(Quasi)Universal Relations for *f*-mode oscillations in Compact stars

Ishfaq Ahmad Rather

ITP, Goethe University

Compact Stars in the QCD Phase Diagram

CSQCD 2026, Barcelona

18 - 23 May 2026

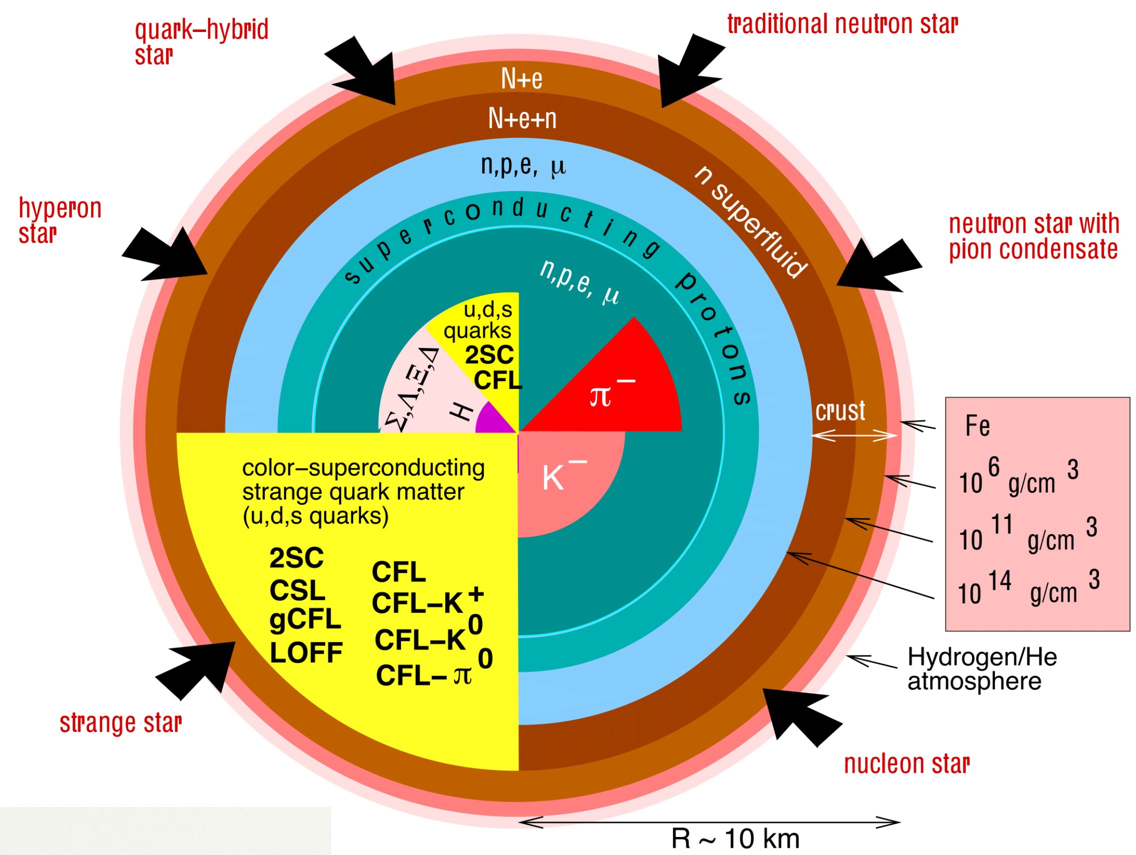
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC–TR 211 Strong-interaction matter under extreme conditions

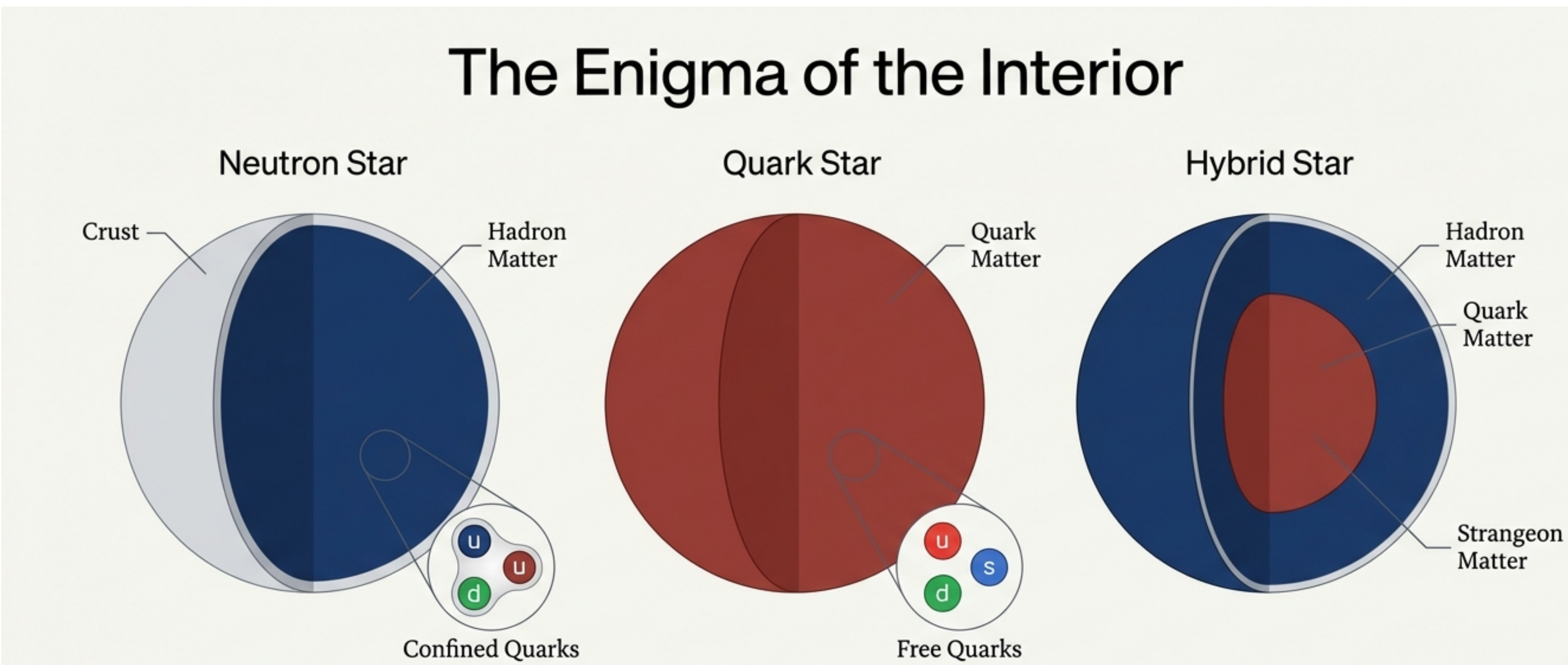
Alexander von HUMBOLDT STIFTUNG

Neutron Star

- Formed in: Type II, Ib or Ic SN (remnant of $M \approx 8 - 30 M_{\odot}$)
- **Mass:** $M \approx 2\ M_{\odot}$ and more
- **Radius:** $R \approx 10 - 13 km$ **(NICER)**
- **Density:** $\rho \approx 10^{14} - 10^{15} g/cm^3$

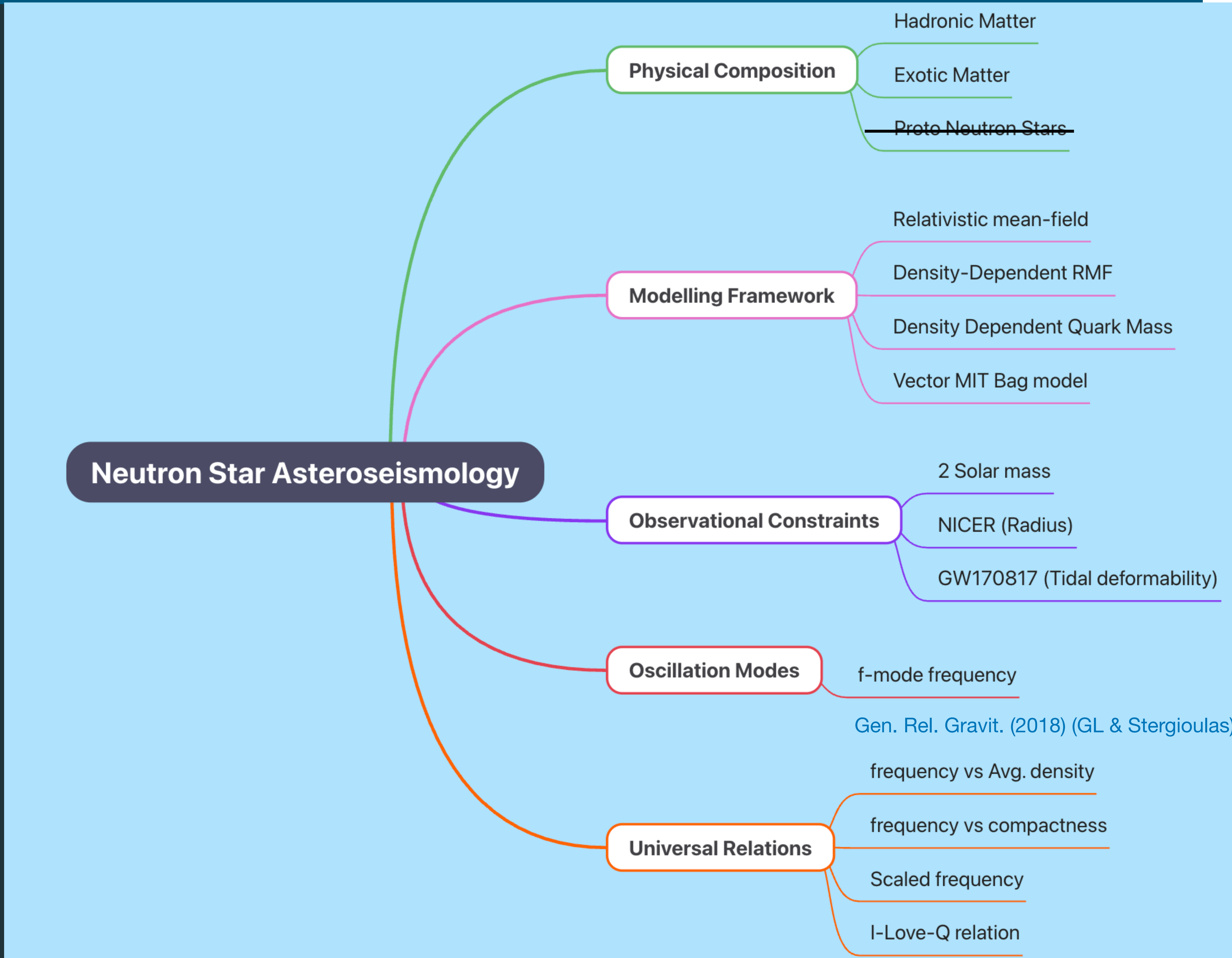


Weber 1999



Compact Stars N. K. Glendenning
Compact Star Physics Jürgen Schaffner-Bielich

Plan



Neutron Star Asteroseismology
Physical Composition
Hadronic Matter
Exotic Matter
~~Proto Neutron Stars~~
Modelling Framework
Relativistic mean-field
Density-Dependent RMF
Density Dependent Quark Mass
Vector MIT Bag model
Observational Constraints
2 Solar mass
NICER (Radius)
GW170817 (Tidal deformability)
Oscillation Modes
f-mode frequency
Gen. Rel. Gravit. (2018) (GL & Stergioulas)
Universal Relations
frequency vs Avg. density
frequency vs compactness
Scaled frequency
I-Love-Q relation

Universality in Neutron Stars?

- Well known that many physical quantities of NSs depend sensitively on EoS (Good for constraining EoS)
- Exist various approximately EoS-insensitive relations connecting different quantities of NSs————-**Universal Relations** (Definition? EoS-insensitive to ~O(1%) level?)

Measuring one quantity → Inferring others from relation

Measuring both together → (deviation from UR) → modified gravity or exotic matter (quark core, phase transition)?

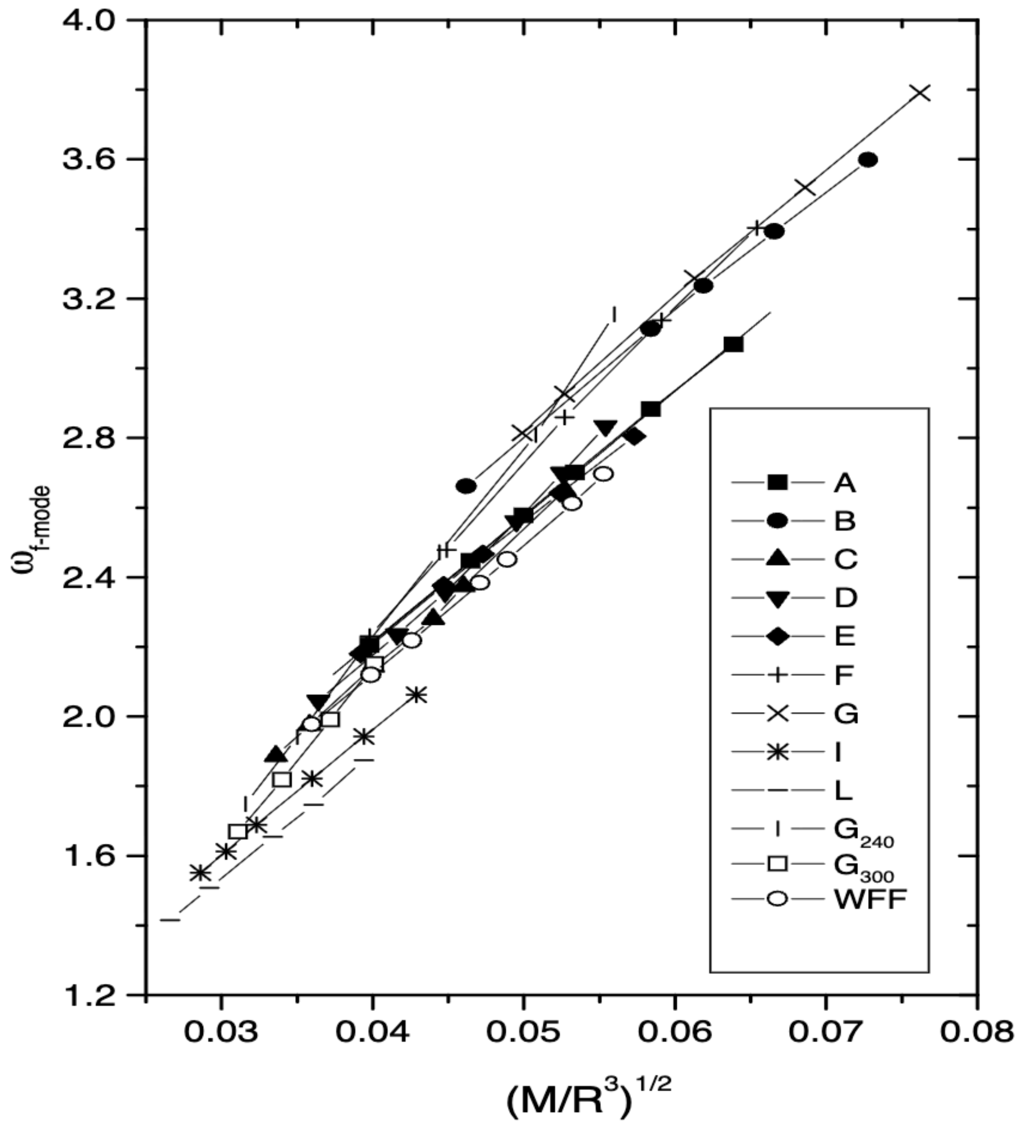
➡**Extract stellar properties from GW data.**

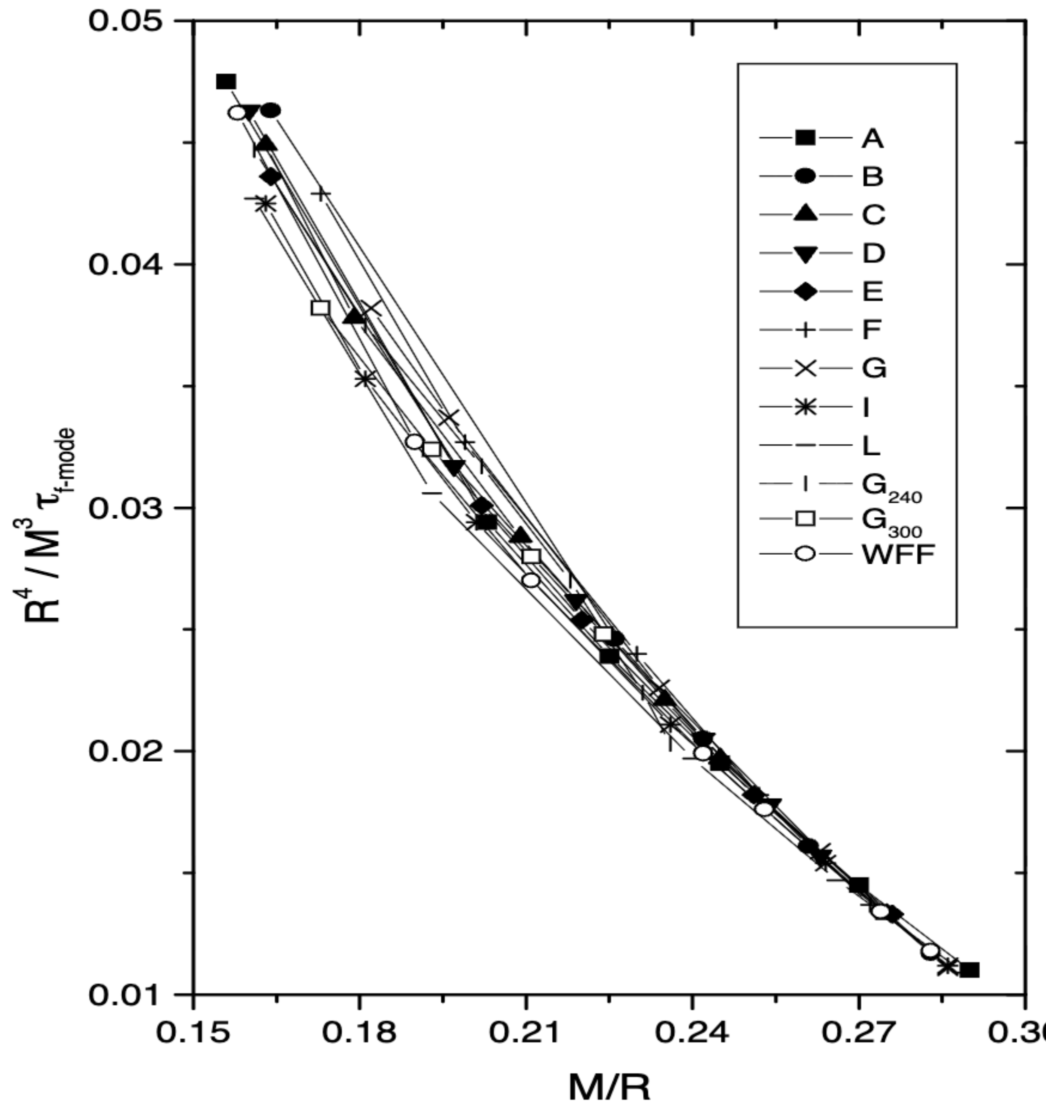
f-mode universal relations

Empirical relations for the f-mode oscillation frequency and damping time.

[Andersson & Kokkotas(1998)]

$$f\,(\mathrm{kHz}) \approx 0.78 + 1.635\left(\frac{M_{1.4}}{R_{10}^3}\right)^{1/2} \qquad \frac{1}{\tau}(\mathrm{s}) \approx \frac{M_{1.4}^3}{R_{10}^4}\left[22.85 - 14.65\left(\frac{M_{1.4}}{R_{10}}\right)\right]$$





- 12 hadronic EoSs, **Cowling approximation** **Not all satisfying necessary constraints**

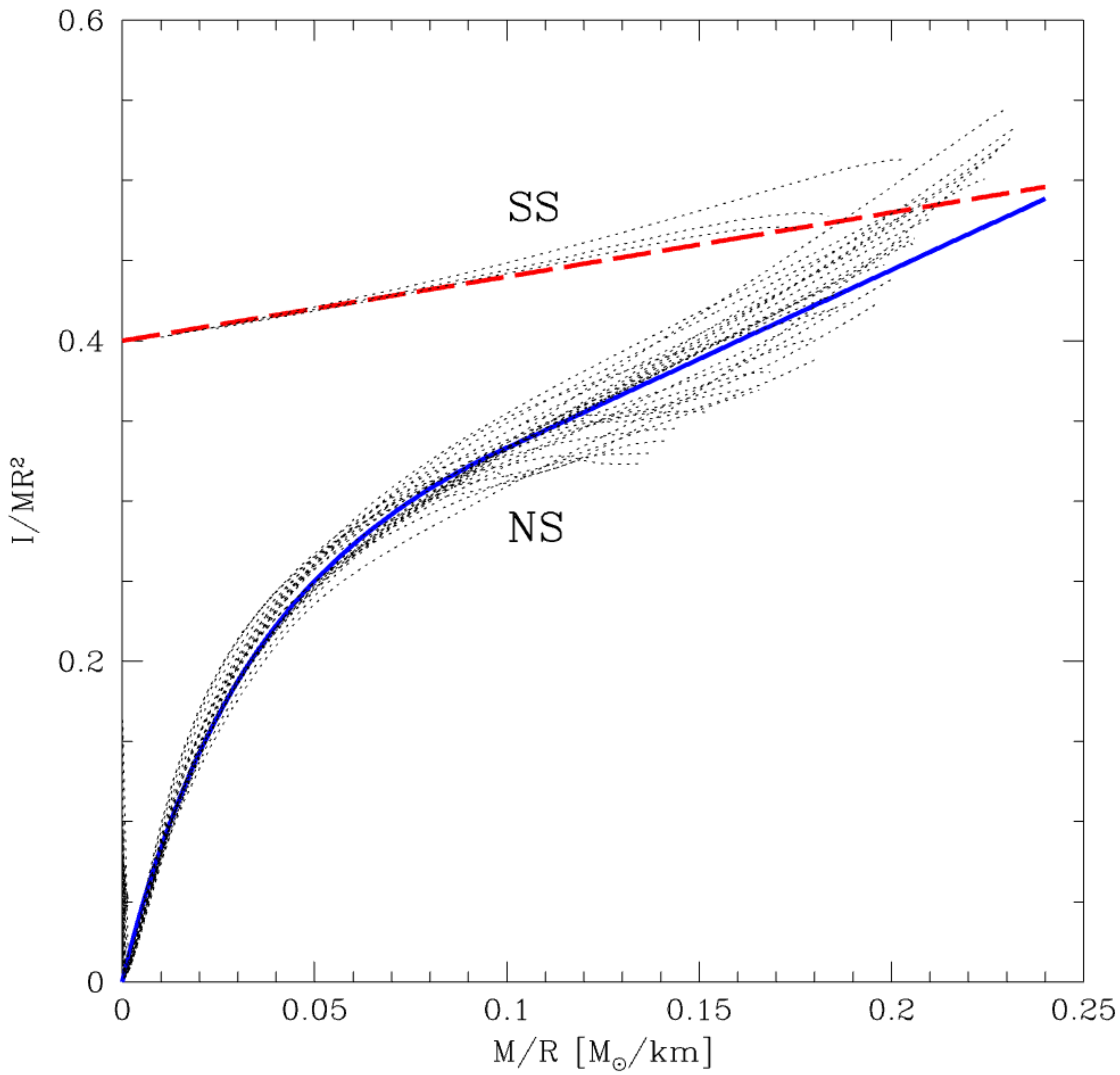
I-MR and I-M-η URs

I-MR and I-M-η URs

Empirical relations between NS's moment of inertia (I) and compactness (M/R).

[Bejger & Haensel (2002); Lattimer & Schutz (2005)]

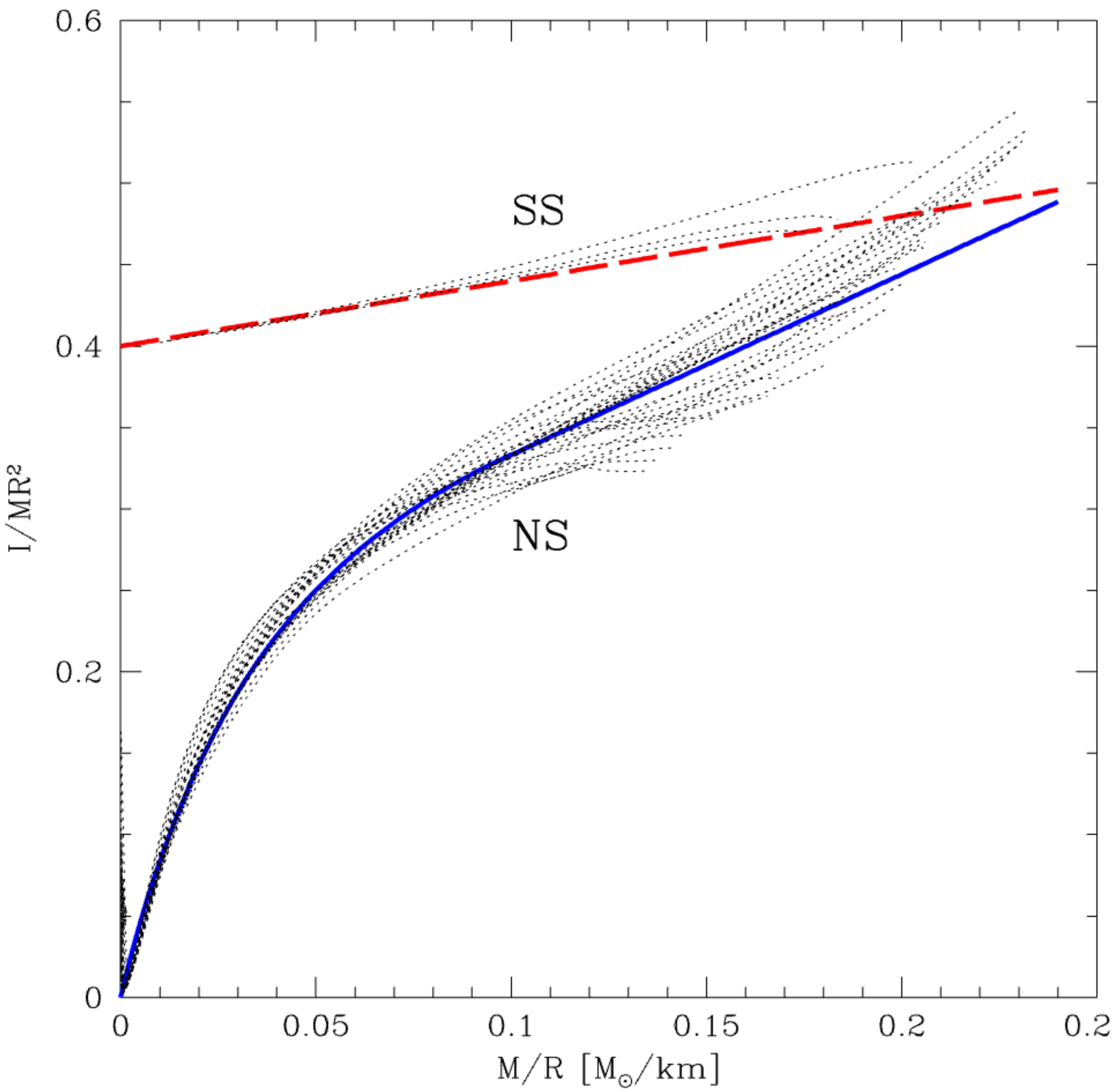
$$\frac{I}{MR^2} = \begin{cases} x_{GR}/(0.295 + 2x_{GR}), & \text{for} x_{GR} \leq 0.3, \\ 2/9(1 + 1.69x_{GR}), & \text{for} x_{GR} \geq 0.3. \end{cases}$$



I-MR and I-M-η URs

Empirical relations between NS's moment of inertia (I) and compactness (M/R).
[Bejger & Haensel (2002); Lattimer & Schutz (2005)]

$$\frac{I}{MR^2} = \begin{cases} x_{GR}/(0.295 + 2x_{GR}), & \text{for } x_{GR} \le 0.3, \\ 2/9(1 + 1.69x_{GR}), & \text{for } x_{GR} \ge 0.3. \end{cases}$$

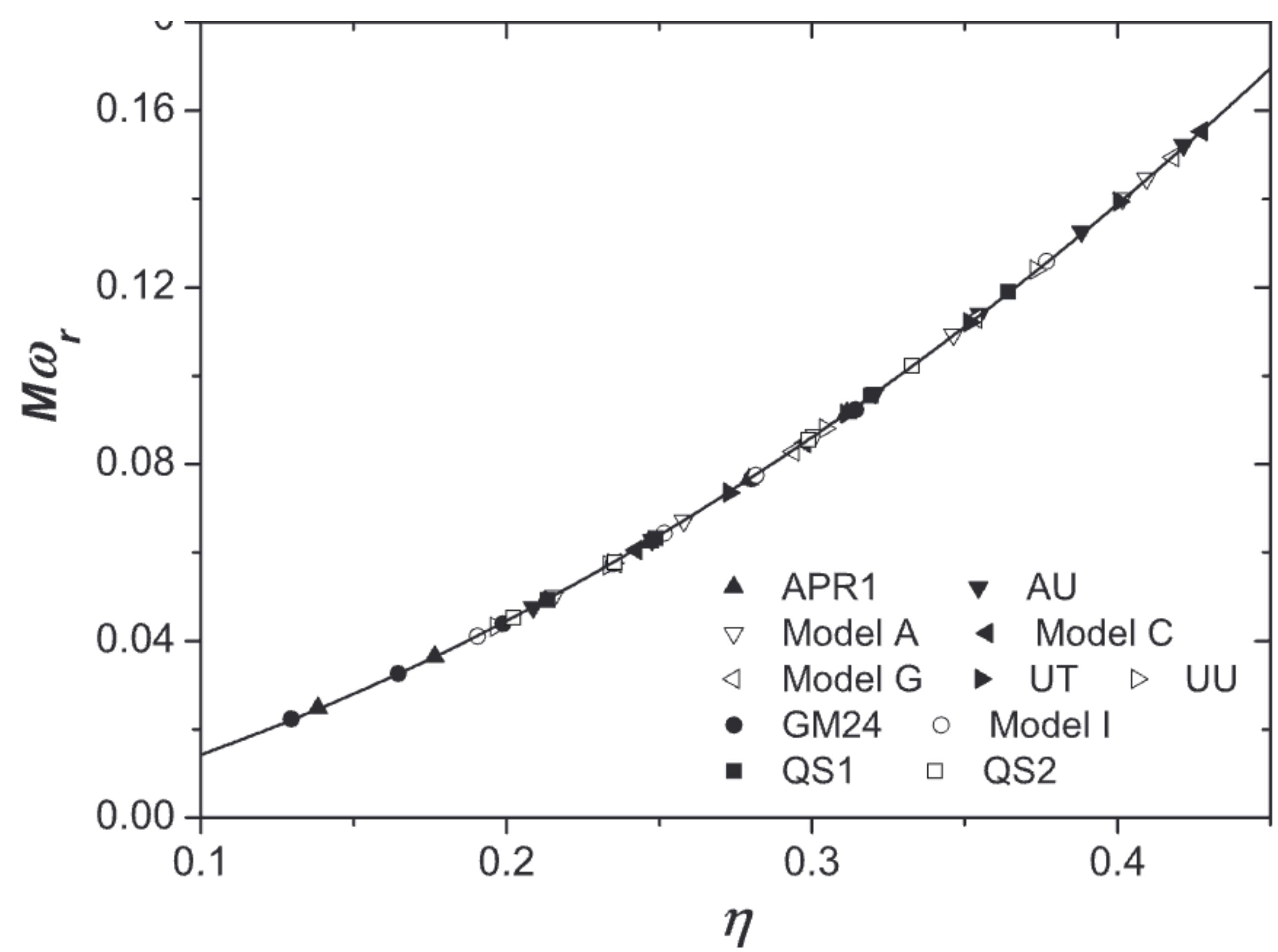


A more robust universal relation for the f-mode can be obtained by I:
I carries richer information about the mass distribution
[Lau, Leung and LML (2010)]

$$\eta \equiv \sqrt{\frac{M^3}{I}}$$

Scaled f-mode UR

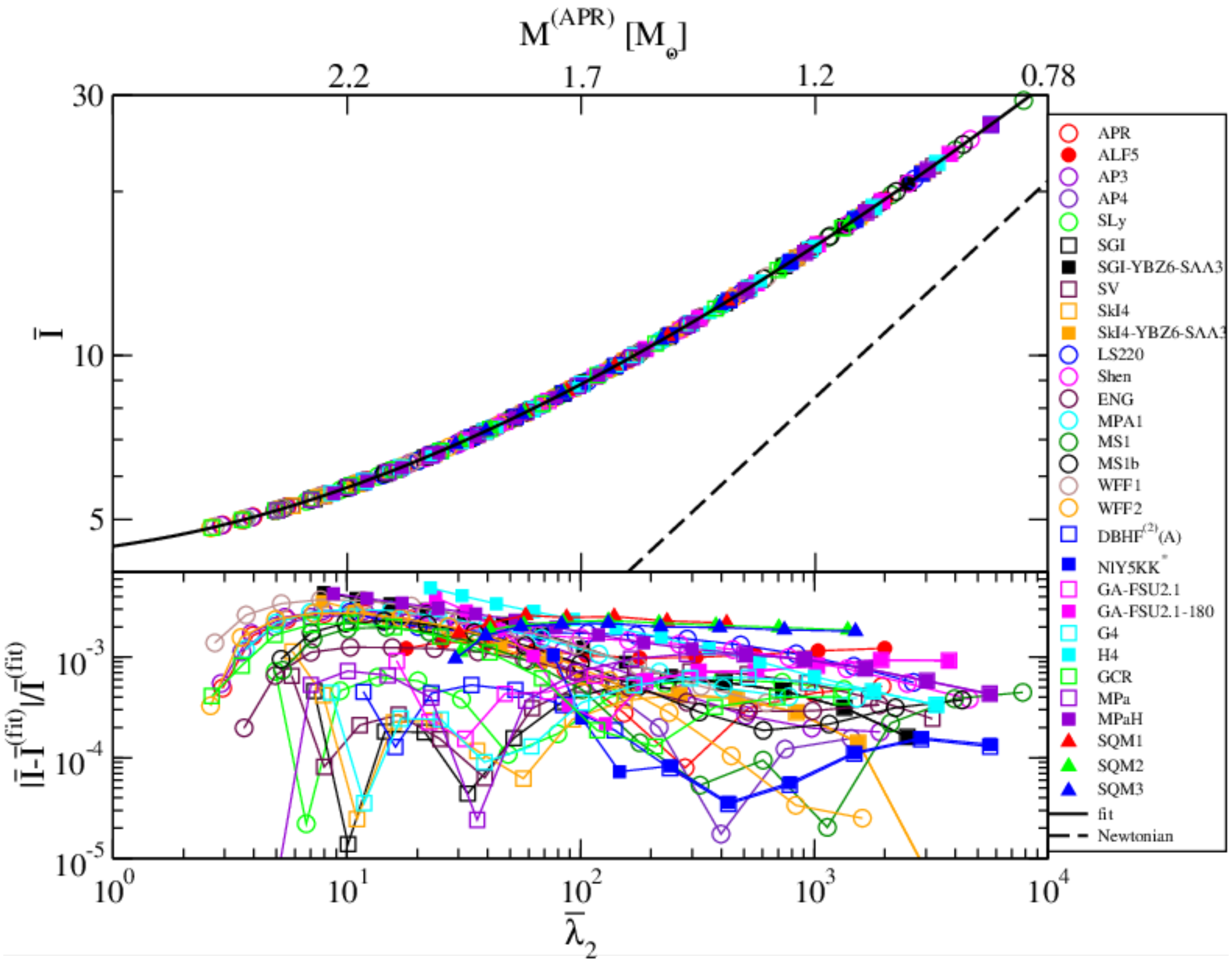
$$M\omega_r = -0.0047 + 0.133\eta + 0.575\eta^2$$

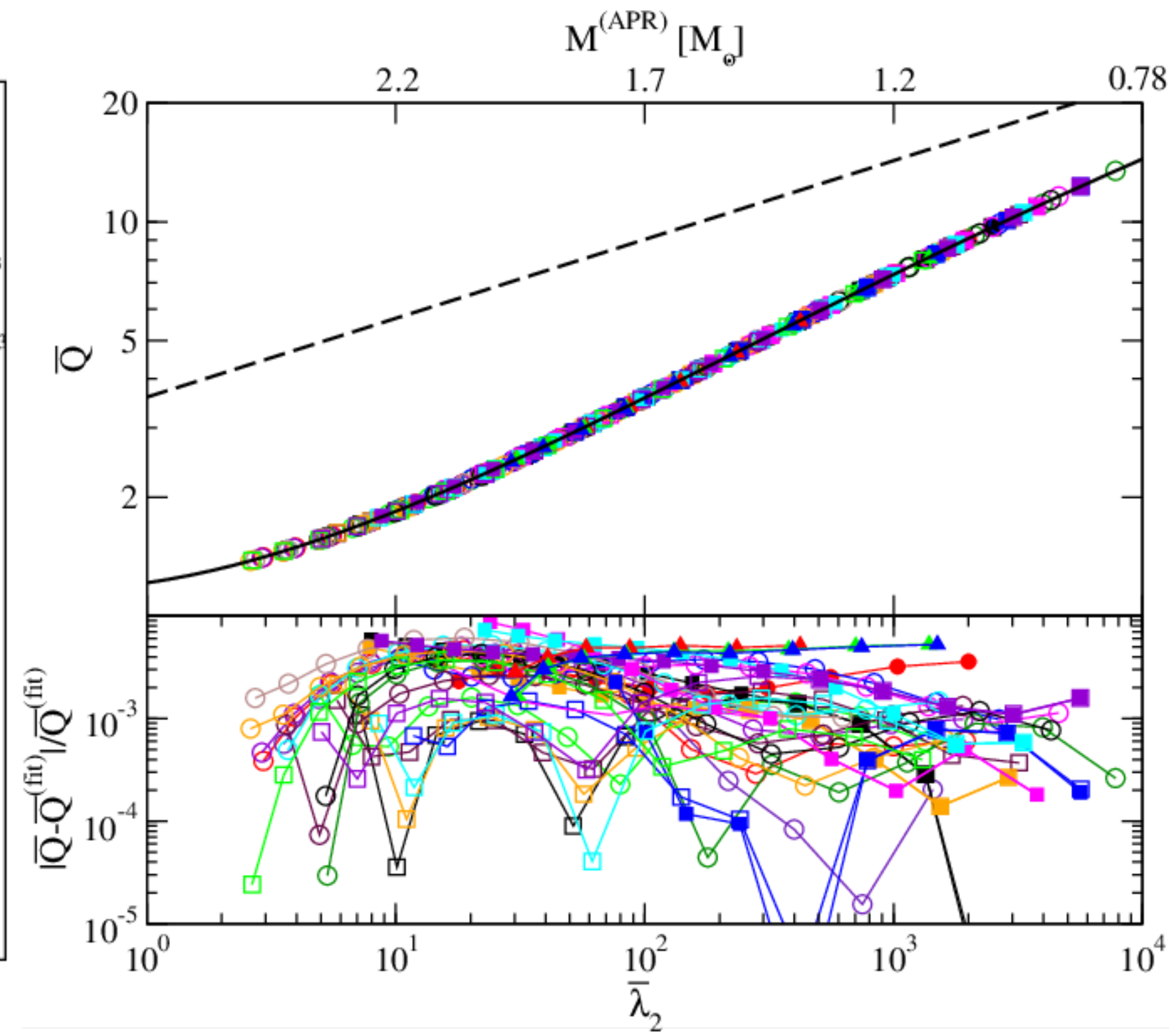


I-Love-Q URs

I-Love-Q relations [Yagi and Yunes 2013, 2017]

$$\bar{I} \equiv \frac{I}{M^3}, \bar{Q} \equiv \frac{Q}{M^3 j^2}, \lambda \equiv \frac{\lambda}{M^5}$$





The moment of inertia, quadrupole moment and Love number satisfy Universal, EoS-independent relations!

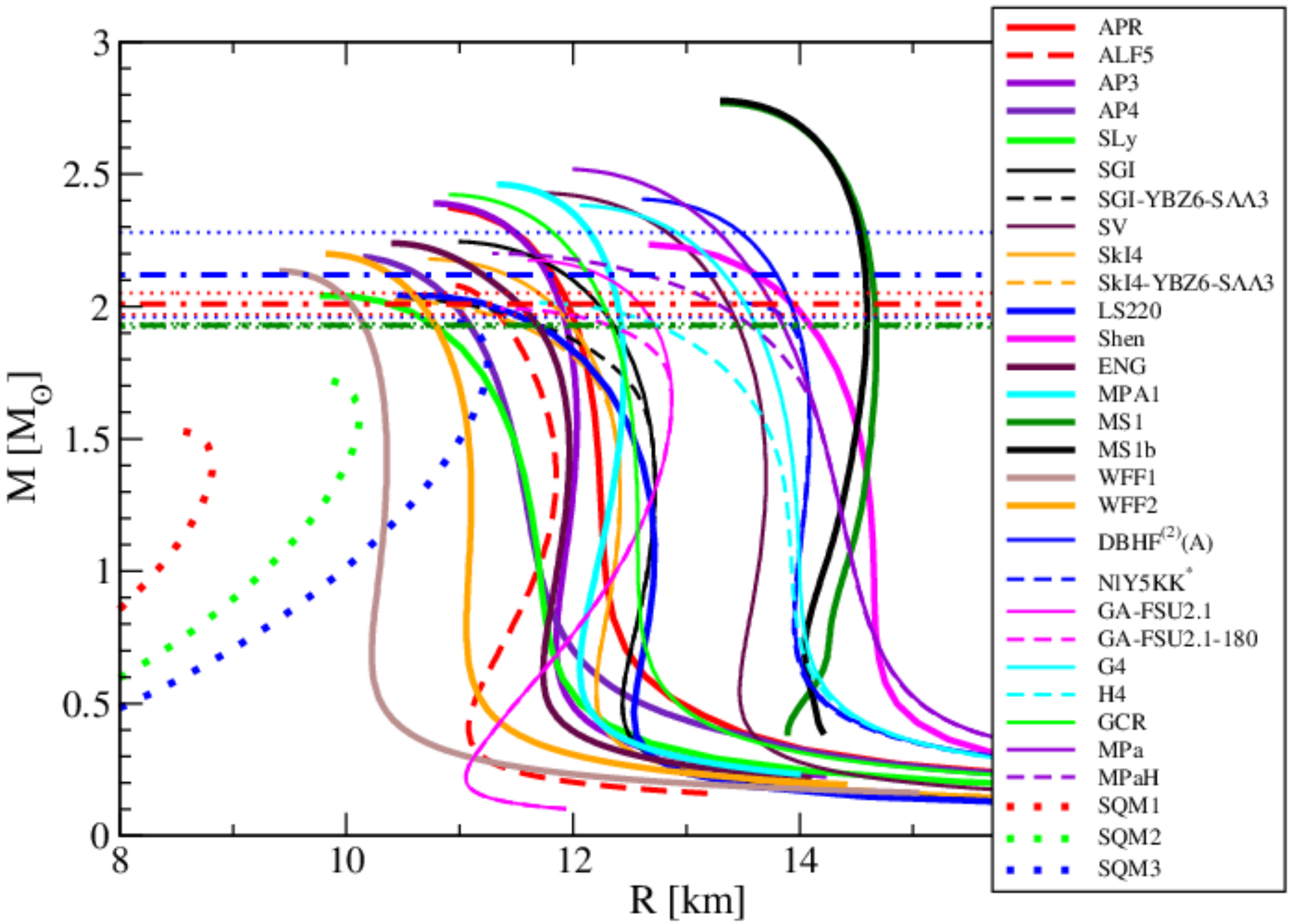
UR EoSs

I-Love-Q relations [Yagi and Yunes 2013] → **Few EoSs only**

Updated in 2017

Phys.Rept. 681 (2017) 1-72

30 EoSs including exotic matter



I-Love-Q relations [Yagi and Yunes 2013] → **Few EoSs only**

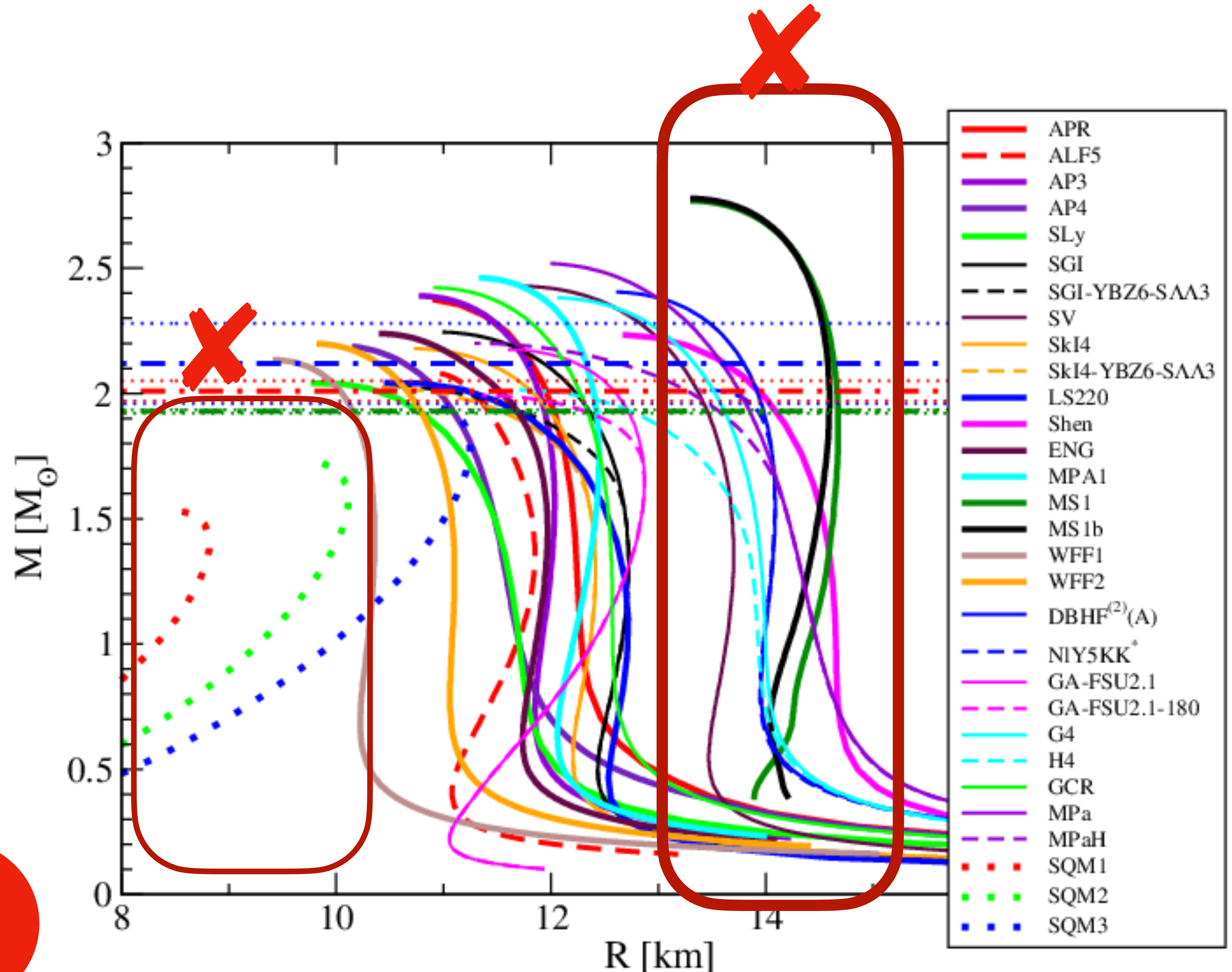
Updated in 2017

Phys.Rept. 681 (2017) 1-72

30 EoSs including exotic matter

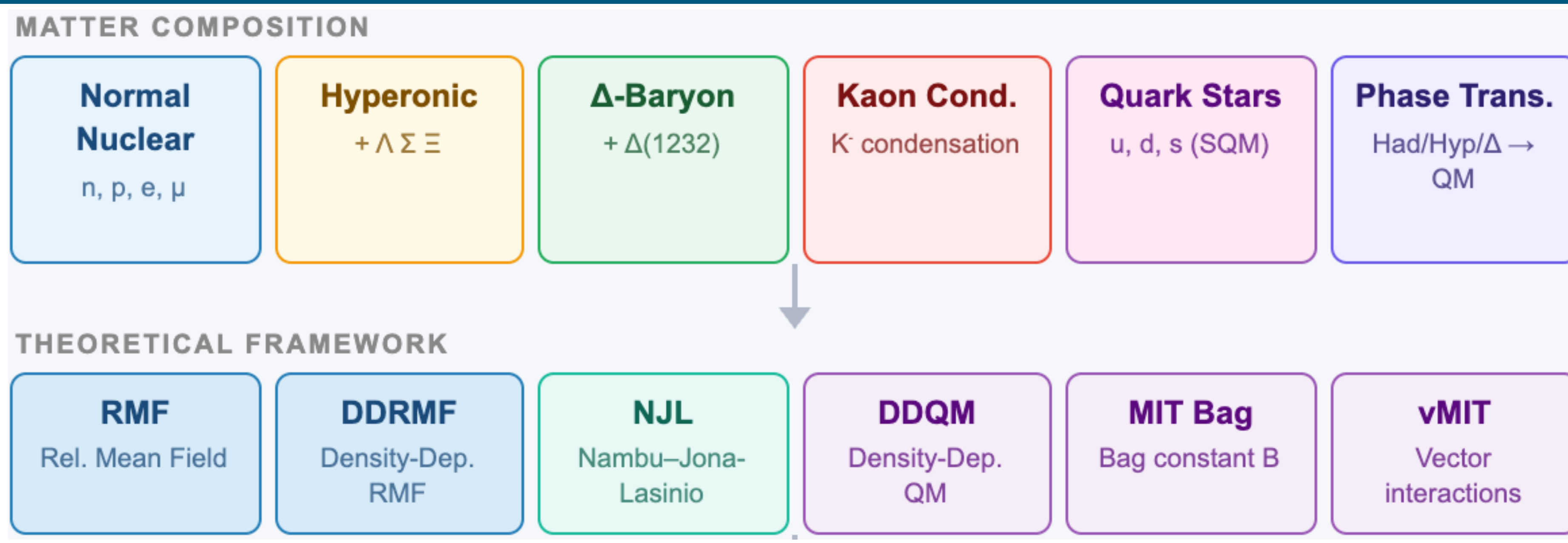
Not all satisfying Astro constraints

Not all within the nuclear parameters range at n_0



Pipeline

Pipeline


MATTER COMPOSITION
Normal Nuclear
n, p, e, μ
Hyperonic
+ Λ Σ Ξ
Δ-Baryon
+ Δ(1232)
Kaon Cond.
K⁻ condensation
Quark Stars
u, d, s (SQM)
Phase Trans.
Had/Hyp/Δ → QM
THEORETICAL FRAMEWORK
RMF
Rel. Mean Field
DDRMF
Density-Dep. RMF
NJL
Nambu–Jona-Lasinio
DDQM
Density-Dep. QM
MIT Bag
Bag constant B
vMIT
Vector interactions

Pipeline

CRC–TR 211

MATTER COMPOSITION

Normal Nuclear	Hyperonic	Δ-Baryon	Kaon Cond.	Quark Stars	Phase Trans.
n, p, e, μ	+ Λ Σ Ξ	+ Δ(1232)	K^- condensation	u, d, s (SQM)	Had/Hyp/Δ → QM

THEORETICAL FRAMEWORK

RMF	DDRMF	NJL	DDQM	MIT Bag	vMIT
Rel. Mean Field	Density-Dep. RMF	Nambu–Jona-Lasinio	Density-Dep. QM	Bag constant B	Vector interactions

NUCLEAR MATTER CONSTRAINTS

Symmetry energy *J*	**30 – 32 MeV**	Satisfied ✓	Outside ✗
Slope param. *L*	**40 – 60 MeV**	Satisfied ✓	Outside ✗
Effective mass m^*/M	**0.55 – 0.75**	Satisfied ✓	Outside ✗
Incompressibility K_0	**240 MeV**	Satisfied ✓	Outside ✗

Pipeline

CRC-TR 211

MATTER COMPOSITION

Normal Nuclear	Hyperonic	Δ-Baryon	Kaon Cond.	Quark Stars	Phase Trans.
n, p, e, μ	+ Λ Σ Ξ	+ Δ(1232)	K^- condensation	u, d, s (SQM)	Had/Hyp/Δ → QM

THEORETICAL FRAMEWORK

RMF	DDRMF	NJL	DDQM	MIT Bag	vMIT
Rel. Mean Field	Density-Dep.	Nambu–Jona-	Density-Dep.	Bag constant B	Vector

OBSERVATIONAL CONSTRAINTS

Max. mass M_{max}	≥ 2.0 M⊙ (PSR J0740+6620)	Supported	✓	< 2.0 M⊙	✗
Radius $R_{1.4}$	NICER bands (J0030, J0437, J0614, J0740)	Within 90% CI	✓	Excluded	✗
Tidal def. $\Lambda_{1.4}$	≤ 580 (GW170817)	Satisfied	✓	Too stiff	✗
Causality & stability	$v_s \leq c$ throughout	Physical	✓	Violated	✗

Pipeline

CRC–TR 211

MATTER COMPOSITION

Normal Nuclear	Hyperonic	Δ-Baryon	Kaon Cond.	Quark Stars	Phase Trans.
n, p, e, μ	+ Λ Σ Ξ	+ Δ(1232)	K^- condensation	u, d, s (SQM)	Had/Hyp/Δ → QM

THEORETICAL FRAMEWORK

RMF	DDRMF	NJL	DDQM	MIT Bag	vMIT
Rel. Mean Field	Density-Dep.	Nambu–Jona-	Density-Dep.	Bag constant B	Vector

OBSERVATIONAL CONSTRAINTS

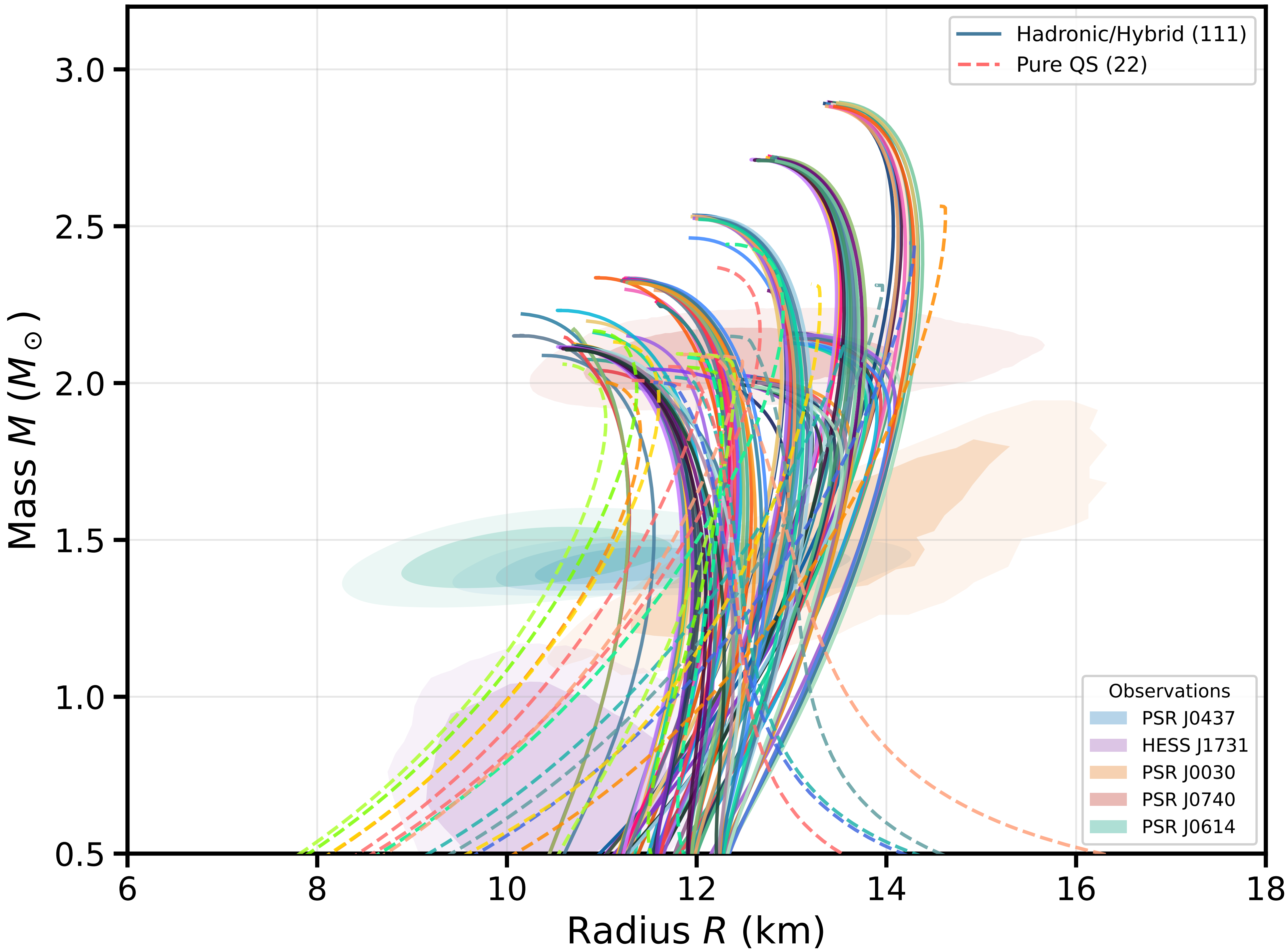
Max. mass M_{max}	≥ 2.0 M⊙ (PSR J0740+6620)	Supported	✓	< 2.0 M⊙	✗
Radius $R_{1.4}$	NICER bands (J0030, J0437, J0614, J0740)	Within 90% CI	✓	Excluded	✗
Tidal def. $\Lambda_{1.4}$	≤ 580 (GW170817)	Satisfied	✓	Too stiff	✗
Causality & stability	$v_s \leq c$ throughout	Physical	✓	Violated	✗

✅ **133 Observationally Vetted EoS Models**

111 hadronic/hybrid + 22 quark stars = 133 total

All satisfy: $M_{max} \geq 2.0\ M_\odot$ · NICER R bands · GW170817 $\Lambda_{1.4} \leq 580$ · Nuclear saturation parameters

Mass-Radius



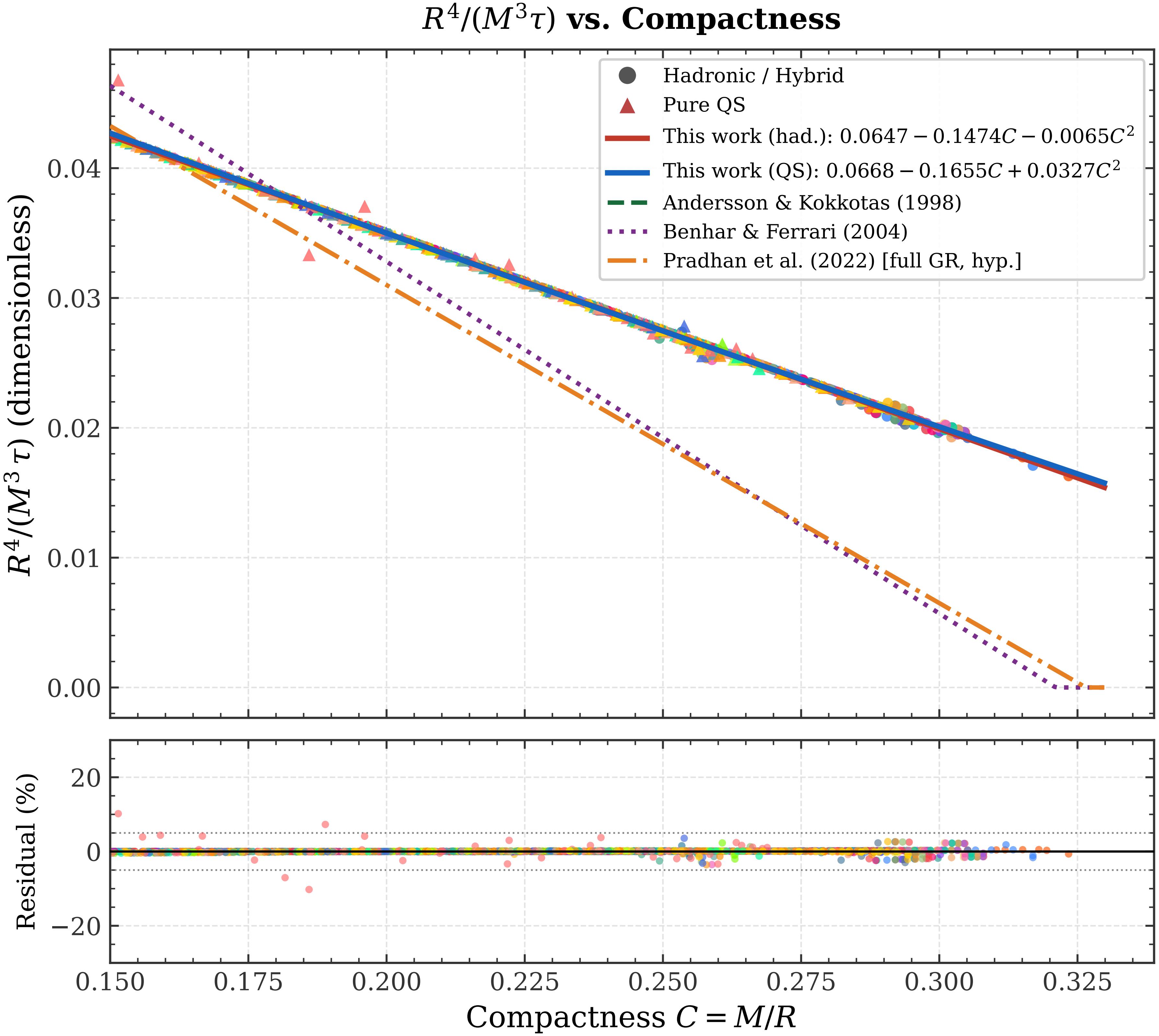
✔ **All EoS lie within the limits of m*, S, L, at n_0**

J. M. Lattimer - Particles, 2023

✔ **Different models with exotic phases**

✔ **All satisfying observational constraints: 2 $M_\odot$ + NICER + GW170817**

UR-1



Agreement with Andersson & Kokkotas (1998)

MNRAS 299 (1998) 1059-1068

Deviation from Behnar (2004) and Pradhan et al. (2022)

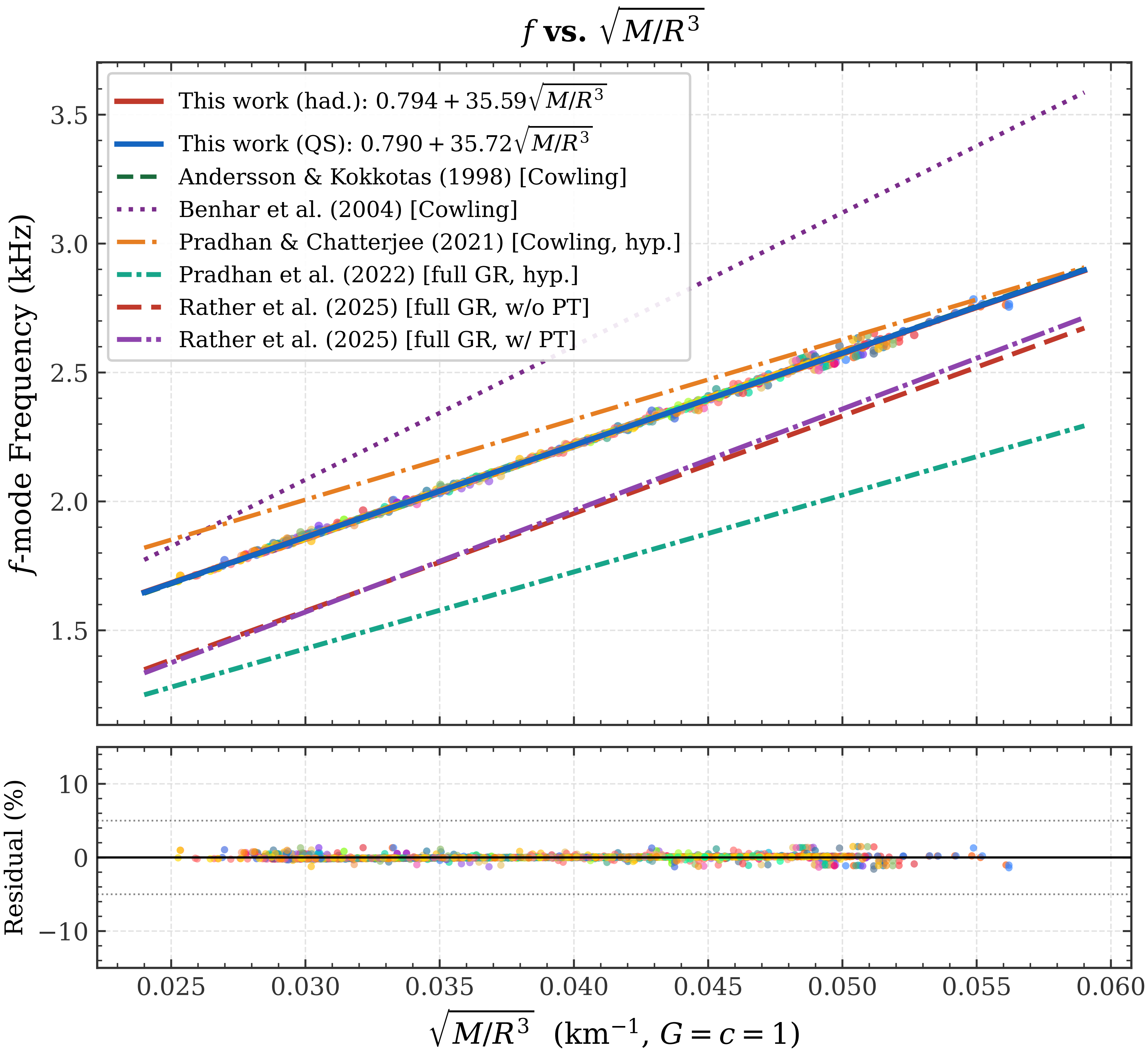
Phys.Rev.D 70 (2004) 124015

PR*C* 106 (2022) 1, 015805

Small EoS set and linear fit drops much faster,

***f*-mode damping time scaling remains robust**

CRC-TR 211



✓ The AK UR fits perfectly

MNRAS 299 (1998) 1059-1068

✗ Deviation from *Benhar et al. (2004), Pradhan et al. (2022), Rather et al. (2025)*

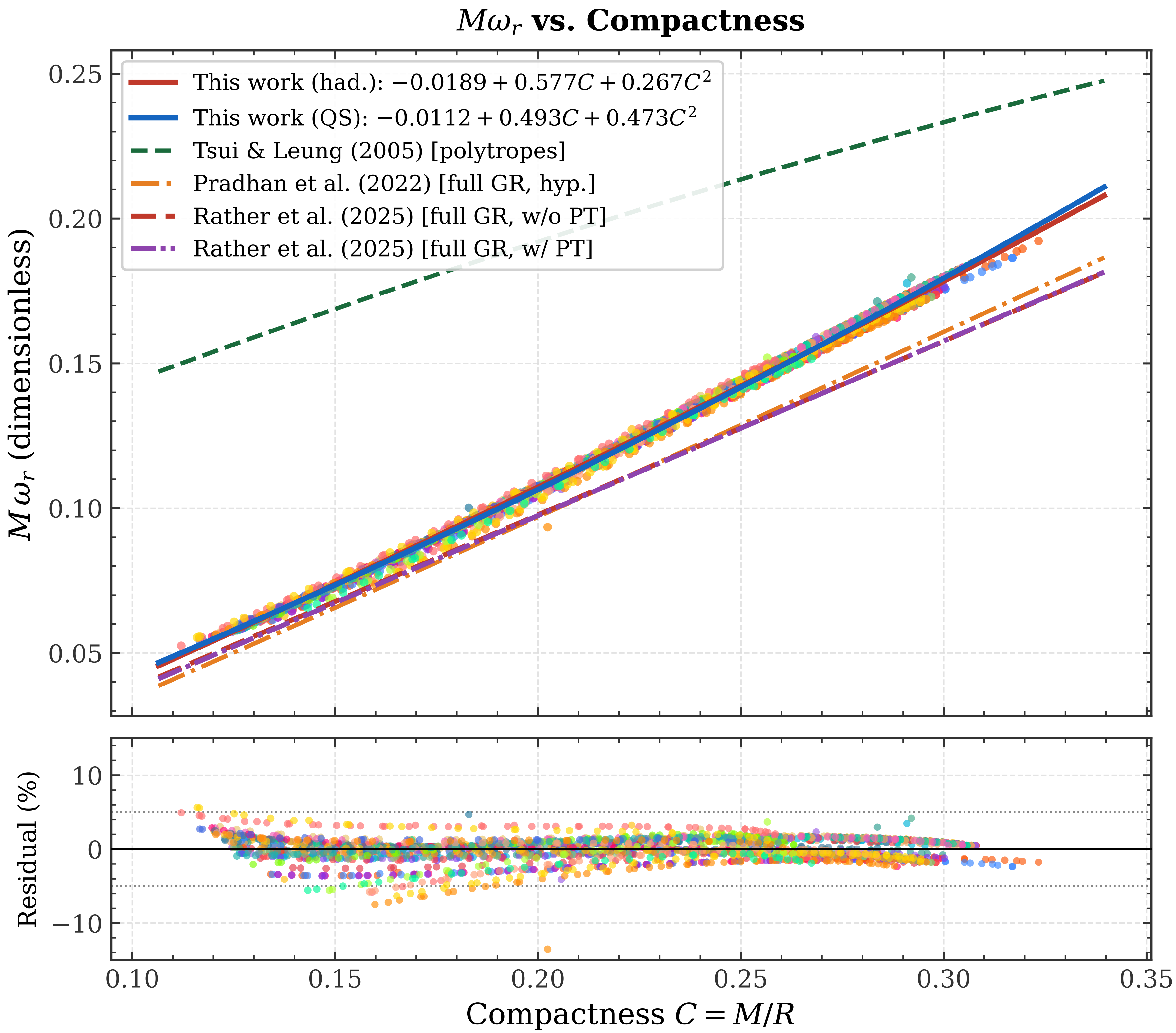
Phys.Rev.D 70 (2004) 124015

PR*C* 106 (2022) 1, 015805

PRD 112 (2025) 2, 023013

few selected EoSs

Cowling approximation overestimates f-mode frequency by 20–30% — all Cowling literature fits systematically high



✗ Deviation from Tsui et al. (2005)

MNRAS 357:1029–1037,2005

Polytropes are too stiff

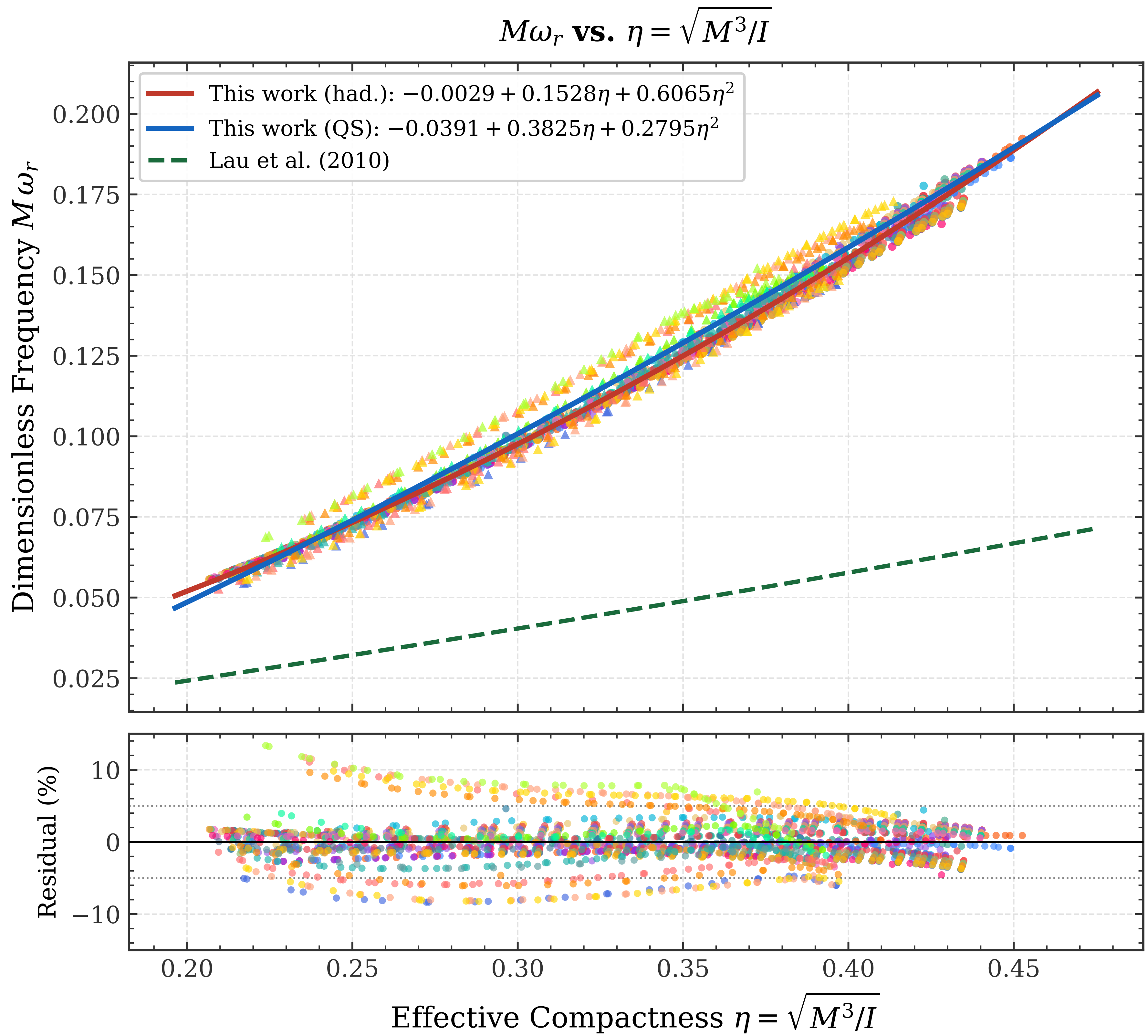
Deviation from Pradhan et al. (2022)

PRC 106 (2022) 1, 015805

5–10% offset

Deviation from Rather et al. (2025)

PRD 112 (2025) 2, 023013



Deviation from Lau et al. (2010)

ApJ, 714:1234–1238, 2010

Cowling approximation

with ≤10 EoSs

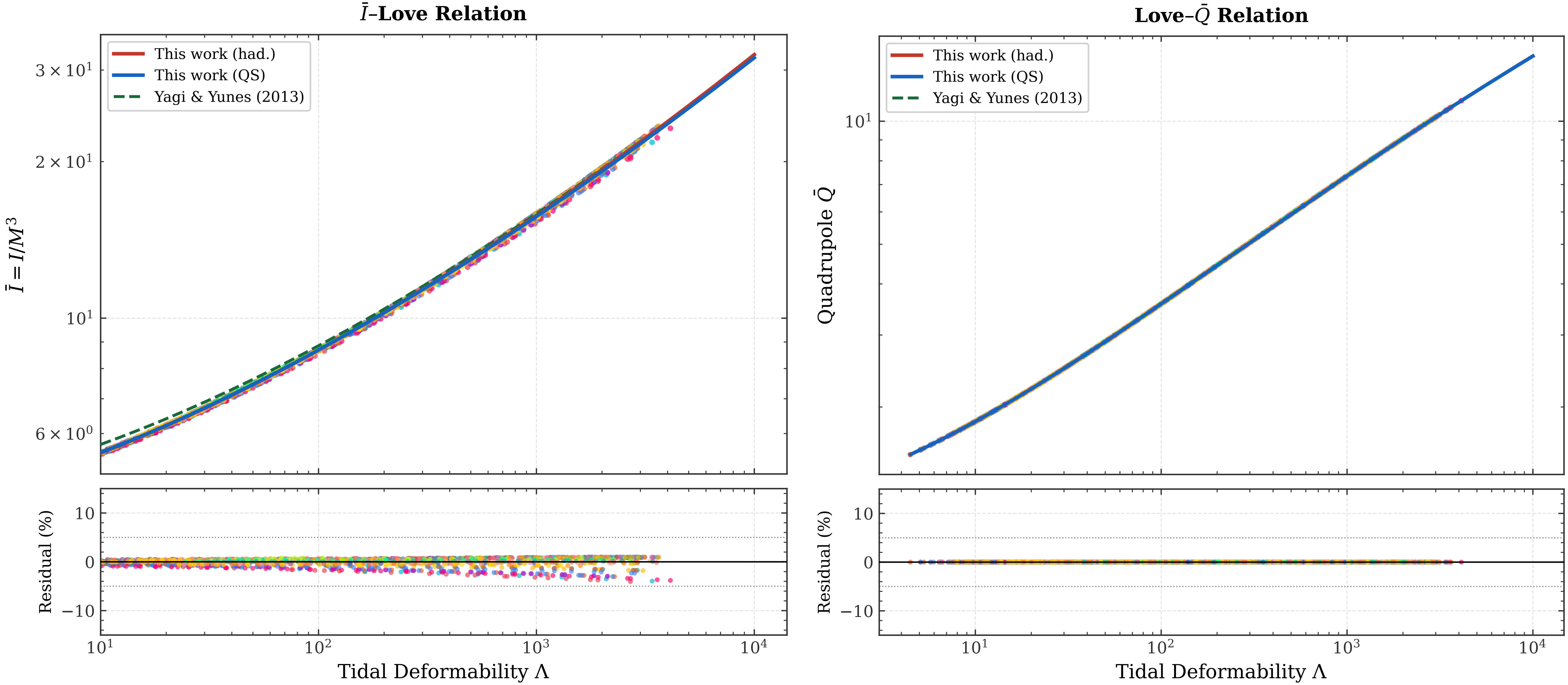
few EoSs. ruled out by astro constraints

~10% scatter, driven by QS outliers at high η

I-Love-Q

✔ **Stiffness of EoS plays a more fundamental role in Universality.**

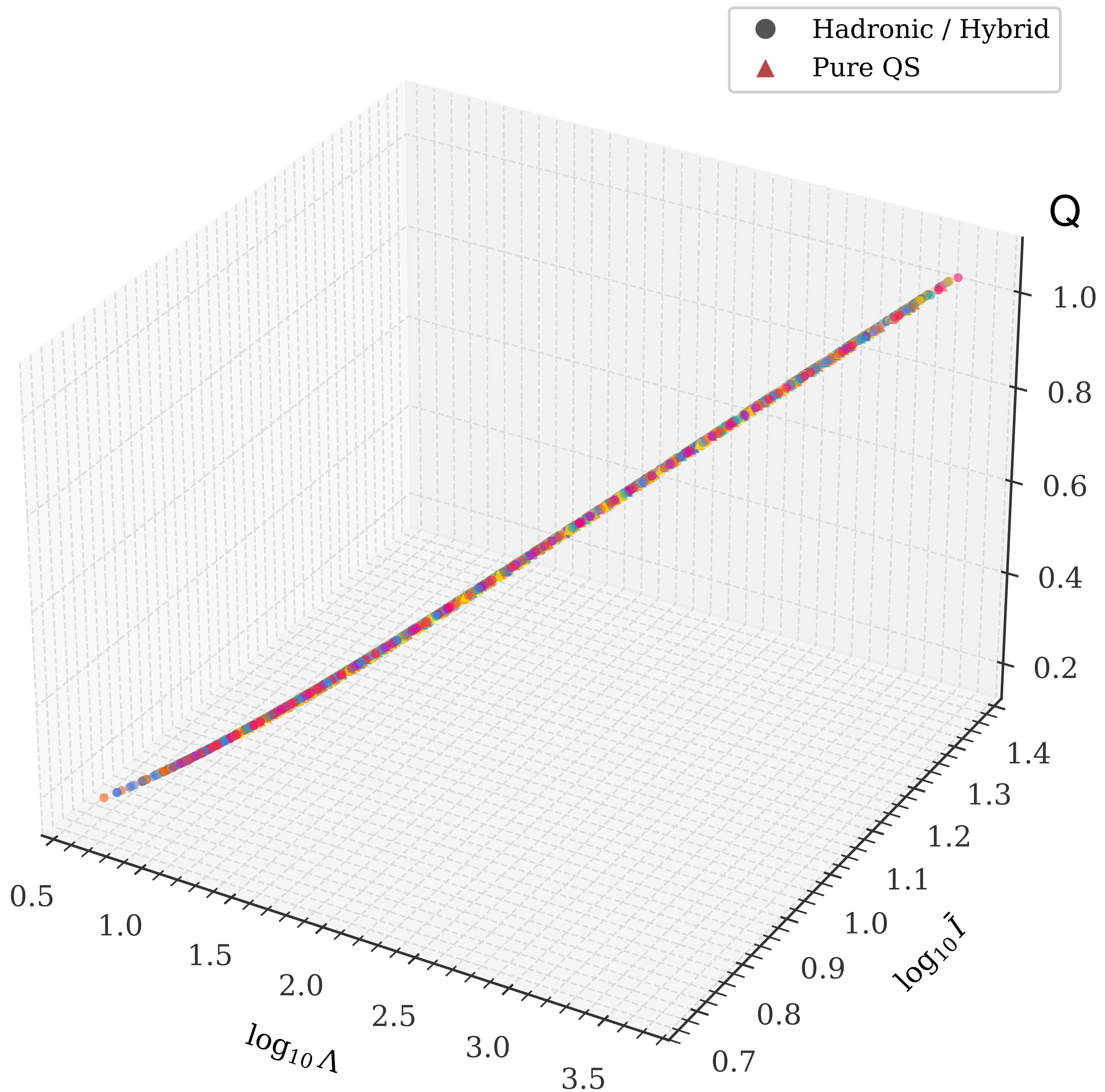
- **133 EoS sample** — hadronic, hybrid, pure QS — all follow Yagi & Yunes to <5%.
- EoS-independence survives exotic matter



Full GR perturbation equations reduce effective stiffness relative to Cowling

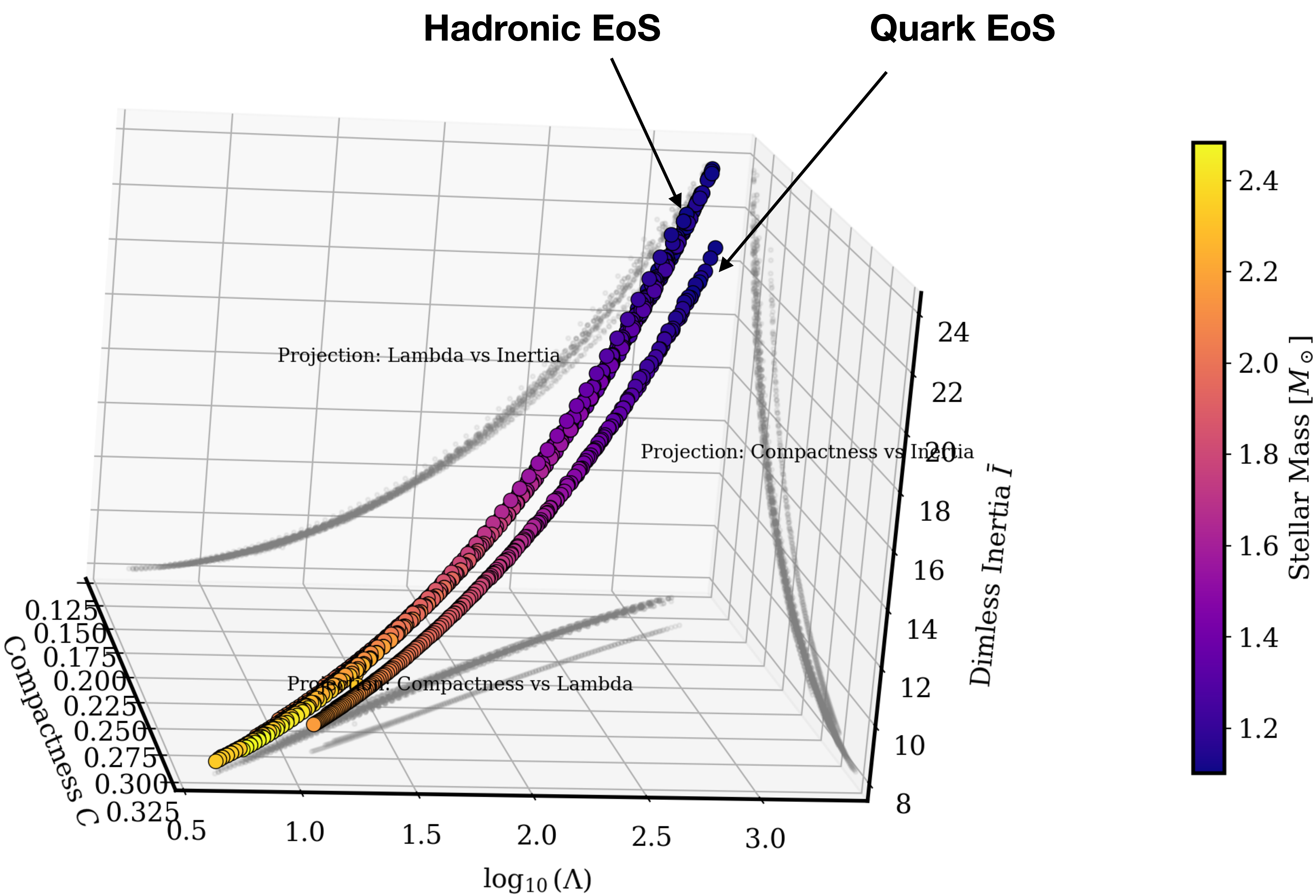
f-mode URs are more EoS-sensitive than I-Love-Q

I-Love-Q Triangle



- All compositions trace a single 1D curve in (Λ, $\bar{I}$, $\bar{Q}$) space
- I-Love-Q universality is geometry, not matter physics.
- f-mode URs offer better discrimination of exotic matter.

Universal Golden Curve



Summary & Outlook

Summary & Outlook

- 133 constraint-filtered EoSs (2Mo + NICER + GW170817)
- New full-GR ***f***-mode URs — larger, modern EoS sample
- I-Love-Q: universality survives even for pure quark stars
- **f**-mode URs → couple to c_s(r) → feel the interior
- UR-3 / UR-4: QS offsets 5–10% → exotic matter fingerprint

Summary & Outlook

CRC-TR 211

- 133 constraint-filtered EoSs (2Mo + NICER + GW170817)
- New full-GR ***f***-mode URs — larger, modern EoS sample
- I-Love-Q: universality survives even for pure quark stars
- **f**-mode URs → couple to c_s(r) → feel the interior
- UR-3 / UR-4: QS offsets 5–10% → exotic matter fingerprint

***f*-mode asteroseismology does what I-Love-Q cannot:**

discriminate hadronic from quark matter.

Summary & Outlook

- 133 constraint-filtered EoSs (2Mo + NICER + GW170817)
- New full-GR ***f***-mode URs — larger, modern EoS sample
- I-Love-Q: universality survives even for pure quark stars
- **f**-mode URs → couple to c_s(r) → feel the interior
- UR-3 / UR-4: QS offsets 5–10% → exotic matter fingerprint

***f*-mode asteroseismology does what I-Love-Q cannot:**

discriminate hadronic from quark matter.

Outlook

Summary & Outlook

- 133 constraint-filtered EoSs (2Mo + NICER + GW170817)
- New full-GR ***f***-mode URs — larger, modern EoS sample
- I-Love-Q: universality survives even for pure quark stars
- **f**-mode URs → couple to c_s(r) → feel the interior
- UR-3 / UR-4: QS offsets 5–10% → exotic matter fingerprint

***f*-mode asteroseismology does what I-Love-Q cannot:**

discriminate hadronic from quark matter.

Outlook

- ET / CE: detect f-modes from BNS post-merger remnants
- Bayesian inversion: URs → extract R, M, I from GW data
- Proto-neutron stars: finite-T EoSs
- RG-consistent NJL quark matter → sharper QS URs

Open question: How much can QS deviate before universality breaks?

→ threshold = exotic matter signature in GW data

Models

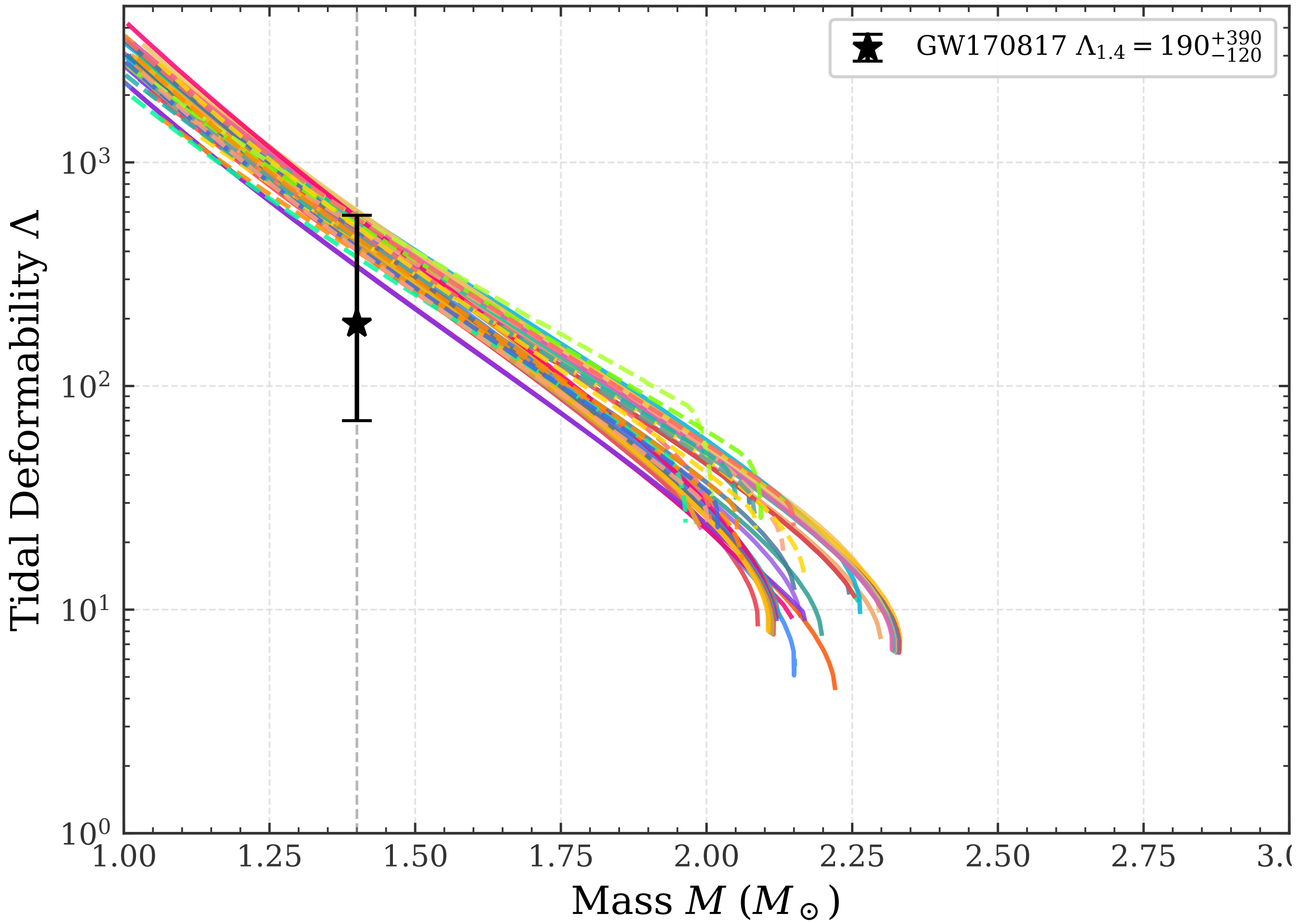
Label	M_max (M_sun)	R_1.4 (km)	Lam_1.4	Reference
HS1	1.93	13.09	572	This Work
HS10	2.08	12.63	431	This Work
HS11	2.05	12.63	430	This Work
HS2	1.99	12.62	429	This Work
HS3	1.98	12.62	429	This Work
HS4	1.95	13.09	572	This Work
HS5	2.32	13.09	572	This Work
HS6	2.00	13.09	572	This Work
HS7	2.30	13.09	572	This Work
HS8	2.25	12.63	430	This Work
HS9	2.25	12.63	430	This Work
N2	2.23	12.03	354	This Work
N3	2.34	12.19	394	This Work
NH2	2.04	12.03	354	This Work
NH3	2.08	12.18	392	This Work

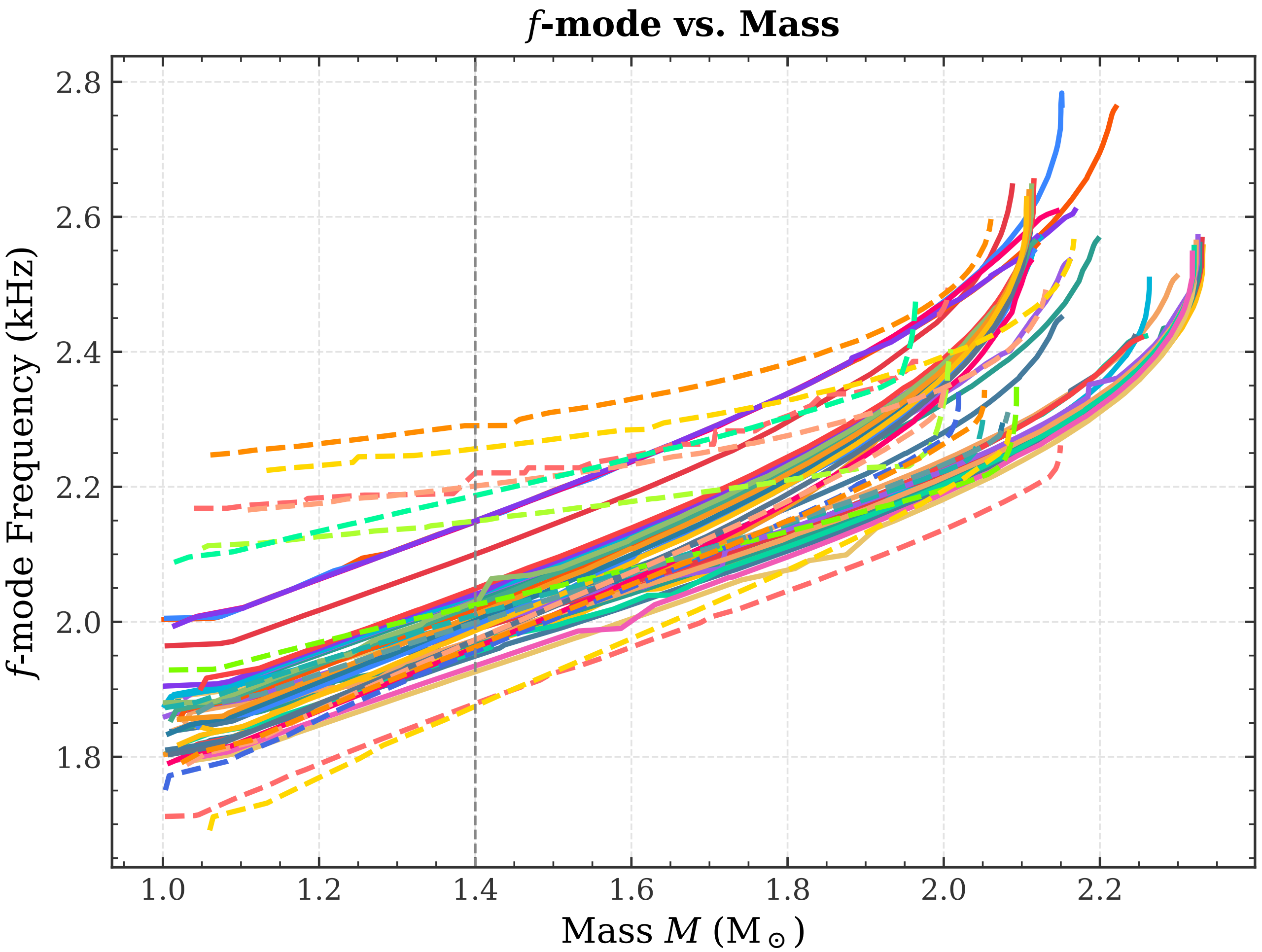
Label	M_max (M_sun)	R_1.4 (km)	Lam_1.4	Reference
NL+DM1	2.09	11.61	281	This Work
NL+DM2	2.20	11.99	349	This Work
NL1	2.15	12.20	365	This Work
NL2	2.16	11.97	343	This Work
NL3	2.30	12.25	399	This Work
NL4	2.30	12.20	405	This Work
PNS N	2.48	12.75	551	This Work
PNS NH	2.26	12.75	551	This Work
QS1	2.13	10.98	501	This Work
QS2	2.17	10.71	429	This Work
QS3	2.06	10.53	378	This Work
QS3	2.44	11.79	821	This Work
QS5	1.54	8.36	95	This Work
RMF1	2.26	12.22	418	This Work
RMF2	2.34	12.01	371	This Work
RMF3	2.78	12.99	659	This Work

Models used

Hadronic **Quark** **Hyperons** **Deltas** **With Phase transition**

M-Lambda





✓ Tidal well within the GW170817 limit

$$\Lambda \leq 580$$

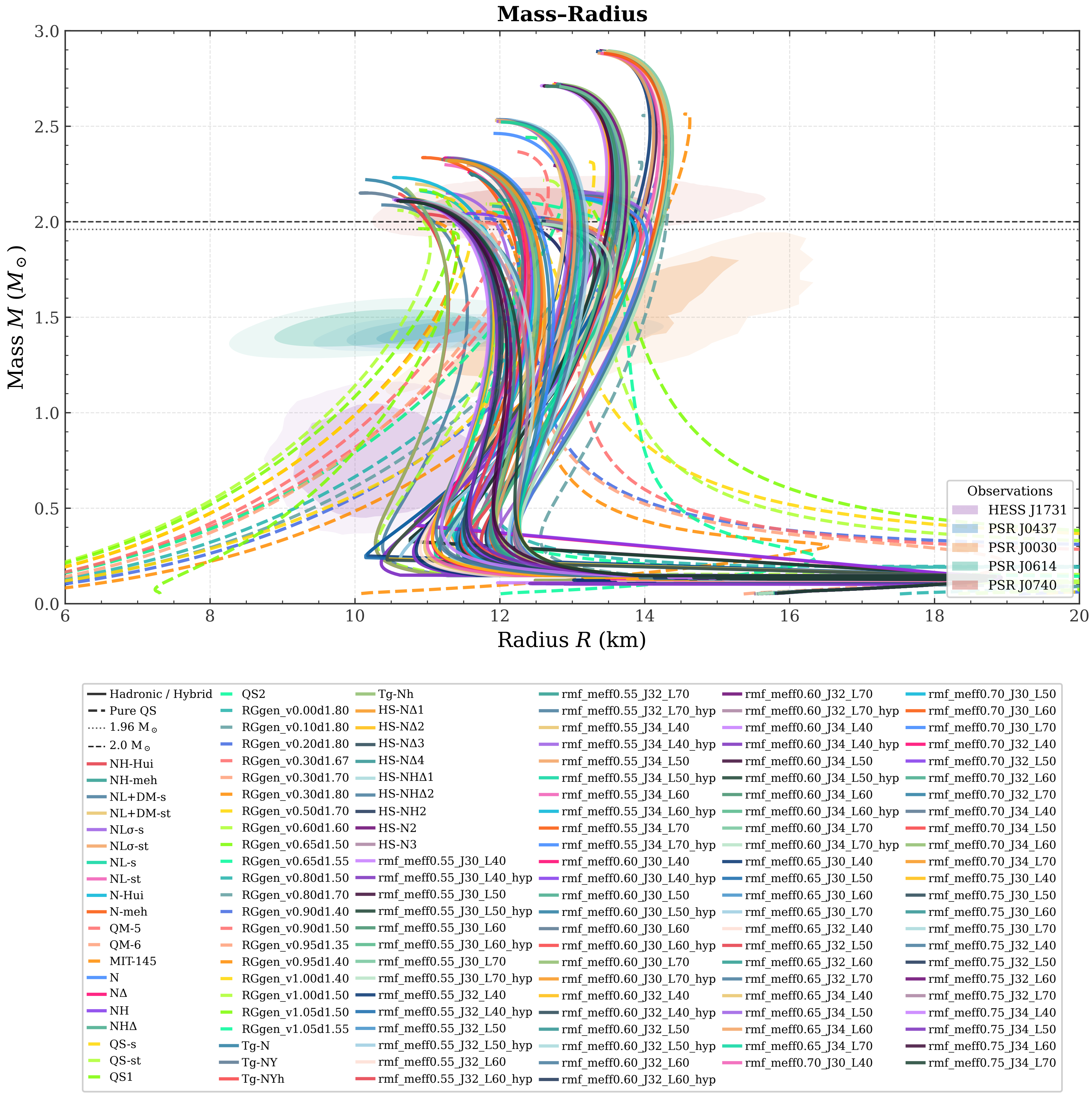
B. P. Abbott *et al.* PRL, 2018

✓ *f*-mode frequency between 1.8 - 2.1 kHz

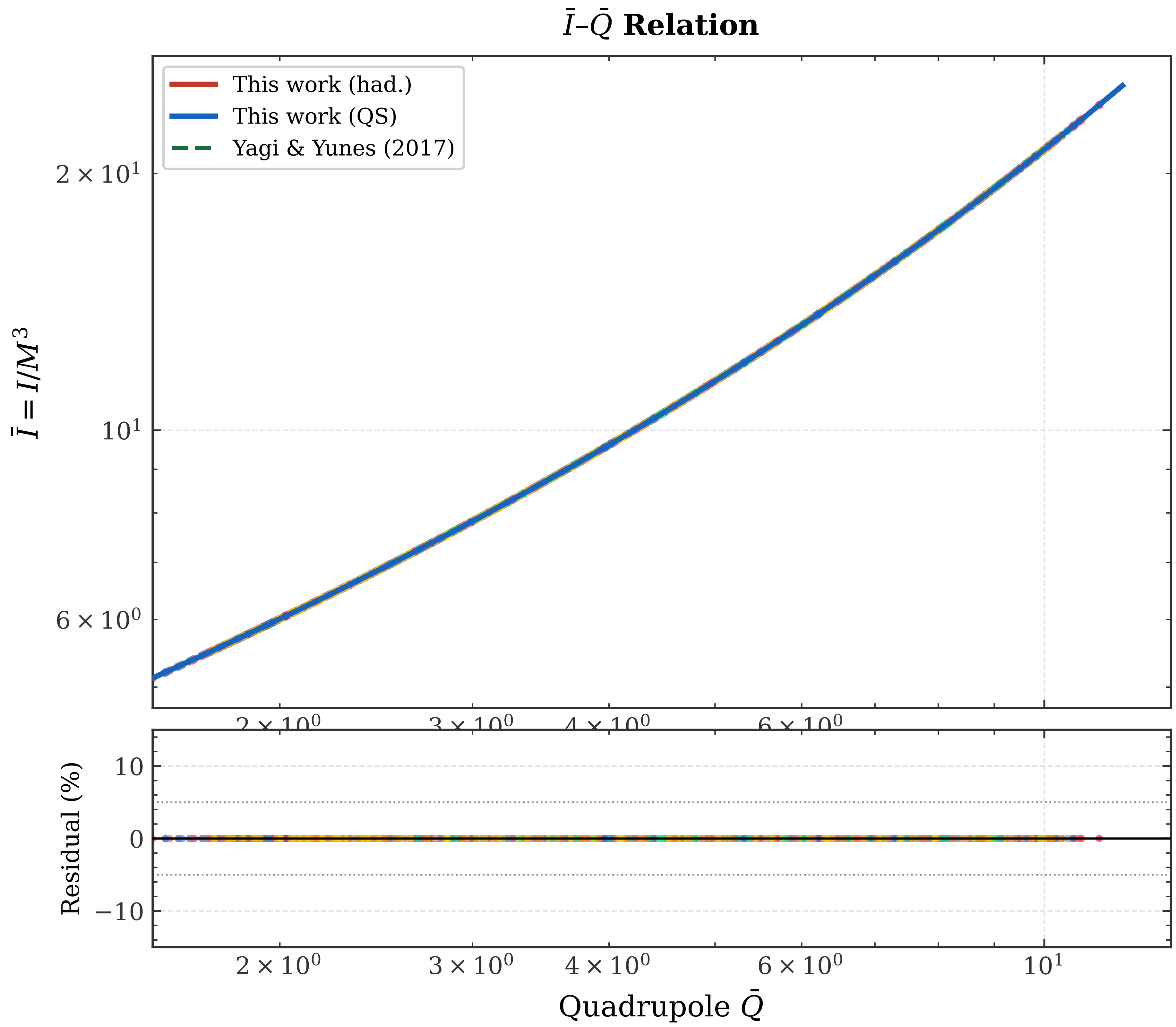
Well within the limit from PSR + GW + NICER analysis

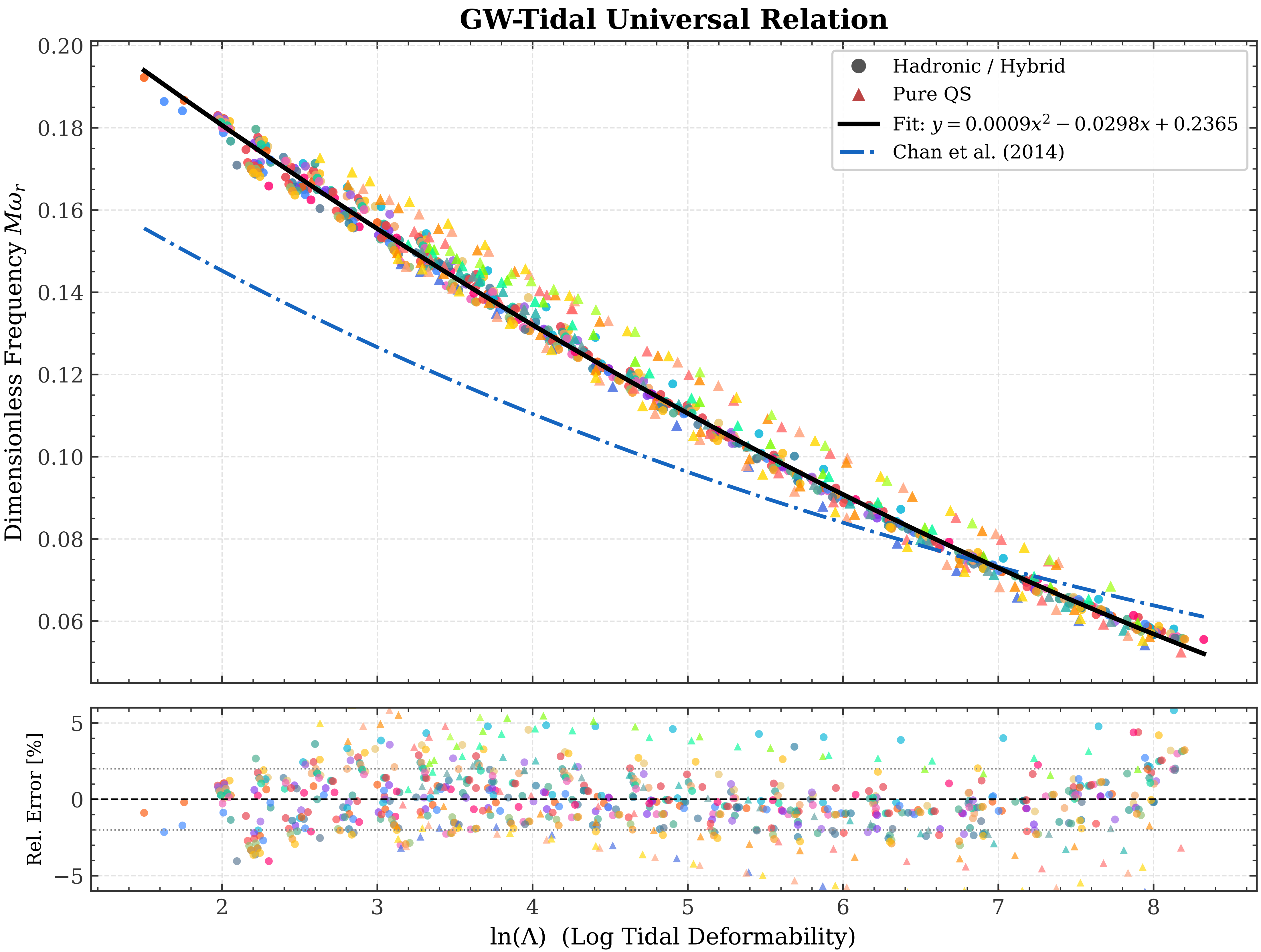
PRD 112, 123043 (2025)

✓ *For* SQM 1.8- 2.3 kHz


Mass–Radius
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Mass M ($M_\odot$)
6
8
10
12
14
16
18
20
Radius R (km)
Observations
HESS J1731
PSR J0437
PSR J0030
PSR J0614
PSR J0740
Hadronic / Hybrid
Pure QS
1.96 $M_\odot$
2.0 $M_\odot$
NH-Hui
NH-meh
NL+DM-s
NL+DM-st
NLσ-s
NLσ-st
NL-s
NL-st
N-Hui
N-meh
QM-5
QM-6
MIT-145
N
NΔ
NH
NHΔ
QS-s
QS-st
QS1
QS2
RGgen_v0.00d1.80
RGgen_v0.10d1.80
RGgen_v0.20d1.80
RGgen_v0.30d1.67
RGgen_v0.30d1.70
RGgen_v0.30d1.80
RGgen_v0.50d1.70
RGgen_v0.60d1.60
RGgen_v0.65d1.50
RGgen_v0.65d1.55
RGgen_v0.80d1.50
RGgen_v0.80d1.70
RGgen_v0.90d1.40
RGgen_v0.90d1.50
RGgen_v0.95d1.35
RGgen_v0.95d1.40
RGgen_v1.00d1.40
RGgen_v1.00d1.50
RGgen_v1.05d1.50
RGgen_v1.05d1.55
Tg-N
Tg-NY
Tg-NYh
Tg-Nh
HS-NΔ1
HS-NΔ2
HS-NΔ3
HS-NΔ4
HS-NHΔ1
HS-NHΔ2
HS-NH2
HS-N2
HS-N3
rmf_meff0.55_J30_L40
rmf_meff0.55_J30_L40_hyp
rmf_meff0.55_J30_L50
rmf_meff0.55_J30_L50_hyp
rmf_meff0.55_J30_L60
rmf_meff0.55_J30_L60_hyp
rmf_meff0.55_J30_L70
rmf_meff0.55_J30_L70_hyp
rmf_meff0.55_J32_L40
rmf_meff0.55_J32_L40_hyp
rmf_meff0.55_J32_L50
rmf_meff0.55_J32_L50_hyp
rmf_meff0.55_J32_L60
rmf_meff0.55_J32_L60_hyp
rmf_meff0.55_J32_L70
rmf_meff0.55_J32_L70_hyp
rmf_meff0.55_J34_L40
rmf_meff0.55_J34_L40_hyp
rmf_meff0.55_J34_L50
rmf_meff0.55_J34_L50_hyp
rmf_meff0.55_J34_L60
rmf_meff0.55_J34_L60_hyp
rmf_meff0.55_J34_L70
rmf_meff0.55_J34_L70_hyp
rmf_meff0.60_J30_L40
rmf_meff0.60_J30_L40_hyp
rmf_meff0.60_J30_L50
rmf_meff0.60_J30_L50_hyp
rmf_meff0.60_J30_L60
rmf_meff0.60_J30_L60_hyp
rmf_meff0.60_J30_L70
rmf_meff0.60_J30_L70_hyp
rmf_meff0.60_J32_L40
rmf_meff0.60_J32_L40_hyp
rmf_meff0.60_J32_L50
rmf_meff0.60_J32_L50_hyp
rmf_meff0.60_J32_L60
rmf_meff0.60_J32_L60_hyp
rmf_meff0.60_J32_L70
rmf_meff0.60_J32_L70_hyp
rmf_meff0.60_J34_L40
rmf_meff0.60_J34_L40_hyp
rmf_meff0.60_J34_L50
rmf_meff0.60_J34_L50_hyp
rmf_meff0.60_J34_L60
rmf_meff0.60_J34_L60_hyp
rmf_meff0.60_J34_L70
rmf_meff0.60_J34_L70_hyp
rmf_meff0.65_J30_L40
rmf_meff0.65_J30_L50
rmf_meff0.65_J30_L60
rmf_meff0.65_J30_L70
rmf_meff0.65_J32_L40
rmf_meff0.65_J32_L50
rmf_meff0.65_J32_L60
rmf_meff0.65_J32_L70
rmf_meff0.65_J34_L40
rmf_meff0.65_J34_L50
rmf_meff0.65_J34_L60
rmf_meff0.65_J34_L70
rmf_meff0.70_J30_L40
rmf_meff0.70_J30_L50
rmf_meff0.70_J30_L60
rmf_meff0.70_J30_L70
rmf_meff0.70_J32_L40
rmf_meff0.70_J32_L50
rmf_meff0.70_J32_L60
rmf_meff0.70_J32_L70
rmf_meff0.70_J34_L40
rmf_meff0.70_J34_L50
rmf_meff0.70_J34_L60
rmf_meff0.70_J34_L70
rmf_meff0.75_J30_L40
rmf_meff0.75_J30_L50
rmf_meff0.75_J30_L60
rmf_meff0.75_J30_L70
rmf_meff0.75_J32_L40
rmf_meff0.75_J32_L50
rmf_meff0.75_J32_L60
rmf_meff0.75_J32_L70
rmf_meff0.75_J34_L40
rmf_meff0.75_J34_L50
rmf_meff0.75_J34_L60
rmf_meff0.75_J34_L70

IQ UR


$\bar{I}$–$\bar{Q}$ Relation
This work (had.)
This work (QS)
Yagi & Yunes (2017)
$\bar{I} = I/M^3$
2×10^1
10^1
6×10^0
Residual (%)
10
0
−10
2×10^0
3×10^0
4×10^0
6×10^0
10^1
Quadrupole $\bar{Q}$



GW-Tidal Universal Relation
Hadronic / Hybrid
Pure QS
Fit: $y = 0.0009x^2 - 0.0298x + 0.2365$
Chan et al. (2014)
Dimensionless Frequency $M\omega_r$
Rel. Error [%]
ln(Λ) (Log Tidal Deformability)