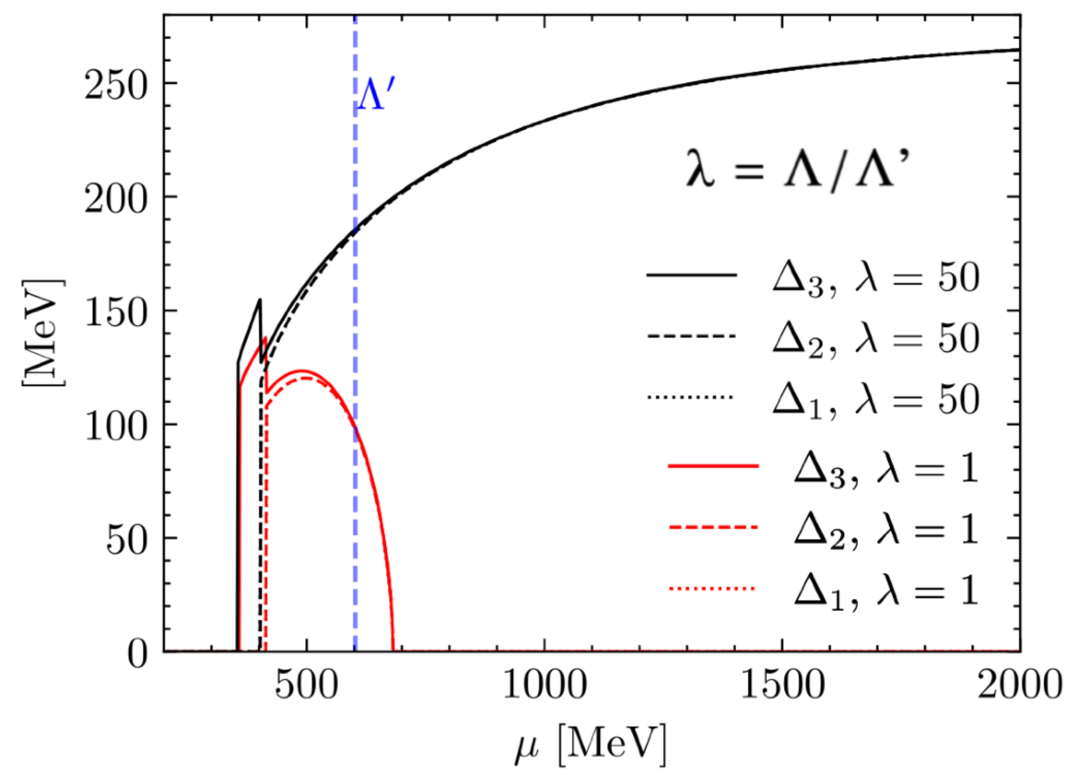
2SC CSC

NJL+MSS X NJL + RG X QMM

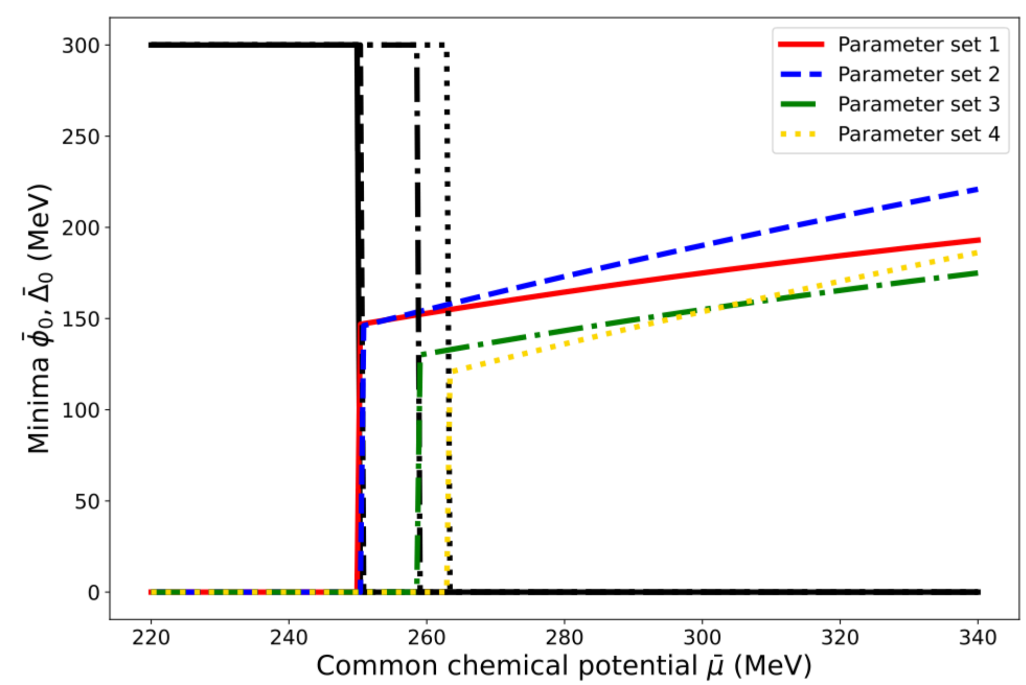




RLSF, F.X.Azeredo and D.C.Duarte, Phys. Rev. D **113** (2026) 5, 056032



H. Gholami, M. Hofmann and M. Buballa, PRD 111, 014006 (2025)



J.O. Andersen and M. P. Nødtvedt, PRD 111, 034031 (2025)

UFSM–Darmstadt Collaboration

Motivation

UFSM and Technische Universität Darmstadt are collaborating to understand why NJL+MSS and RGNJL lead to the same qualitative behavior for color-superconducting gaps at finite density.

$$\text{NJL} + \text{MSS} \quad \Longleftrightarrow \quad \text{RGNJL}$$

Observation

- Similar density dependence
- Gap survives at large density
- Consistent medium behavior

Main Question

- Why do different regularizations agree?
- Is the agreement connected to the separation between vacuum and medium effects?

Two color QCD

Two color QCD in NJL model

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + G_d(\bar{\psi}i\gamma_5\tau_2 t_2 C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2 t_2\psi),$$

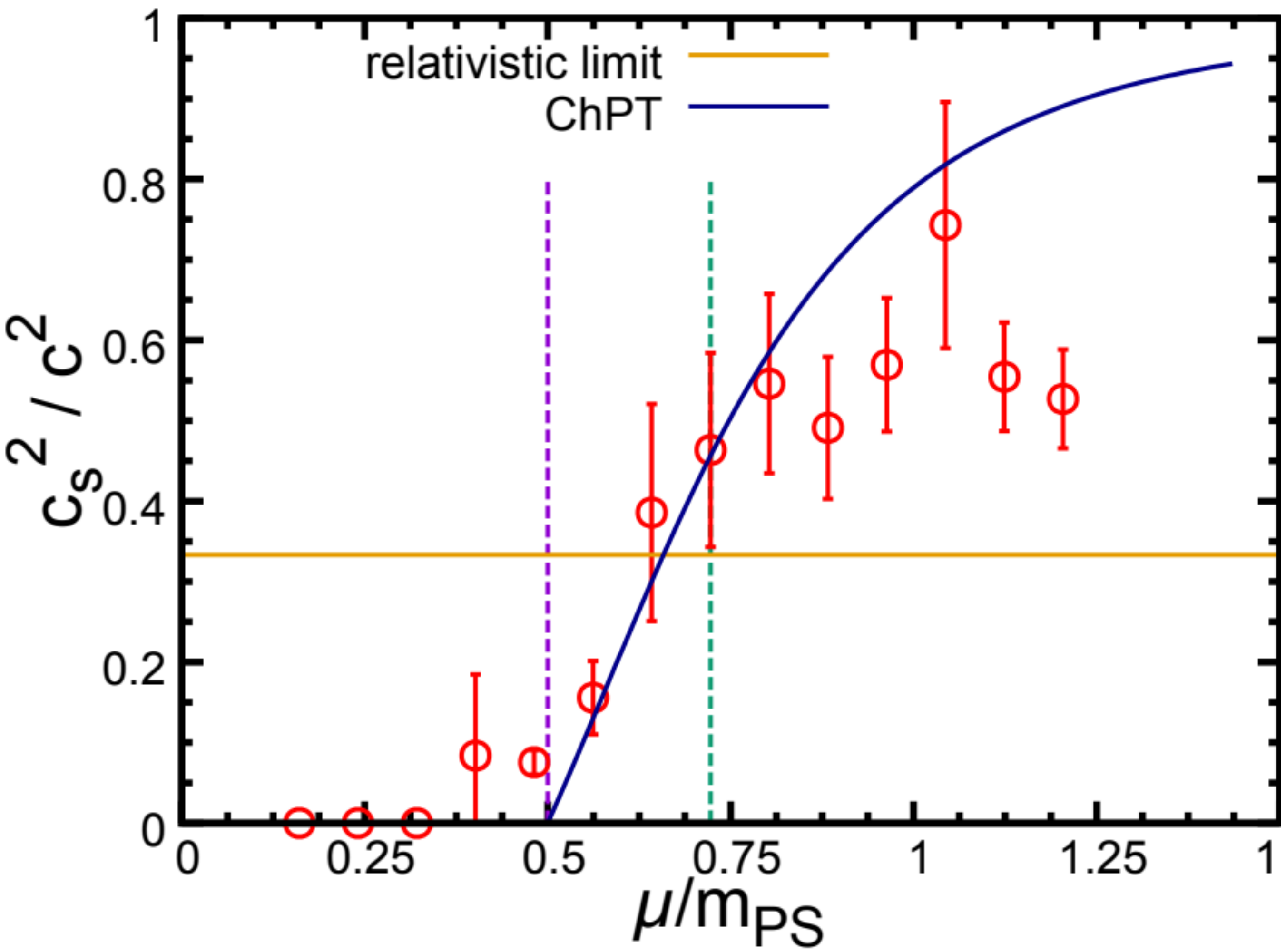
$$\Omega_{T=0} = \Omega_0 + \frac{(M-m)^2 + \Delta^2}{4G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi^3)} E^s_\Delta$$

$$E^s_\Delta = \sqrt{\left(\sqrt{p^2 + M^2} + s\mu\right)^2 + \Delta^2}.$$

R.L.S.F, A.E.B. Pasqualotto, D.C.Duarte and R.O.Ramos, Phys. Rev. D **113**, 036007 (2026)

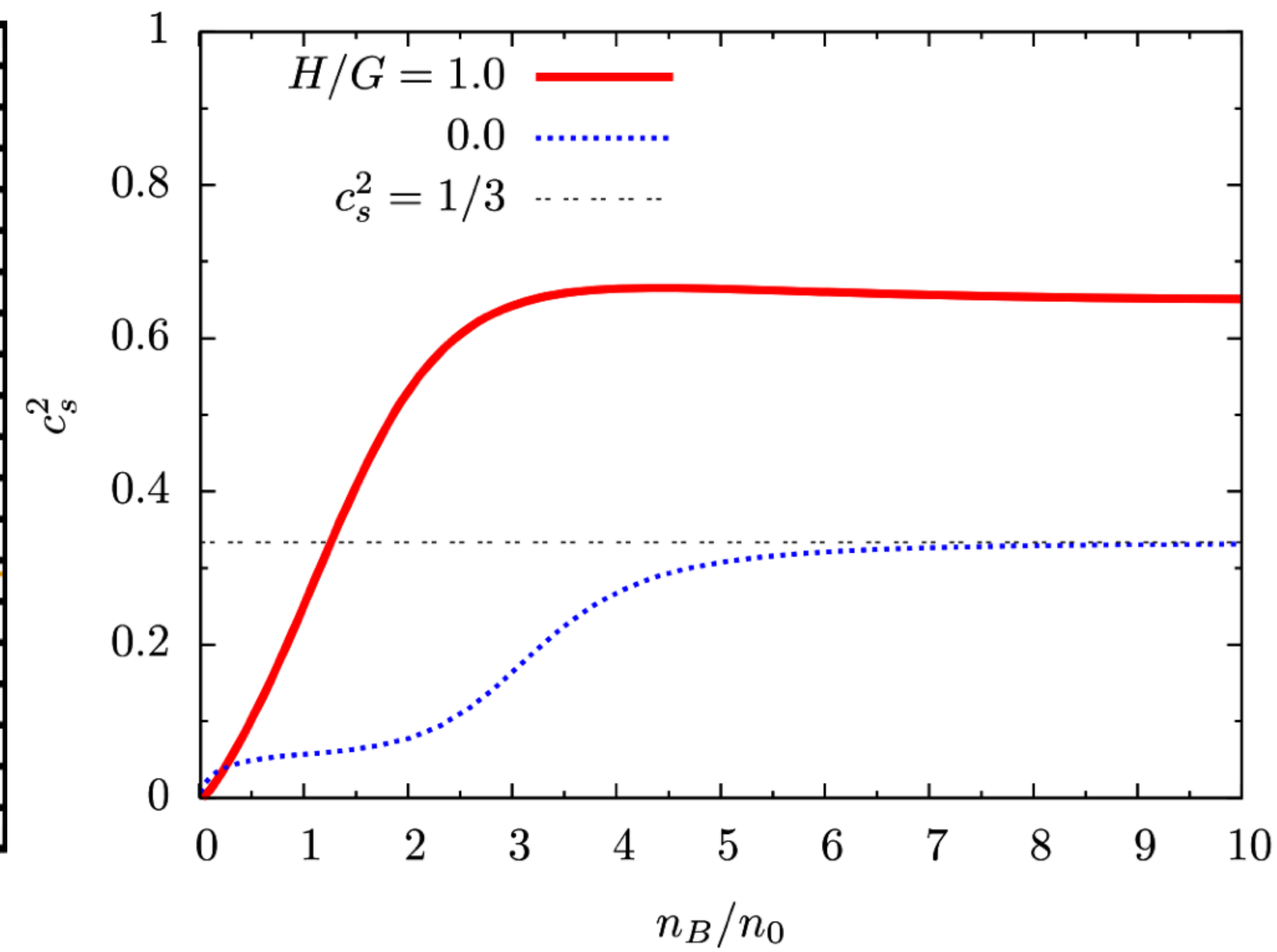
Two color QC2D lattice simulations

Bump of sound velocity in dense 2-color QCD



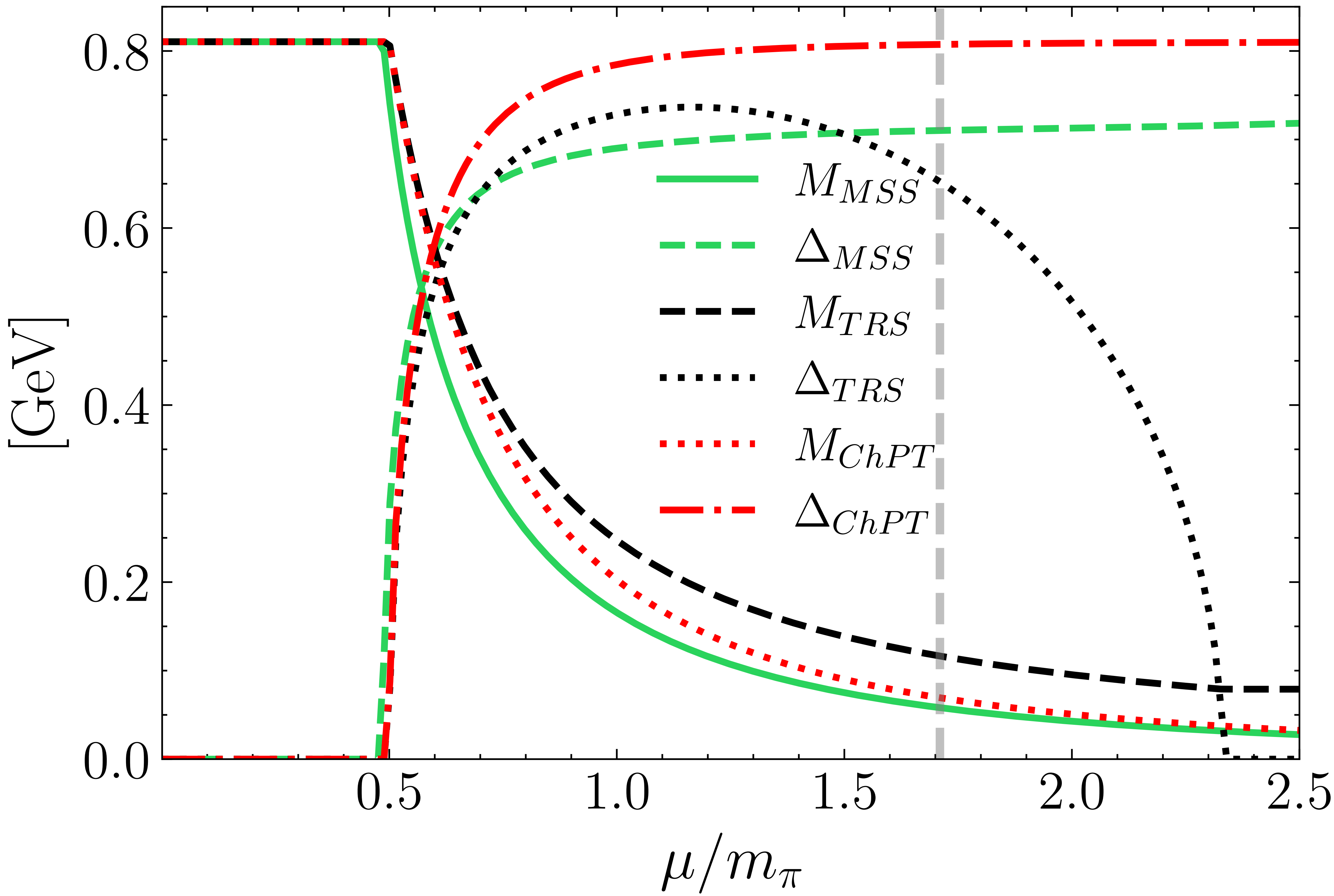
K. Lida and E. Itou, PTEP 2022 (2022) **11**, 111B01
K. Lida, E. Itou, K. Murakami, D. Suenaga, JHEP **10** (2024) 022

2-color NJL model



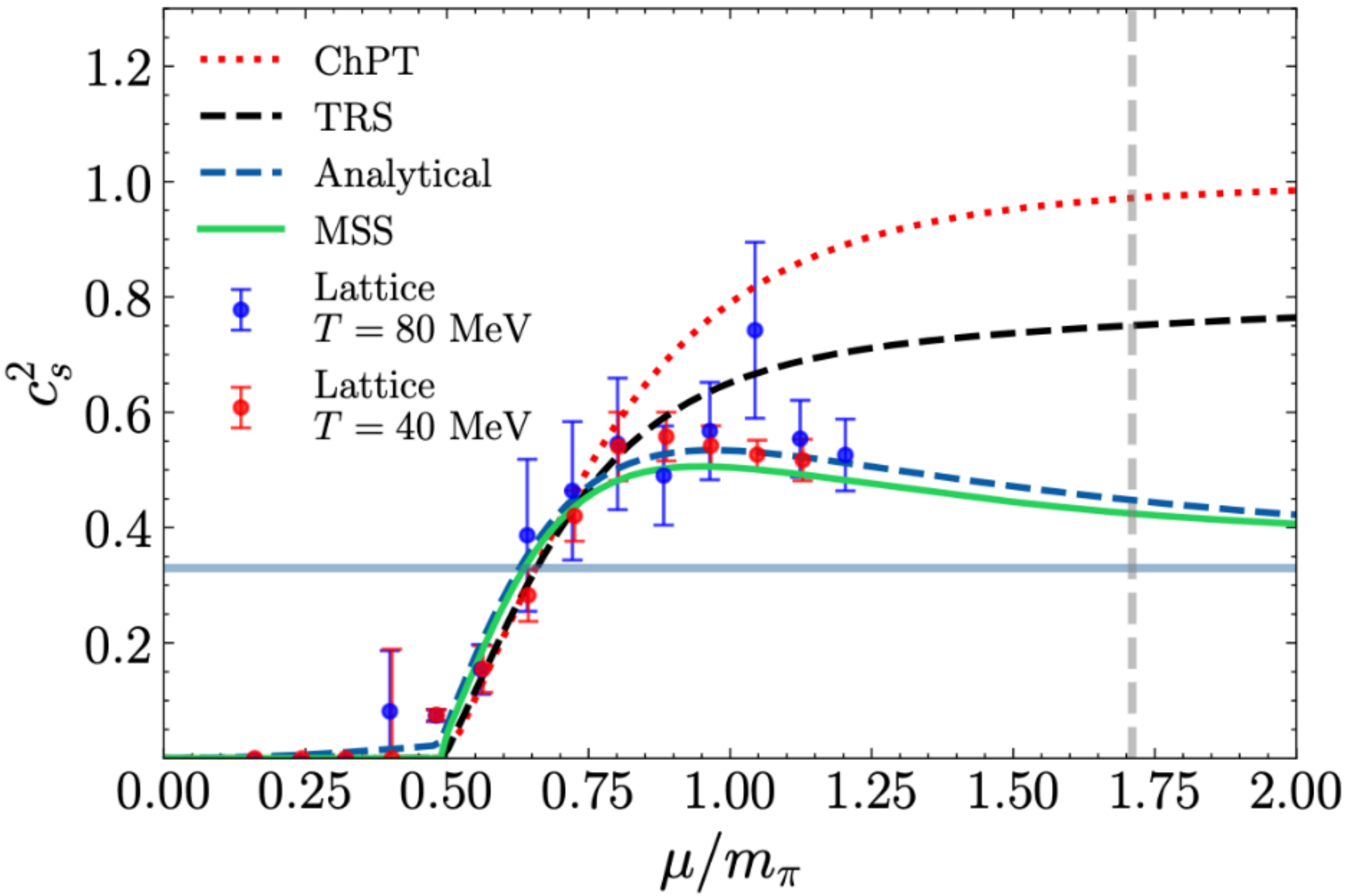
T. Kojo and D. Suenaga, PRD **105**, 076001 (2022)

Two color QCD in NJL model



R.L.S.F, A.E.B. Pasqualotto, D.C.Duarte and R.O.Ramos, Phys. Rev. D 113, 036007 (2026)

Two color QCD in NJL model



R.L.S.F, A.E.B. Pasqualotto, D.C.Duarte and R.O.Ramos, Phys. Rev. D **113**, 036007 (2026)
Lattice: K. Lida, E. Itou, K. Murakami, D. Suenaga, JHEP **10** (2024) 022

Conclusions:

- MSS results are consistent with LQCD across different contexts and with QMM and FRG methods (CSC).
- LQCD + effective model building + astrophysical observations will continue to play a crucial role in clarifying the structure of dense QCD matter.
- Disentangling vacuum and medium contributions appears to be essential for dense QCD

Perspectives:

- Comparison with RG approach
- Regulator dependence of inhomogeneous phases
- Finite densities + confinement effects
- Quarkyonic phase?

Thank you for your attention!