李政道研究所
TSUNG-DAO LEE INSTITUTE

Measuring the moment of inertia and ellipticity of neutron stars with gravitational waves

Zhiqiang Miao (T. D. Lee Institute)

Collaborator： Huan Yang (THU)

Based on arXiv: 2511.08895

CSQCD2026 @ Barcelona

2026.05.19

Outline

- ➡Motivation
- ➡Neutron stars are generically elliptical
- ➡Spin-orbit resonance in neutron star binaries
- ➡Detection and physical implication
- ➡Summary

Motivation

Morras+2025
arXiv:2503.15393

Orbital Eccentricity in a Neutron++ Star–Black Hole Merger

CrossMark

Gonzalo Morras[1,2,3], Geraint Pratten[1], and Patricia Schmidt[1]
[1] School of Physics and Astronomy and Institute for Gravitational Wave Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK; P.Schmidt@bham.ac.uk
[2] Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco 28049 Madrid, Spain
[3] Max Planck Institute for Gravitational Physics (Albert Einstein Institute), D-14467 Potsdam, Germany
Received 2025 September 3; revised 2026 January 28; accepted 2026 February 1; published 2026 March 11

Abstract

The observation of gravitational waves (GWs) from merging black holes and neutron stars provides a unique opportunity to discern information about their astrophysical environment. Two signatures that are considered powerful tracers to distinguish between different binary formation channels are general relativistic spin-induced orbital precession and orbital eccentricity. Both effects leave characteristic imprints in the GW signal that can be extracted from observations. To date, neither precession nor eccentricity has been confidently discerned in merging neutron star–black hole binaries. Here we report the measurement of orbital eccentricity in a neutron star–black hole merger. Using, for the first time, a waveform model that incorporates precession and eccentricity, we perform Bayesian inference on the GW event GW200105 and infer a median orbital eccentricity of $e_{20} \sim 0.145$ at an orbital period of 0.1 s, ruling out eccentricities smaller than 0.028 with 99.5% confidence. We find inconclusive evidence for the presence of precession, consistent with previous, noneccentric results, but a more unequal mass ratio. Our result implies a fraction of these binaries will exhibit orbital eccentricity even at small separations, suggesting formation through mechanisms involving dynamical interactions beyond isolated binary evolution. Future observations will reveal the contribution of eccentric neutron star–black hole binaries to the total merger rate across cosmic time.

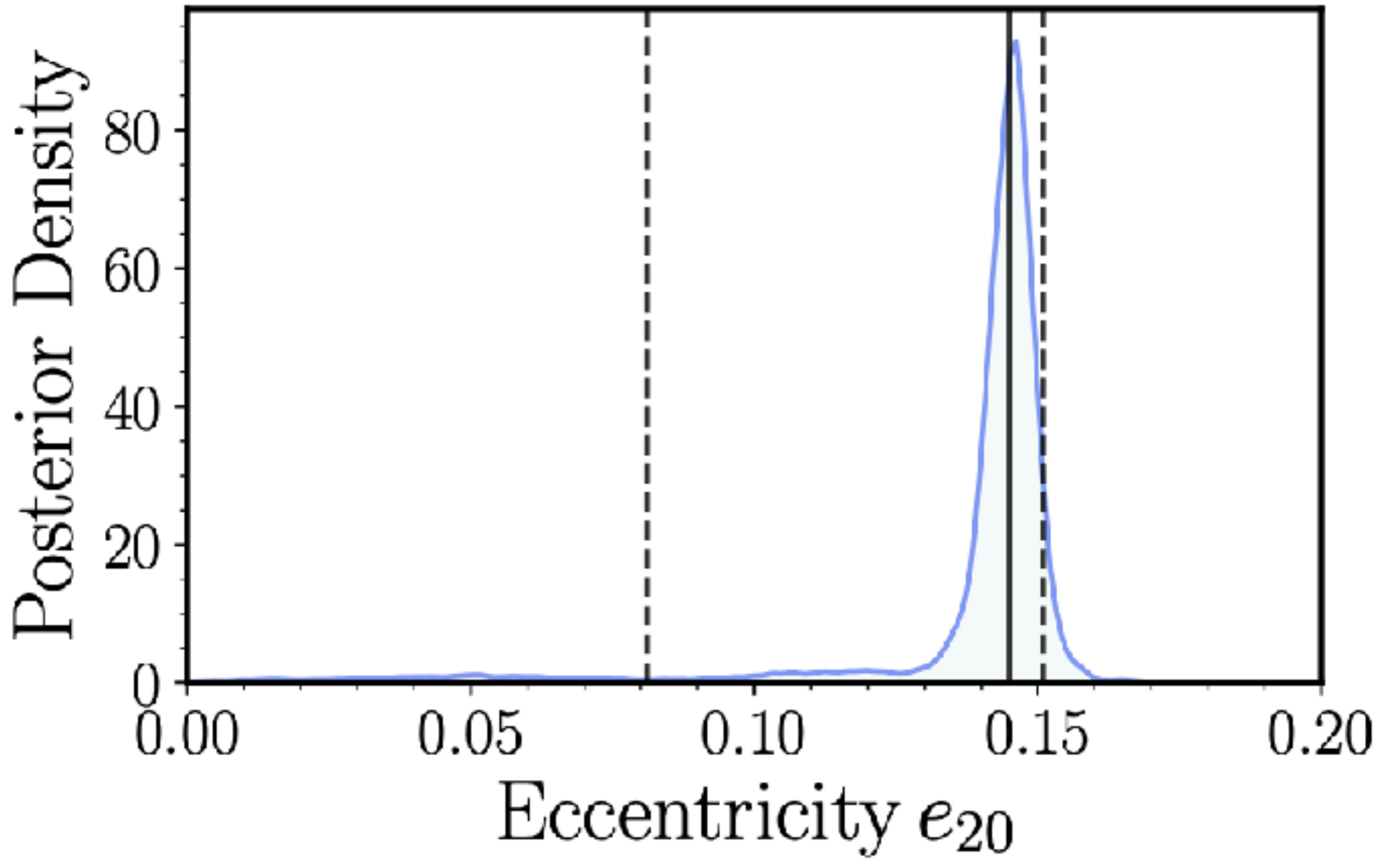


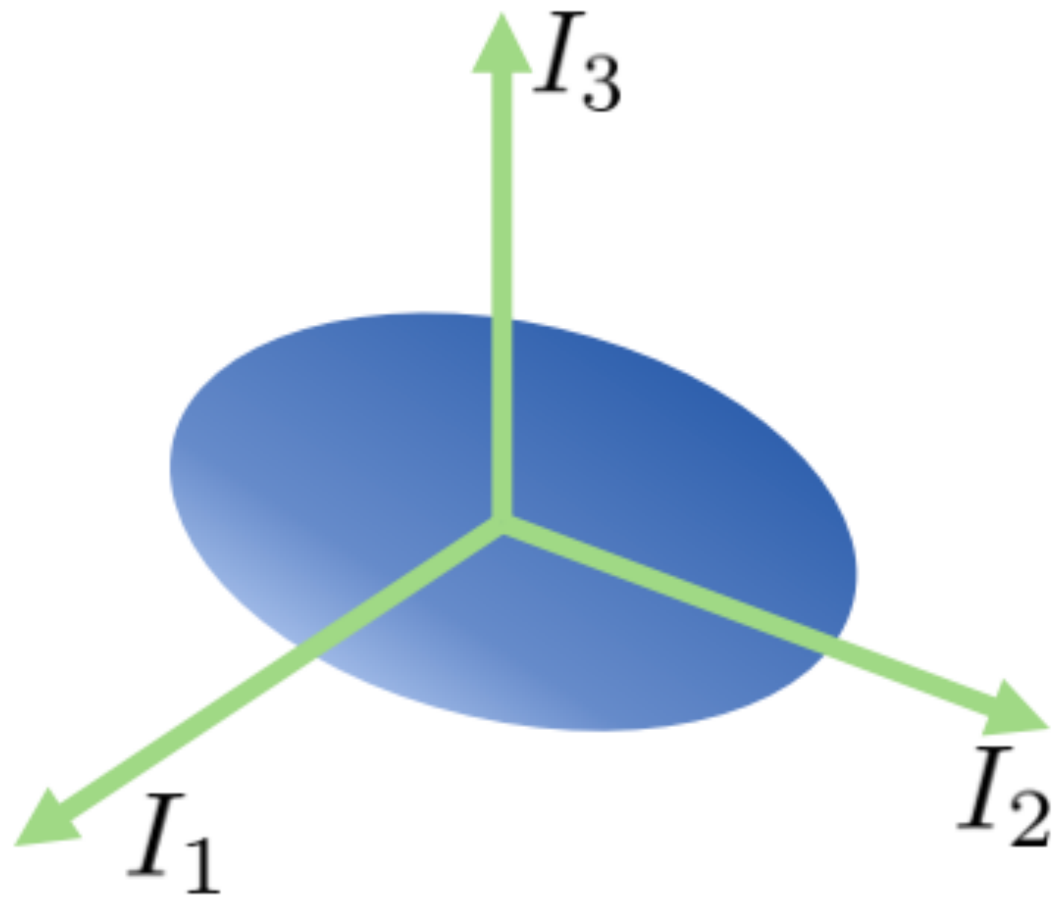
Figure 1. Measured eccentricity distribution in GW200105. One-dimensional posterior probability distribution for our measurement of the orbital eccentricity at a GW frequency of 20 Hz. The solid vertical line indicates the median and the two dashed lines the 90% credible interval.

Dynamical formation channel means large orbit eccentricity, short delay time

Is the NS in GW200105 a magnetar? What would happen if it were a magnetar?

Magnetar (with permanent deformation)+ companion similar to Earth-Moon/ Sun-Mercury?

Neutron stars are generically elliptical



Definition $\epsilon := \dfrac{I_2 - I_1}{I_3}$

Origin：

- Strong B field (magnetars)

$$\epsilon \sim \frac{\bar{B}^2 R^3}{GM^2/R} \sim 4 \times 10^{-6} (\frac{1.4 M_\odot}{M})^2 (\frac{R}{12\,\mathrm{km}})^2 (\frac{\bar{B}}{10^{15}\mathrm{G}})^2$$

(Haskell+2008, Akgun+2008, Lander+2009)

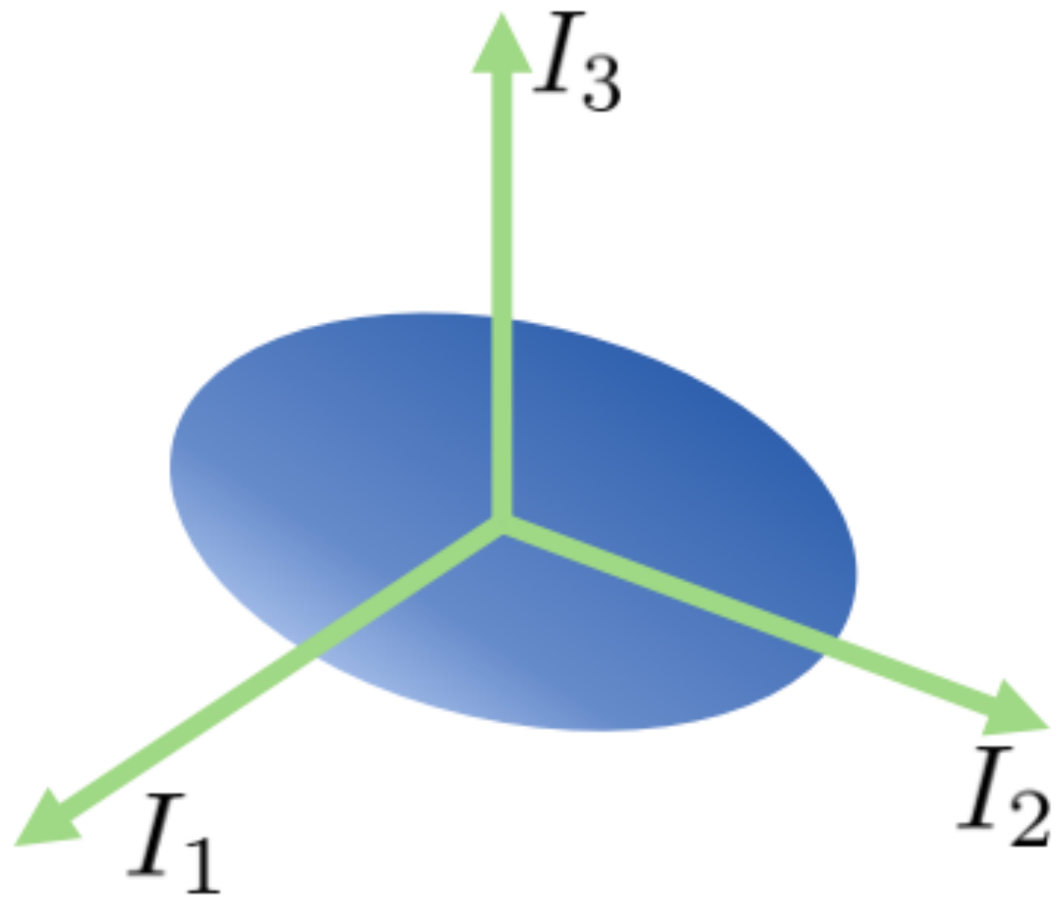
- Solid crust

$$\epsilon \approx 10^{-6} (\frac{\sigma_{\mathrm{br}}}{0.1})$$

(Ushomirsky+2000, Haskell+20006)

Neutron stars are generically elliptical

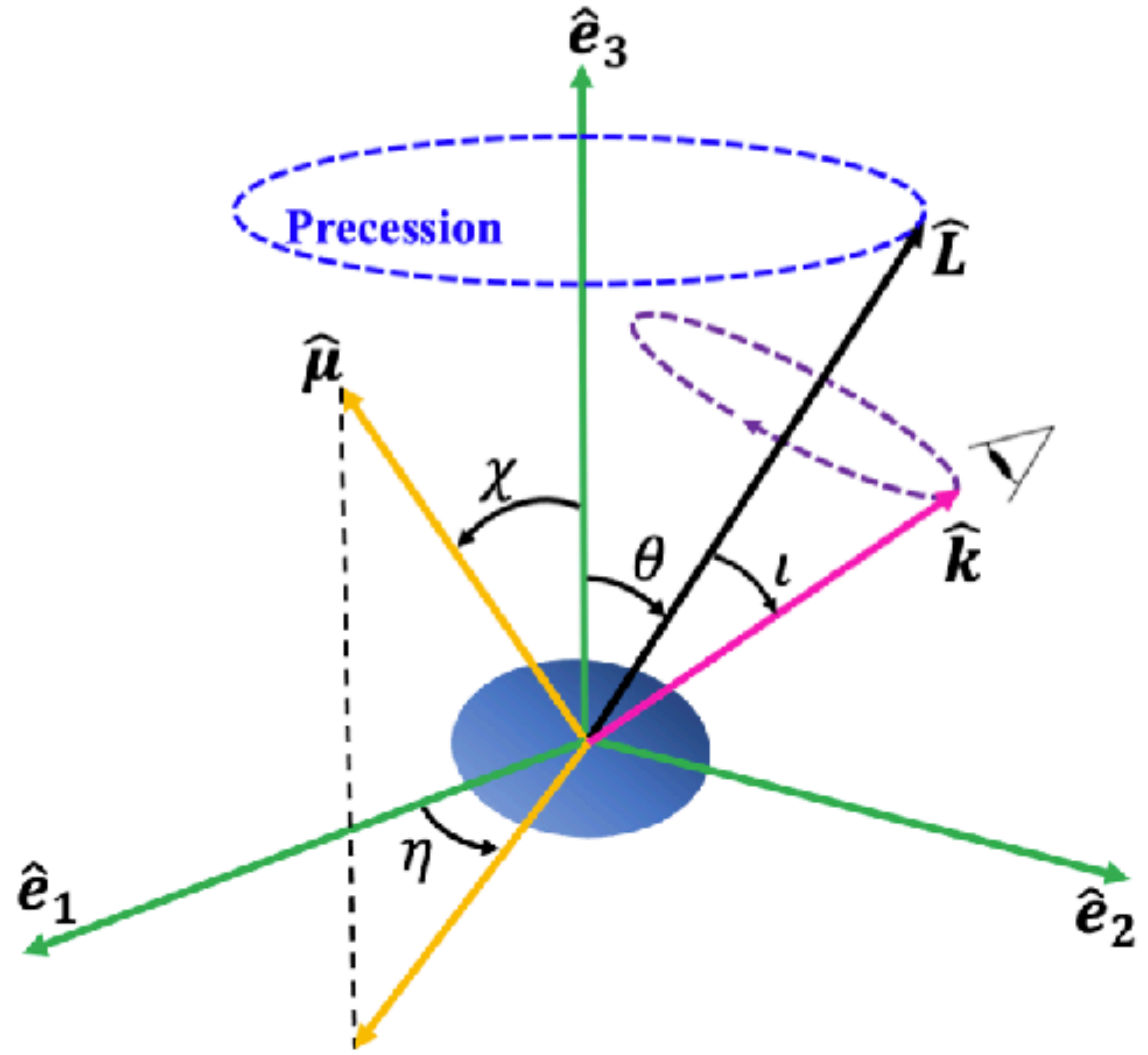
Definition $\epsilon := \frac{I_2 - I_1}{I_3}$



Detection:

→ Precession of neutron star, modulate the EM signal

(Gao+2022, Desvignes+2024)



→ Emit continuous GW,

$$h_0 \sim \frac{\epsilon I f^2}{d} \approx 10^{-28}\left(\frac{\epsilon}{10^{-6}}\right)\left(\frac{M}{1.4M_\odot}\right)\left(\frac{R}{12\mathrm{km}}\right)^2\left(\frac{f}{1\mathrm{Hz}}\right)^2\left(\frac{1\mathrm{kpc}}{d}\right)$$

Outline

➡Neutron stars are generically elliptical

➡**Spin-orbit resonance in neutron star binaries**

➡Detection and physical implication

➡Summary

Spin-orbit resonance in NS binarie

- **Equations of motion (for circular orbit)**

$$\ddot{f} + \frac{3}{2}\epsilon \frac{M'}{a^3}\frac{I}{I_{\rm orb}} \sin[2(f-\theta)] = \dot{\Omega}_{\rm orb}\,,$$

$$\ddot{\theta} - \frac{3}{2}\epsilon \frac{M'}{a^3} \sin[2(f-\theta)] = \dot{\Omega}_{\rm s} \approx 0$$

Define $\gamma = 2(f-\theta)$

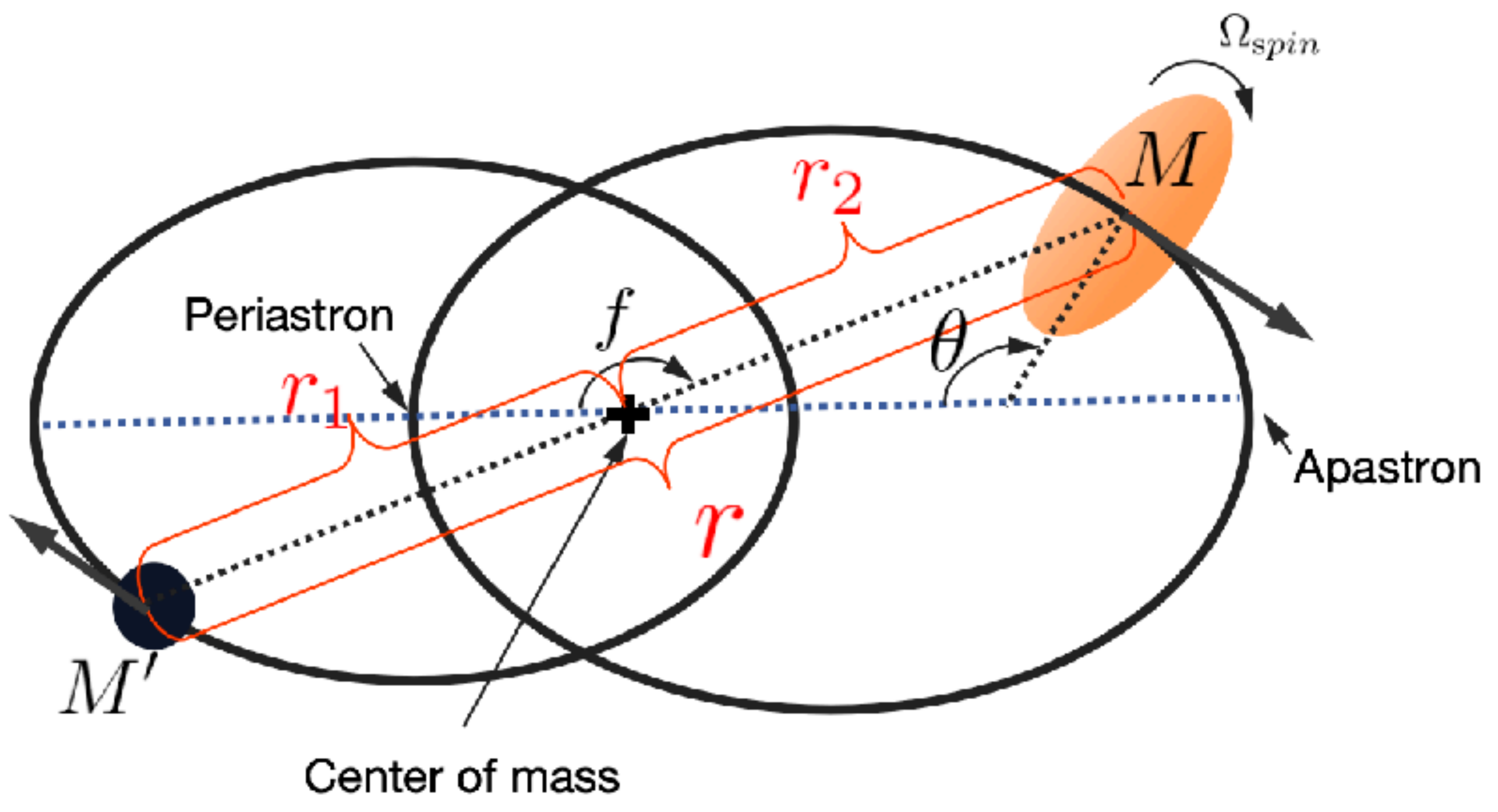
$$\ddot{\gamma} + \mathcal{B}\sin\gamma + \mathcal{C} = 0\,,$$

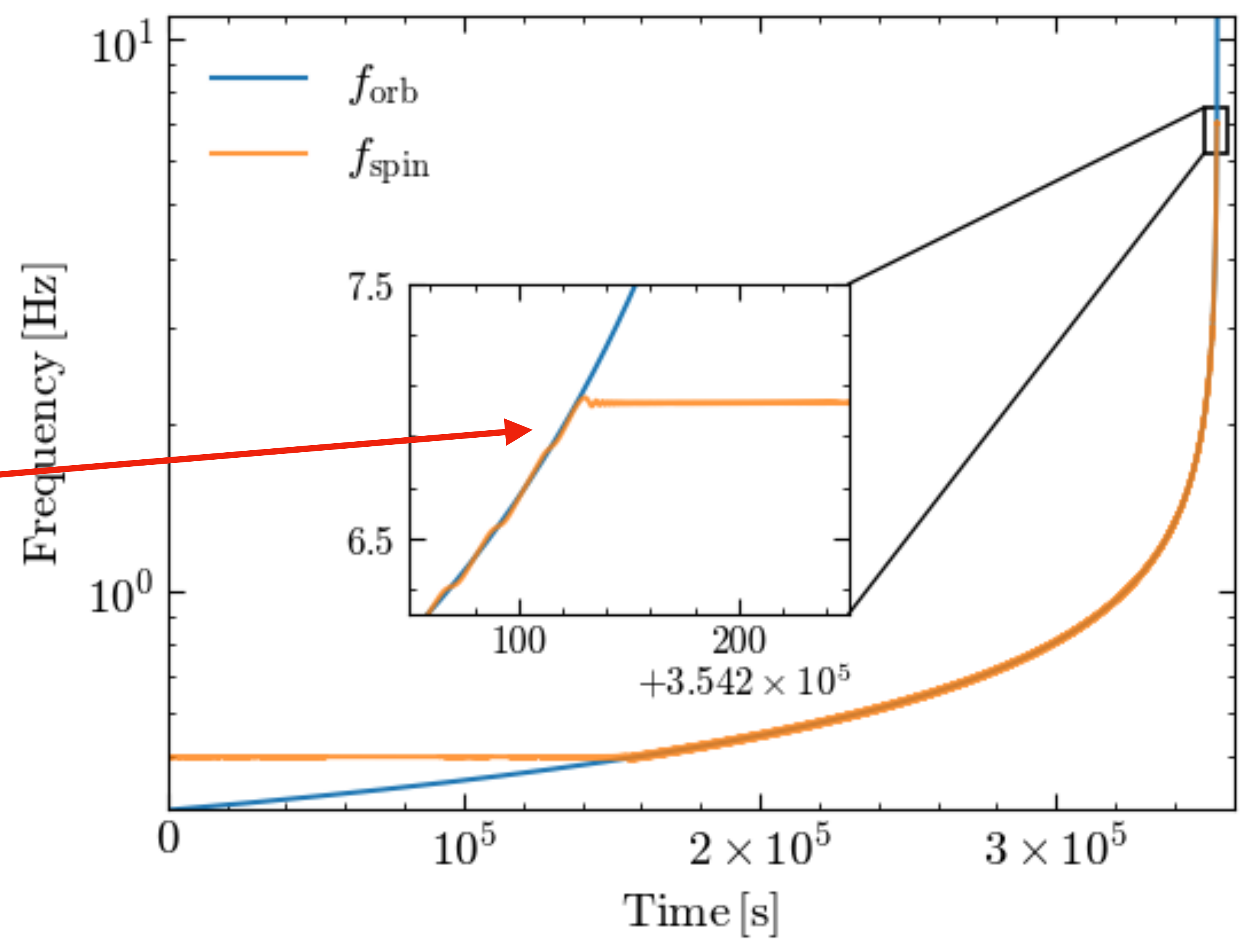
$$\mathcal{B} = 3\epsilon G M'/a^3 \text{ and } \mathcal{C} = 2\dot{\Omega}_{\rm orb}.$$

Locking breaks down when $\mathscr{B} < \mathscr{C}$,

$$f_{\rm br} = \frac{\Omega_{\rm br}}{\pi} = \frac{1}{\pi(M+M')}\left(\frac{5}{64}\frac{M+M'}{M}\right)^{3/5}\epsilon^{3/5}$$

$$\approx 10\left(\frac{1}{1+q}\right)^{2/5}\left(\frac{1.4\,M_\odot}{M}\right)\left(\frac{\epsilon}{10^{-5}}\right)^{3/5}\ \mathrm{Hz}\,.$$





Can resonance happen in reality?

- **Can NS in inspiral stage have non-negligible ellipticity?**

- **If NS has ellipticity, what's the possibility of capturing into resonance?**

Can NS in inspiral stage have non-negligible ellipticity

- **The timescale of magnetic field decay for a magnetar is**

$$T_d \sim 10^5 - 10^6\,\mathrm{yr}$$ Goldreich+1992

- **Usually, merger time should be very long for isolate binary evolution**

$$T_{\mathrm{merger},0} \simeq 6\times 10^8 \left(\frac{2.8\,M_\odot}{M+M'}\right)^2 \left(\frac{0.7\,M_\odot}{\mu}\right)\left(\frac{a}{1\,\mathrm{AU}}\right)^4\,\mathrm{yr}$$

- **For highly eccentric orbit,**

$$T_{\mathrm{merger}} = T_{\mathrm{merger},0}\frac{768}{429}(1-e^2)^{7/2}$$

  - Dynamical capture in dense stellar environments

    $$R_{\mathrm{NSBH}} \sim 0.1 - 1\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

  - Hierarchical triples (Lidov-Kozai oscillation)

    $$R_{\mathrm{NSBH}} \sim 1 - 23\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

(Stegmann+2025)

Evidence: orbital eccentricity found in a neutron star–black hole merger, GW200105

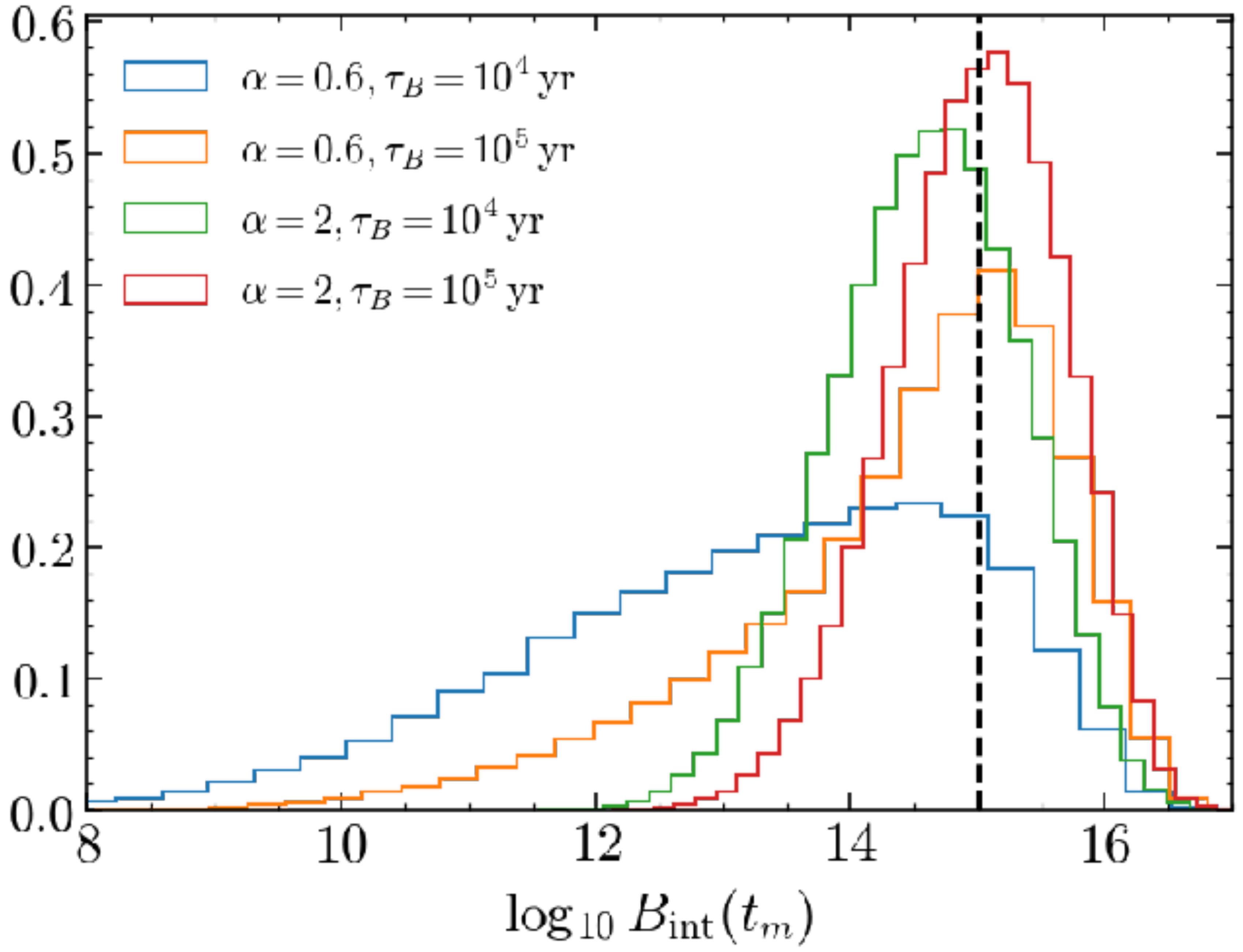
(Morras+2025)

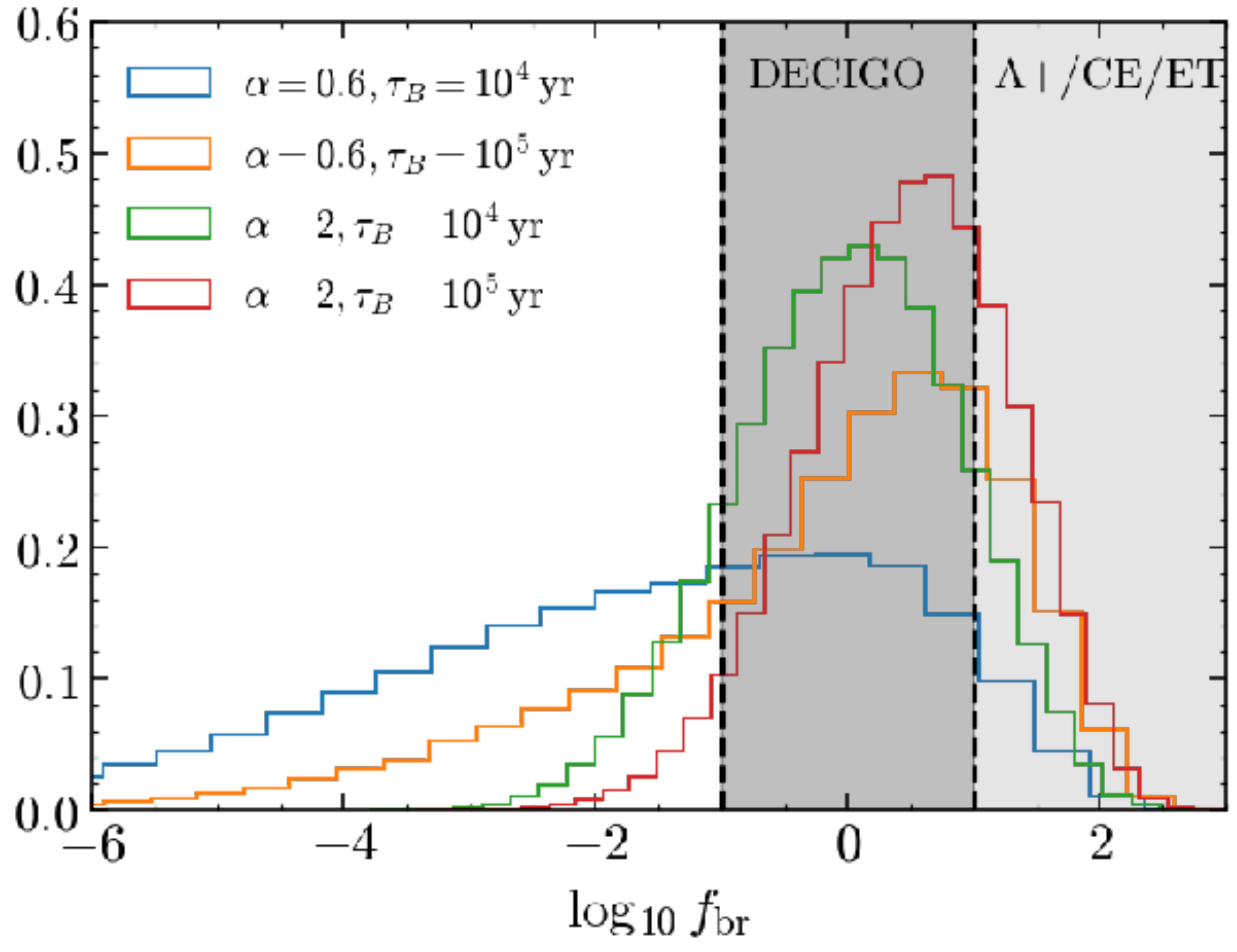
Can NS in inspiral stage have non-negligible ellipticity

Magnetic field evolution

$$B_{\rm int}(t) = B_{\rm int,0}\left(1+\frac{\alpha t}{\tau_B}\right)^{-1/\alpha},$$

$$B_{\rm int,0} = \xi\, B_{p,0}\,, \quad \log_{10} B_{p,0} \sim \mathcal{N}(14.5, 0.25^2)$$





Can resonance happen in reality?

- **Can NS in inspiral stage have non-negligible ellipticity?**

Yes $f_{\rm mag,merge} = f_{\rm mag,0} P_{\rm mag,merge}(B_{\rm int}(t_{\rm m}) > B_{\rm th}) \sim f_{\rm mag,0} \times \mathcal{O}(0.1)$

- **If NS has ellipticity, what's the possibility of capturing into resonance?**

Can resonance happen in reality?

- **Can NS in inspiral stage have non-negligible ellipticity?**

Yes $f_{\rm mag,merge} = f_{\rm mag,0} P_{\rm mag,merge}(B_{\rm int}(t_{\rm m}) > B_{\rm th}) \sim f_{\rm mag,0} \times \mathcal{O}(0.1)$

- **If NS has ellipticity, what's the possibility of capturing into resonance?**

For circular orbit

$$\ddot{\gamma} + \mathcal{B} \sin\gamma + \mathcal{C} = 0\,,$$

$$\mathcal{B} = 3\epsilon G M'/a^3 \text{ and } \mathcal{C} = 2\dot{\Omega}_{\rm orb}.$$

$$\mathcal{P}_{\rm lock} = \frac{2}{1 + \frac{\pi}{2}\frac{\mathcal{B}^{1/2}\mathcal{C}}{\dot{\mathcal{B}}}} \approx \frac{4}{\pi}\frac{\dot{\mathcal{B}}}{\mathcal{B}^{1/2}\mathcal{C}} = \frac{4\sqrt{3}}{\pi}\left(\frac{q}{1+q}\right)^{1/2}\epsilon^{1/2}$$

$\sim 10^{-2}$ for $\epsilon \sim 10^{-5}$

For Eccentric orbit

$$\mathcal{P}(1,e) = \frac{4\sqrt{3}}{\pi}\left(\frac{q}{1+q}\right)^{1/2}\epsilon^{1/2}\left(1+\frac{25}{18}e^2\right),$$

$$\mathcal{P}(\frac{3}{2},e) = \frac{4\sqrt{3}}{\pi}\left(\frac{q}{1+q}\right)^{1/2}\epsilon^{1/2}\frac{17}{54}\sqrt{\frac{7}{2}}e^{1/2}.$$

Outline

- ➡Neutron stars are generically elliptical
- ➡Spin-orbit resonance in neutron star binaries
- ➡**Detection and physical implication**
- ➡Summary

How to detect?

- Locking occurs when the orbital frequency sweeps through the neutron star spin frequency, typically occurring in the range of 0.1 – 1 Hz for magnetars.
- The locking can persist up to O(10) Hz or higher, depends on the ellipticity.

Change the gravitational waveform

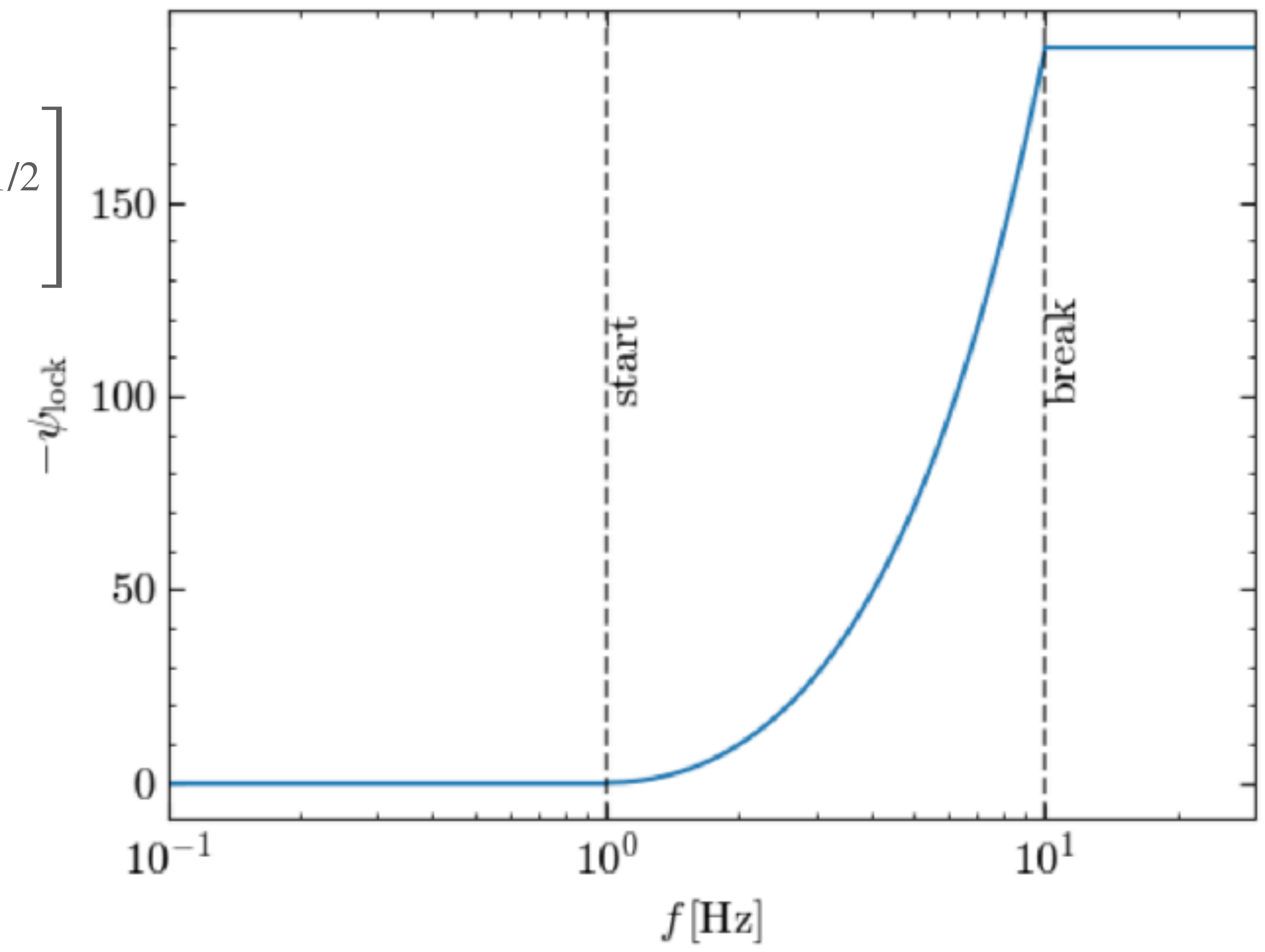
$$\dot{a} = -\frac{64}{5}\frac{\mu M_t^2}{a^3}\left(1-\frac{3I_2}{\mu a^2}\right)^{-1} = \dot{a}_0(1-\delta)^{-1} \qquad N = \int_{a_i}^{a_f} \frac{\Omega_{\rm orb}}{2\pi \dot{a}} da$$

$$\delta N = \int_{a_i}^{a_f} -\frac{\sqrt{M_t/a^3}}{2\pi\dot{a}_0}\delta da = \frac{5}{128\pi}\frac{3M_t^{1/2}I_2}{\mu^2 M_t^2}\int_{a_i}^{a_f} a^{-1/2}da = -\frac{15}{64\pi}\frac{I_2}{\mu^2 M_t}\left[\left(\frac{a_i}{M_t}\right)^{1/2} - \left(\frac{a_f}{M_t}\right)^{1/2}\right]$$

Frequency domain waveform correction (phase correction)

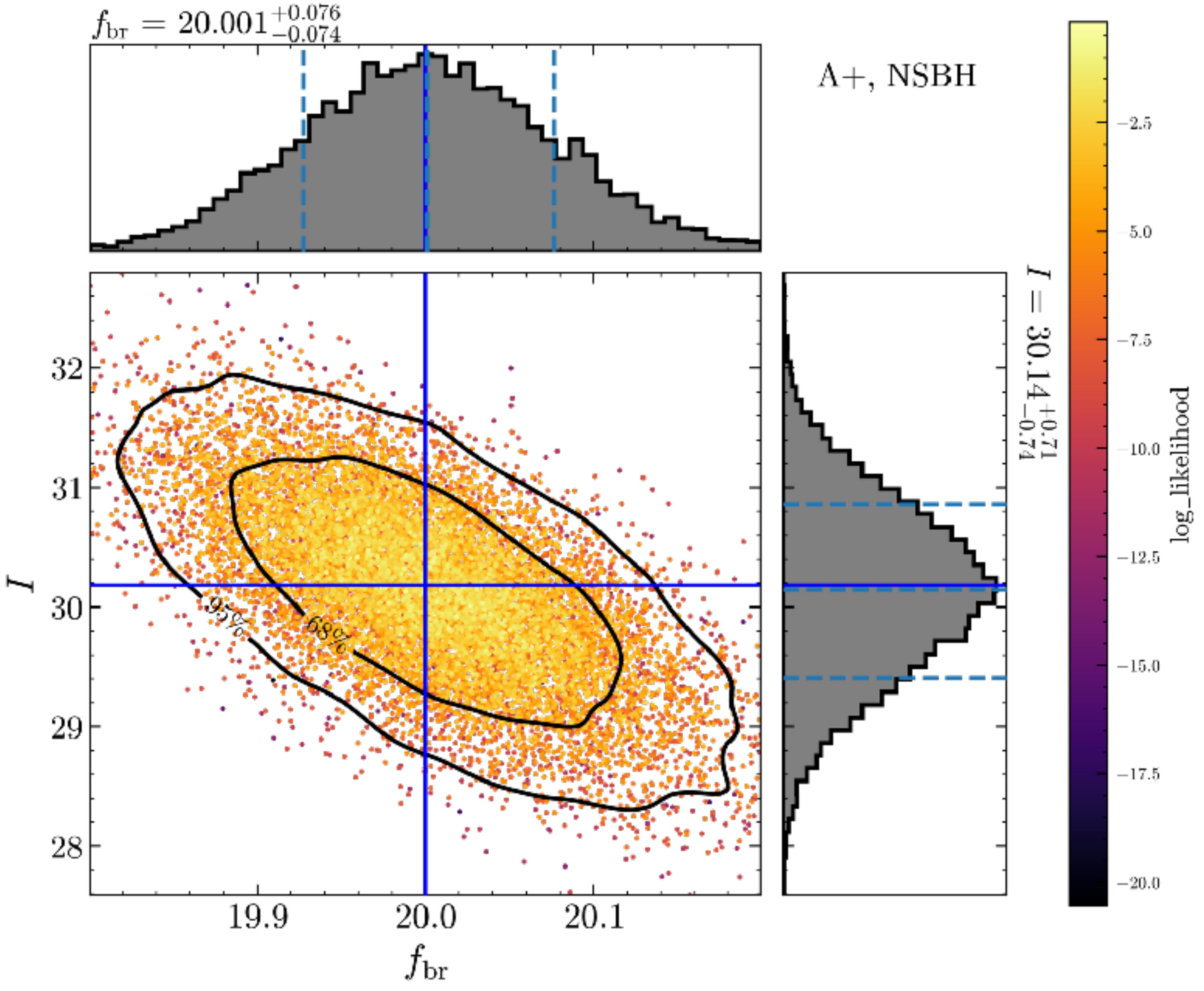
$$h(f) = \mathscr{A} e^{i\psi(f)}$$

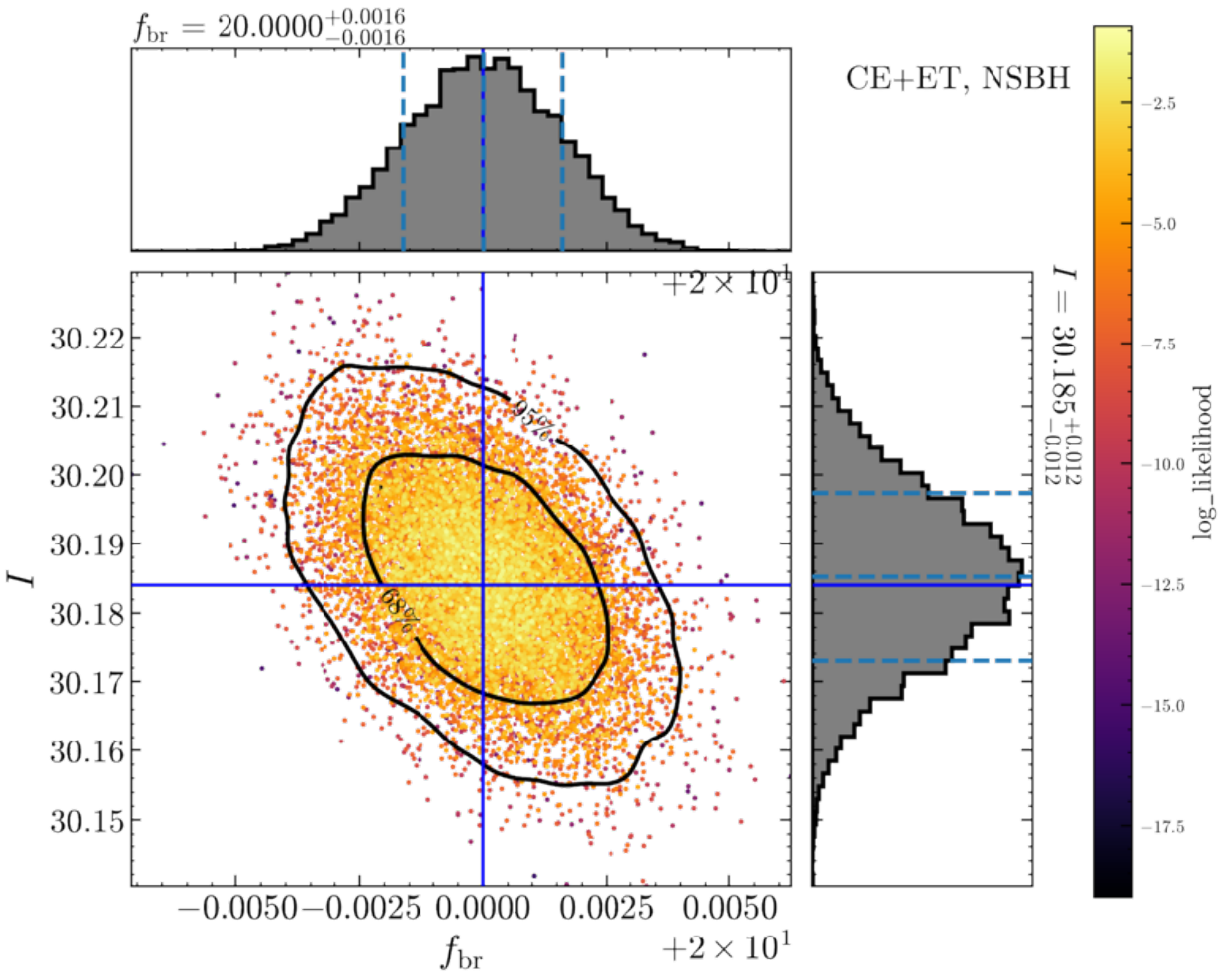
$$\delta\psi(f) = \begin{cases} -\frac{45}{64}\frac{I}{\mu^2 M_t}\left(v^{-1} - \frac{4}{3}v_i^{-1} + \frac{1}{3}\frac{v^3}{v_i^4}\right), & f \le f_{\rm br} \\ -\frac{45}{64}\frac{I}{\mu^2 M_t}\left(v_{\rm br}^{-1} - \frac{4}{3}v_i^{-1} + \frac{1}{3}\frac{v_{\rm br}^3}{v_i^4}\right), & f > f_{\rm br} \end{cases}$$



Injection study - NSBH

NSBH Injection: $m_1 = 7M_\odot, m_2 = 1.4M_\odot, D_L = 100\mathrm{Mpc}$



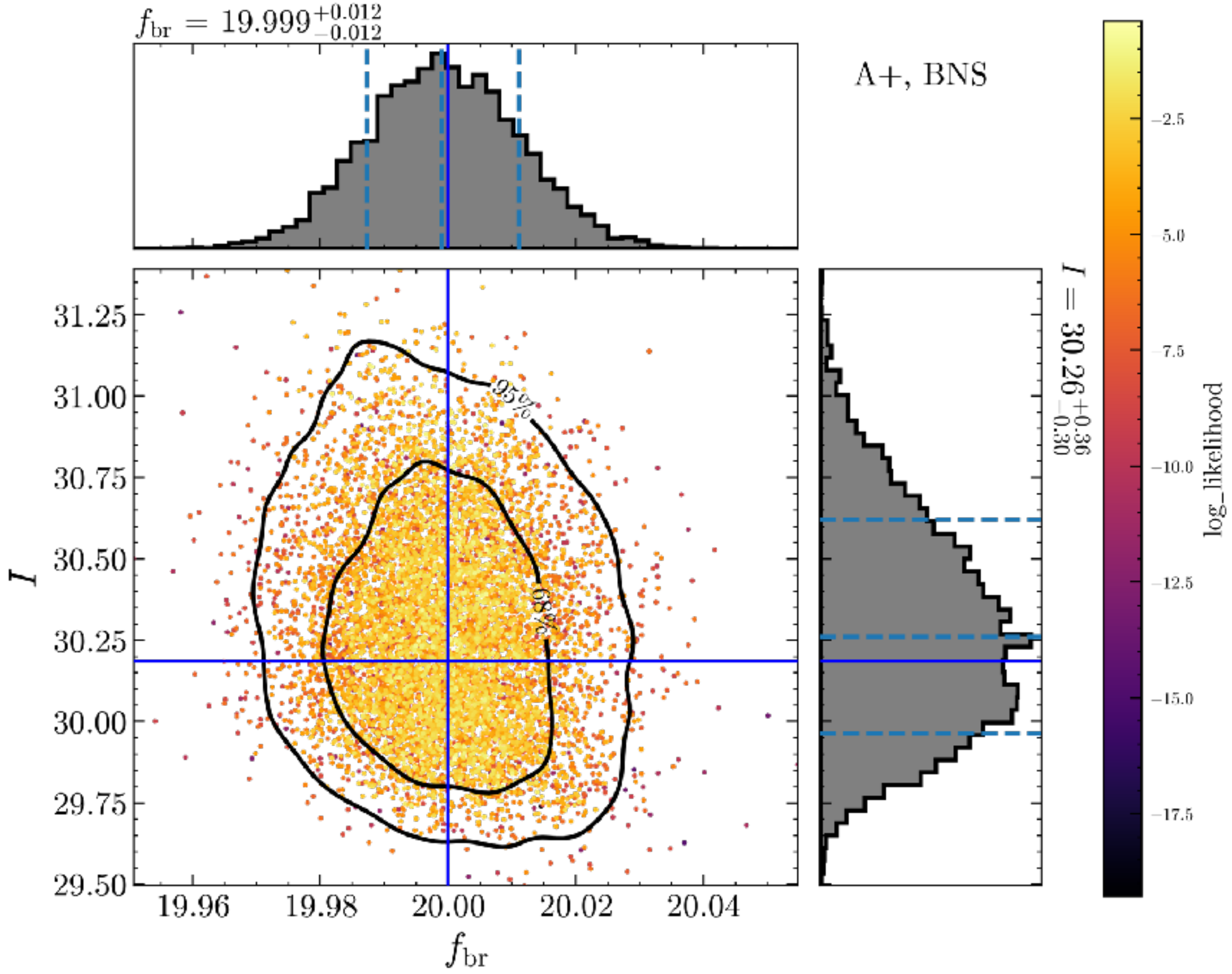


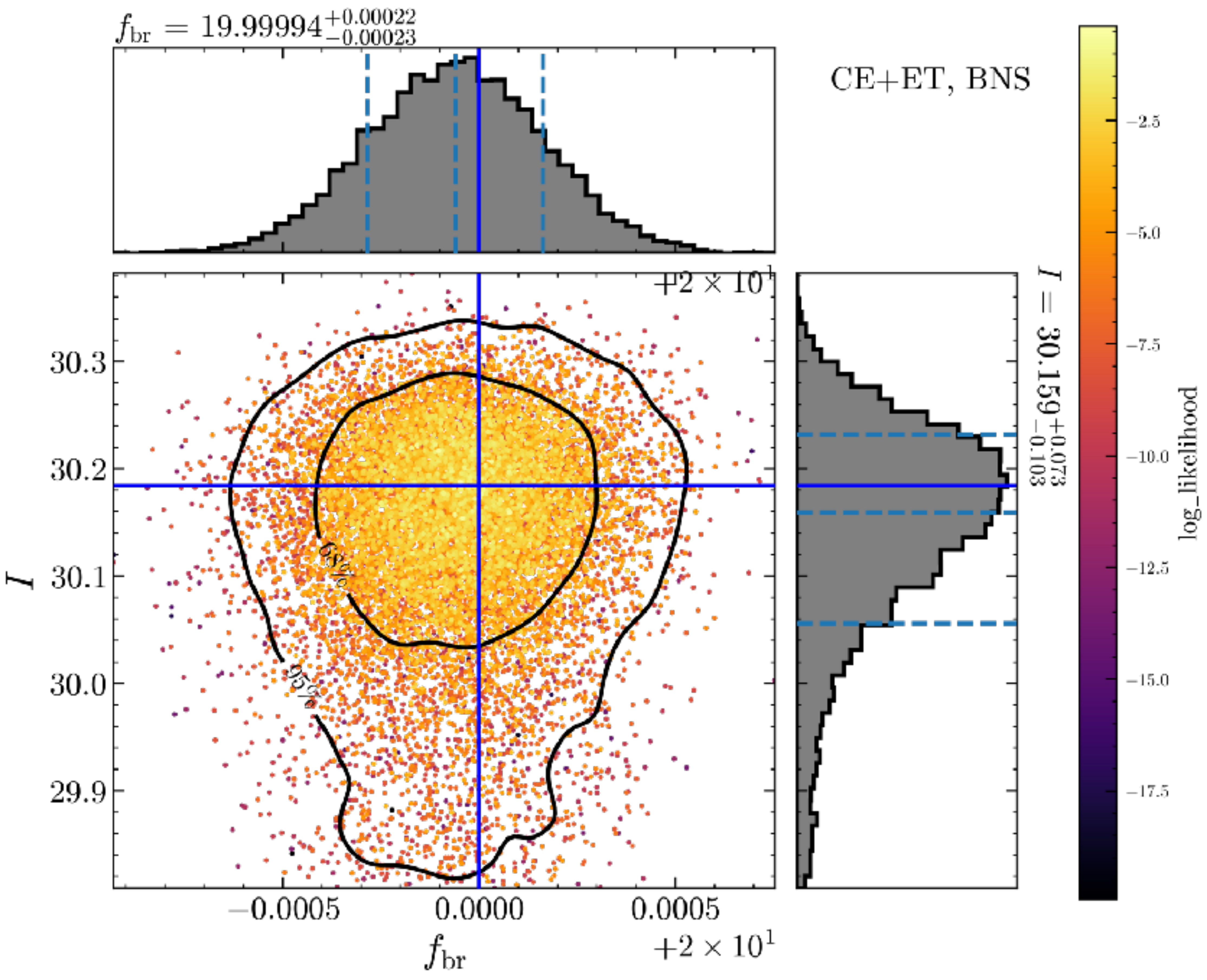
$\Delta f_{\rm br}/f_{\rm br} \sim \mathcal{O}(0.1\%), \quad \Delta I/I \sim \mathcal{O}(1\%)$

$\Delta f_{\rm br}/f_{\rm br} \sim \mathcal{O}(0.01\%), \quad \Delta I_2^{src}/I_2^{src} \sim \mathcal{O}(0.1\%)$

Injection study - BNS

BNS Injection: $m_1 = 1.4M_\odot, m_2 = 1.4M_\odot, D_L = 100\mathrm{Mpc}$

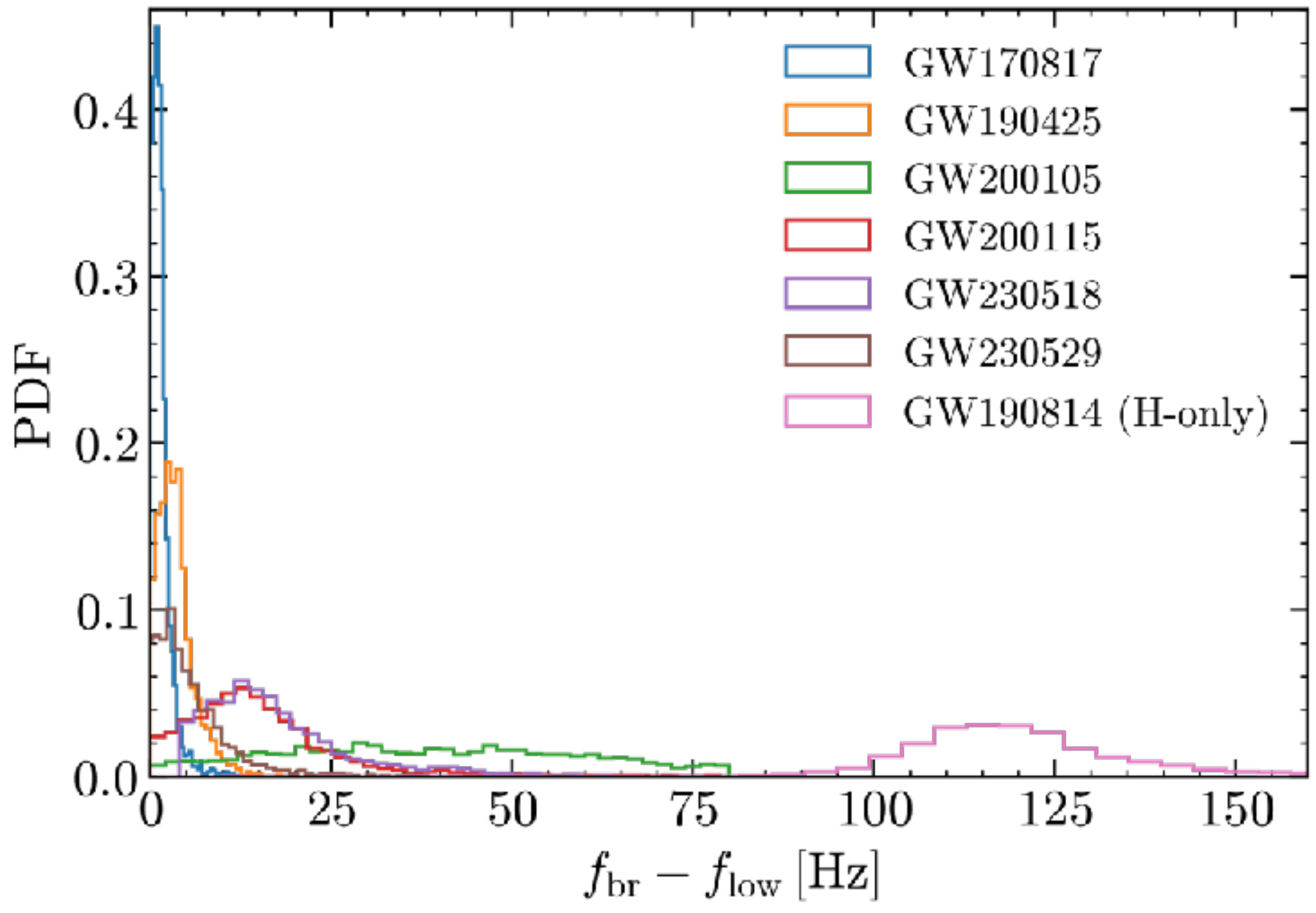




$\Delta f_{\rm br}/f_{\rm br} \sim \mathcal{O}(0.1\%)$, $\Delta I_2^{src}/I_2^{src} \sim \mathcal{O}(1\%)$

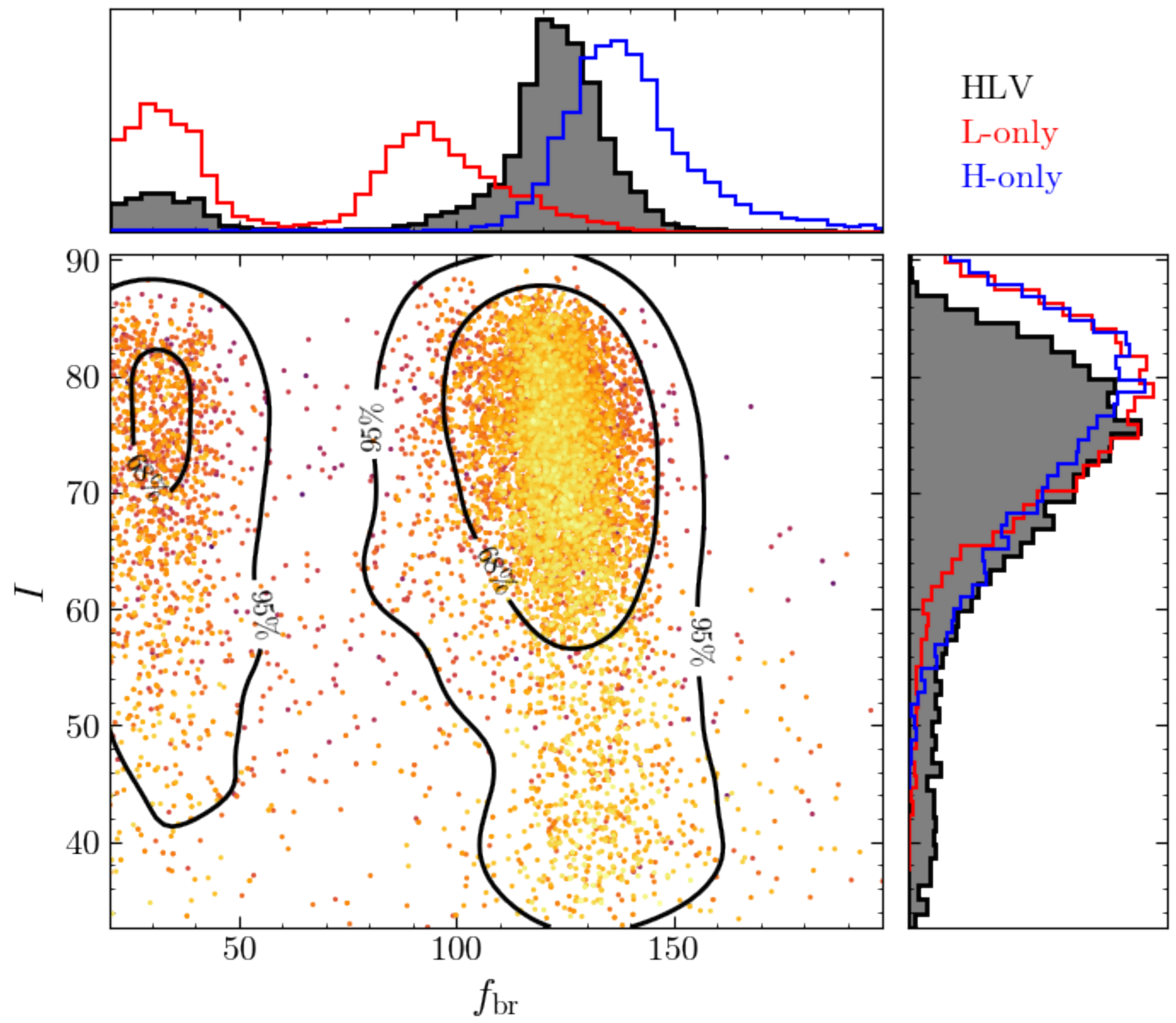
$\Delta f_{\rm br}/f_{\rm br} \sim \mathcal{O}(0.01\%)$, $\Delta I_2^{src}/I_2^{src} \sim \mathcal{O}(0.1\%)$

Search in known events



No positive events, but the mass-gap event GW190814 shows an interesting peak

GW190814



If the $f_{br} \sim 120\,\mathrm{Hz}$ peak is real, it would imply that the secondary object of GW190814 is not a BH, but rather a neutron star with an ellipticity of $\epsilon \sim 8 \times 10^{-3}$, corresponding to an extremely strong internal magnetic field $\sim 8 \times 10^{16}\,\mathrm{G}$

Physical implications

- **Measure the NS ellipticity, possibly a magnetar observed in binaries, can be used to constrain the B field.**

$$\Delta f_{\rm br}/f_{\rm br} \sim \mathcal{O}(0.01\%) - \mathcal{O}(0.1\%)$$

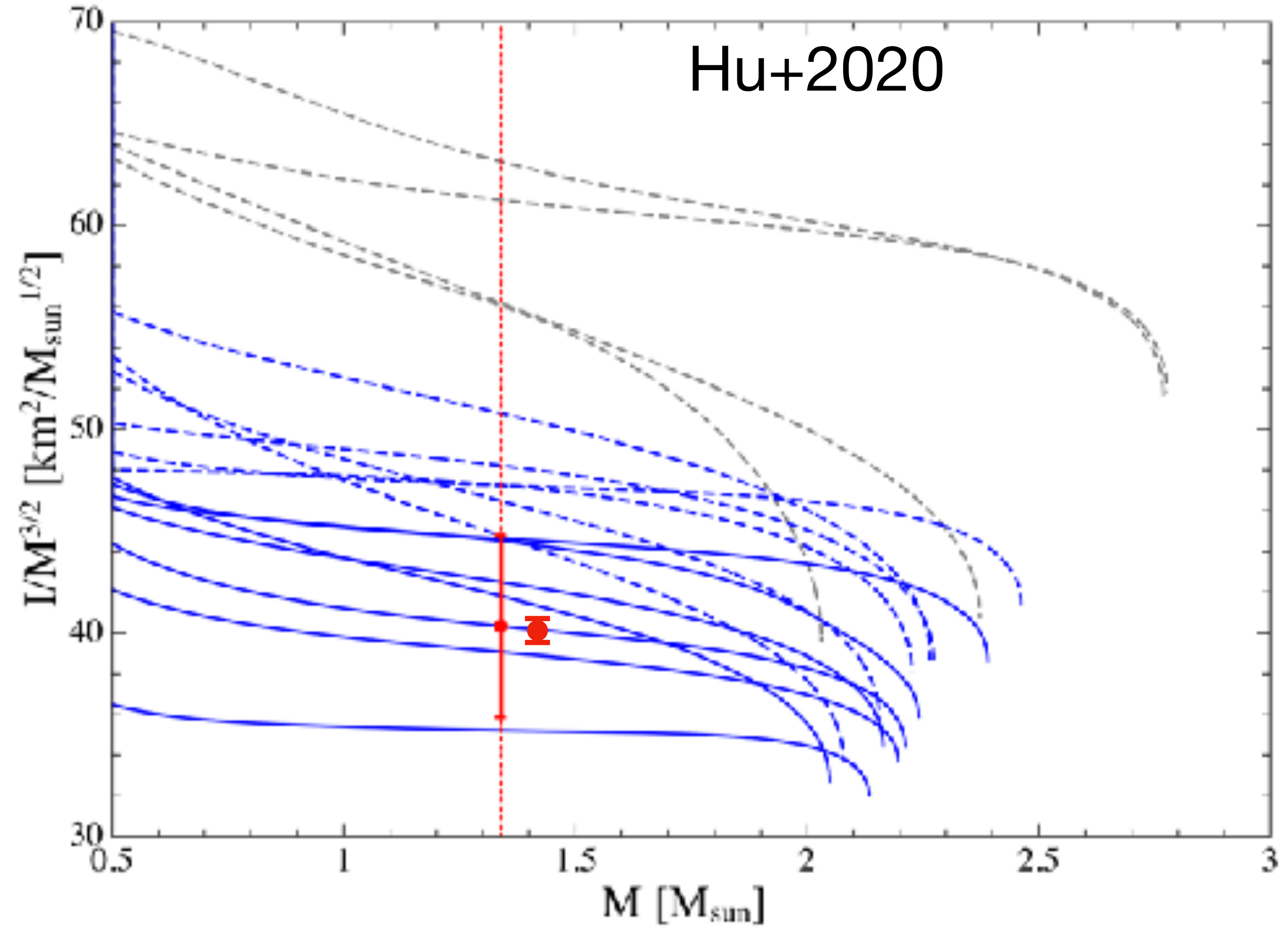
- **Measure the moment of inertia,**

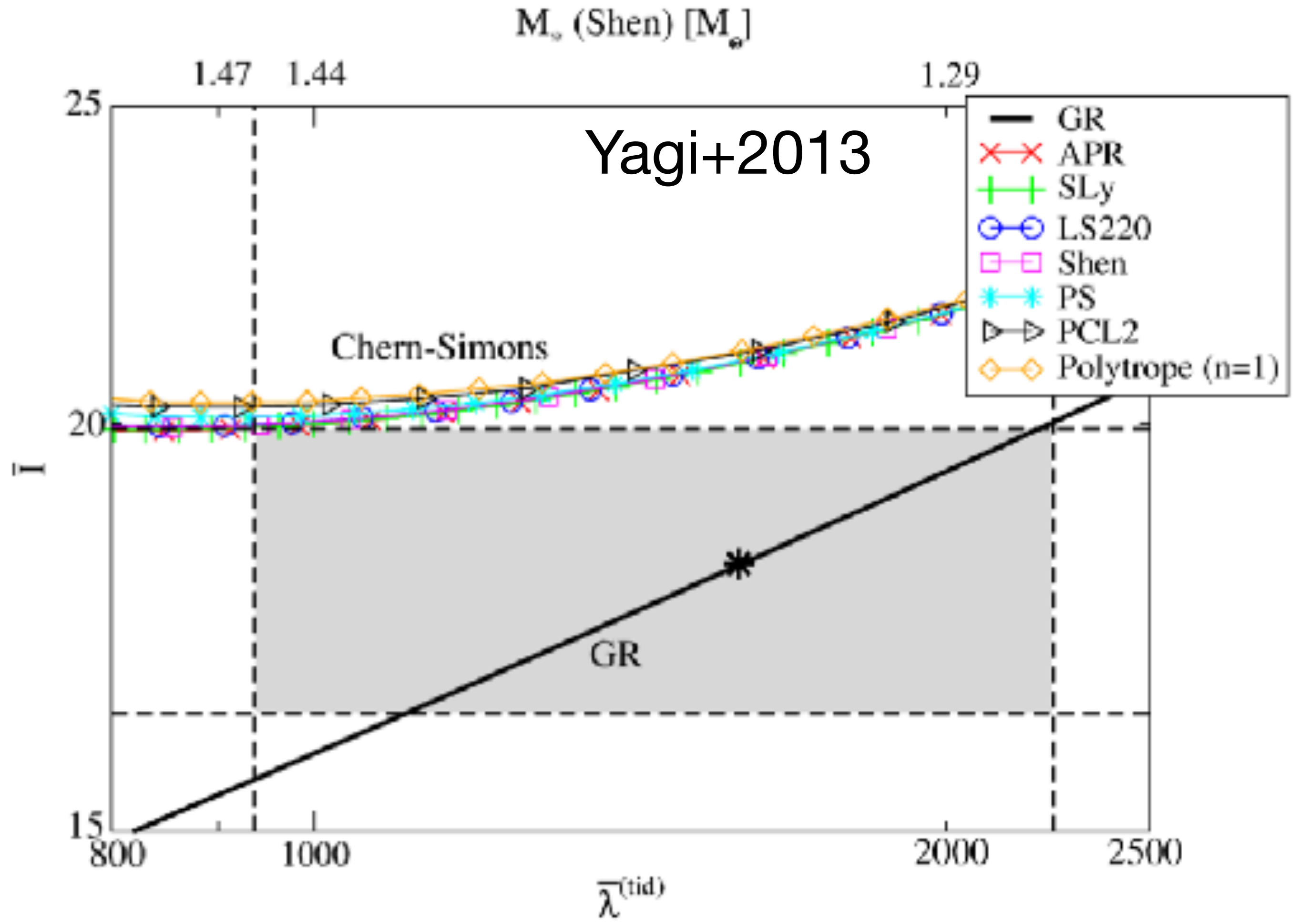
$$\Delta I/I \sim \mathcal{O}(0.1\%) - \mathcal{O}(1\%)$$

  - **Even more precise than tidal Love number, of great help in constraining the equation of state.**

  - **A unique system in which both I and tidal love number can be measured, providing a test of gravity.**

- **Test Exotic Compact Object (ECO)**





What if undetected?

- The expected merger rate for a resonance-locking NSBH system is approximately

$$R_{\rm lock} = f_{\rm mag}\mathscr{P}_{\rm lock}R_{\rm NSBH}$$
$$\approx 0.12 f_{\rm mag}\left(\frac{\epsilon}{10^{-5}}\right)^{1/2}\left(\frac{R_{\rm NSBH}}{20\,{\rm Gpc^{-3}\,yr^{-1}}}\right){\rm Gpc^{-3}\,yr^{-1}}\,,$$

- The total merger rate in the observer frame is written as

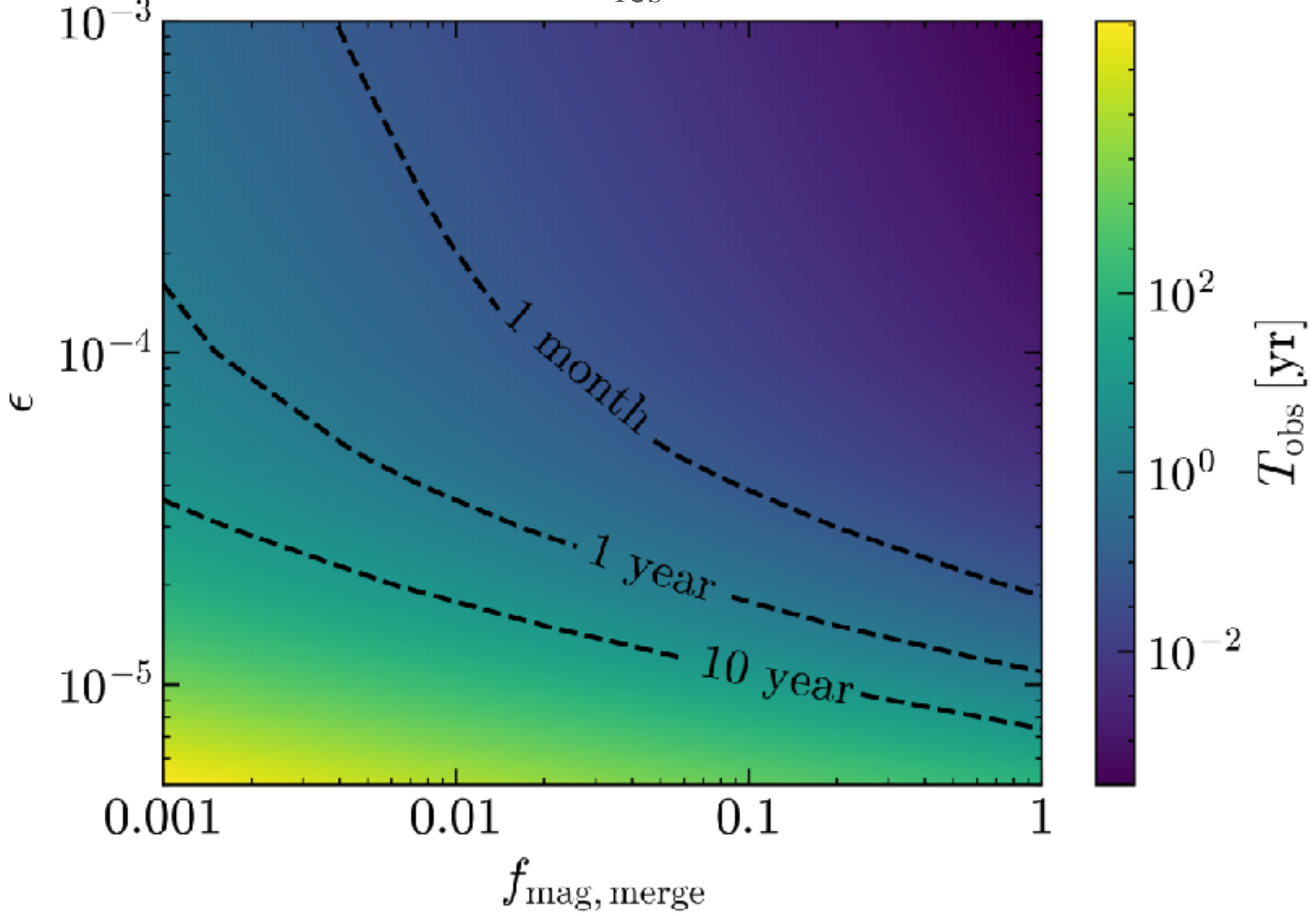
$$\dot{N} = \int \frac{dV_c(z)}{dz}\frac{R(z)}{1+z}dz$$
$$= 400 f_{\rm mag}\left(\frac{\epsilon}{10^{-5}}\right)^{1/2}\left(\frac{R_{\rm NSBH}}{20\,{\rm Gpc^{-3}\,yr^{-1}}}\right){\rm yr^{-1}}$$
$$R(z) = R_{\rm lock}\times(1+z)^{3.2}e^{-z^2/3}$$

- The resolvable merger events

$$\dot{N}_{\rm res} = \int \frac{dV_c(z)}{dz}\frac{R(z)}{1+z}f_{\rm res}(z)dz\,,$$

$$T_{\rm obs} = \frac{1}{\dot{N}_{\rm res}}$$

ET+CE



Summary

- We present a novel resonance mechanism that can naturally occur in neutron star binaries: a spin-rbit resonance.

- Observations by next-generation detectors such as ET and CE, or even the upcoming upgrade of Advanced LIGO, have the potential to detect such locking signals, enabling **precise measurements of both neutron star's ellipticity and moment of inertia.**

- Future searches should be performed to discover this resonance, or, if undetected, to place constraints on the magnetar fraction in neutron star binaries.

Thank you!

Eccentric orbit

$$\ddot{\ell} = \dot{n}\,,$$

$$\ddot{\theta} - \frac{3}{2}\epsilon n^2 \frac{q}{1+q}\frac{a^3}{r^3}\sin\left[2(f-\theta)\right] = 0\,,$$

$$\ddot{\gamma} + \mathcal{B}\sin\gamma + \mathcal{C} = 0\,,$$

$$\mathcal{B} = 3\epsilon n^2 \frac{q}{1+q} H(p,e)\,,$$

$$\mathcal{C} = 2p\dot{n}\,.$$

$$H(1,e) = 1 - \frac{5}{2}e^2 + \frac{13}{16}e^4\,,$$

$$H(\frac{3}{2},e) = \frac{7}{2}e - \frac{123}{116}e^3\,,$$

$$H(2,e) = \frac{17}{2}e^2 - \frac{115}{6}e^4\,,$$

$$H(\frac{5}{2},e) = \frac{845}{48}e^3\,.$$

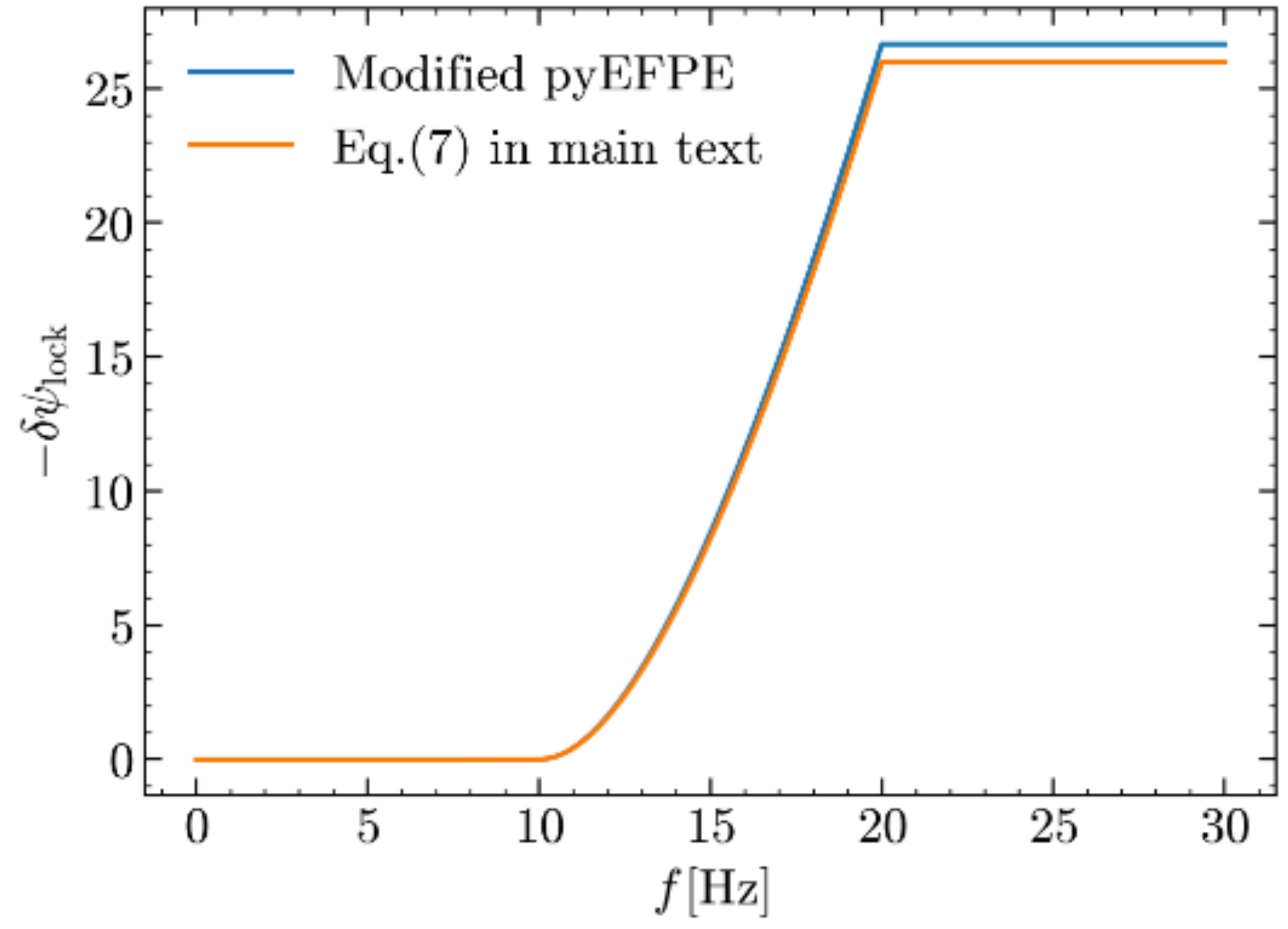
$$\mathcal{P}(p,e) = \frac{2}{1+\frac{\pi}{2}\frac{\mathcal{B}^{1/2}\mathcal{C}}{\dot{\mathcal{B}}}} \approx \frac{4}{\pi}\frac{\dot{\mathcal{B}}}{\mathcal{B}^{1/2}\mathcal{C}}$$

$$= \frac{4\sqrt{3}}{\pi p}\left(\frac{q}{1+q}\right)^{1/2}\epsilon^{1/2}\left(\sqrt{H(p,e)} + \frac{\dot{H}(p,e)}{2\sqrt{H(p,e)}}\frac{n}{\dot{n}}\right)\,.$$

Eccentric orbit: waveform correction

$$\mathcal{D}y = \nu y^9 \left(a_0 + \sum_{n=2}^{6} a_n y^n \right) ,$$

$$\mathcal{D}e^2 = -\nu y^8 \left(b_0 + \sum_{n=2}^{6} b_n y^n \right) ,$$



$$\mathcal{D}y = \mathcal{D}^0 y + \mathcal{D}^1 y ,$$

$$\mathcal{D}e^2 = \mathcal{D}^0 e^2 + \mathcal{D}^1 e^2 ,$$

$$D^1 y = \nu y^{13} c_0 \frac{3(1-e^2)^{1/2} I}{\mu M_t^2 - 3(1-e^2)^2 I y^4} ,$$

$$D^1 e^2 = -\nu y^{12} c_0 \frac{6(1-e^2)(1-e^2-\sqrt{1-e^2}) I}{\mu M_t^2 - 3(1-e^2)^2 I y^4} ,$$

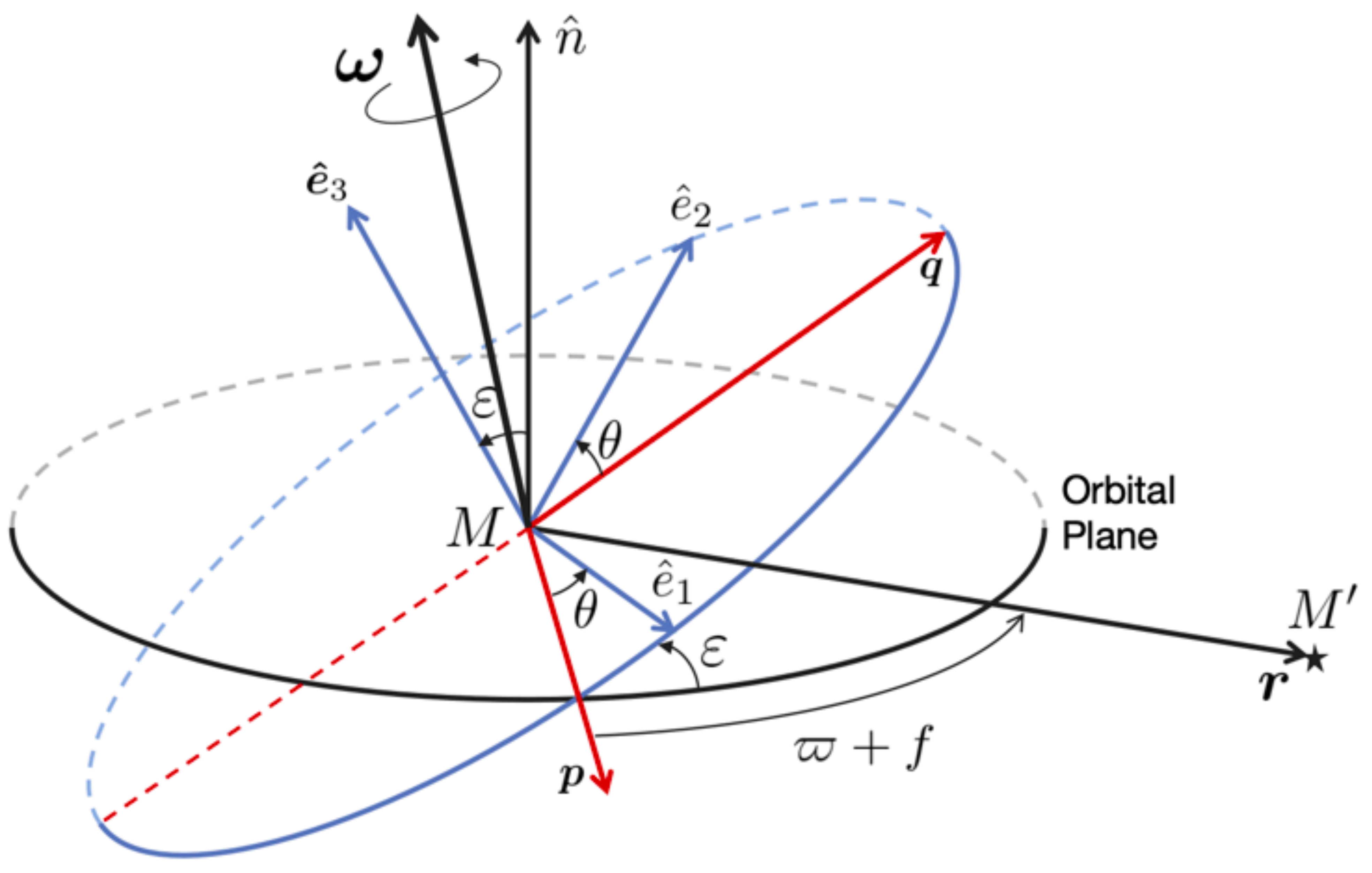
where

$$c_0 = a_0(1-e^2) + \frac{b_0}{2} .$$

General case

$$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = N_1\,,$$
$$I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 = N_2\,,$$
$$I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 = N_3\,,$$

$$N_1 = \frac{3GM'}{r^3}(I_3 - I_2)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_2)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_3)\,,$$
$$N_2 = \frac{3GM'}{r^3}(I_1 - I_3)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_3)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_1)\,,$$
$$N_3 = \frac{3GM'}{r^3}(I_2 - I_1)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_1)(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{e}}_2)\,.$$



$$\begin{aligned}\langle N_3\rangle = -\frac{3GM'}{8a^3}(I_2 - I_1)\Big[&(x+1)^2 H(p,e)\sin 2(\theta - p\ell - \varpi)\\ &+ (x-1)^2 H(-p,e)\sin 2(\theta - p\ell + \varpi)\\ &+ 2(1-x^2)G(p,e)\sin 2(\theta - p\ell)\Big]\,,\end{aligned}$$

$$\dot{\omega}_3 - \epsilon\omega_1\omega_2 = -\frac{\mathcal{B}_x}{2}\sin 2(\theta - p\ell - \varpi_x)\,, \qquad \text{(S59)}$$

where

$$\left(\frac{\mathcal{B}_x}{\mathcal{B}}\right)^2 = 4\big[(g + h^+ + h^-)^2 - 4g(h^+ + h^-)\sin^2\varpi - 4h^+h^-\sin^2 2\varpi\big]\,, \qquad \text{(S60)}$$

$$\tan 2\varpi_x = \frac{(h^+ - h^-)\sin 2\varpi}{g + (h^+ + h^-)\cos 2\varpi}\,, \qquad \text{(S61)}$$

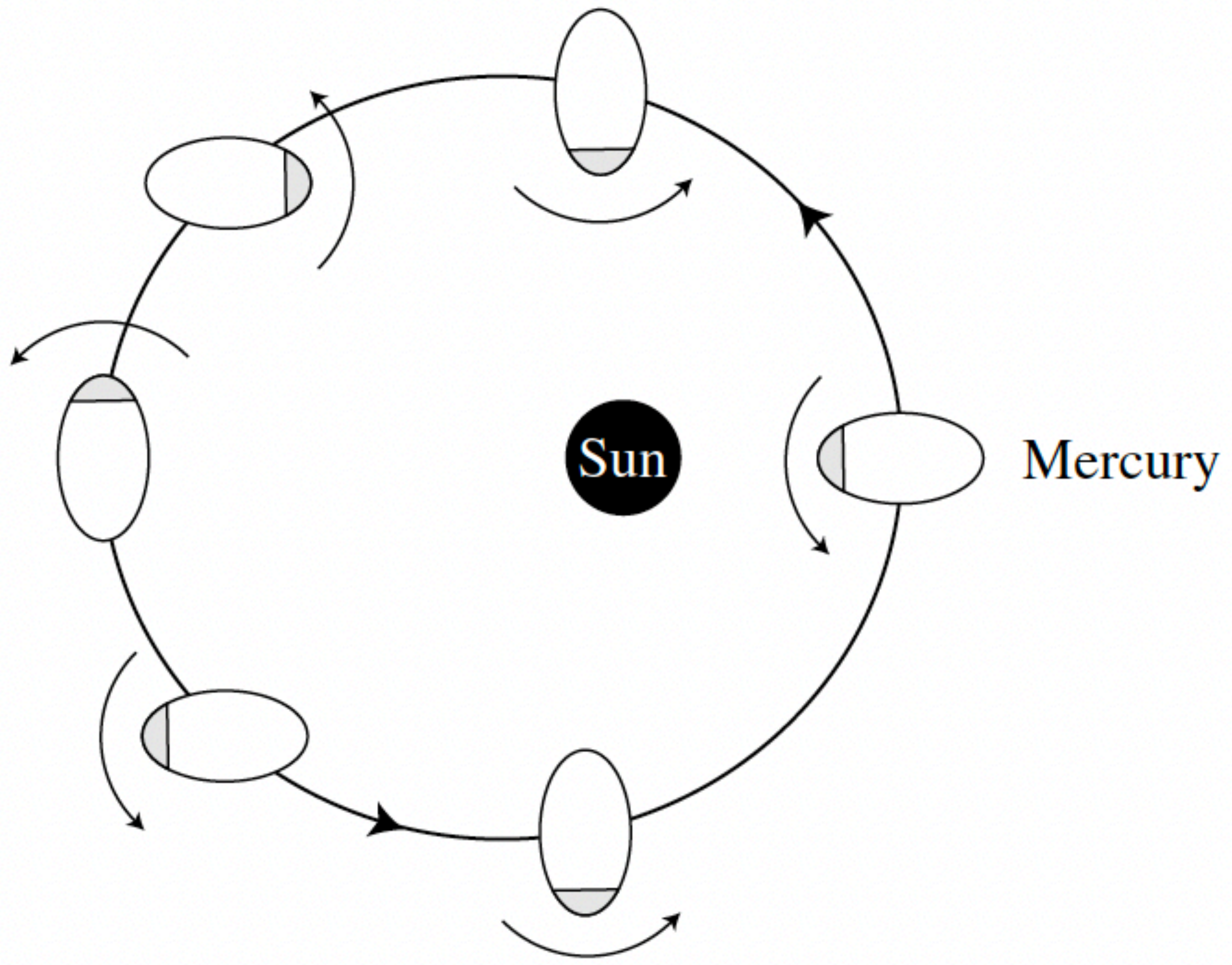
$$\ddot{\gamma} + \mathcal{B}_x \sin\gamma = -2p\dot{n} + 2\epsilon\omega_1\omega_2\,,$$

3:2 spin-orbit resonance of Mercury

- Mercury rotates three times on its axis for every two orbits around the sun.

- First explained by Goldreich & Peale (1966), require a eccentric orbit and a finite ellipticity

- For circular orbit, there is only 1:1 resonance, for eccentric orbit, there could be 3:2, 2:1, 5:2…



Lidov-Kozai effect

$$\sqrt{1 - e_{\rm in}^2} \cos i = \text{constant}$$