

Ab initio finite-temperature effects for neutron star simulations

[Riviuccio, Nadal-Matosas, Rios & Ruiz, *ApJ* **987**, 67 \(2025\)](#)

[D Guerra, Rios et al arxiv:2512.05118](#)

[Kochankovski, Rozalén, Rios & Ramos, *in prep*](#)



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Guerra



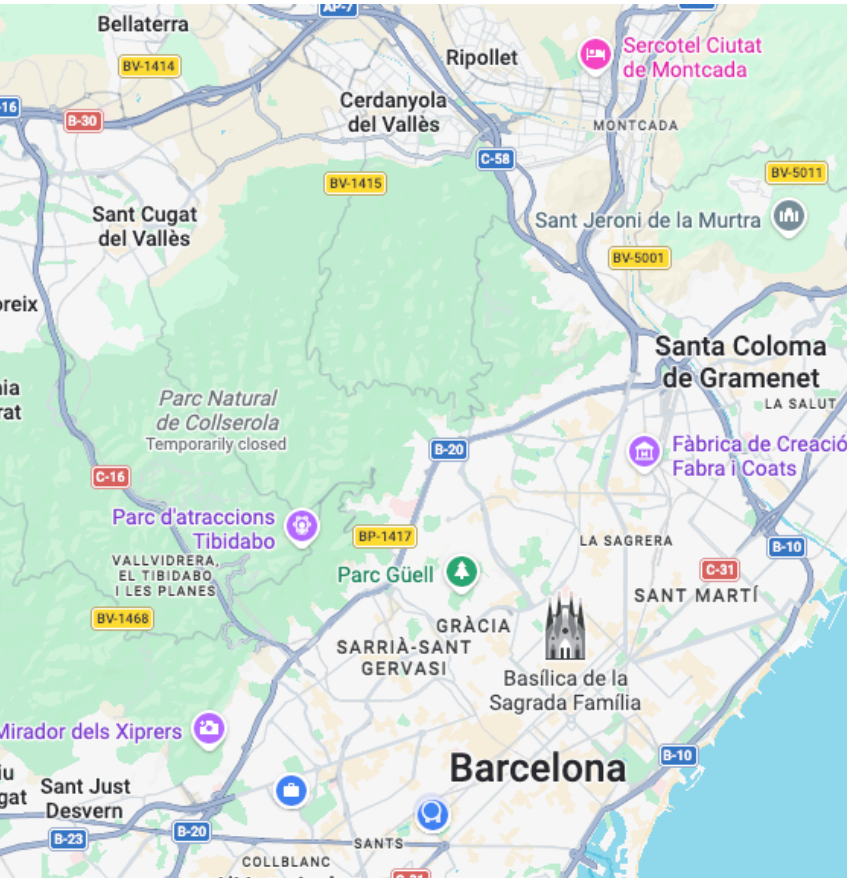
Riviuccio

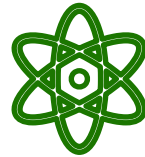
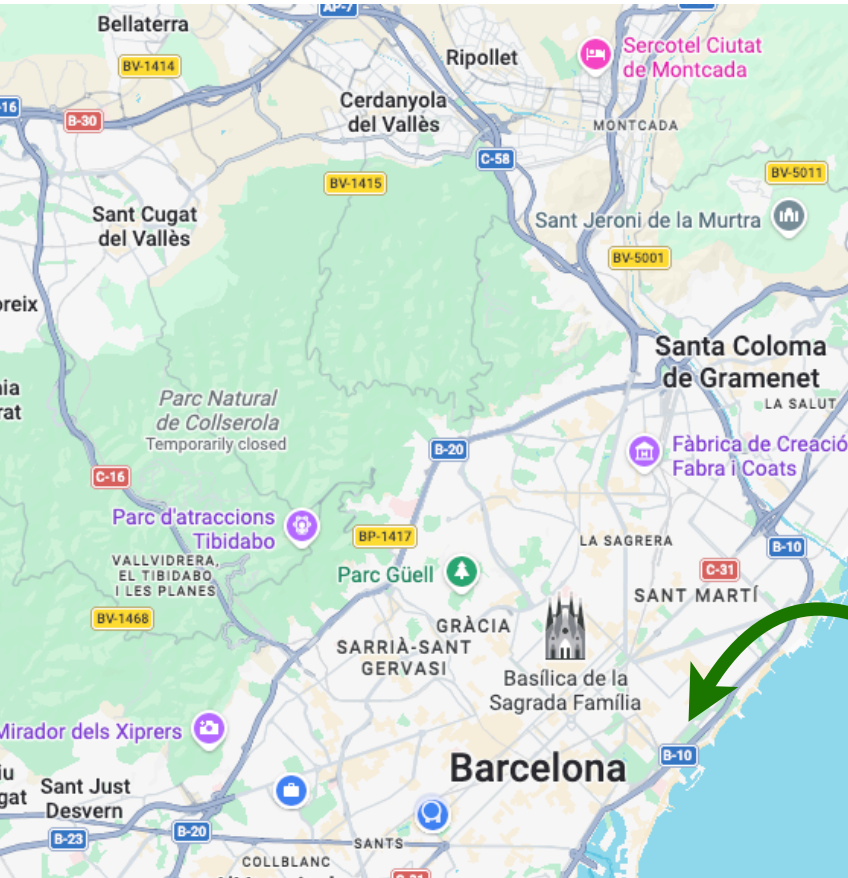


Rozalén Kochankovski



CSQCD
19 May 2026





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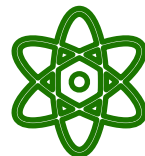
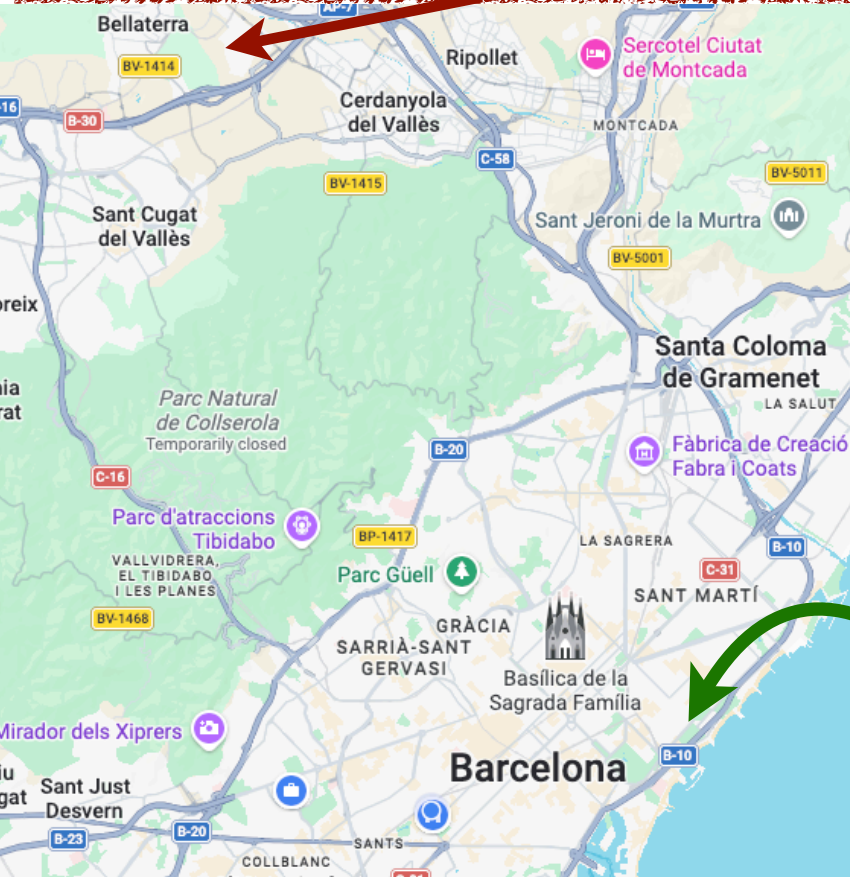
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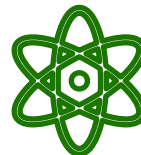
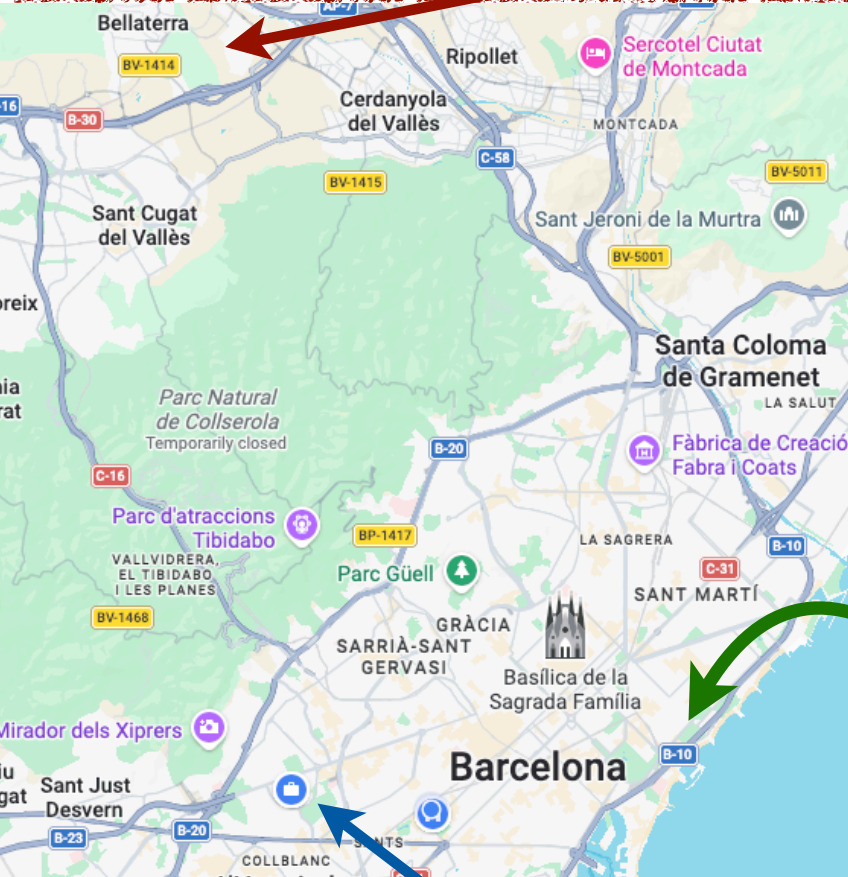
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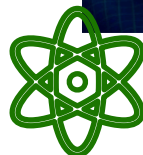
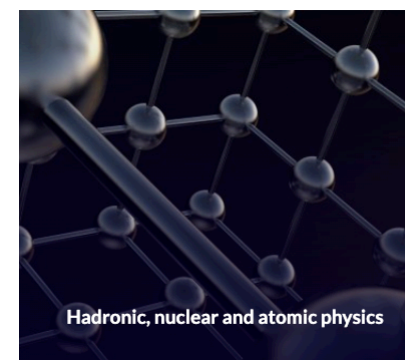
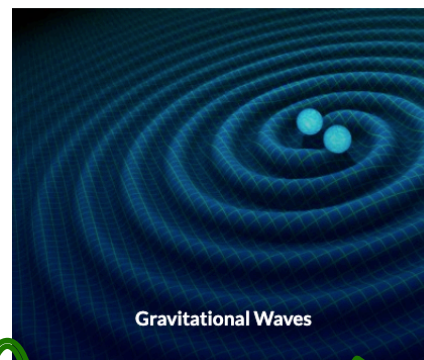
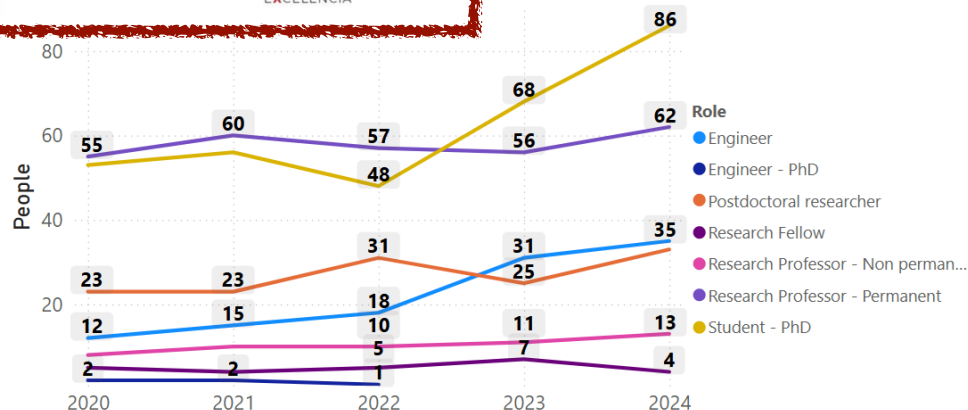
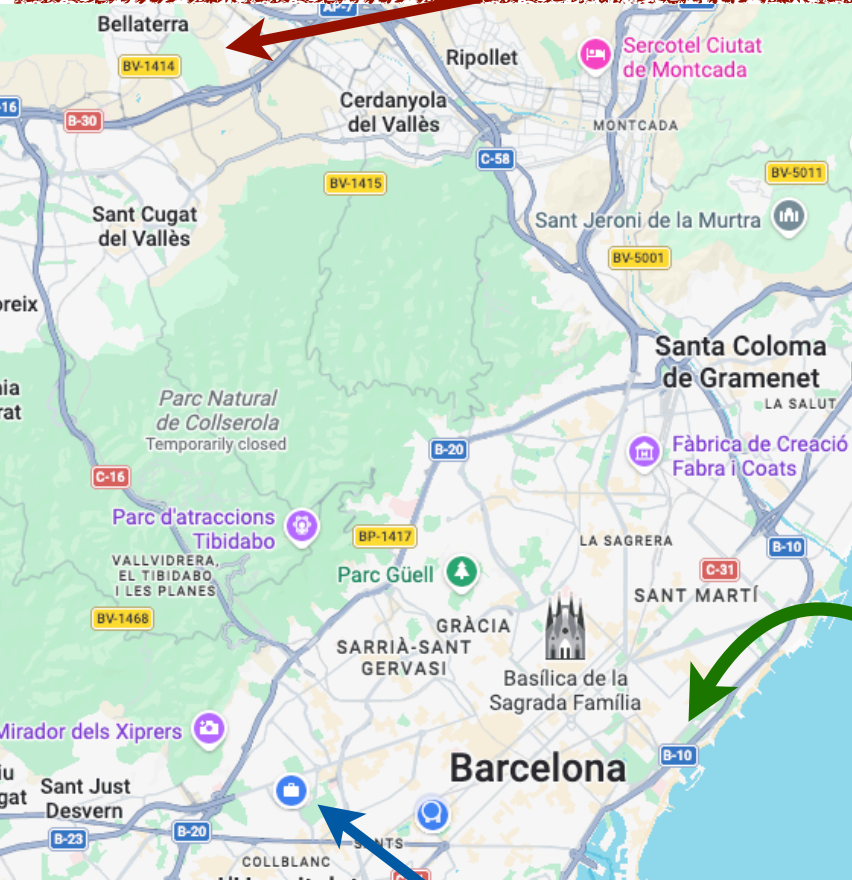


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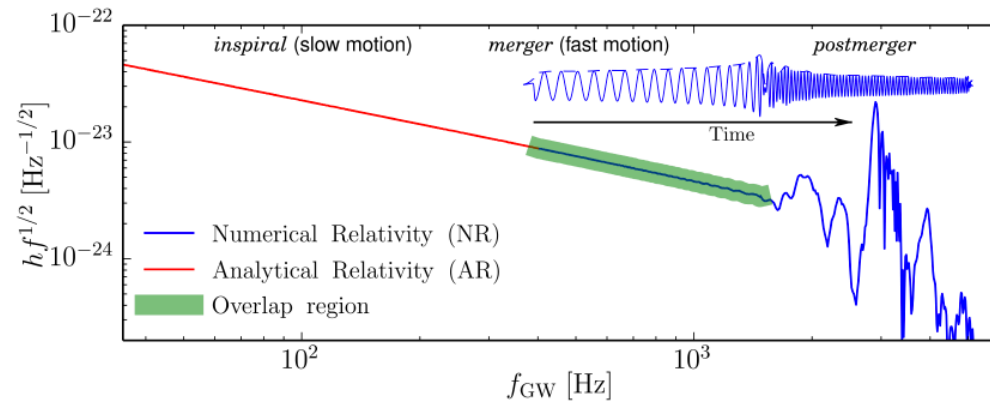


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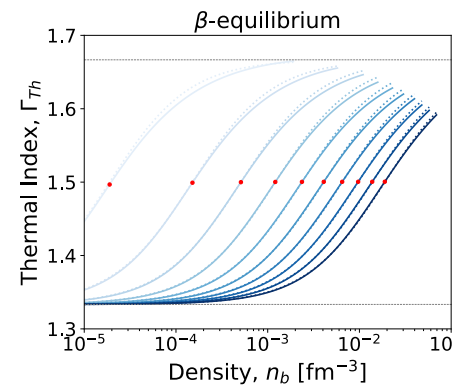
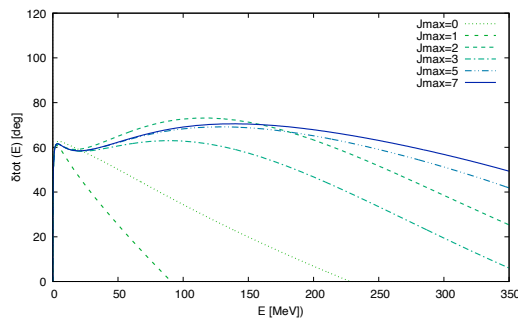


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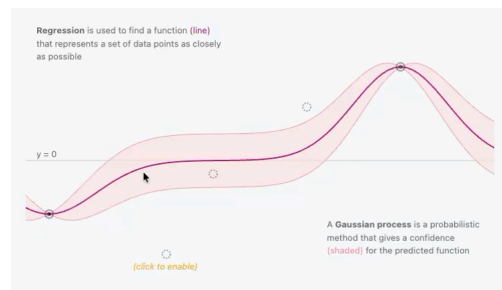
• Motivation



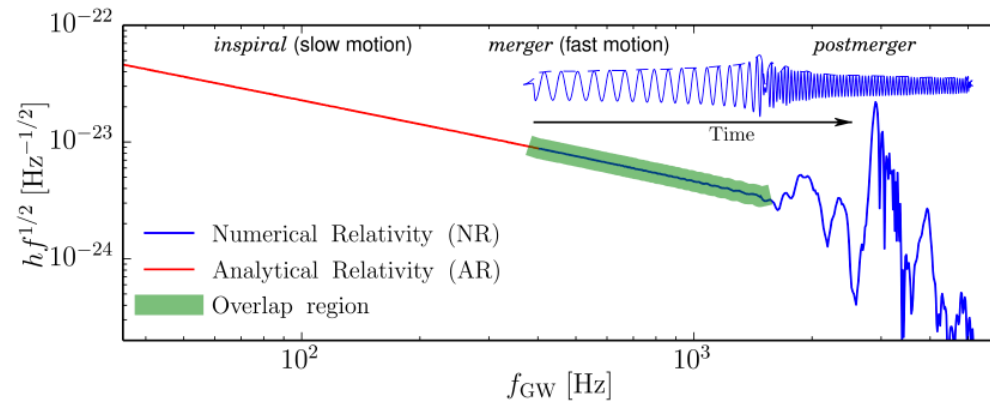
• Virial approximation & thermal effects in BNS



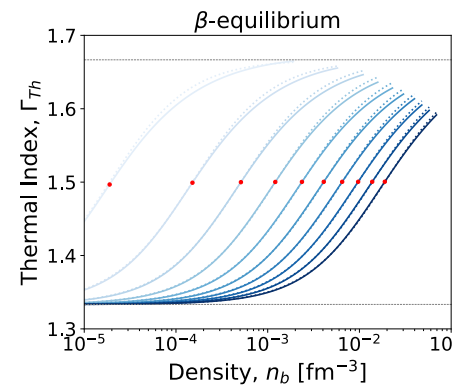
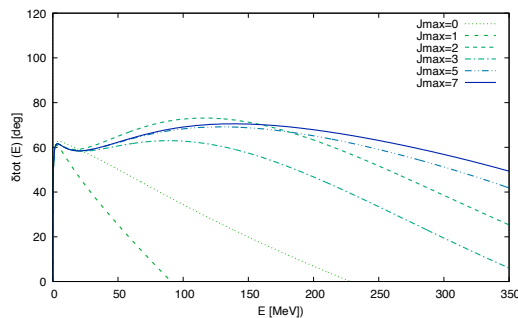
• Gaussian processes



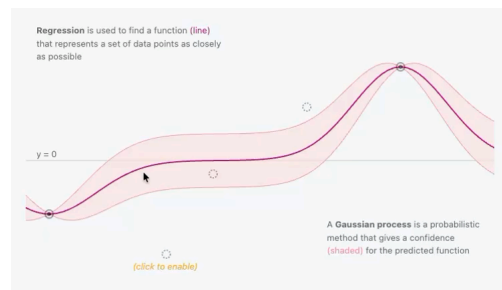
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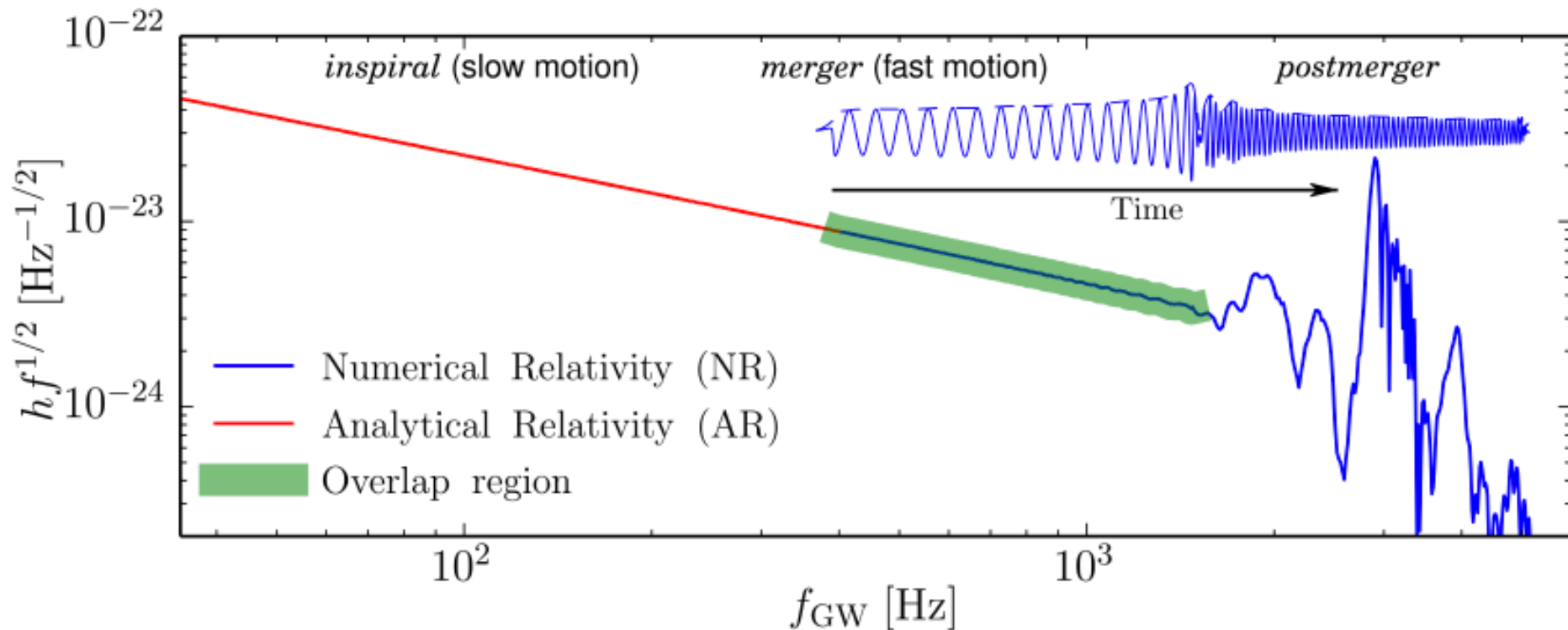


• Virial approximation & thermal effects in BNS



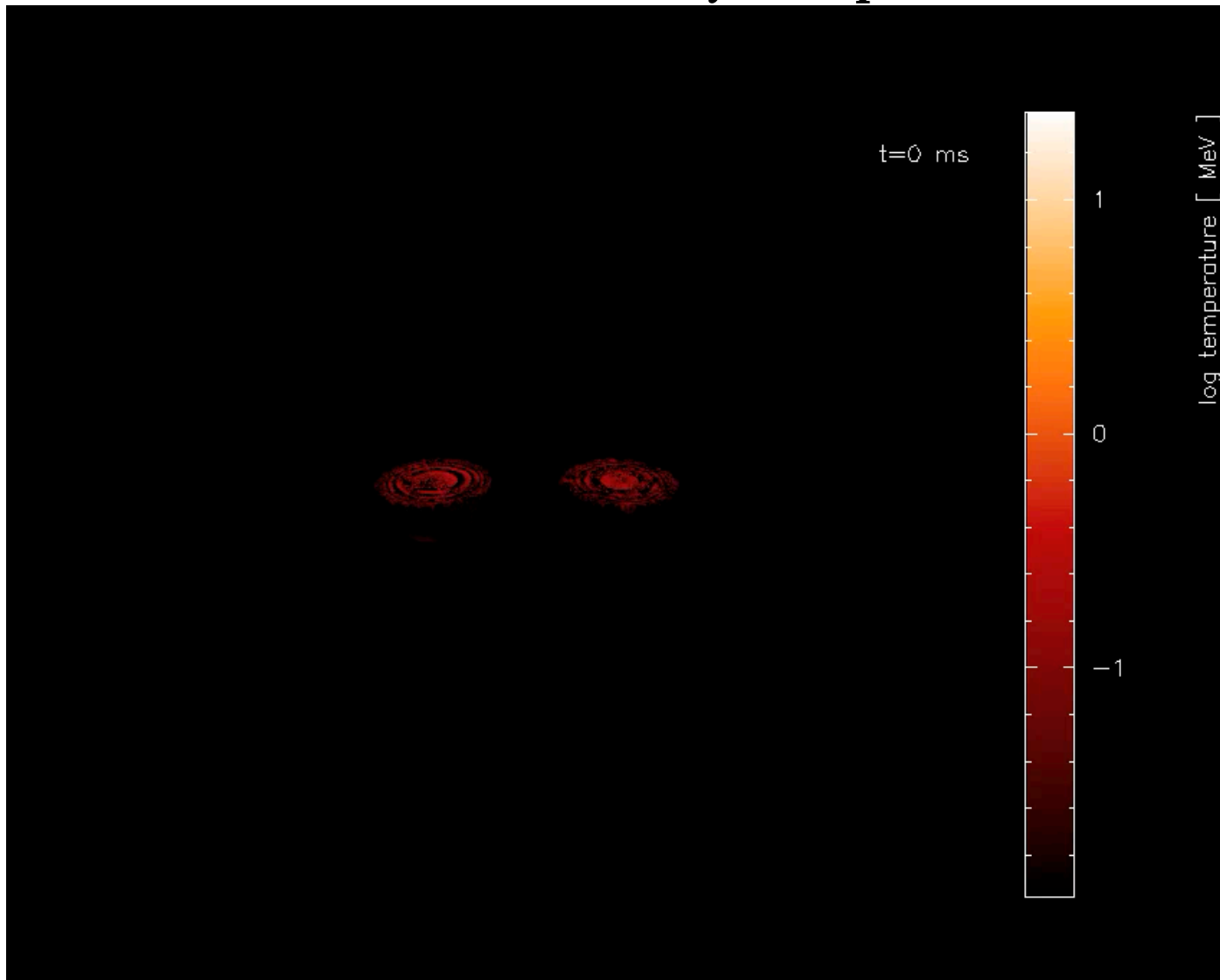
• Gaussian processes

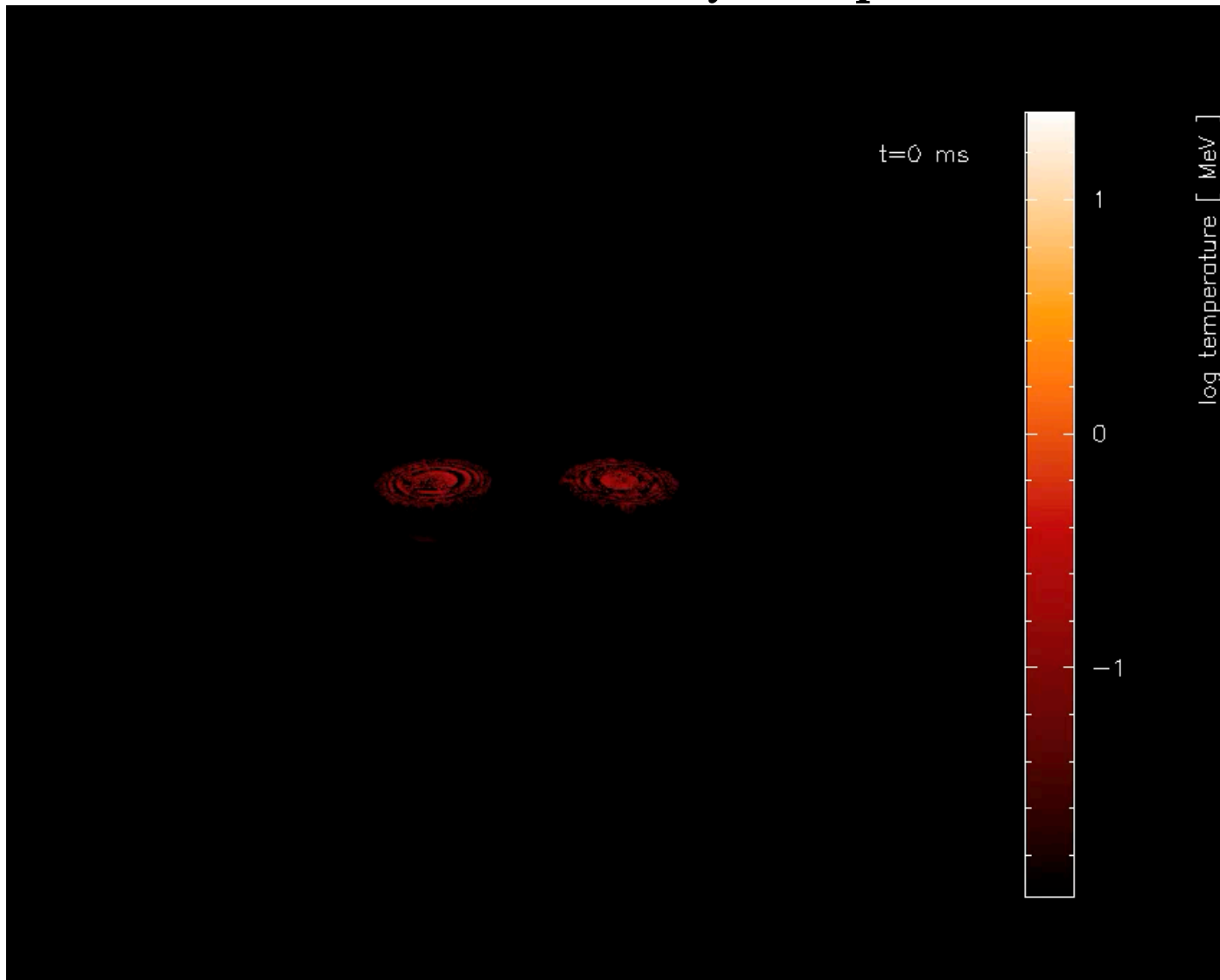




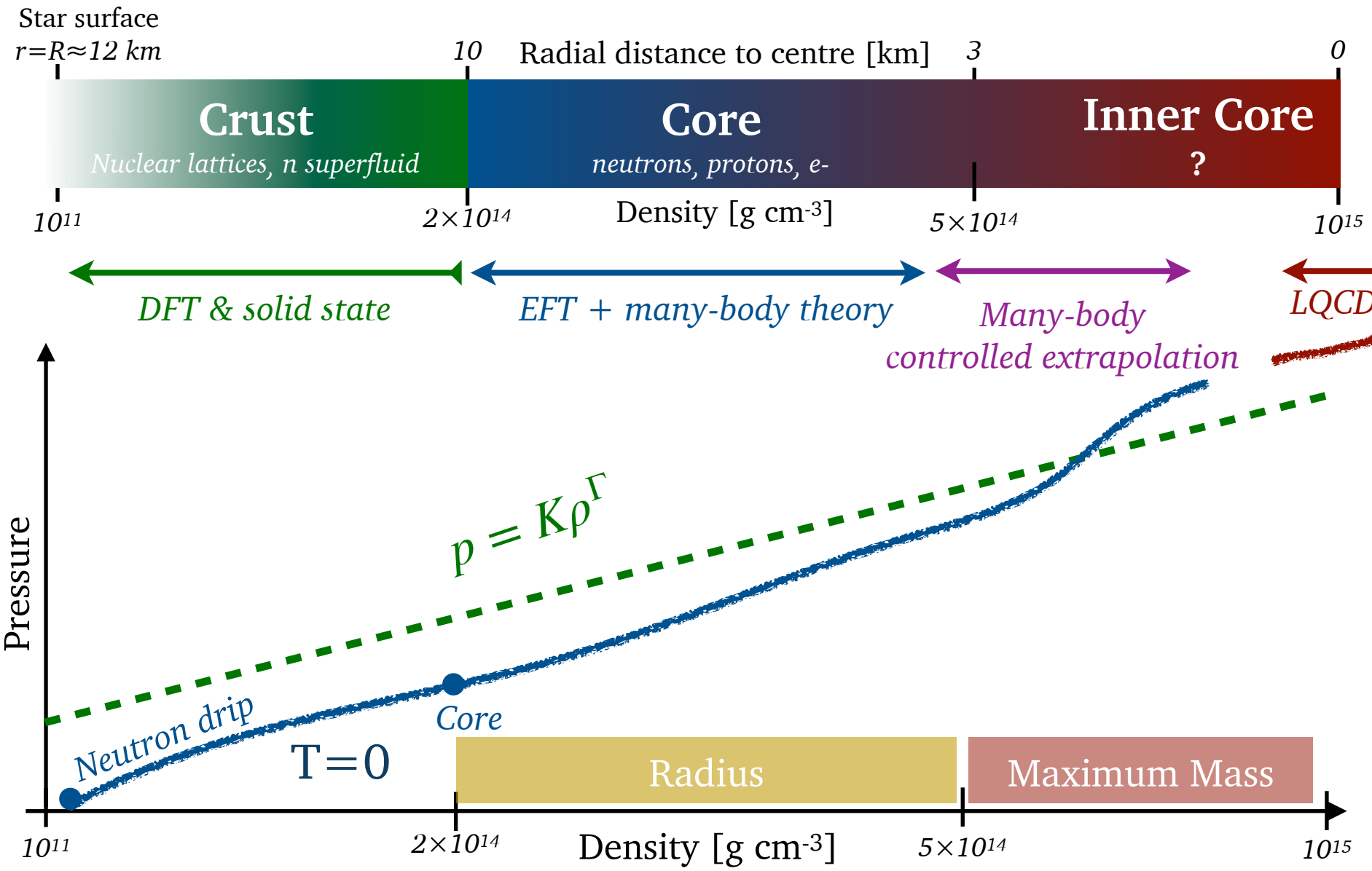
Radice, Bernuzzi & Perego, *Annu. Rev. Nucl. Part. Sci.* **70** 95 (2020)

- **Inspiral:** analytical relativity (eg IB approaches)
- **Post-merger:** numerical relativity.
- **Complete waveform models:** matching the two approaches in the region where both are valid

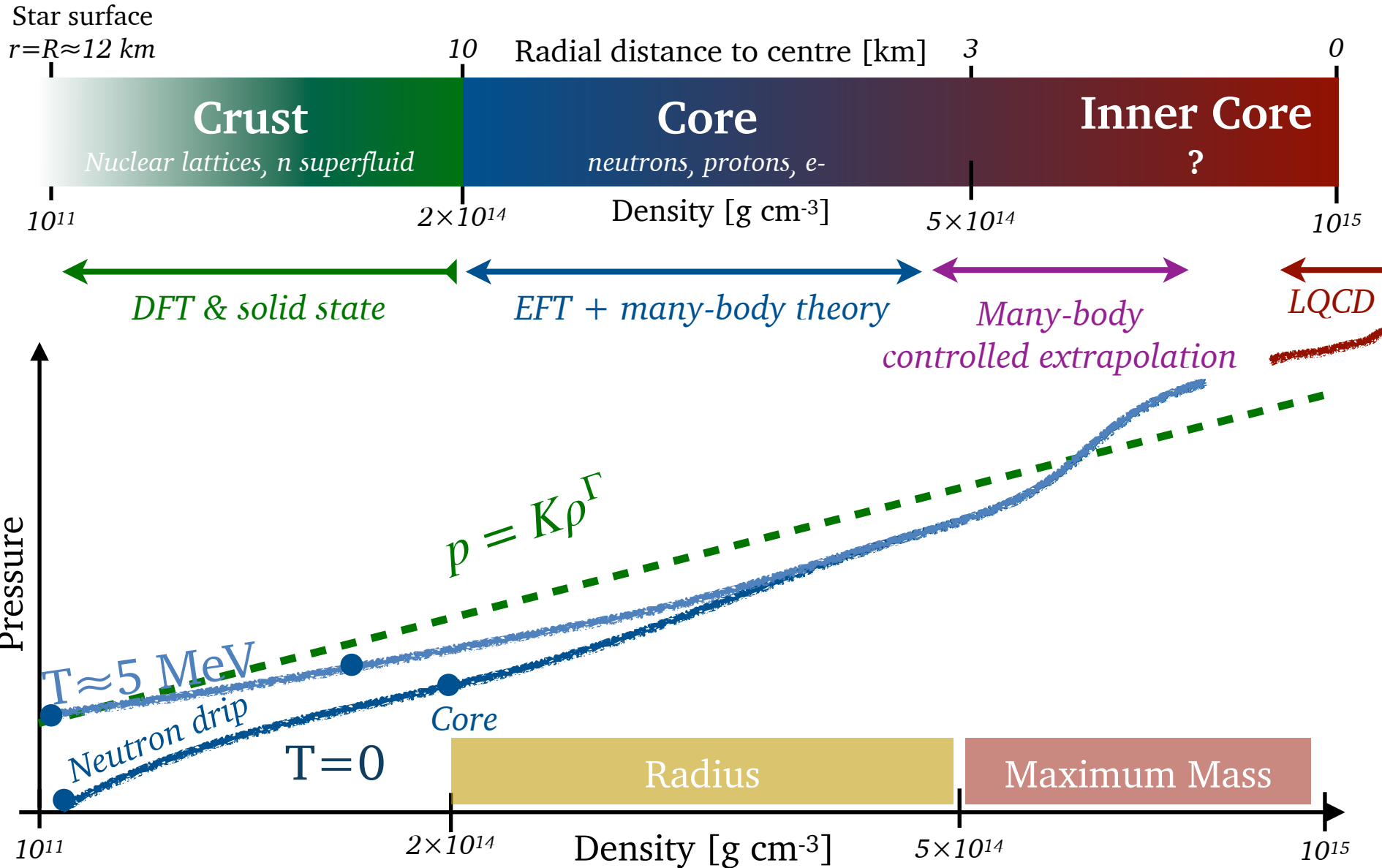
$1.4M_{\odot} + 1.3M_{\odot}$ NS binary: temperature

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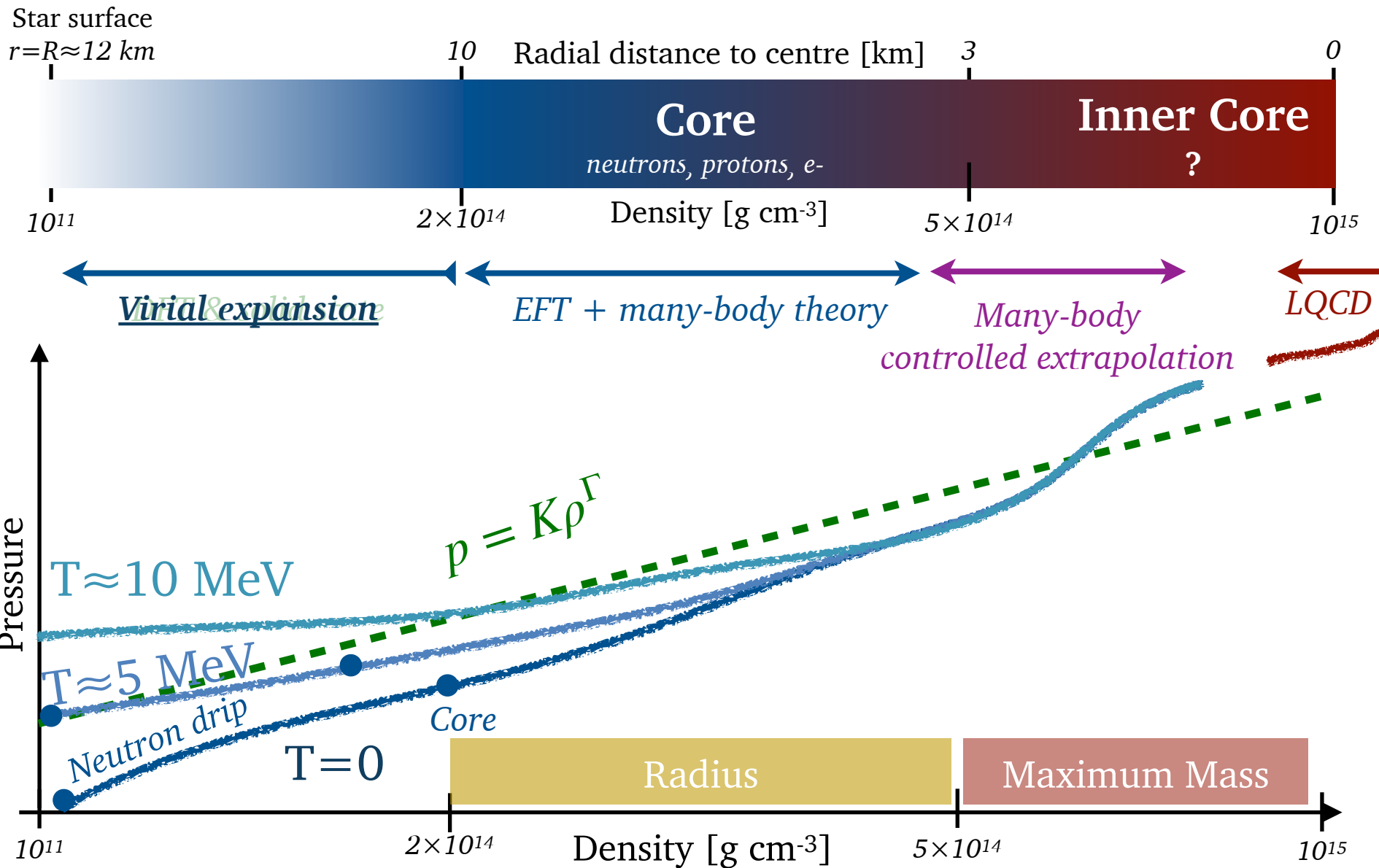
Neutron-star modelling



Neutron-star modelling



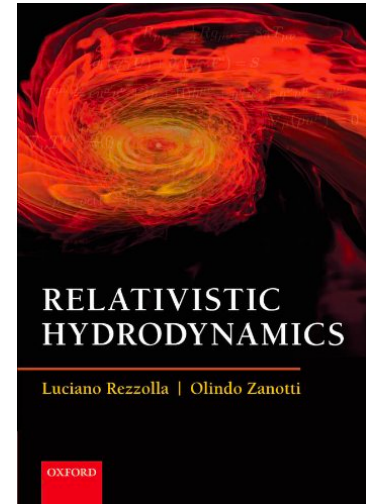
Neutron-star modelling



Simulations require **coupled dynamics** of:

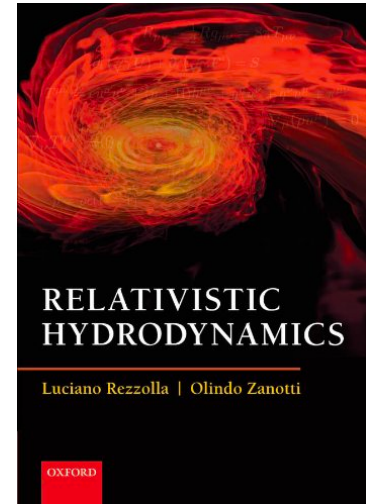
- A fluid which is essentially **cold** (eg NSs before merger)
- A fluid **heated by shocks**, with kinetic energy dissipated into internal energy

$$P(\epsilon, T) = P(\epsilon, T = 0) + P_{\text{th}}(\epsilon_{\text{th}})$$



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$$P(\epsilon, T) = P(\epsilon, T = 0) + P_{\text{th}}(\epsilon_{\text{th}})$$

Cold part

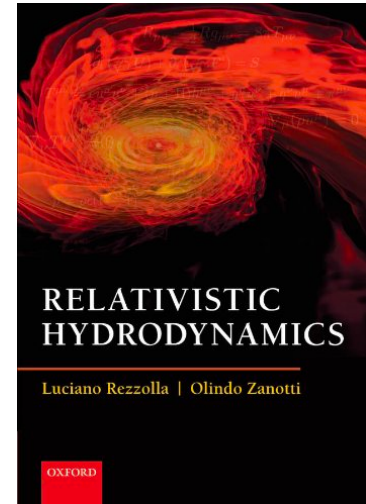
(Piecewise polytrope)

$$P(\epsilon, T = 0) = K \epsilon^{\Gamma_c}$$

No thermal energy

Simulations require **coupled dynamics** of:

- A fluid which is essentially **cold** (eg NSs before merger)
- A fluid **heated by shocks**, with kinetic energy dissipated into internal energy



$$P(\epsilon, T) = P(\epsilon, T = 0) + P_{\text{th}}(\epsilon_{\text{th}})$$

Cold part

(Piecewise polytrope)

$$P(\epsilon, T = 0) = K \epsilon^{\Gamma_c}$$

No thermal energy

Read et al, *Phys Rev D* **79** 124032 (2009)

Thermal part

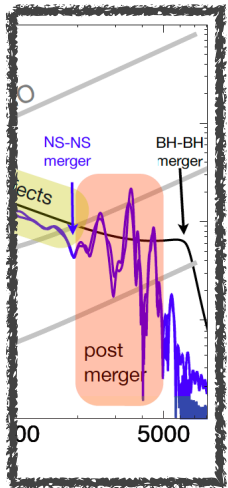
Thermal index

$$P_{\text{th}}(\epsilon_{\text{th}}) \equiv [\Gamma_{\text{th}} - 1] \epsilon_{\text{th}}$$

$$\Gamma_{\text{th}}(\epsilon_{\text{th}}) = 1 + \frac{P_{\text{th}}(\epsilon_{\text{th}})}{\epsilon_{\text{th}}}$$

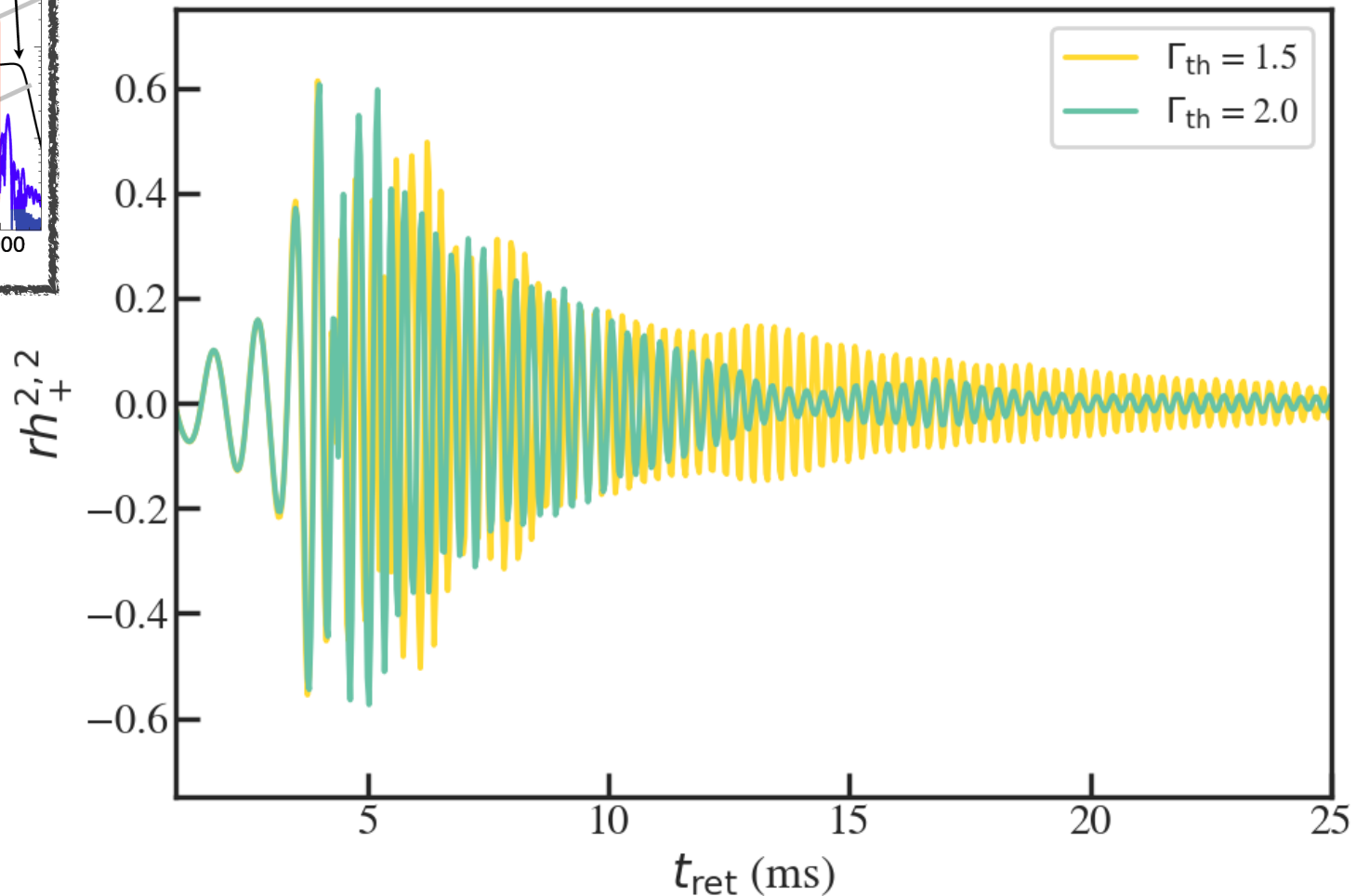
$$\epsilon_{\text{th}} = \epsilon(T) - \epsilon_0$$

Post-merger: from Γ_{th} to GWs



1.4M_⊙+1.4M_⊙ GW spectrum

Same cold component, different constant Γ_{th}



Cold partNon-relativistic

$$\Gamma_c = \frac{5}{3}$$

Thermal part

$$\Gamma_{\text{th}} = \frac{5}{3}$$

Relativistic

$$\Gamma_c = \frac{4}{3}$$



$$\Gamma_{\text{th}} = \frac{4}{3}$$

Cold partThermal partNon-relativistic

$$\Gamma_c = \frac{5}{3}$$



$$\Gamma_{\text{th}} = \frac{5}{3}$$

Relativistic

$$\Gamma_c = \frac{4}{3}$$

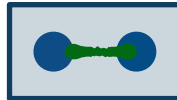


$$\Gamma_{\text{th}} = \frac{4}{3}$$

Unitary

$$k_{FaS} \gg 1$$

$$\Gamma_c = \frac{5}{3}$$



$$\Gamma_{\text{th}} = \frac{5}{3}$$

Cold partThermal partNon-relativistic

$$\Gamma_c = \frac{5}{3}$$



$$\Gamma_{\text{th}} = \frac{5}{3}$$

Relativistic

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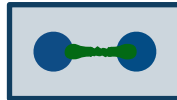


$$\Gamma_{\text{th}} = \frac{4}{3}$$

Unitary

$$k_{FAS} \gg 1$$

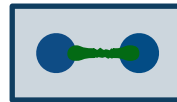
$$\Gamma_c = \frac{5}{3}$$



$$\Gamma_{\text{th}} = \frac{5}{3}$$

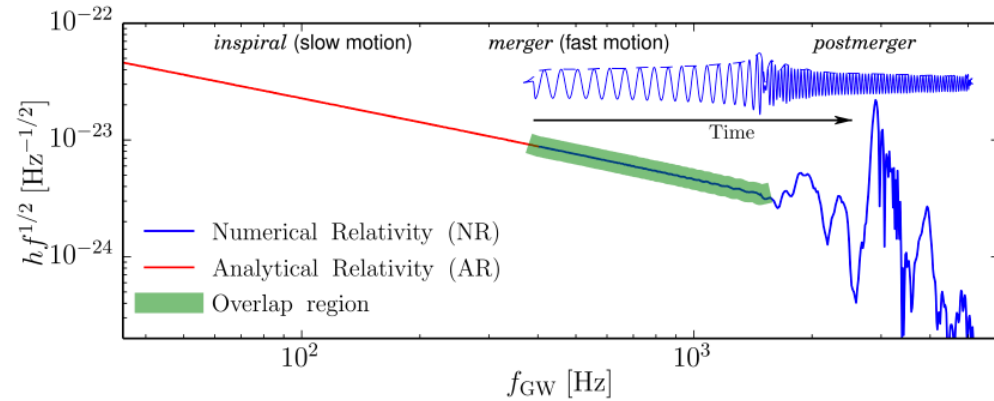
Mean-field/Fermi liquid

$$\Gamma_c = \Gamma_c(\epsilon)$$

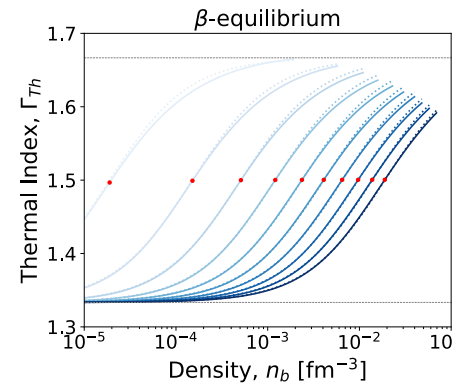
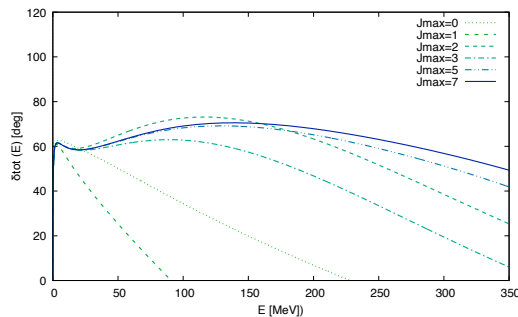


$$\Gamma_{\text{th}}^{m^*} = \frac{5}{3} - \frac{n}{m^*} \frac{\partial m^*}{\partial \rho}$$

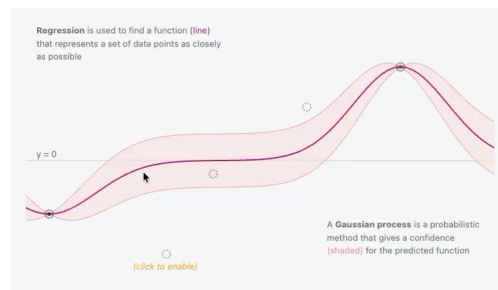
- Motivation



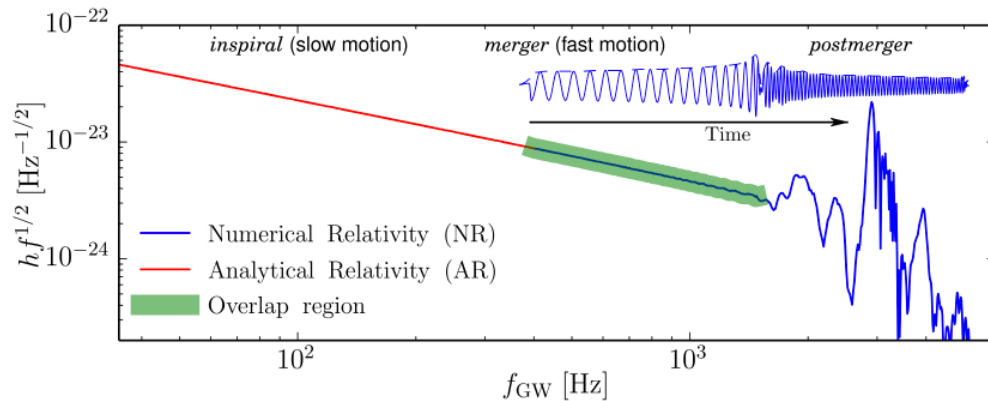
- Virial approximation & thermal effects in BNS



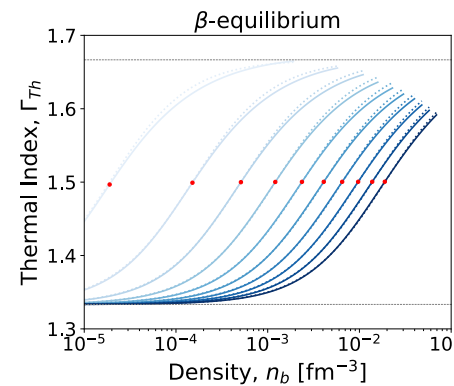
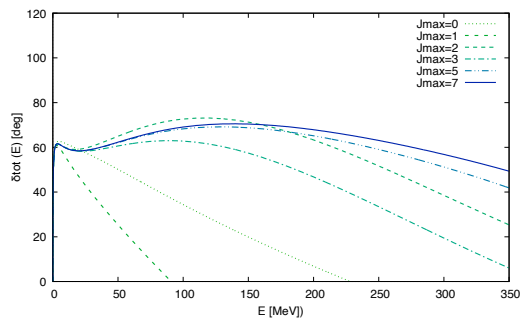
- Gaussian processes



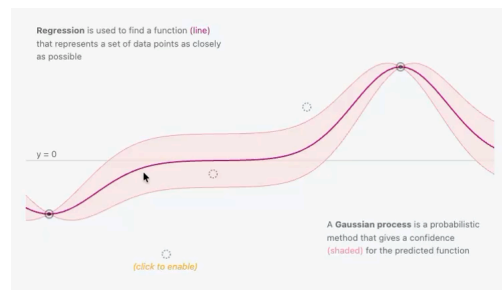
- Motivation



- Virial approximation & thermal effects in BNS



- Gaussian processes



Expansion

$$P = \frac{2T}{\lambda^3} [z + z^2 b_2 + z^3 b_3 + O(z^4)]$$

$$n = \frac{2}{\lambda^3} [z + 2z^2 b_2 + 3z^3 b_3]$$

$$\epsilon = \frac{3}{2}P + \frac{2T^2}{\lambda^3} [z^2 b'_2 + z^3 b'_3]$$

$$z = e^{\mu/T} \quad b'_n = \frac{\partial b_n}{\partial T}$$

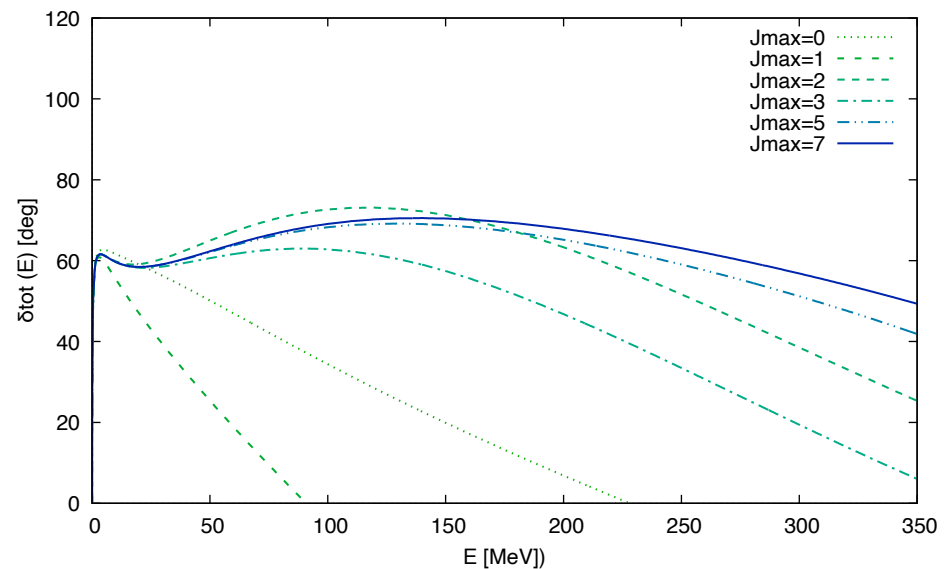
- Systematic expansion
- Improvable order-by-order

Virial coefficients

$$b_2(T) = \frac{1}{\sqrt{2\pi T}} \int_0^\infty dE e^{-E/T} \delta(E) - 2^{-5/2}$$

$$b_3(T) = ?$$

Drut et al. Unitary gas

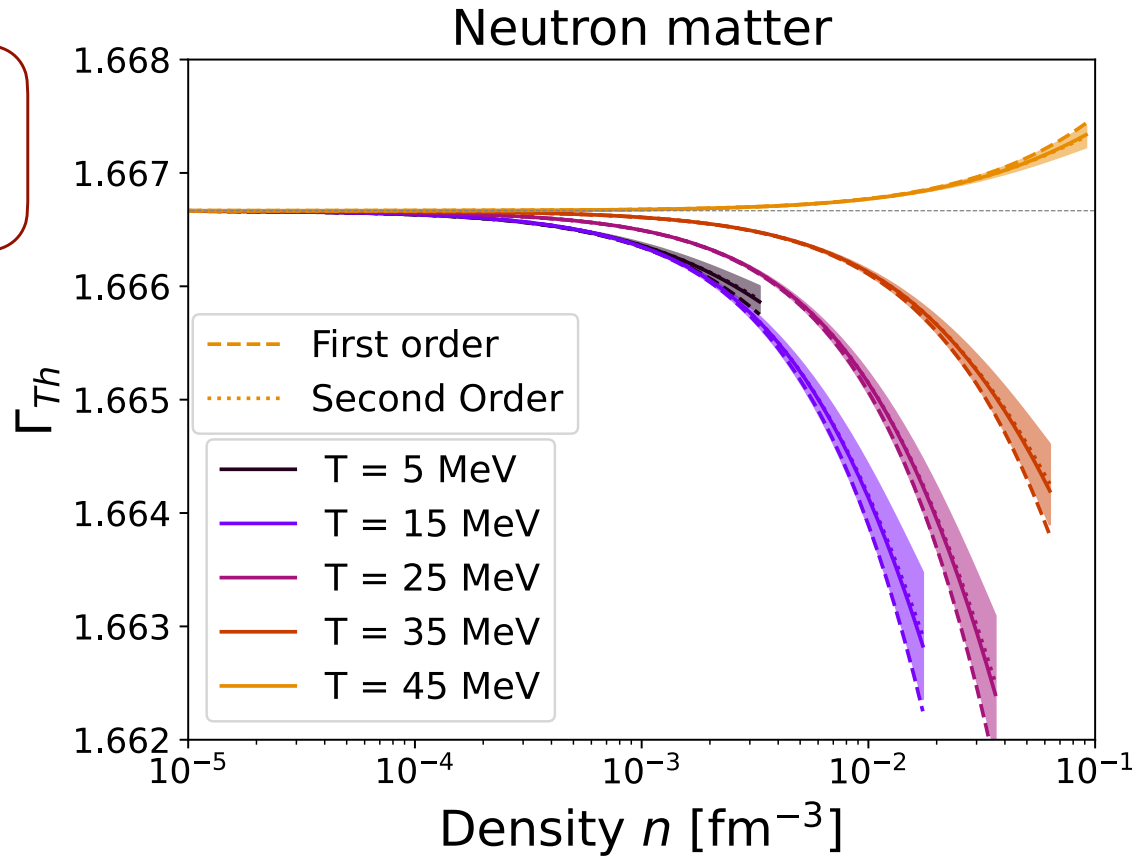


Thermal index: pure neutron matter

$$\Gamma_{th} = \frac{5}{3} + \Gamma_1 z + \Gamma_2 z^2$$

$$\Gamma_1 = -\frac{4}{9} T b'_2 \leq 0$$

$$\Gamma_2 = \frac{2}{3} T b'_2 + b_2 - \frac{b'_3}{b'_2}$$

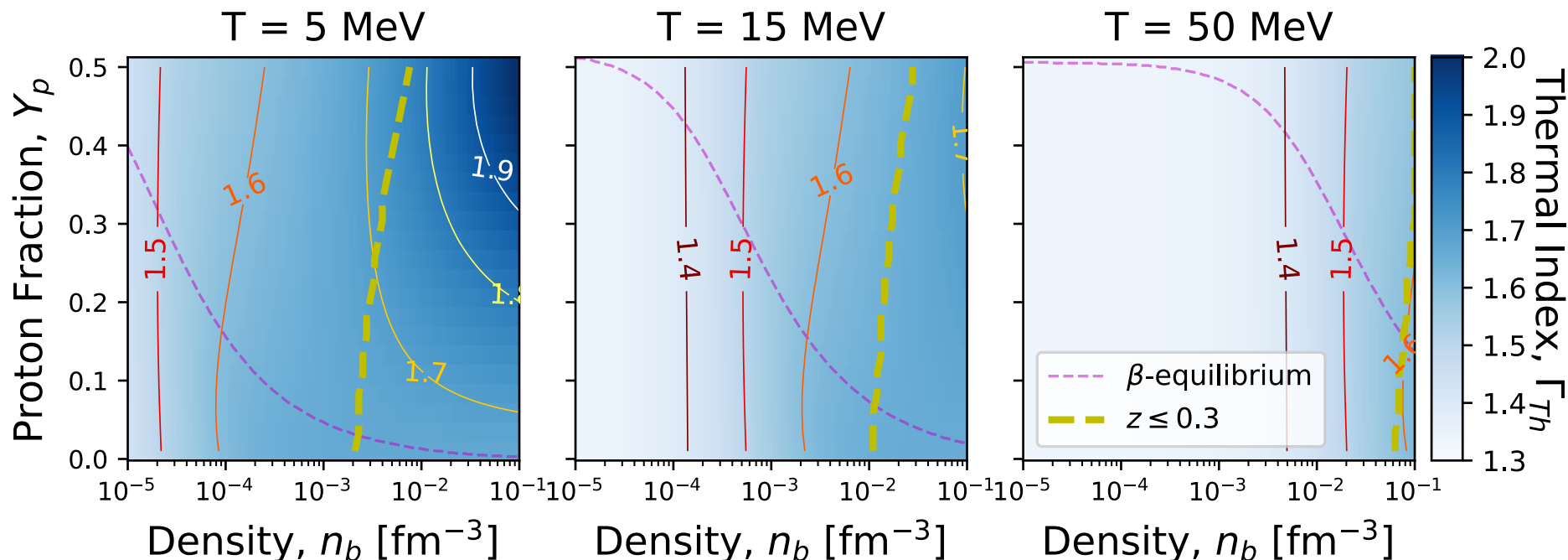


- **Small variation: 0.3%**
- **Small uncertainty: 0.1%**
- **T** dependence mild
- Closeness to **unitary gas**

G Riviuccio @ UV



Thermal index: npe matter



Baryon matter

$$P_{th}^{nuc} = \frac{2T}{\lambda^3} [z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_n z_p b_{np}]$$

Leptons & photons ultrarelativistic

$$\Gamma_{th} = \frac{4}{3}$$

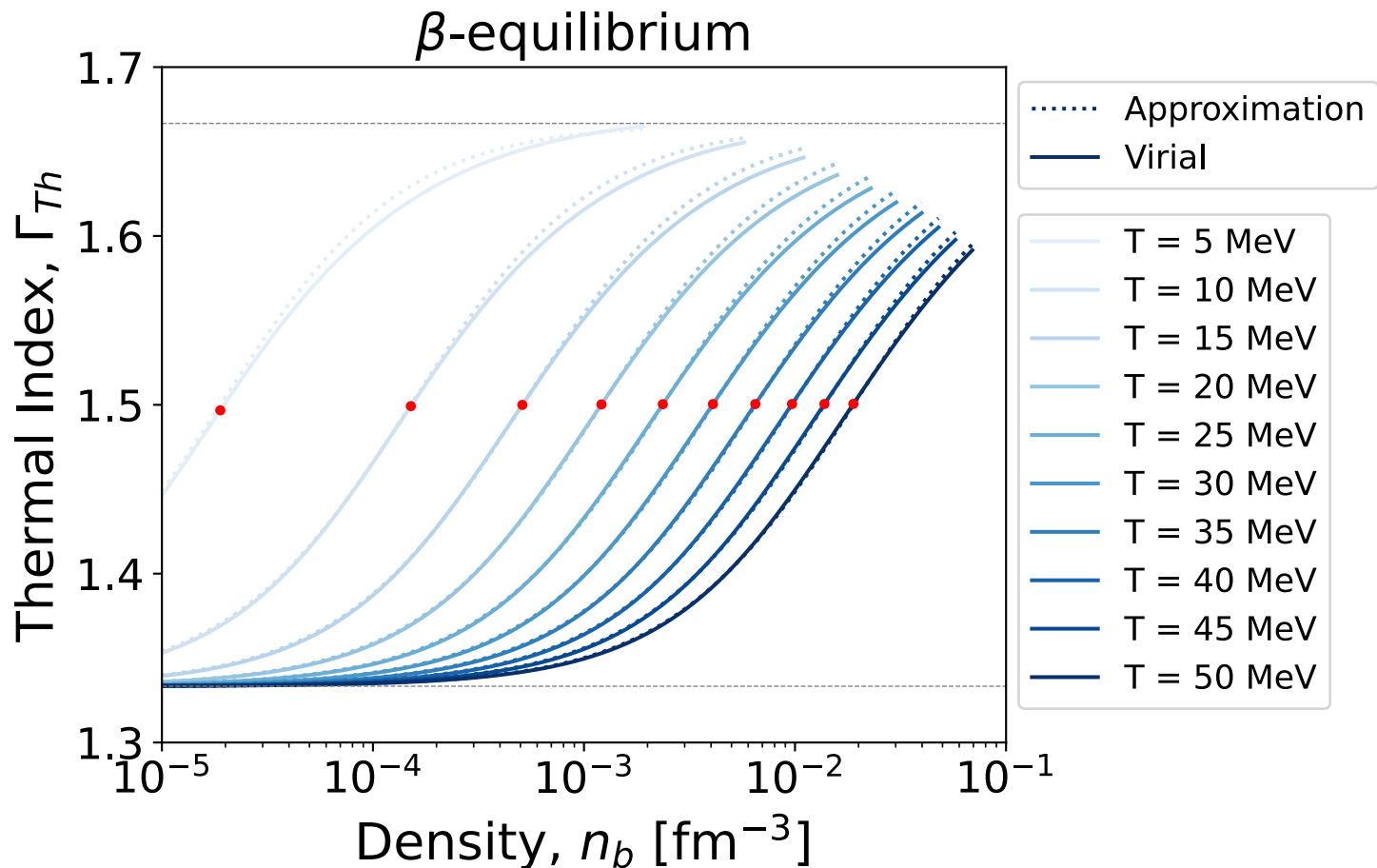
β equilibrium

$$\mu_n = \mu_p + \mu_e$$

Thermal index

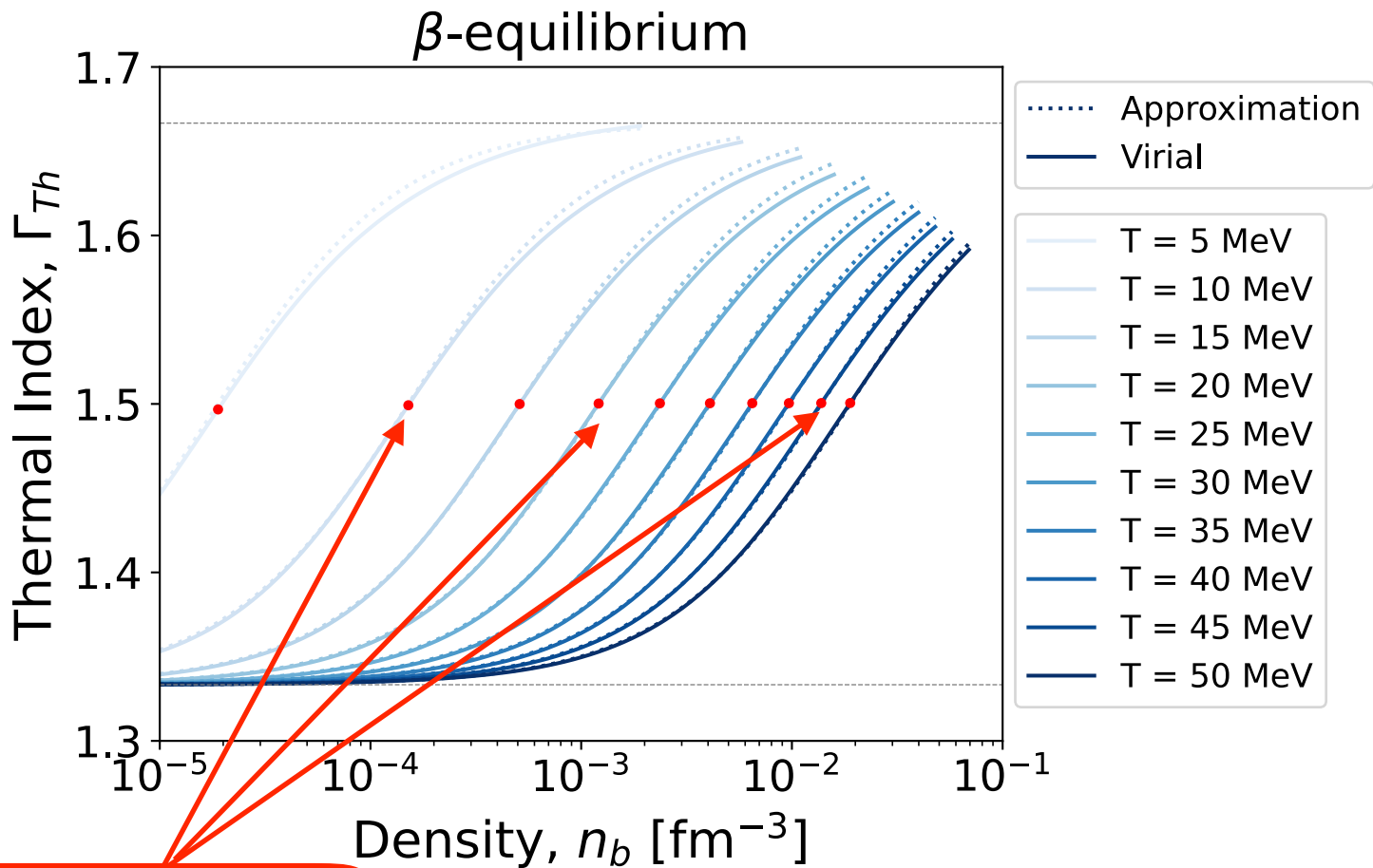
$$\Gamma_{th} = 1 + \frac{P_{th}^{nuc} + P_{th}^{lep} + P_{th}^{\gamma}}{\epsilon_{th}^{nuc} + \epsilon_{th}^{lep} + \epsilon_{th}^{\gamma}}$$

Thermal index: parametrization



$$\Gamma_{th} = \frac{4}{3} + \frac{1}{3} \frac{n_b}{n_b + n_{inf}}$$

Thermal index: parametrization

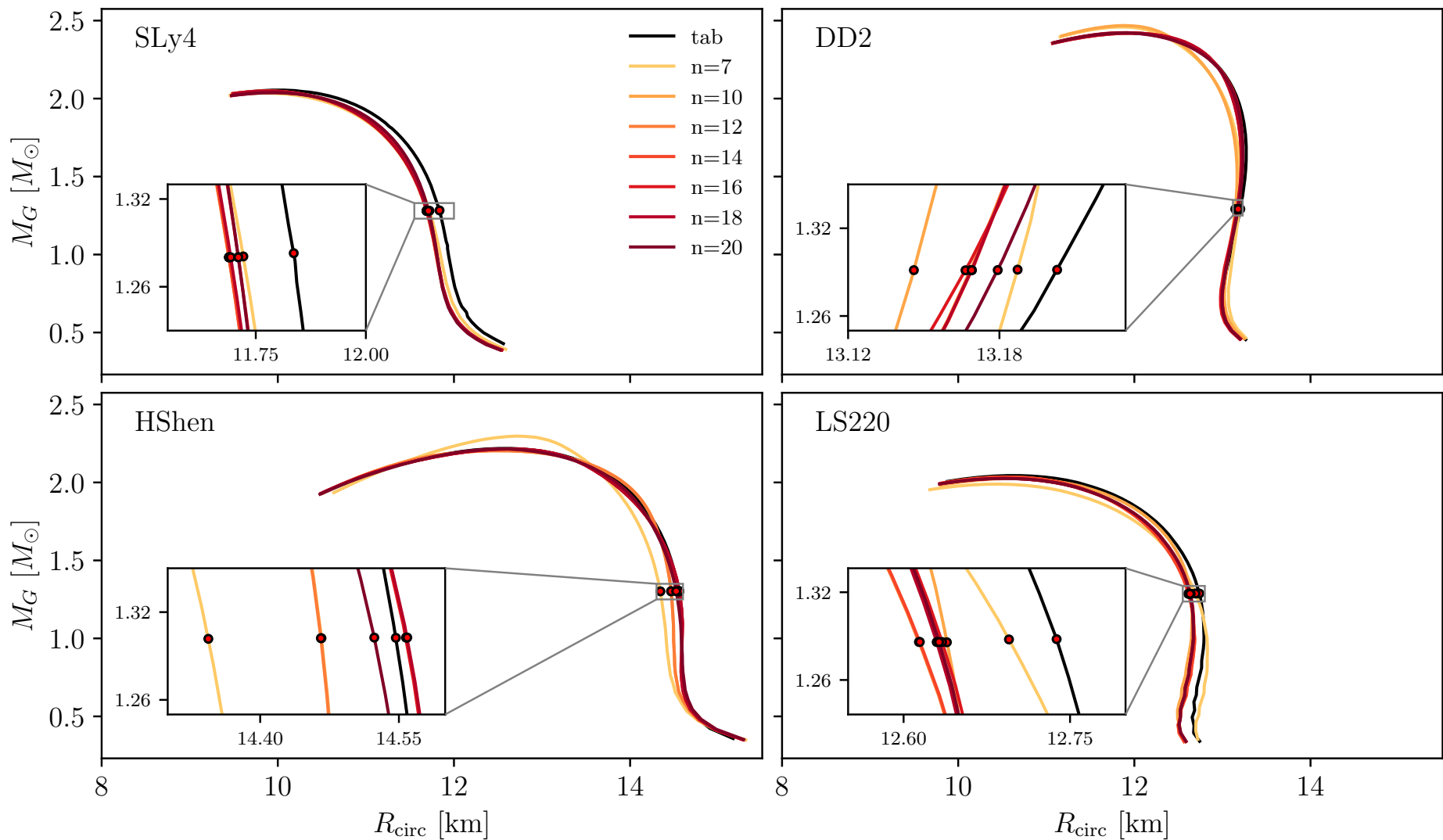


Regime change

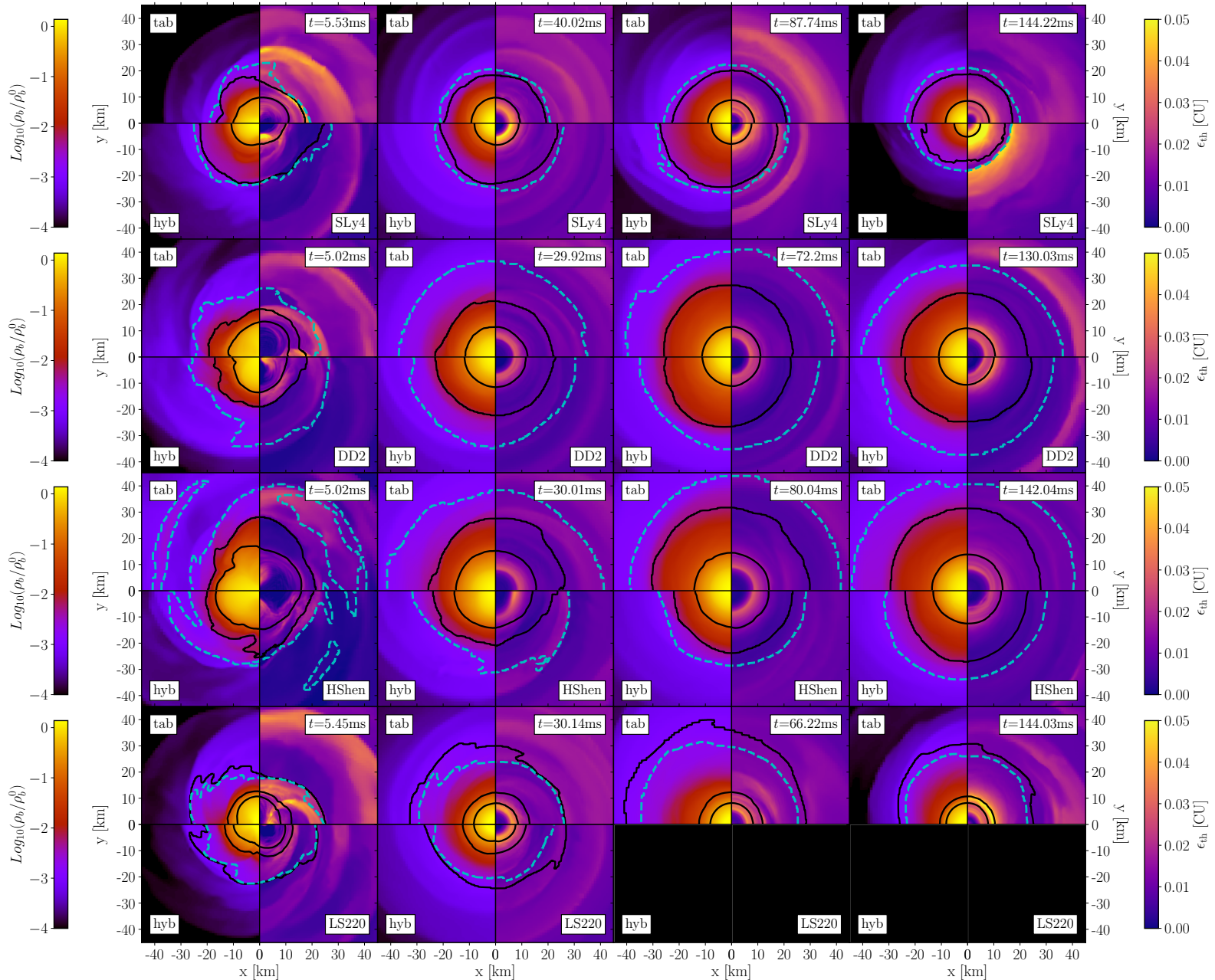
$$P_{th}^{nuc} \approx P_{th}^{lep}$$

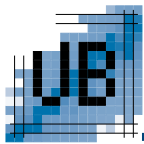
$$n_{inf} = 1.5 \times 10^{-4} \left(\frac{T}{10 \text{ MeV}} \right)^3 \text{ fm}^{-3}$$

$$\Gamma_{th} = \frac{4}{3} + \frac{1}{3} \frac{n_b}{n_b + n_{inf}}$$

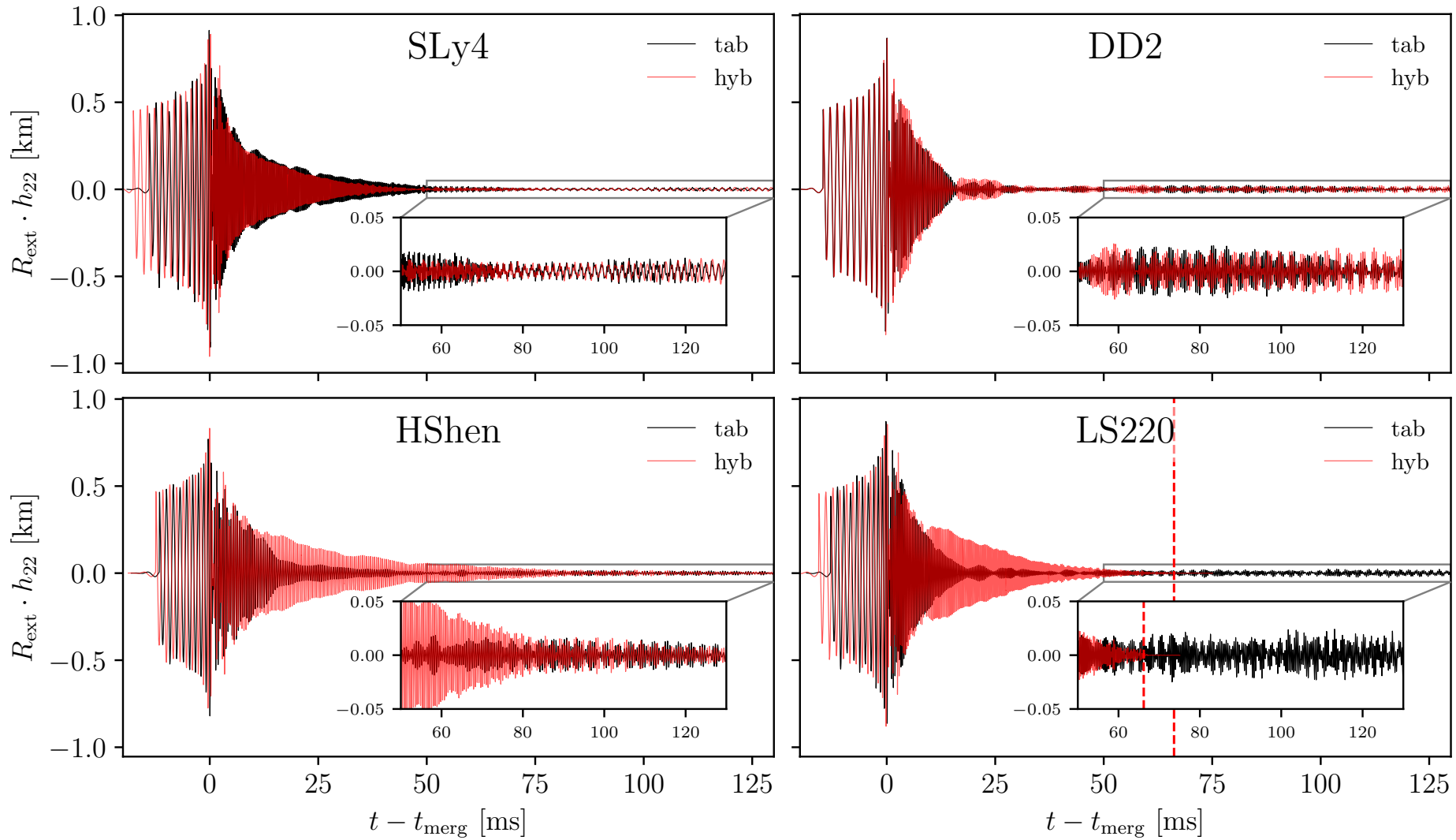


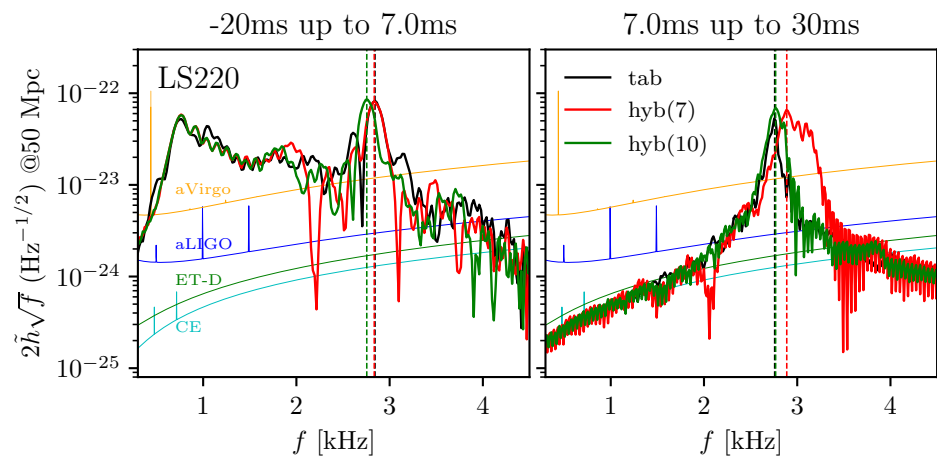
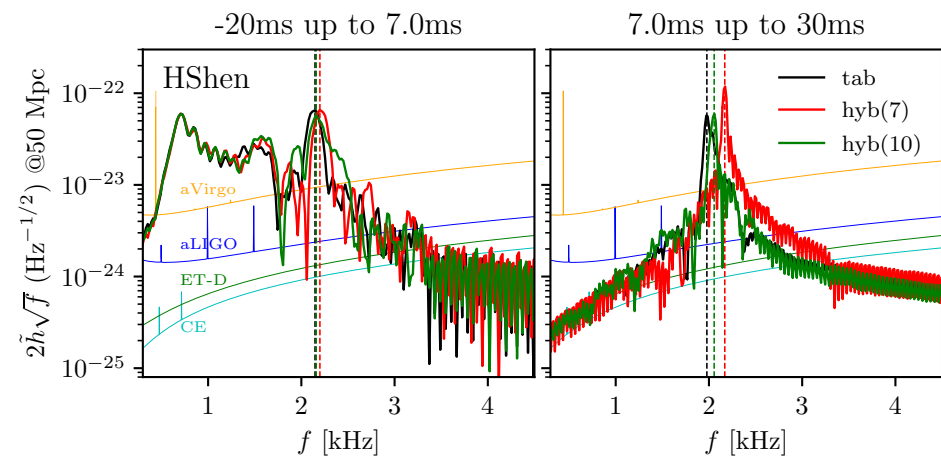
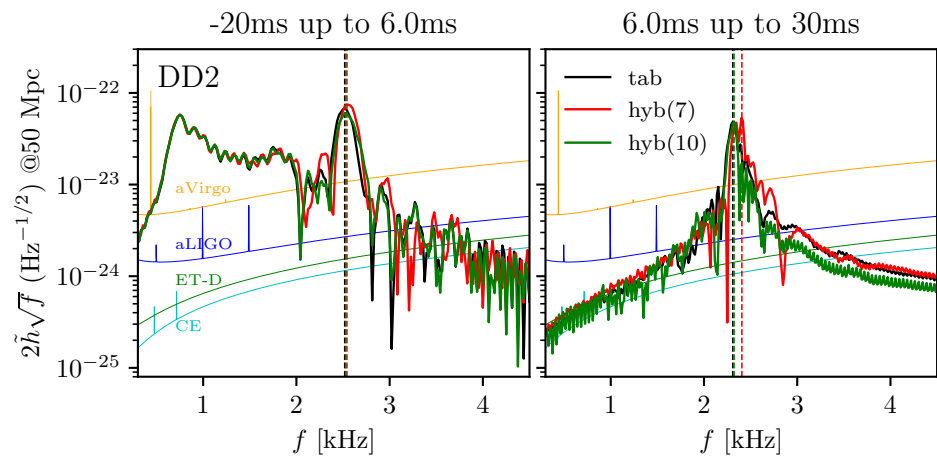
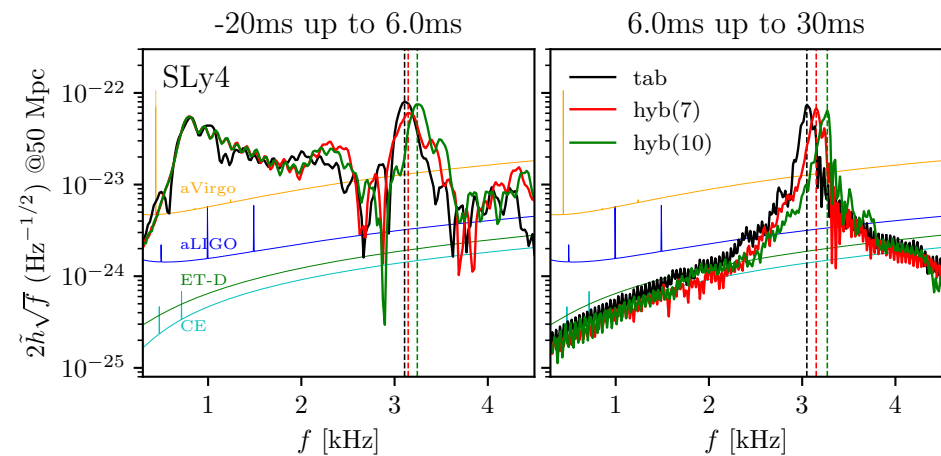
Density and temperature





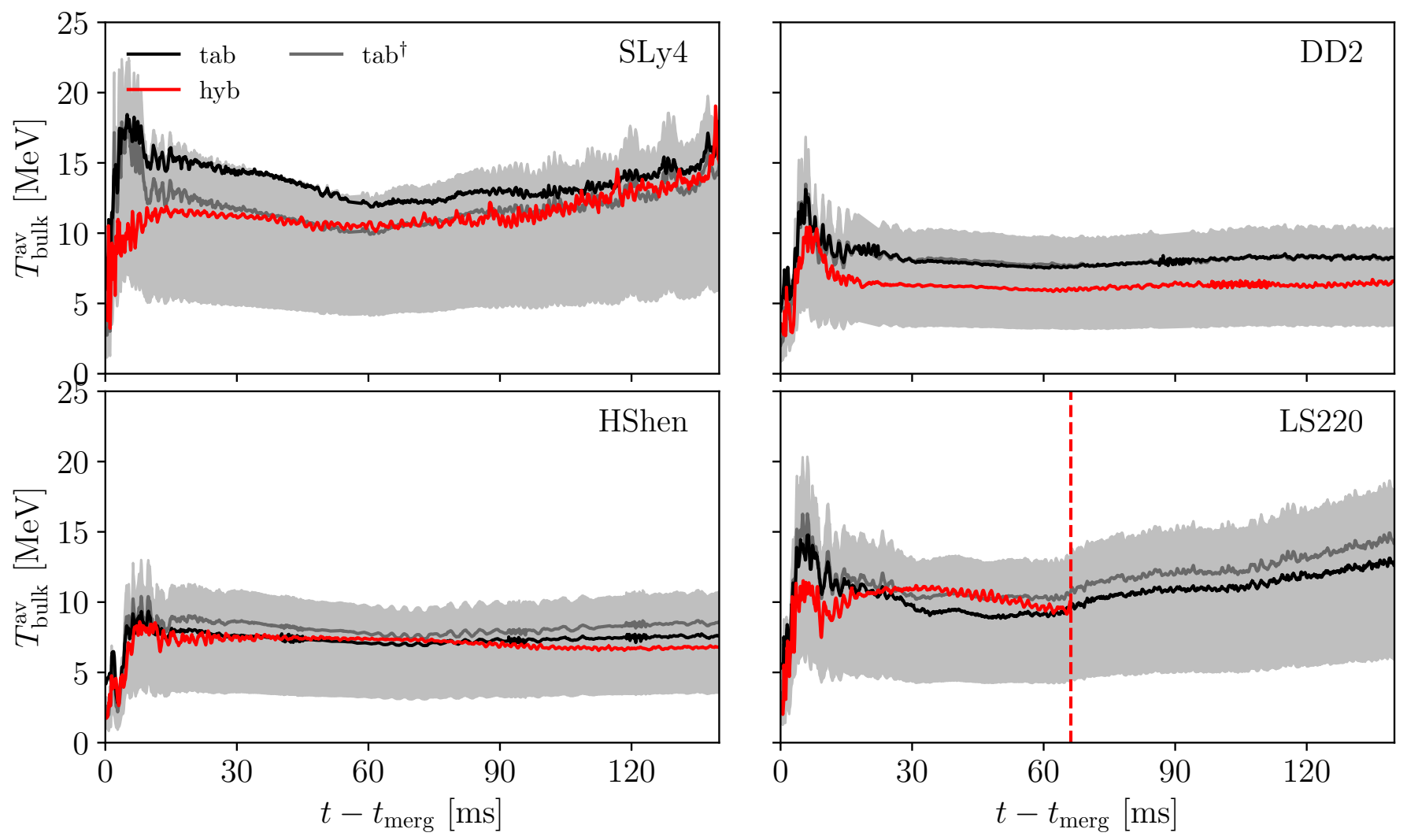
GW emission: late times

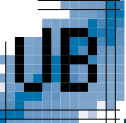




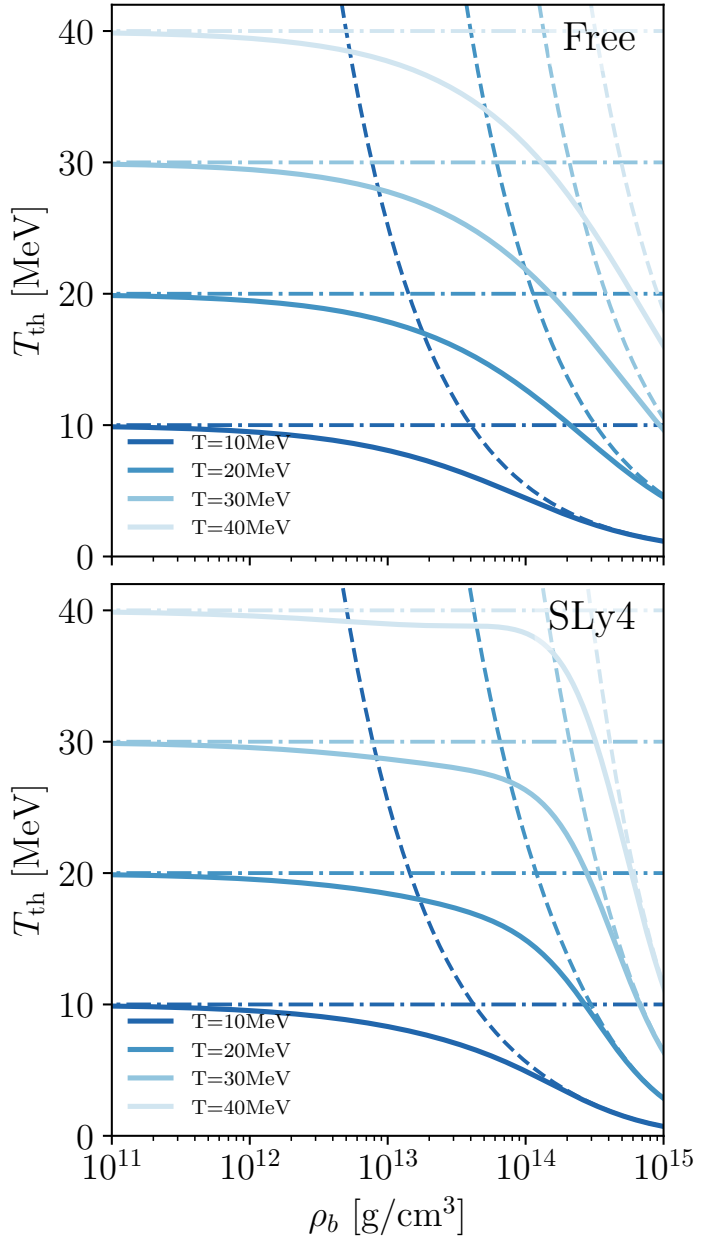
Temperature estimate

$$T_{hyb} = (\Gamma_{th} - 1)\epsilon_{hyb}$$





Estimate error in T



$$e(\rho, T) = e(\rho, T = 0) + e_{th}(\rho, T)$$

$$p(\rho, T) = p(\rho, T = 0) + (\Gamma_{th} - 1) \rho e_{th}(\rho, T)$$

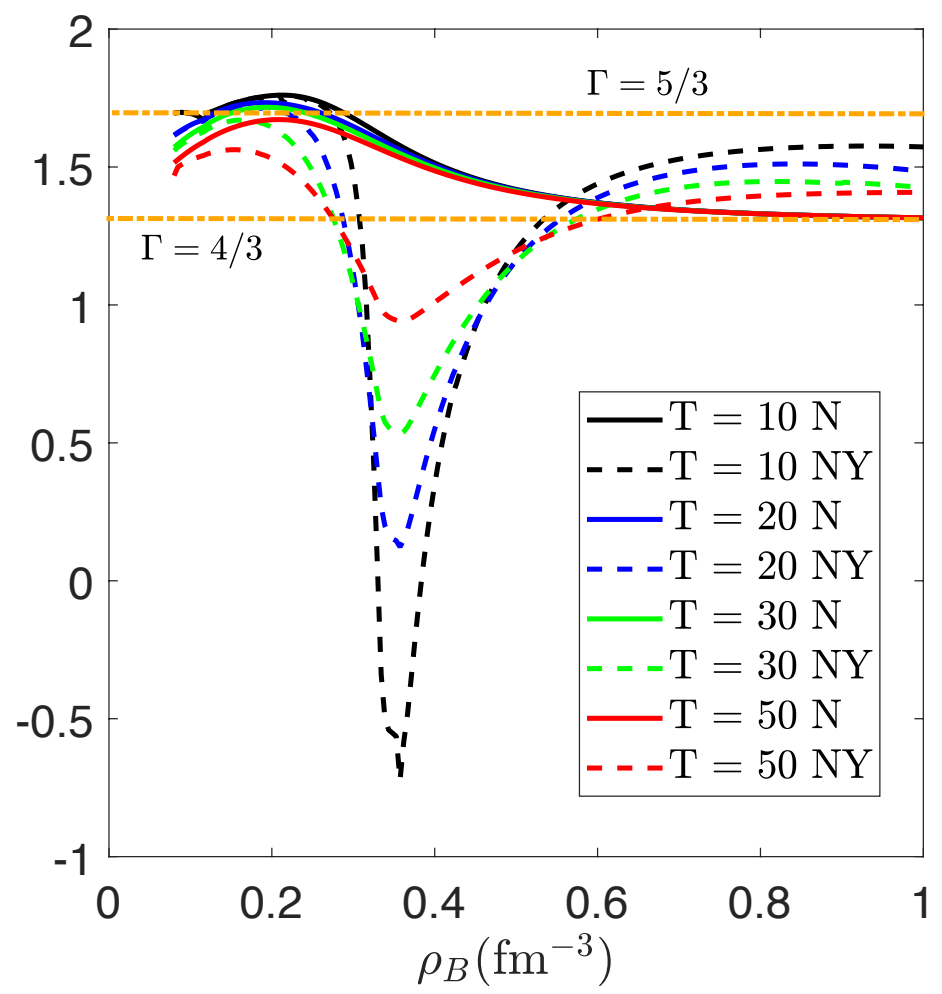
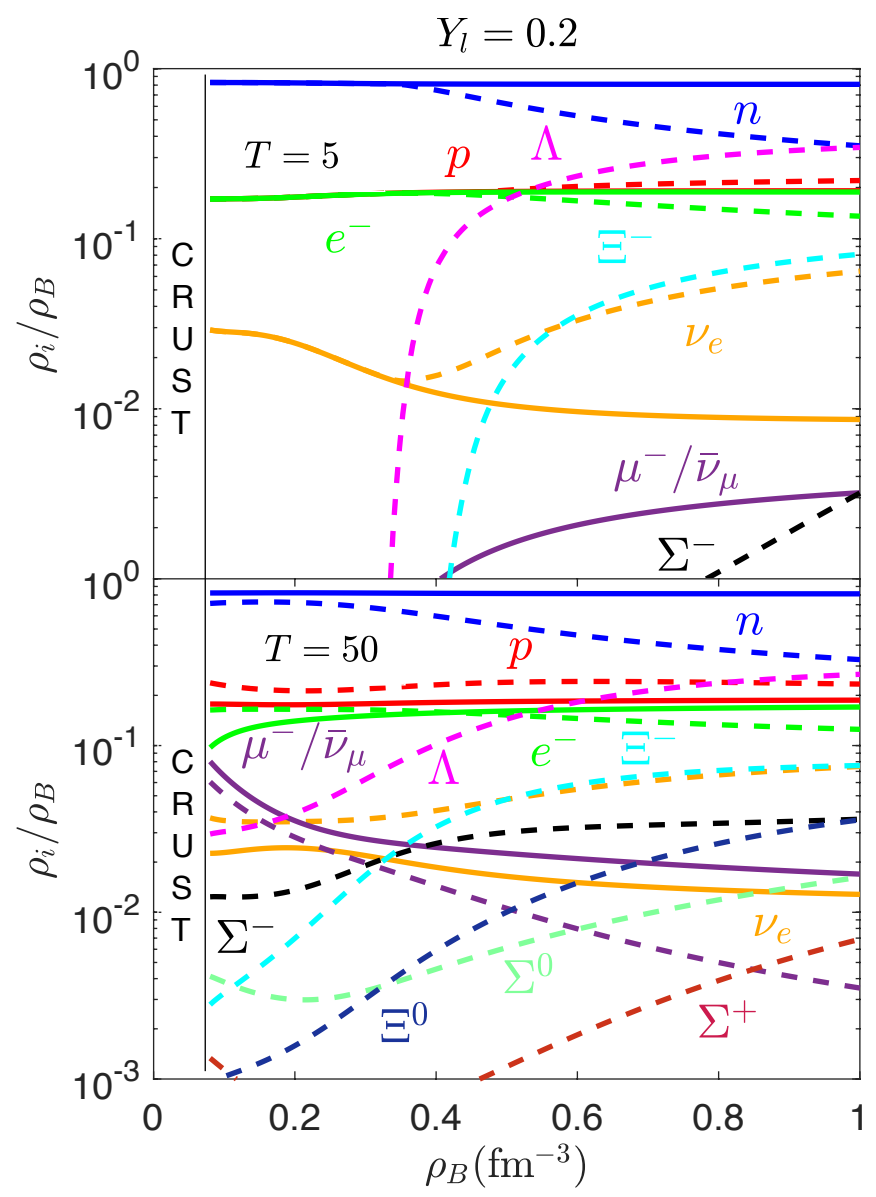
$$T_{th} = (\Gamma_{th} - 1) e_{th}(n, T)$$

Ideal Gas $T_{th} = \left(\frac{5}{3} - 1\right) \frac{3}{2} T = T$

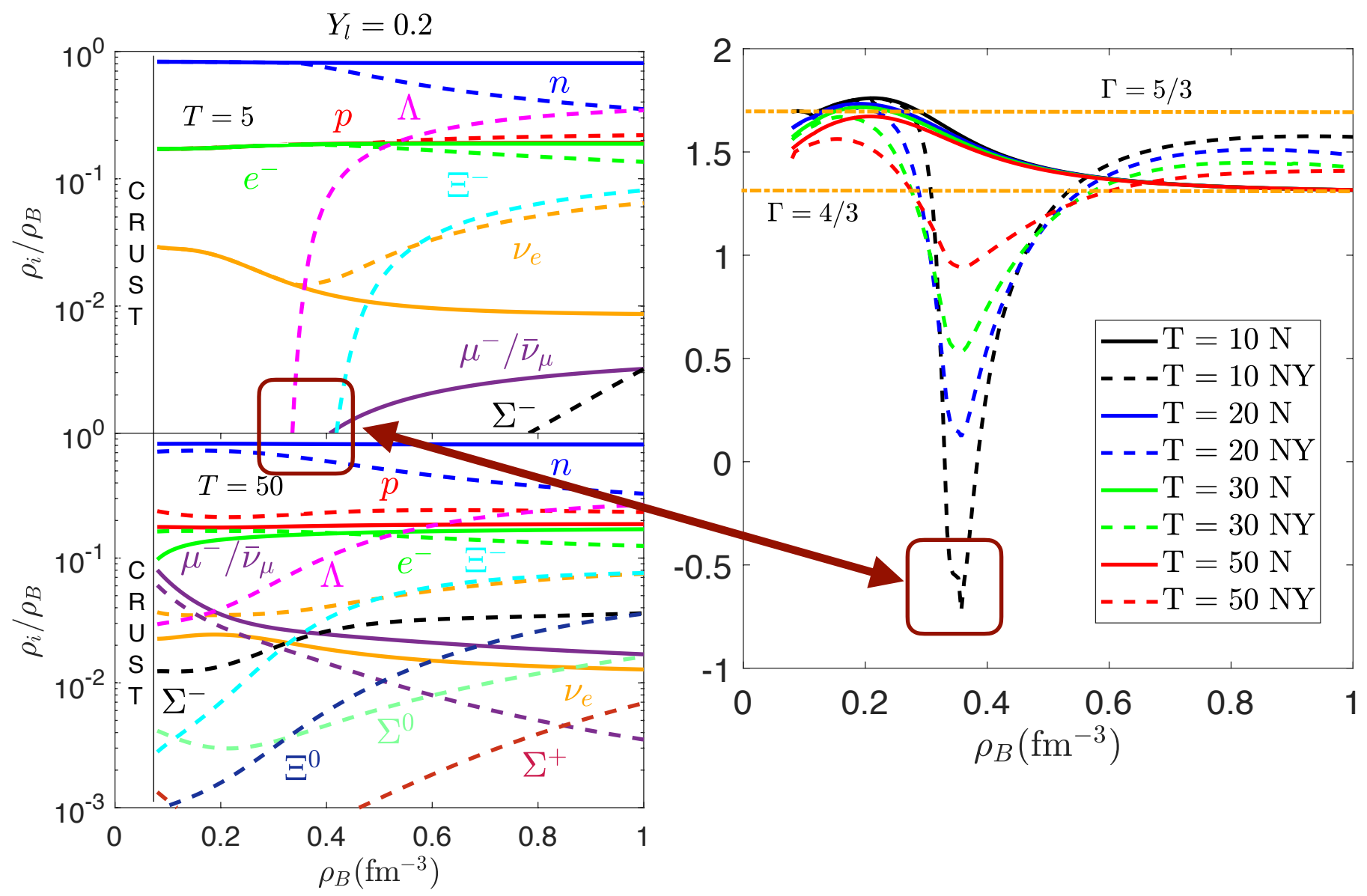
Degenerate Gas

$$T_{th} = \frac{\pi^2 T^2}{6 \varepsilon_F^*} \left[1 - \frac{3}{2} \frac{n}{m_n^*} \frac{\partial m_n^*}{\partial n} \right]$$

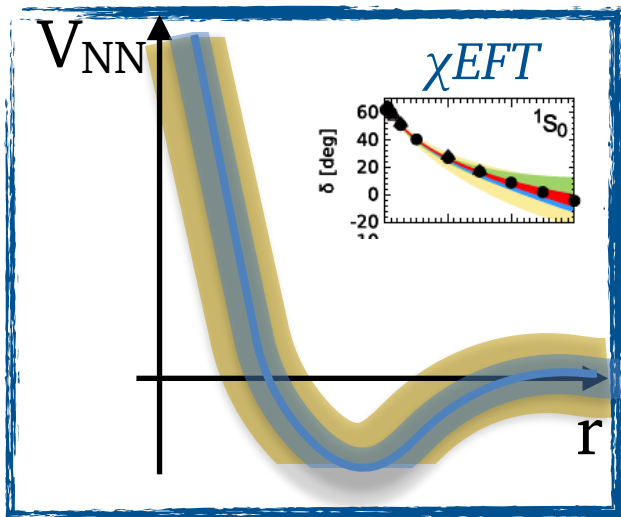
Multicomponent systems: hyperons



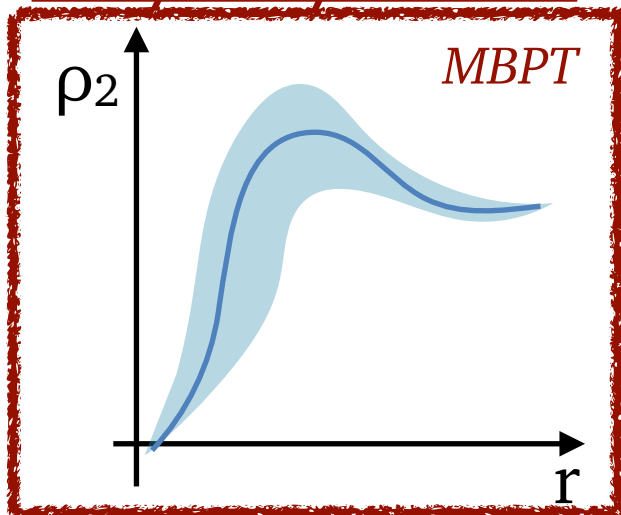
Multicomponent systems: hyperons



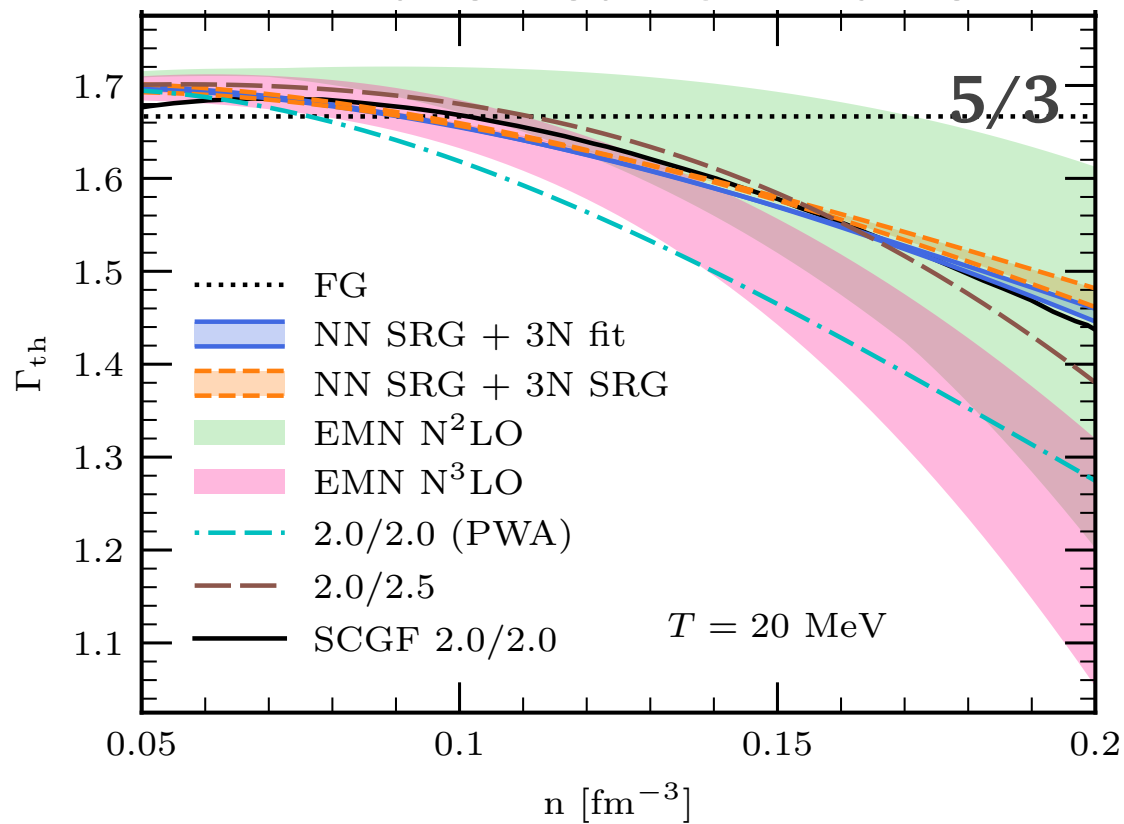
Hamiltonian



Many-body method

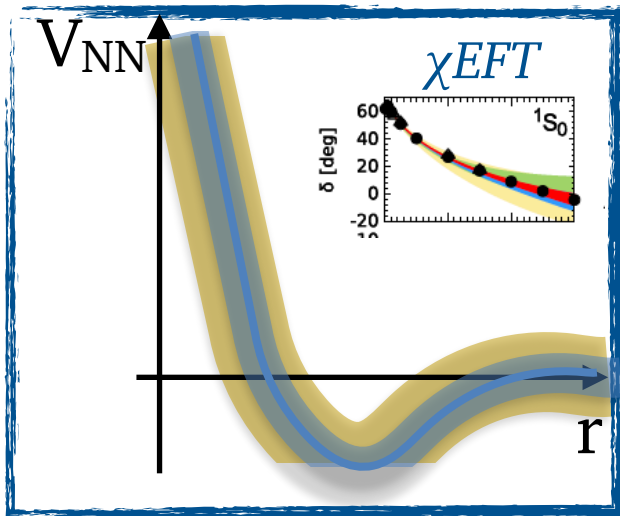


Pure neutron matter

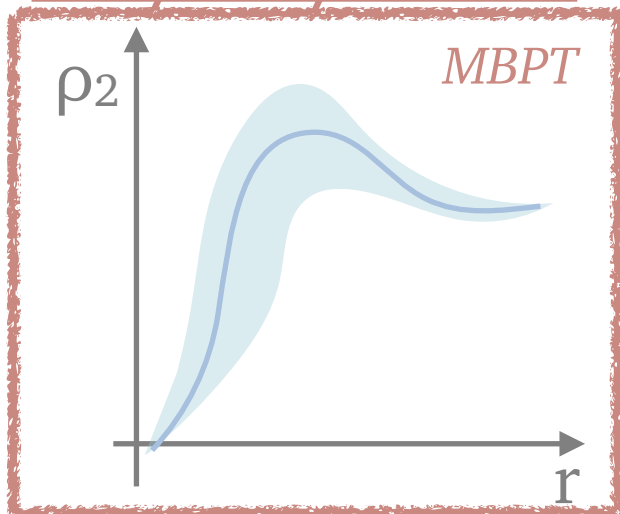


Keller, Wellenhofer, Hebeler & Schwenk *Phys Rev C* **103**, 055806 (2021)

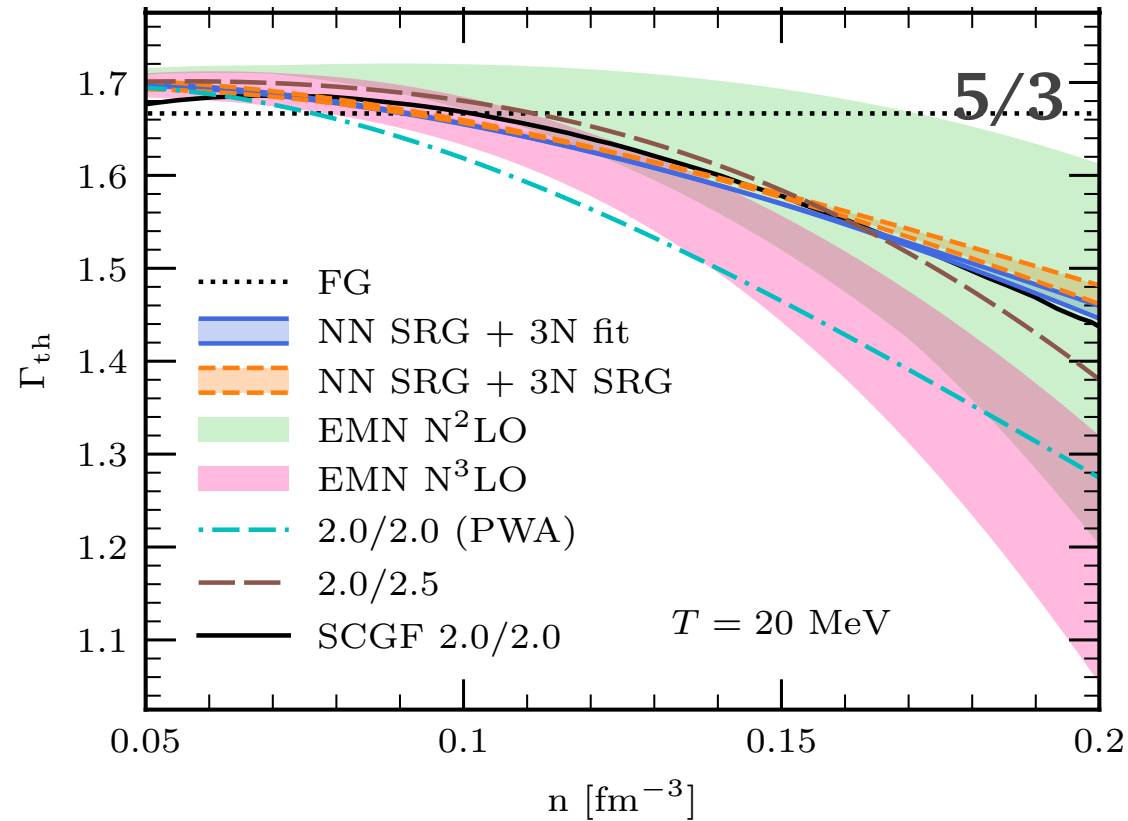
Hamiltonian



Many-body method

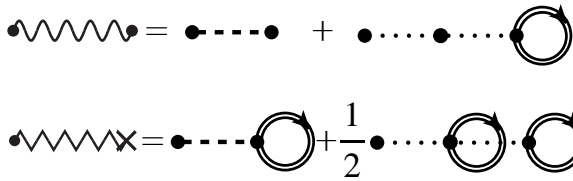


Pure neutron matter

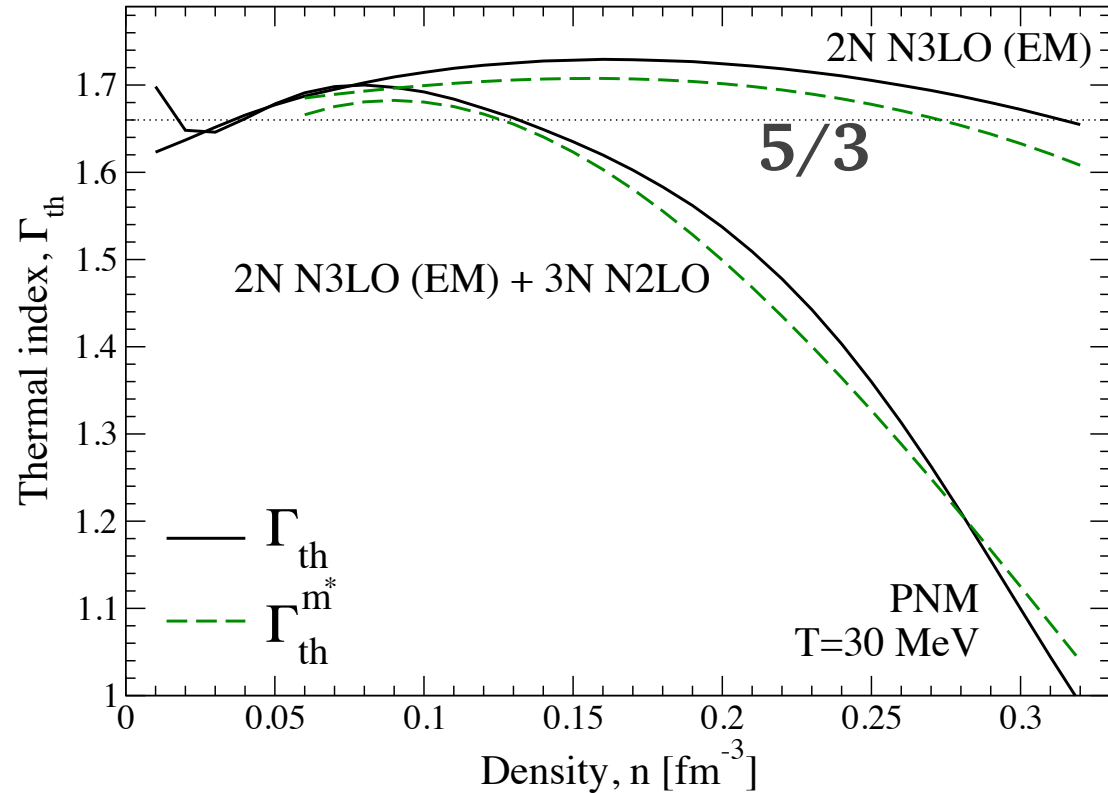


Keller, Wellenhofer, Hebeler & Schwenk *Phys Rev C* **103**, 055806 (2021)

2N & 3N forces



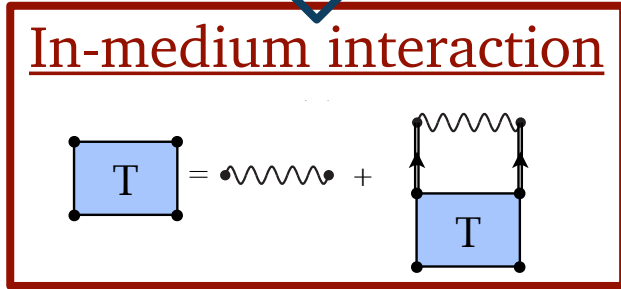
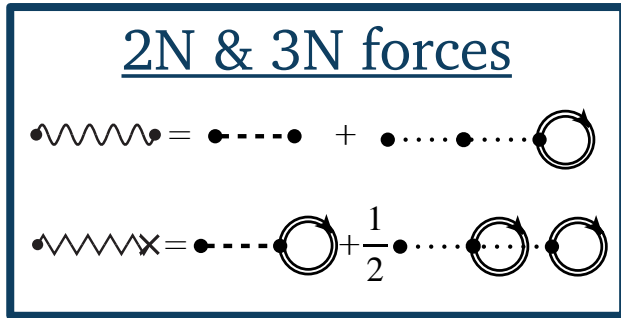
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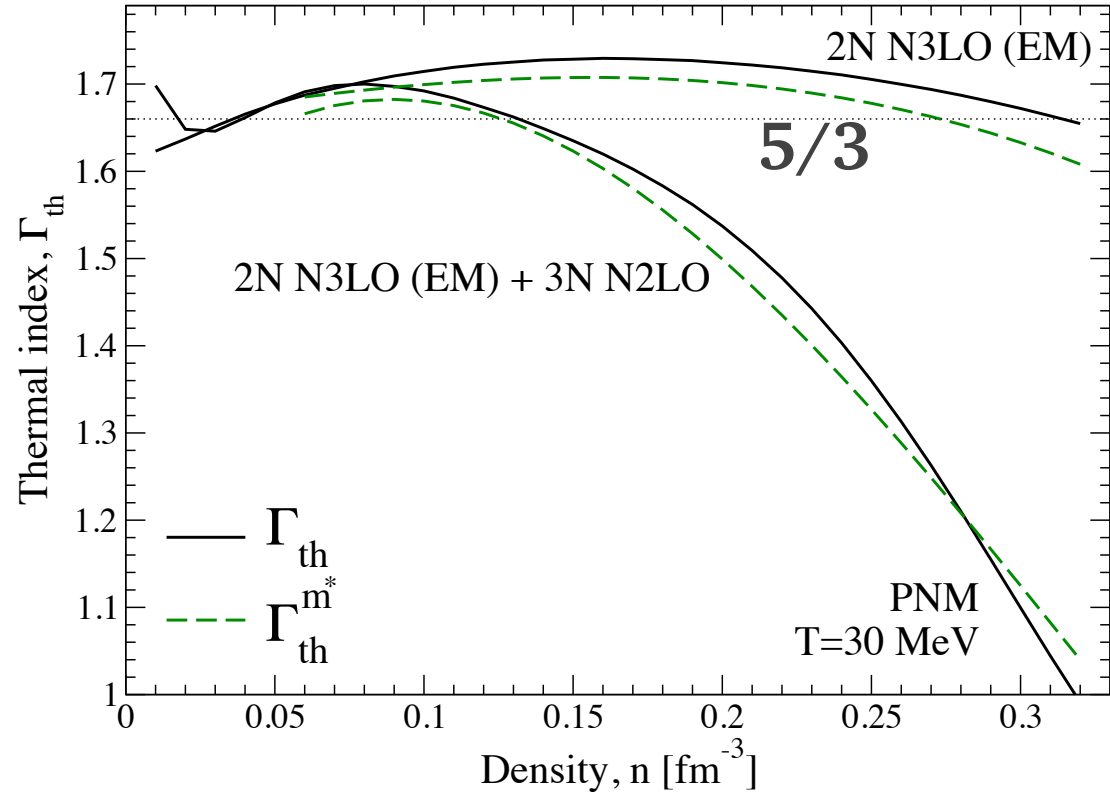
Carbone & Schwenk *Phys Rev C* **100** 025805 (2019)

Carbone, Polls & Rios, *Phys Rev C* **88** 044302 (2014)
 Rios, *Front. Phys.* **8** 387 (2020), arxiv:2006.10610

Ab initio thermal index



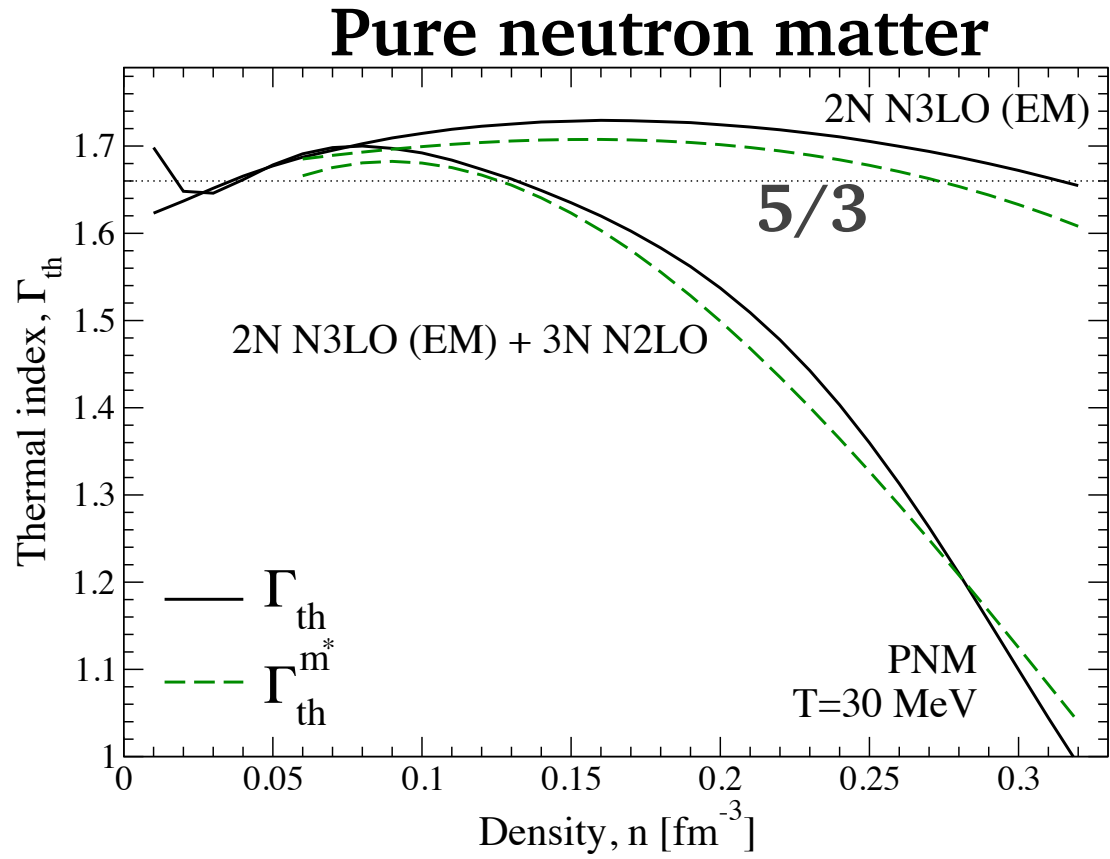
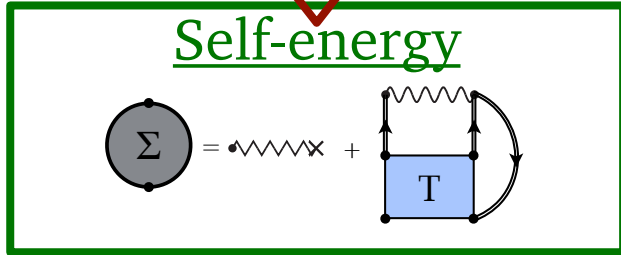
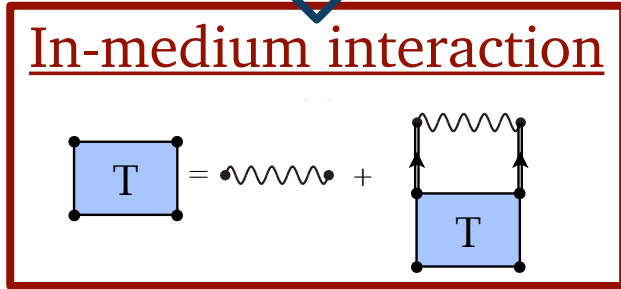
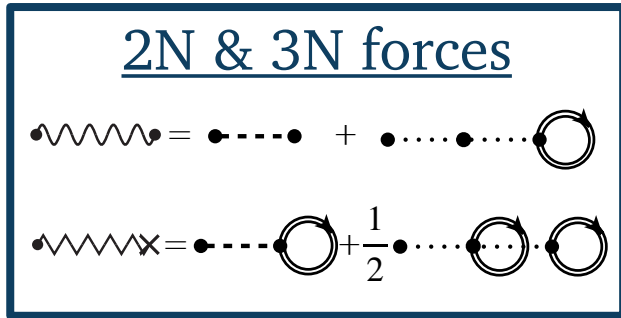
Pure neutron matter



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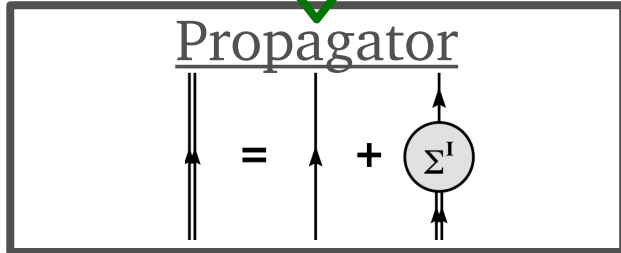
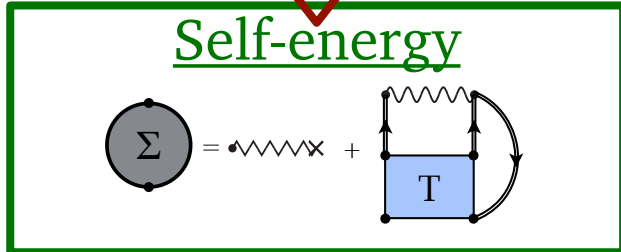
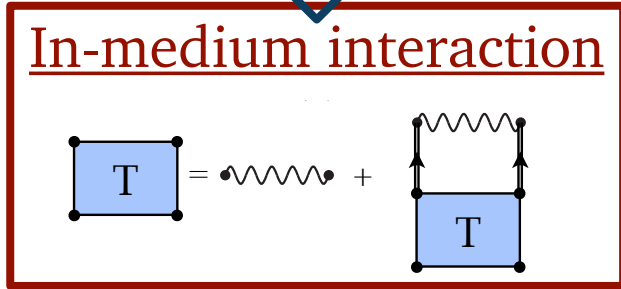
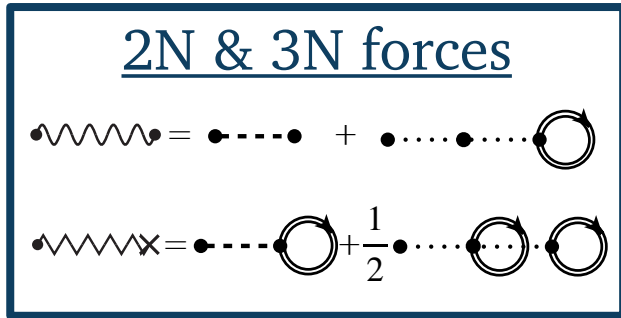
Ab initio thermal index



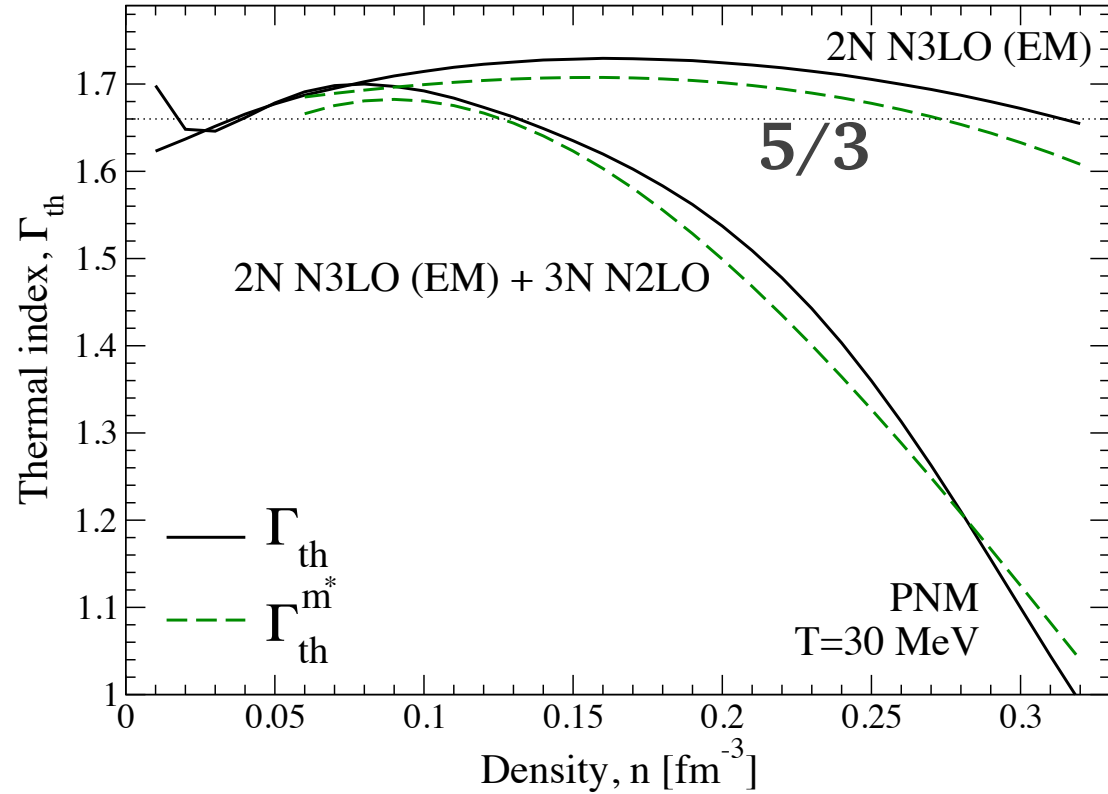
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Ab initio thermal index



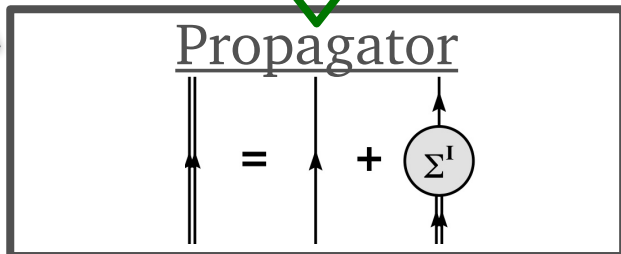
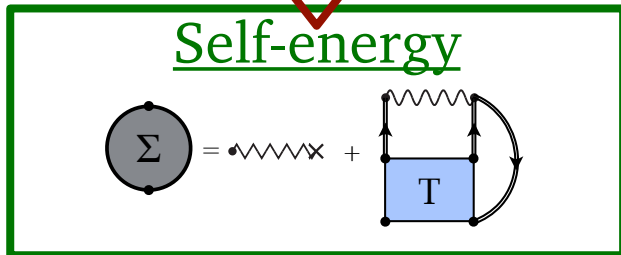
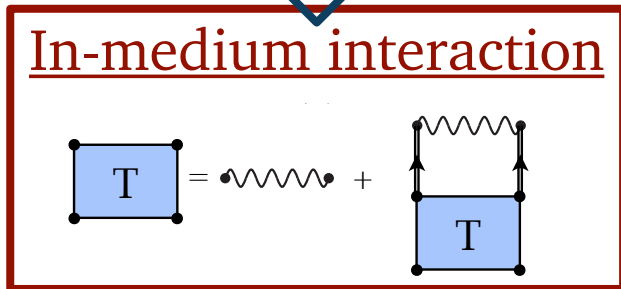
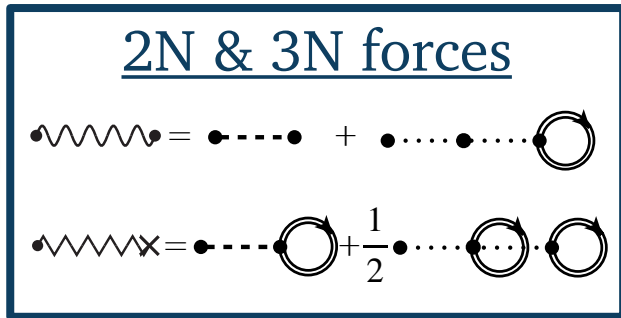
Pure neutron matter



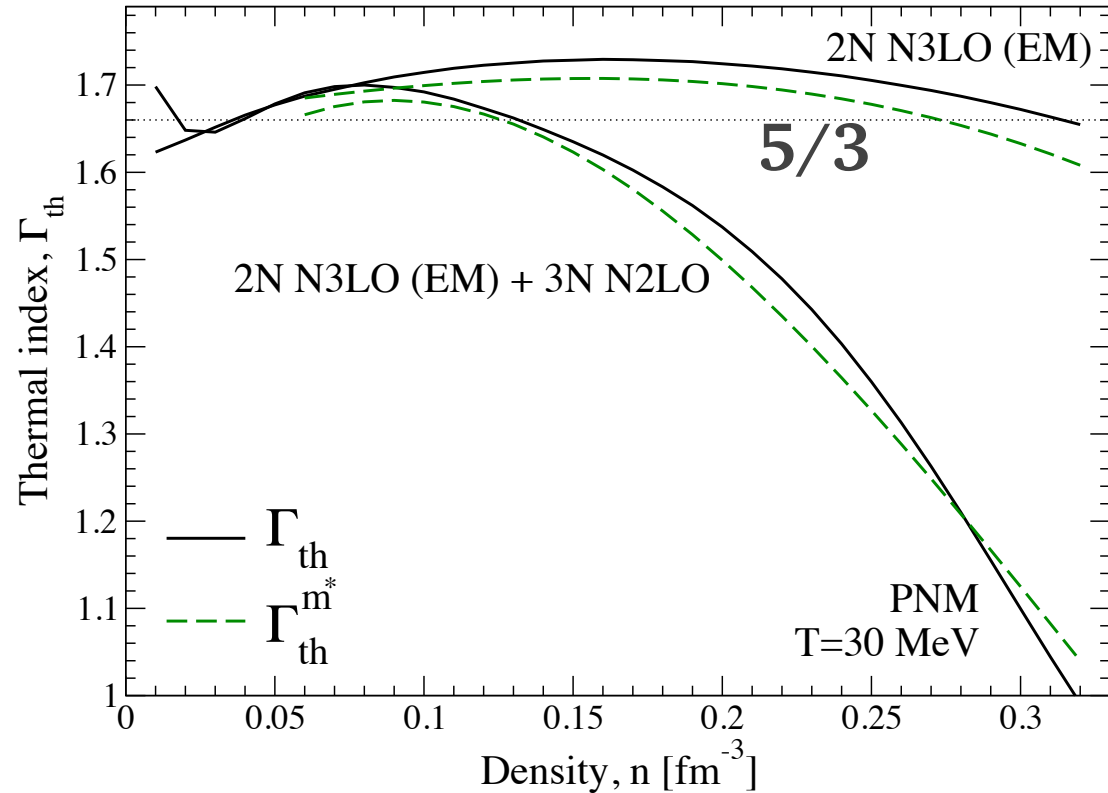
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Ab initio thermal index



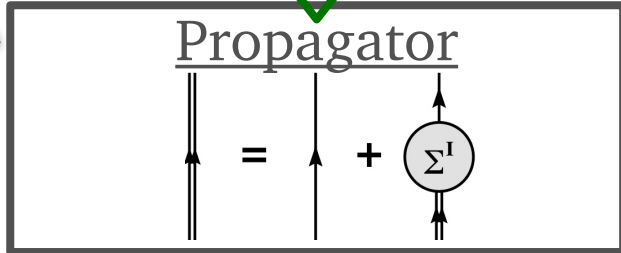
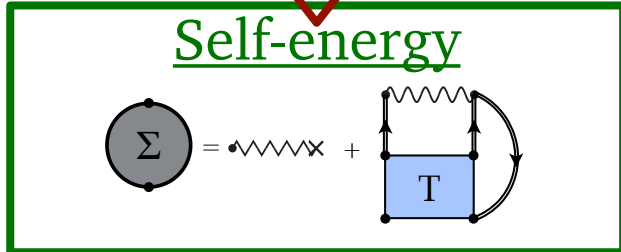
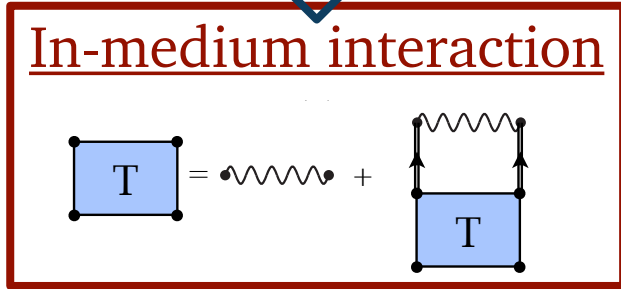
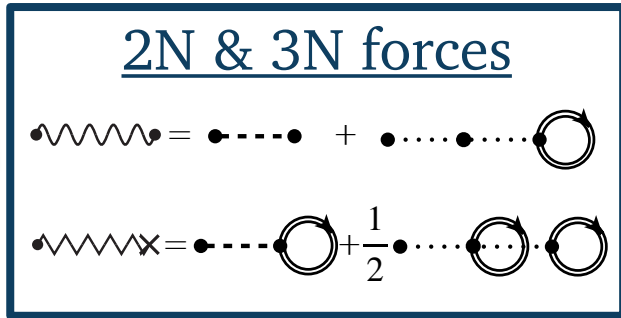
Pure neutron matter



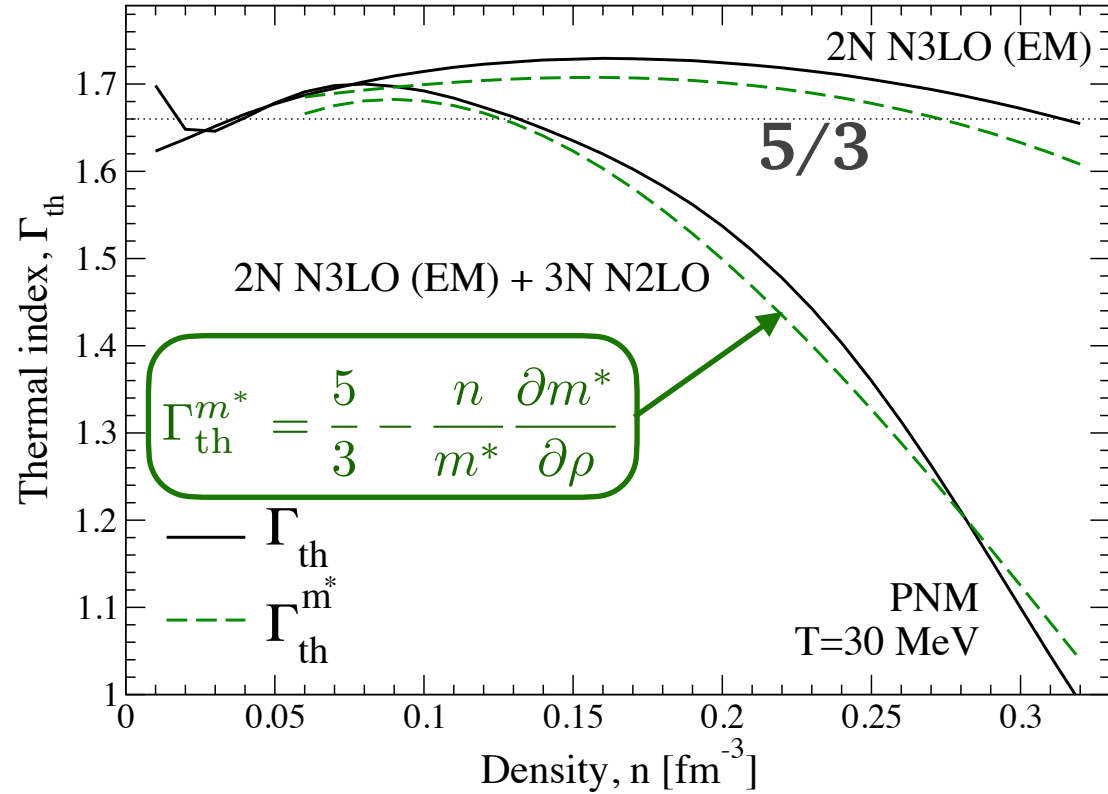
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Ab initio thermal index



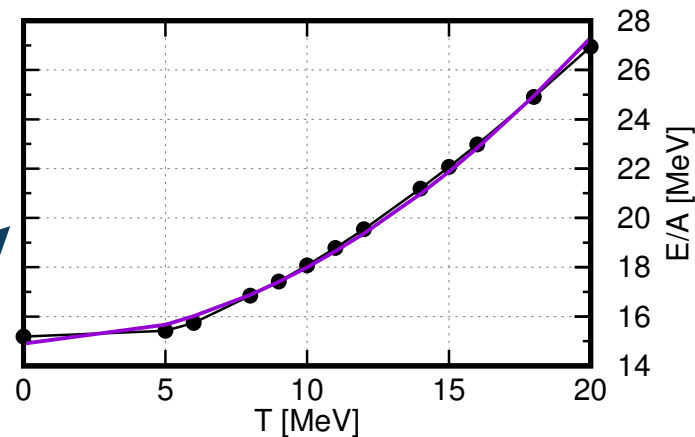
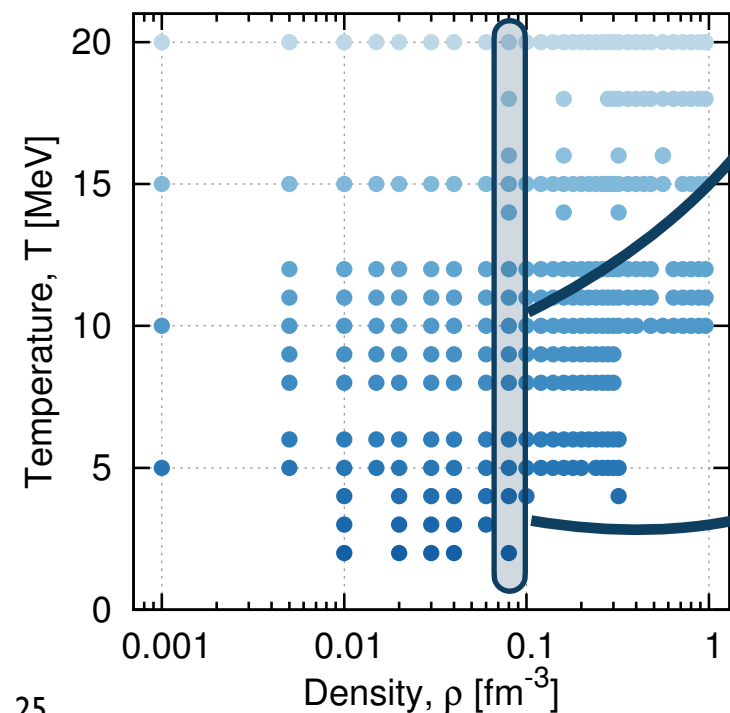
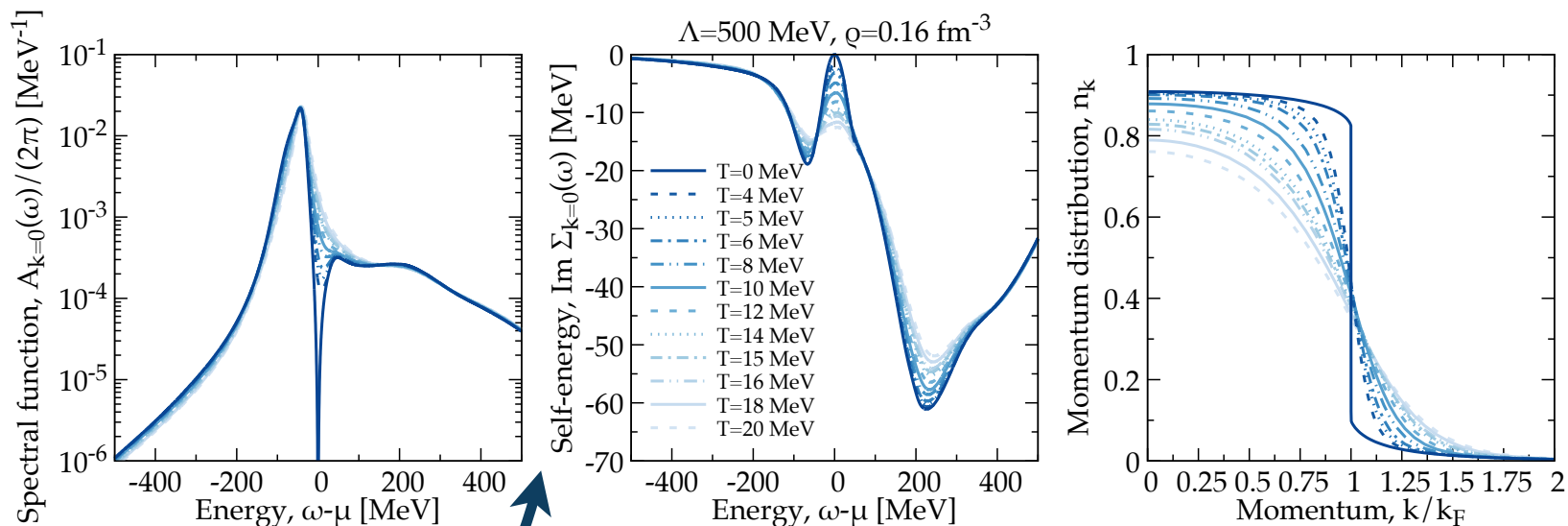
Pure neutron matter

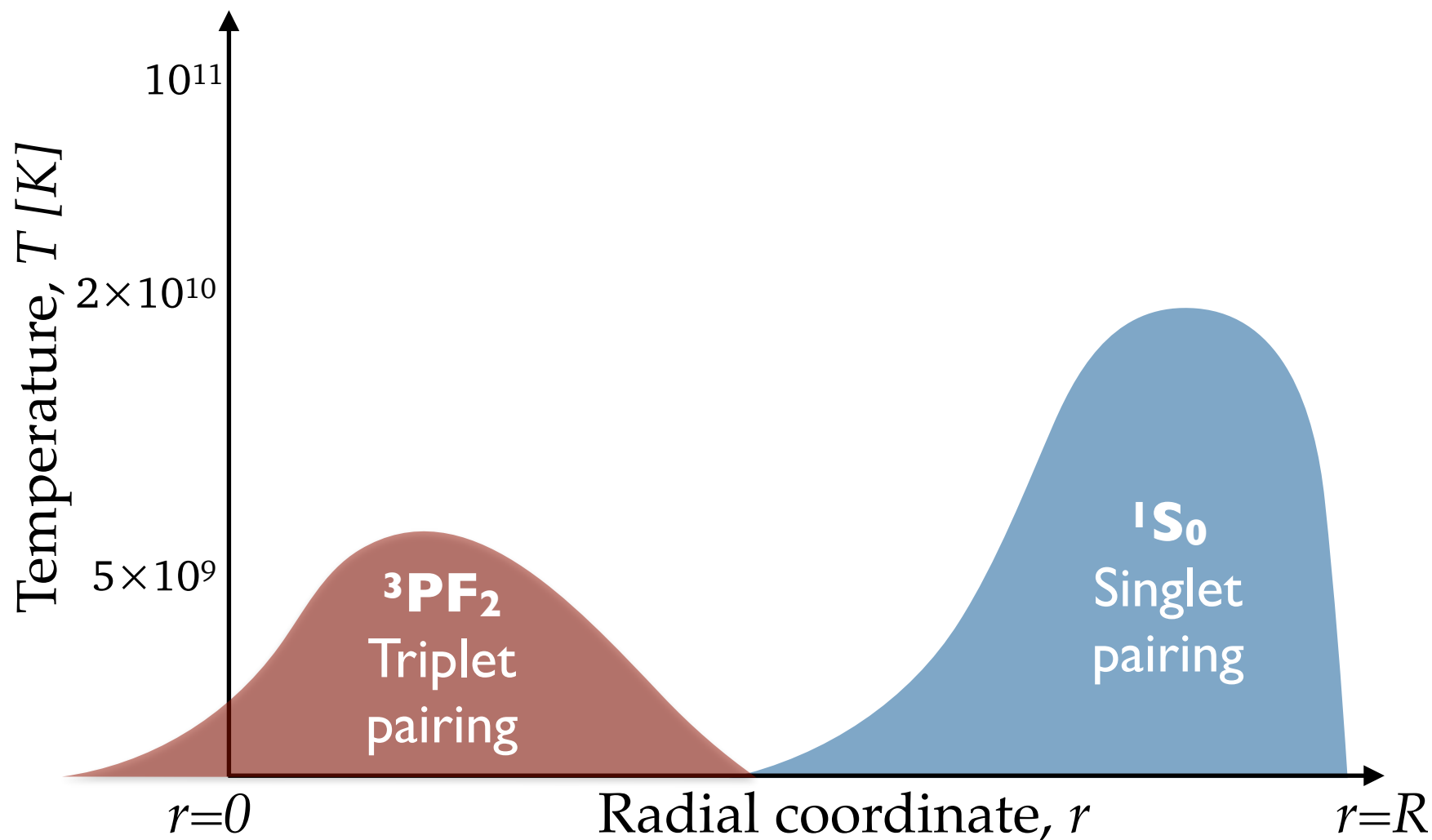


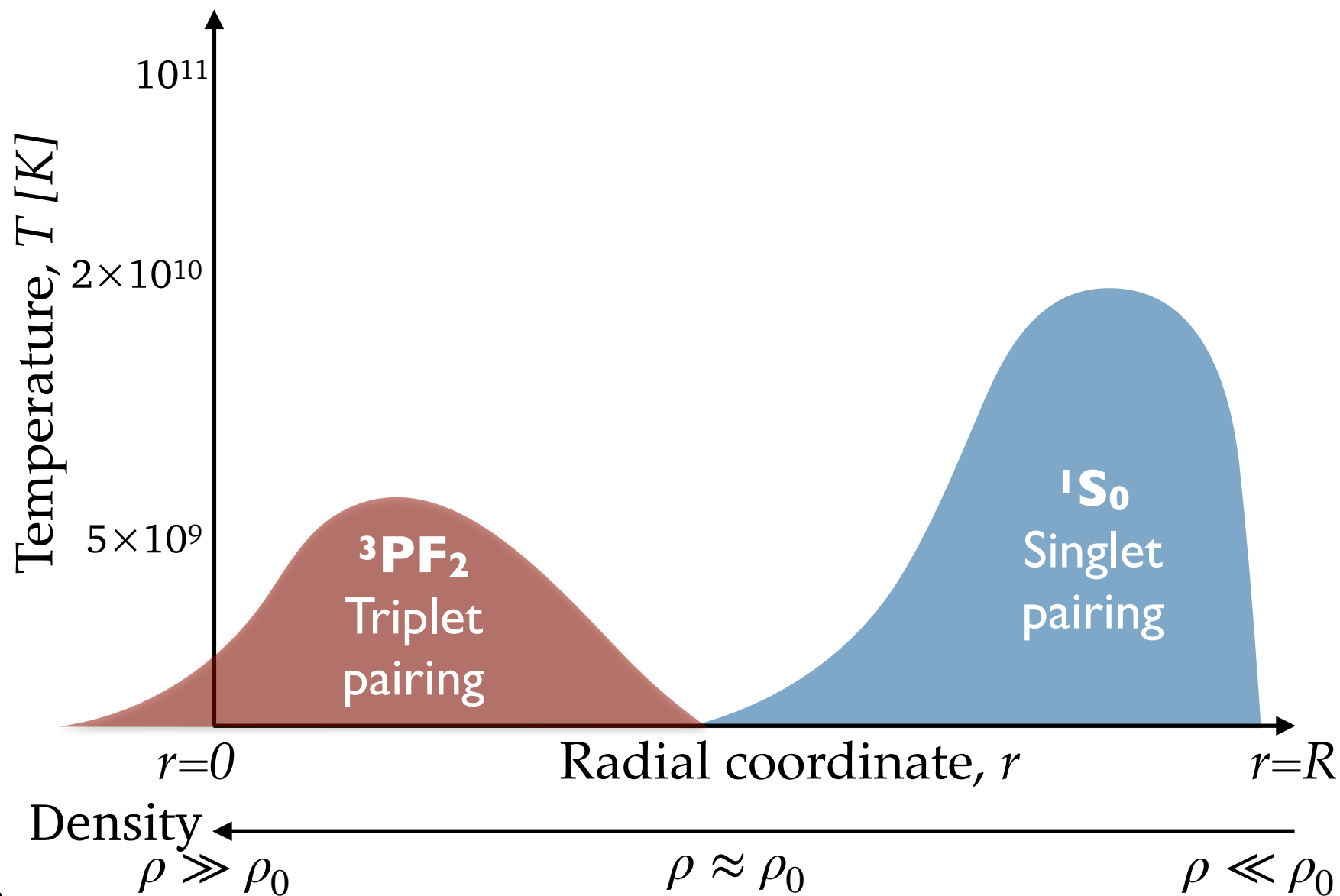
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Zero temperature extrapolation

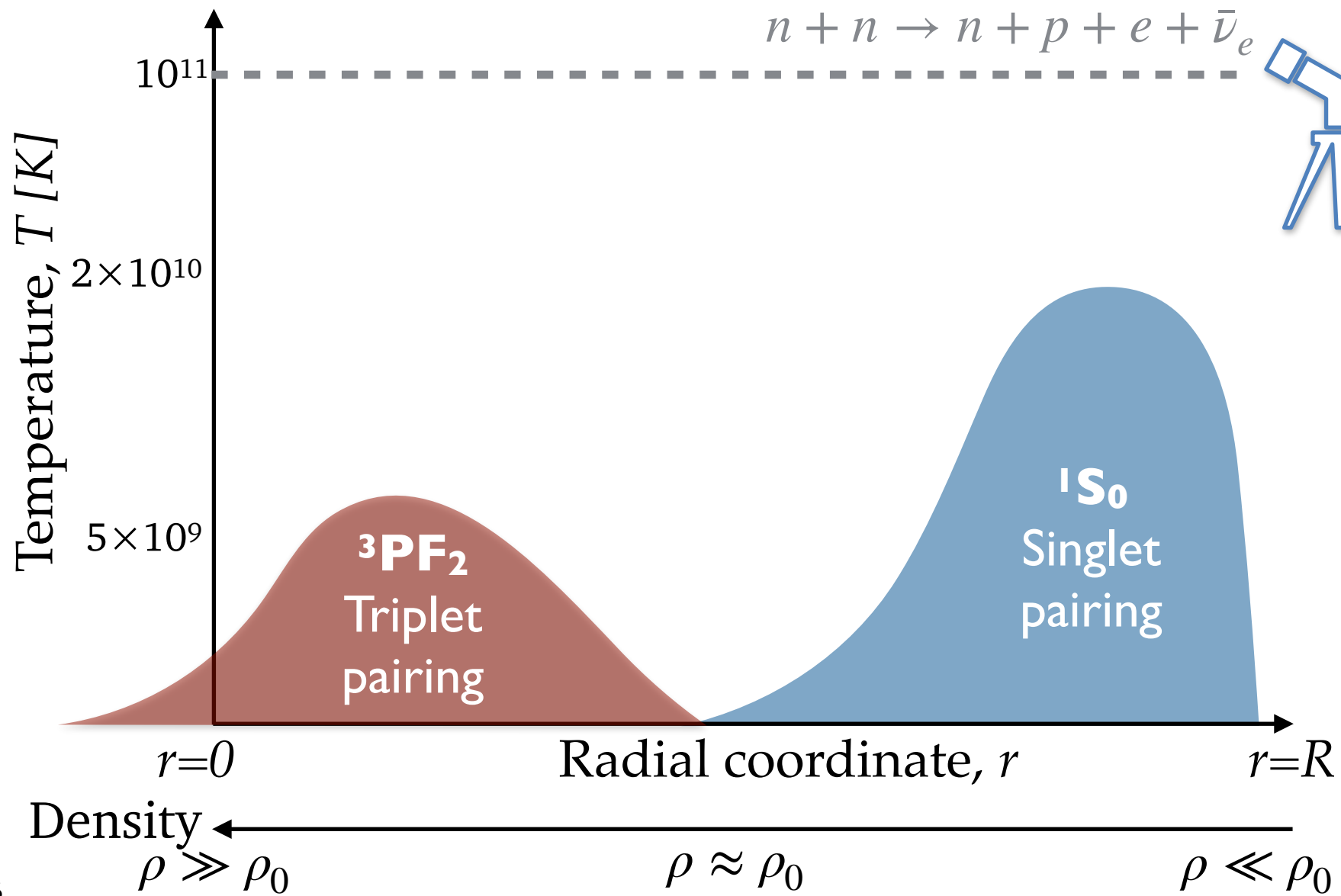




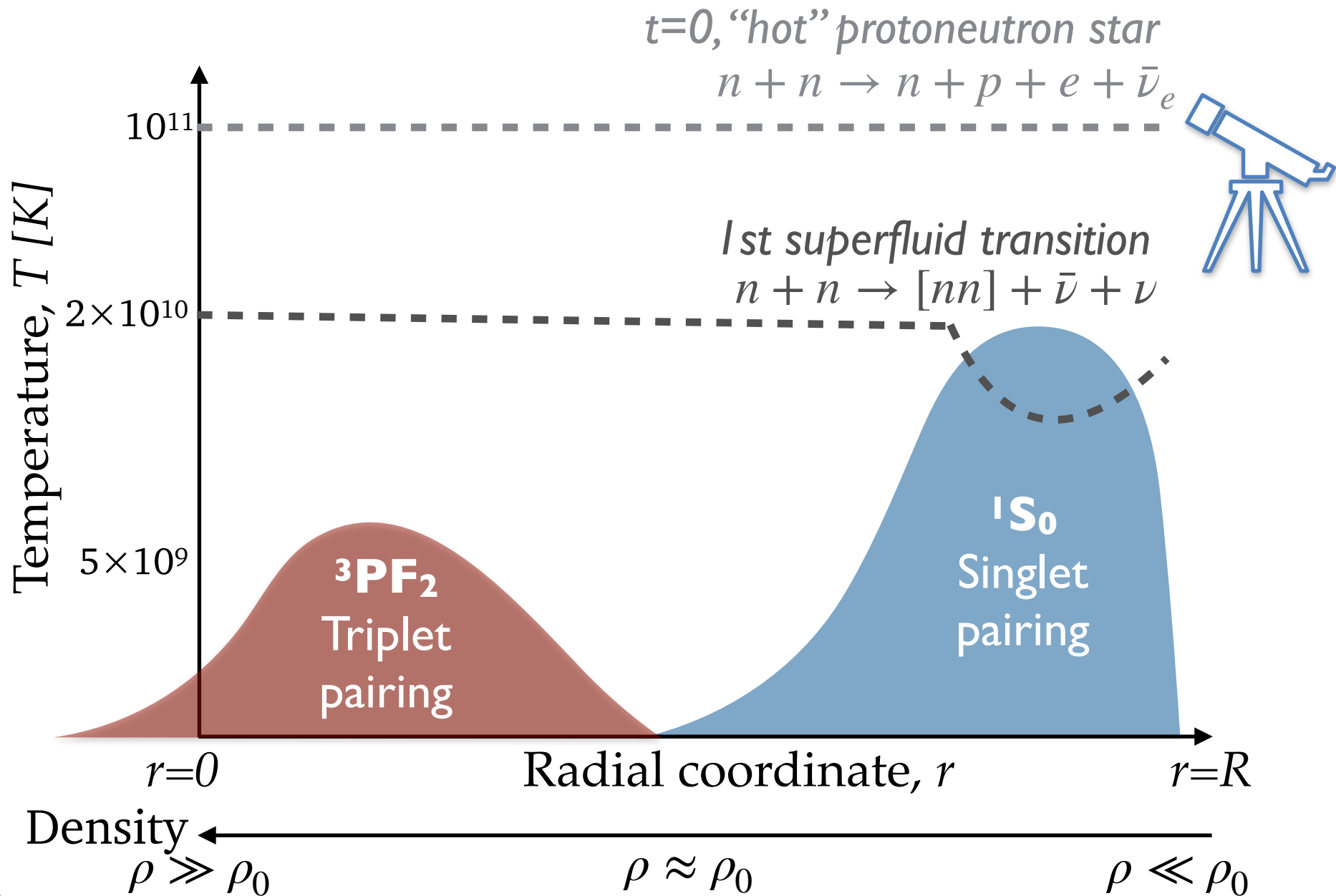


Pairing gaps & cooling

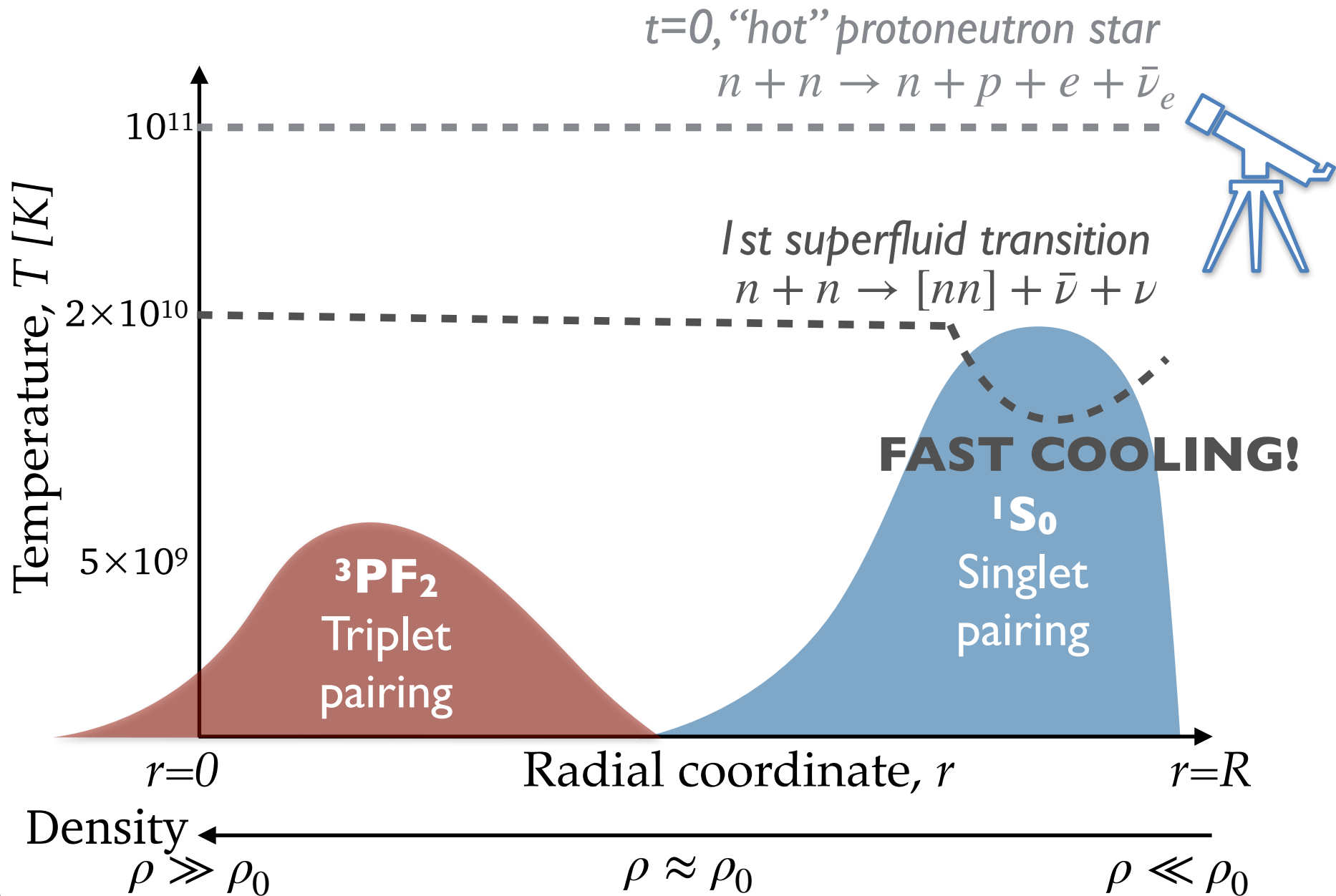
$t=0$, "hot" protoneutron star



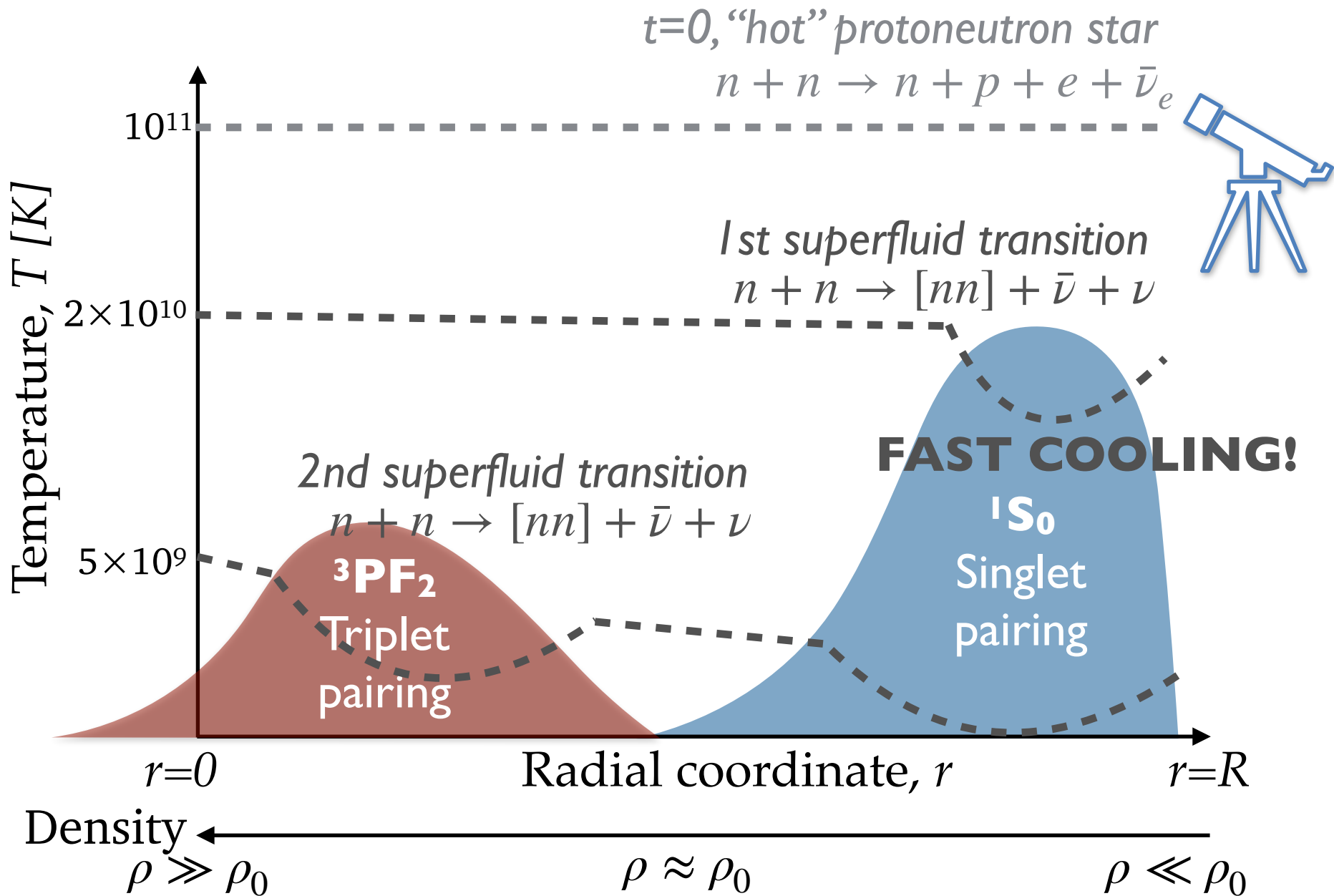
Pairing gaps & cooling



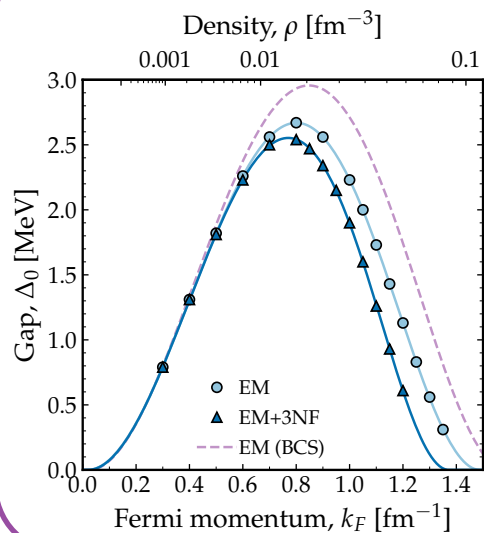
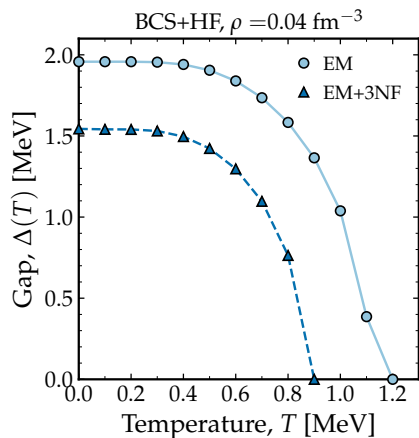
Pairing gaps & cooling



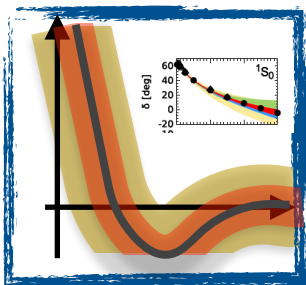
Pairing gaps & cooling



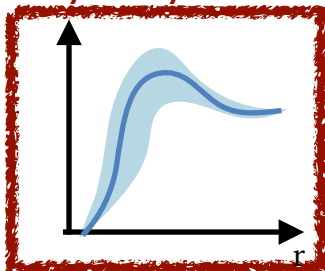
BCS+HF



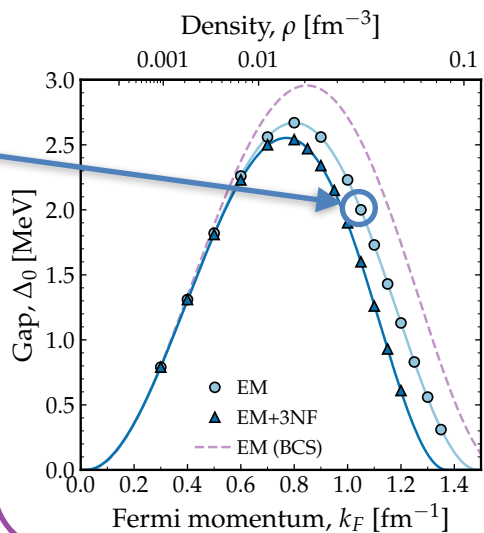
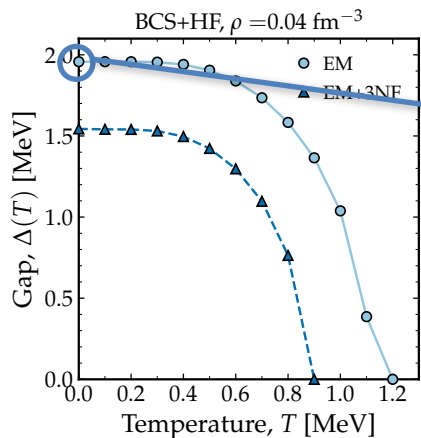
Hamiltonian



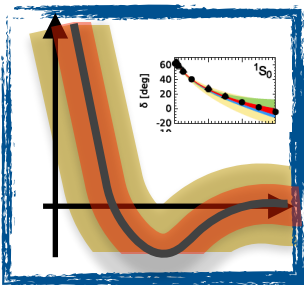
Many-body method



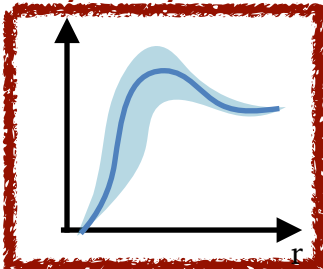
BCS+HF

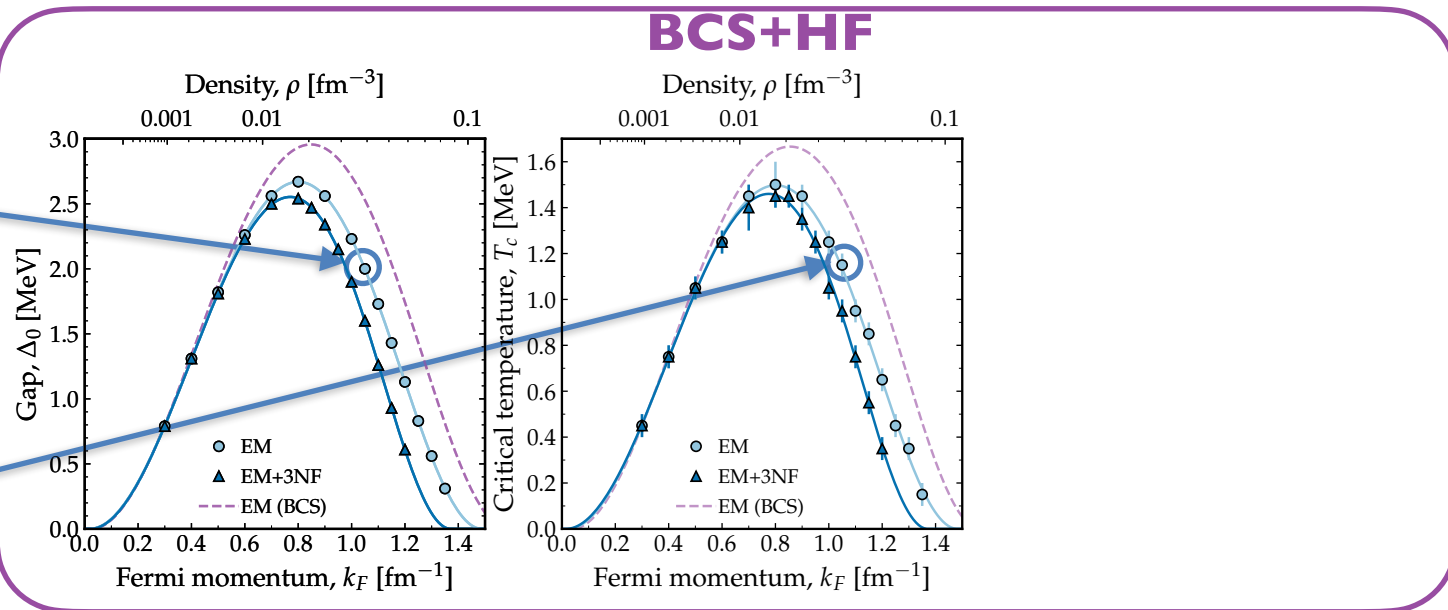
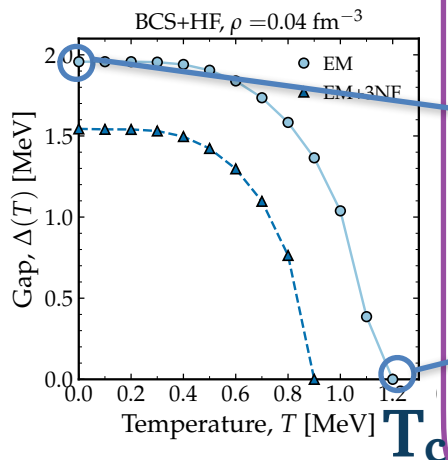


Hamiltonian

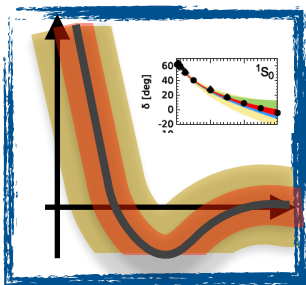


Many-body method

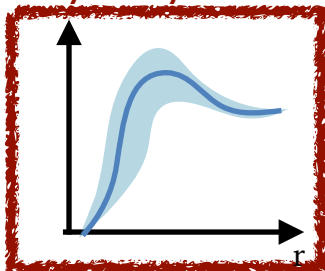




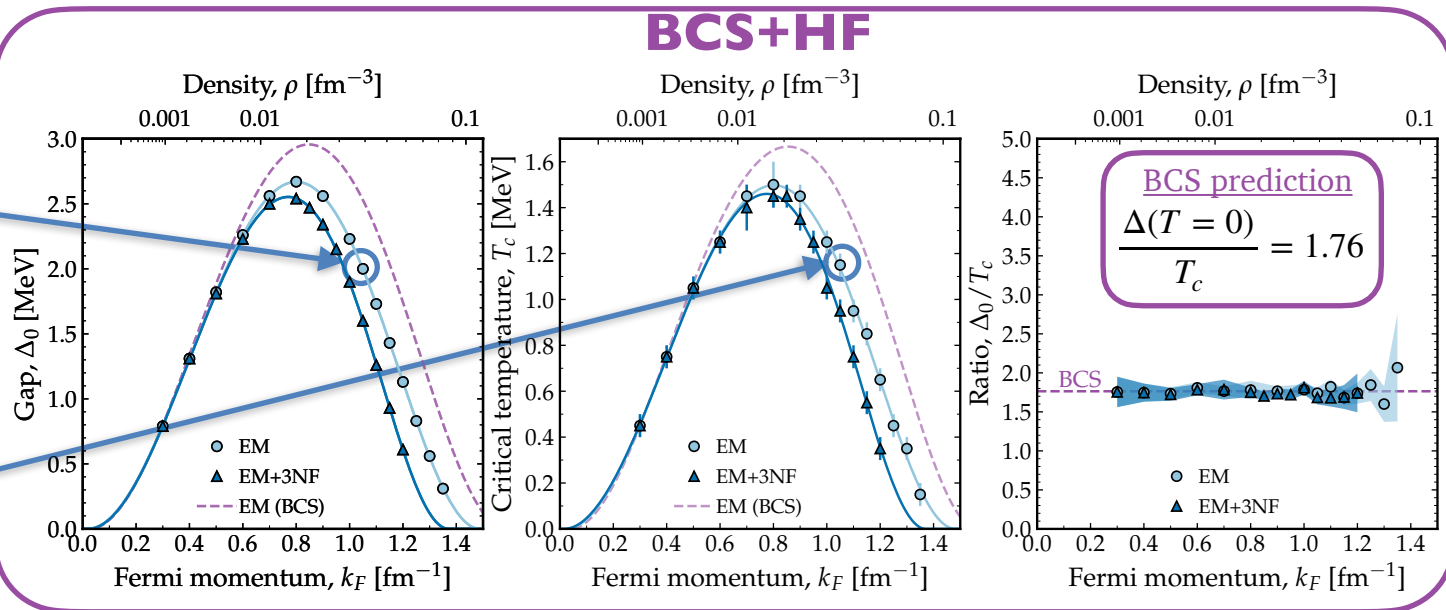
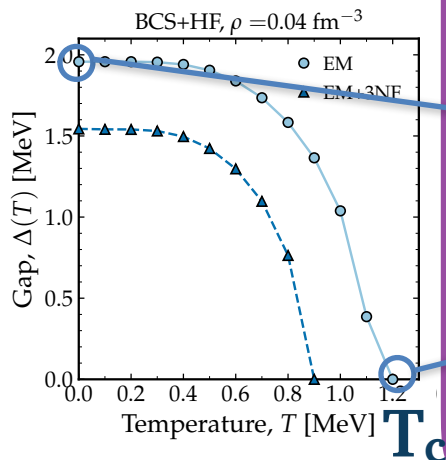
Hamiltonian



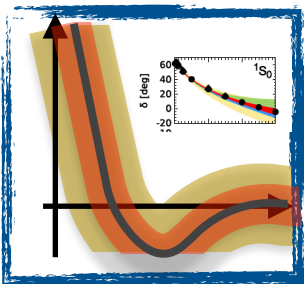
Many-body method



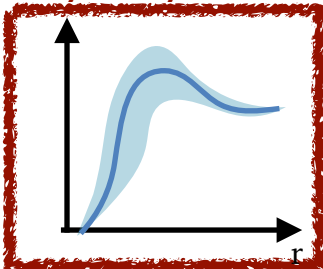
Finite Temperature BCS

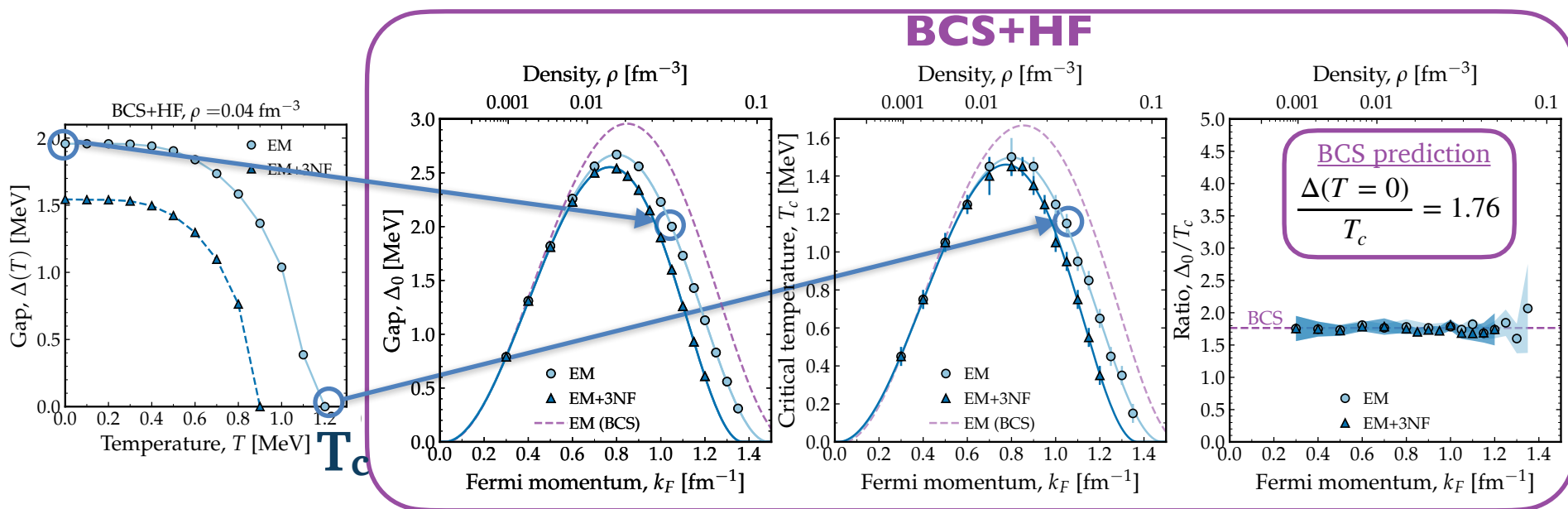


Hamiltonian

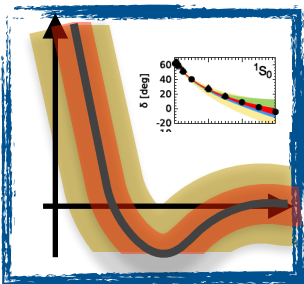


Many-body method

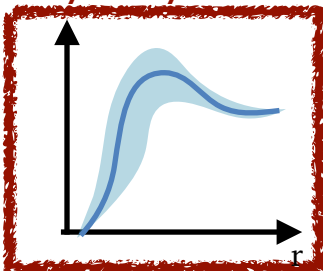




Hamiltonian

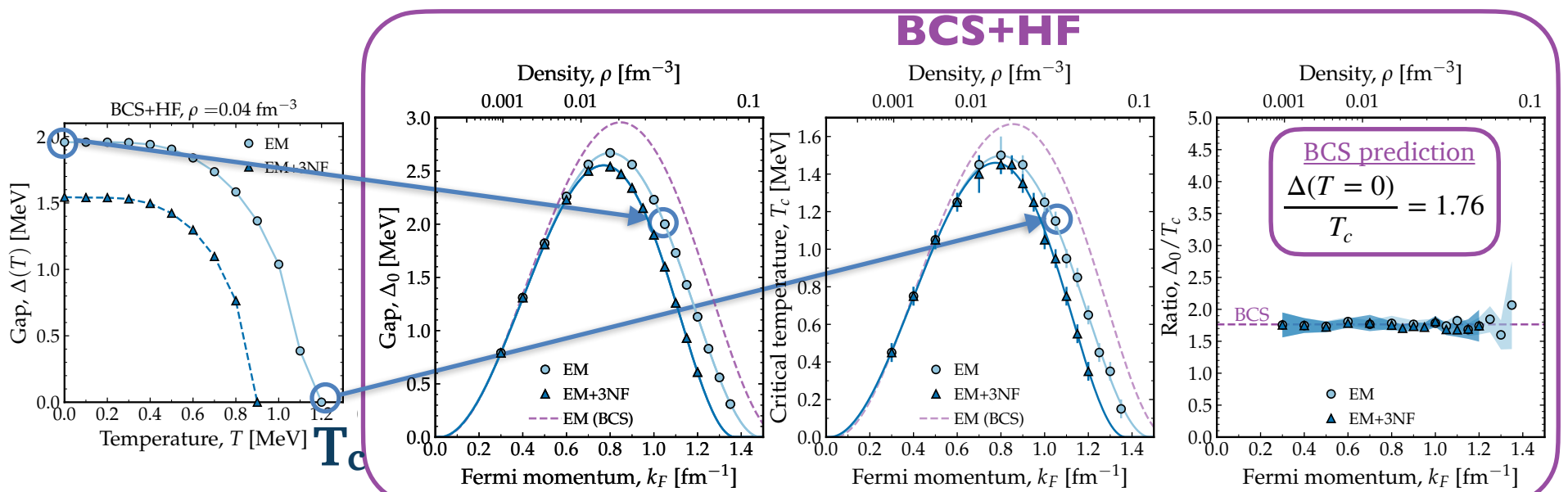


Many-body method

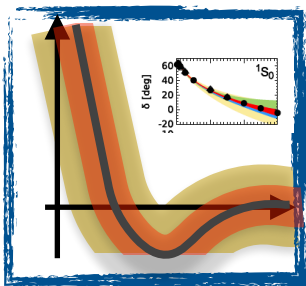


- **BCS** prediction based on **constant V**
- Full chiral **NN** interactions
- Full **HF spectrum**
- 3NFs do not change the ratio!

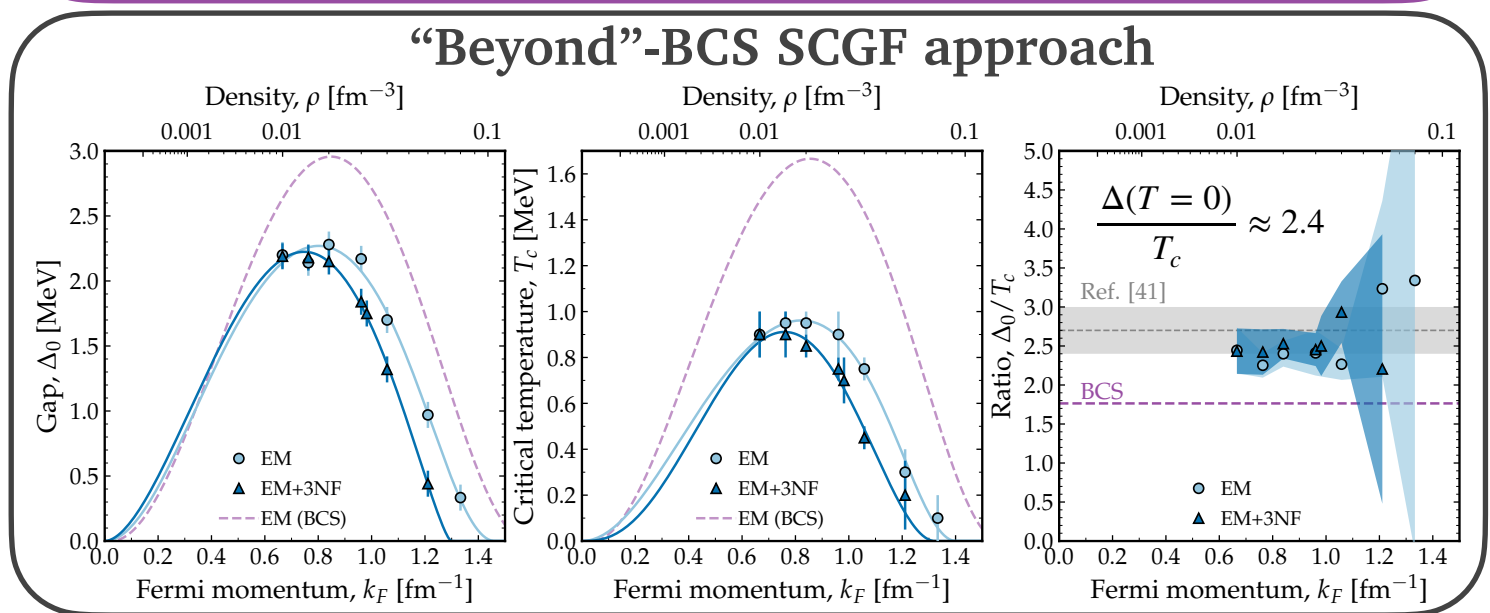
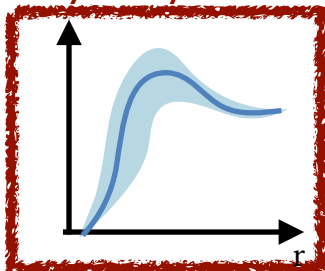
Finite Temperature beyond-BCS

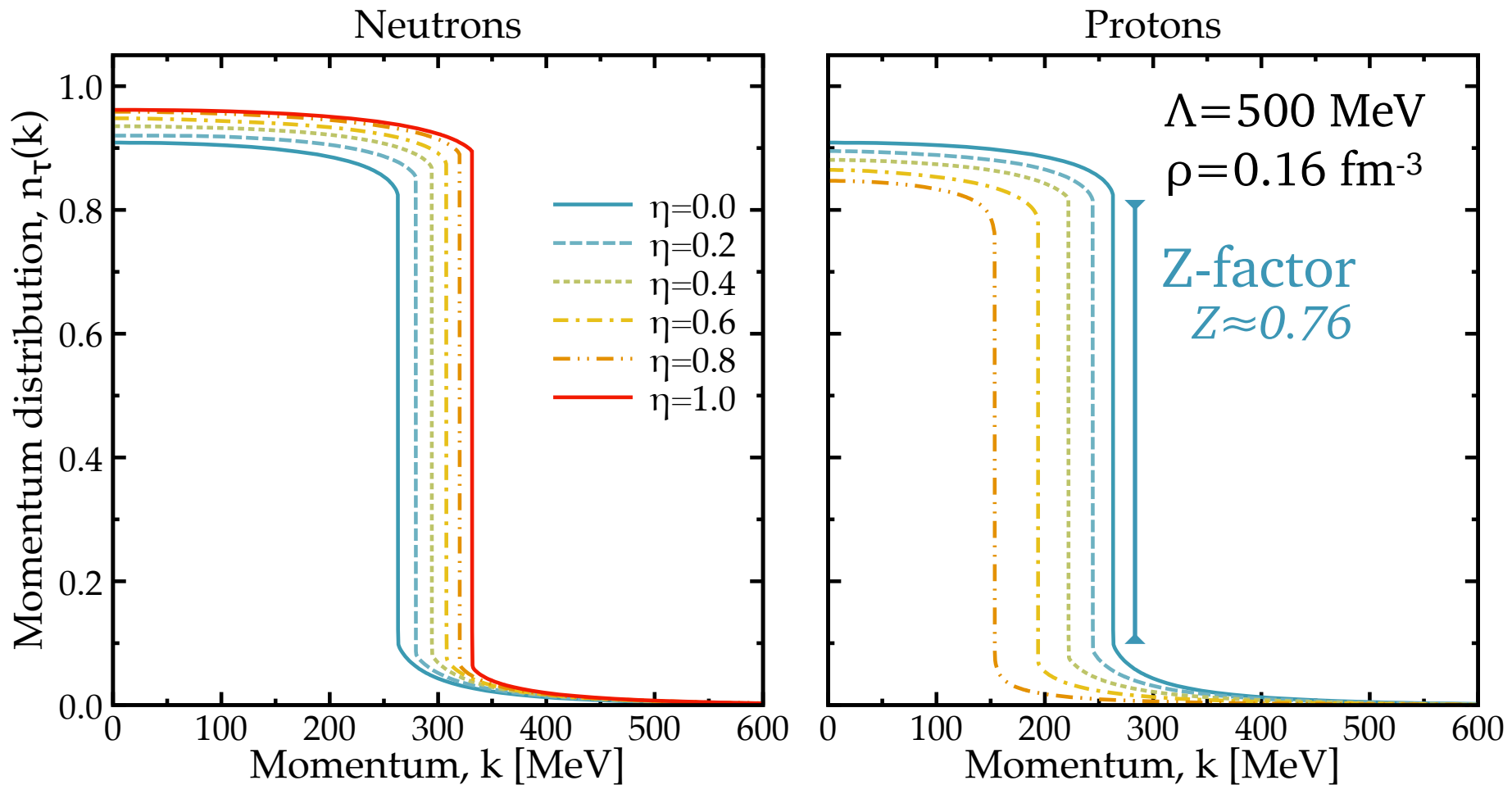


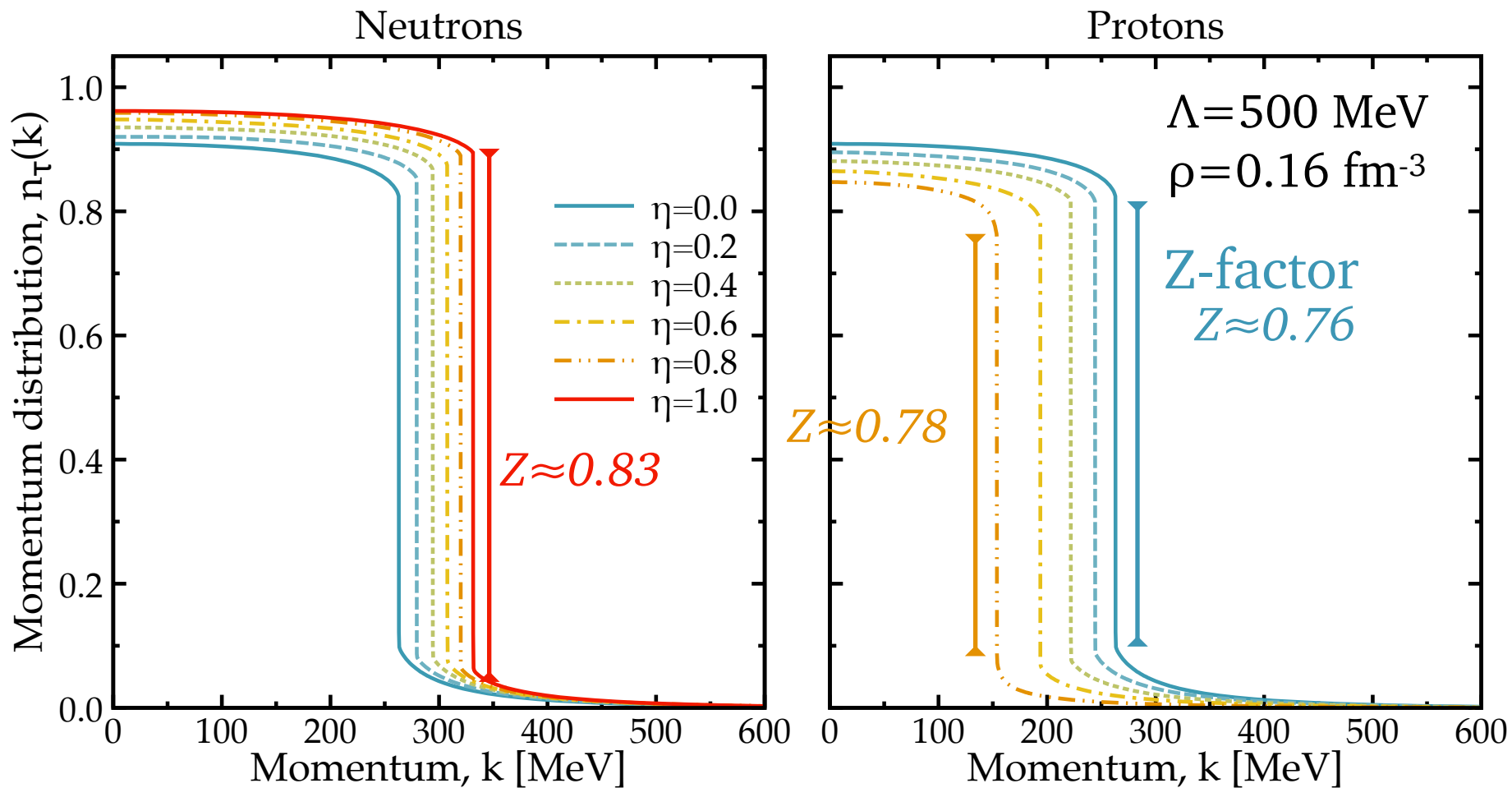
Hamiltonian

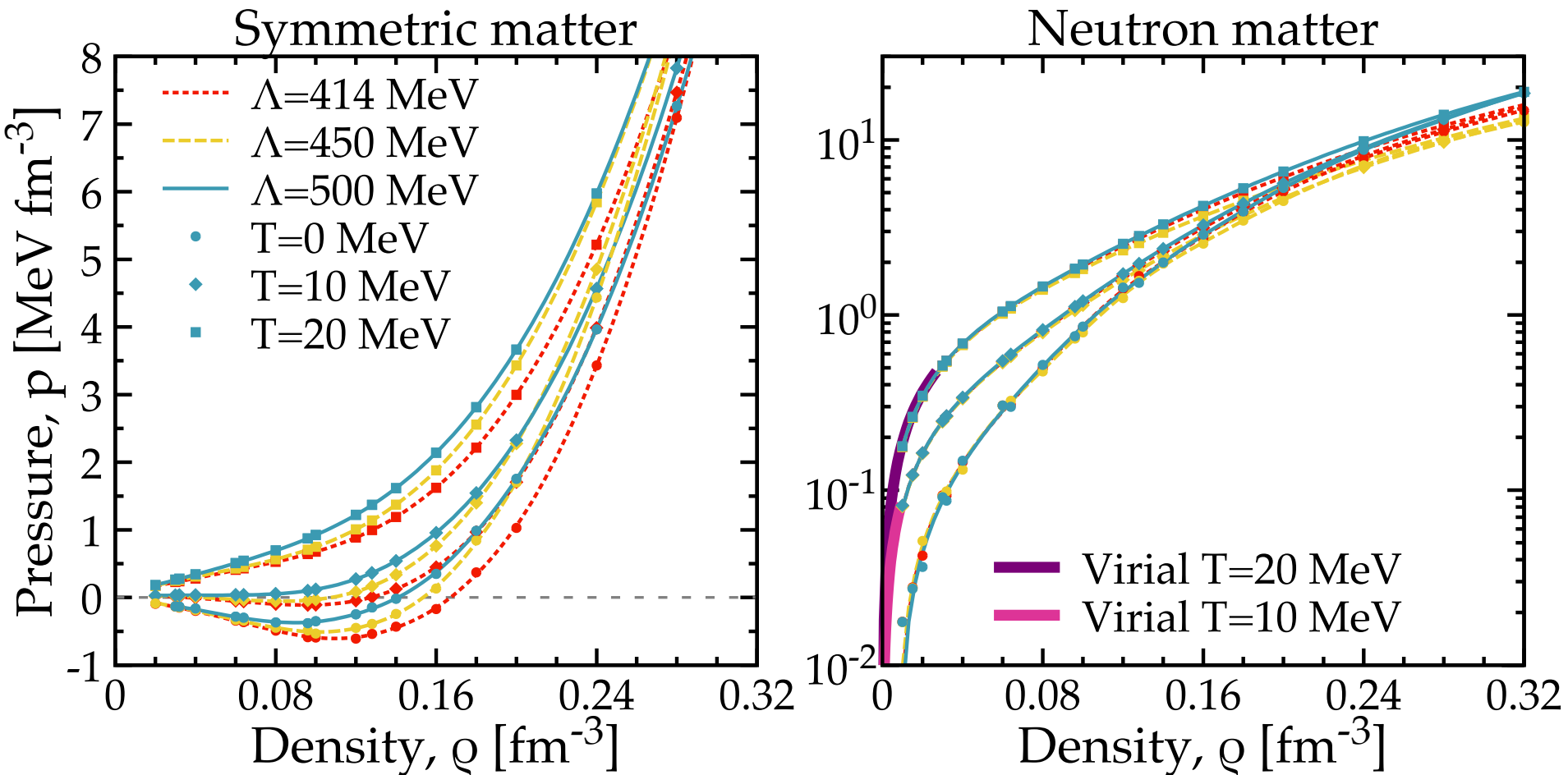


Many-body method



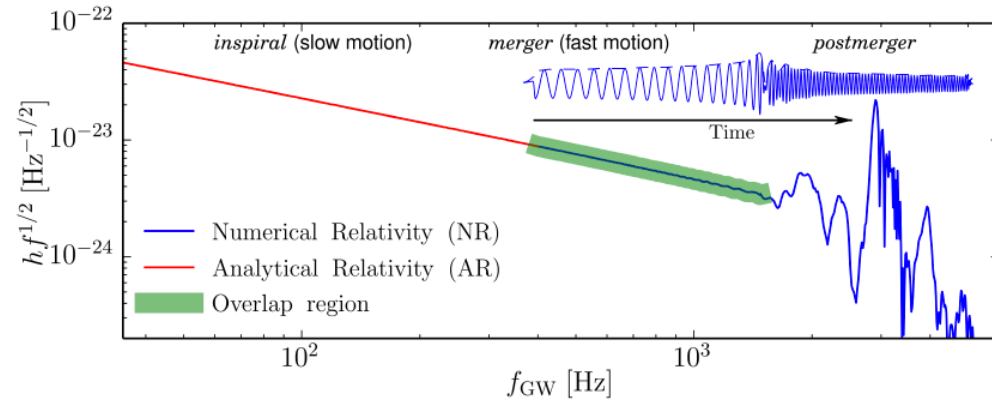




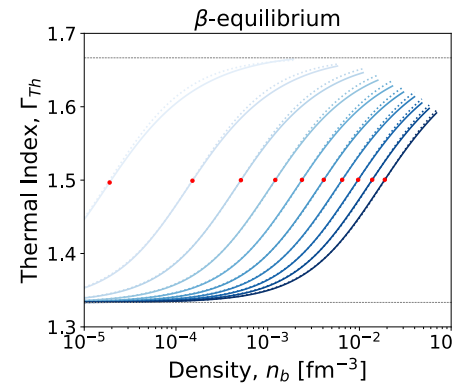
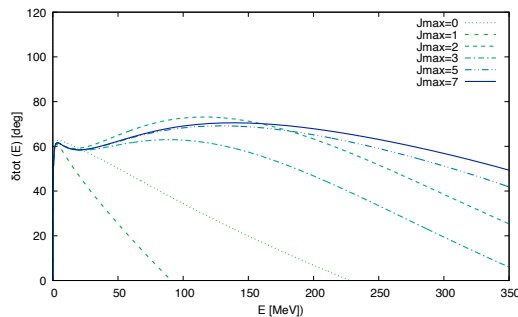


- From a finite set of points to functions?

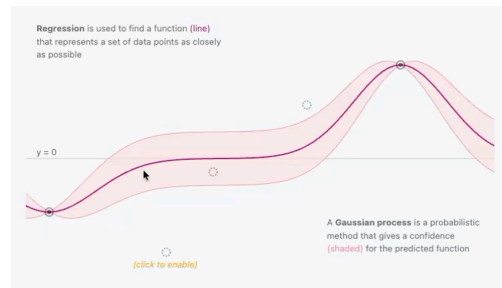
- Motivation



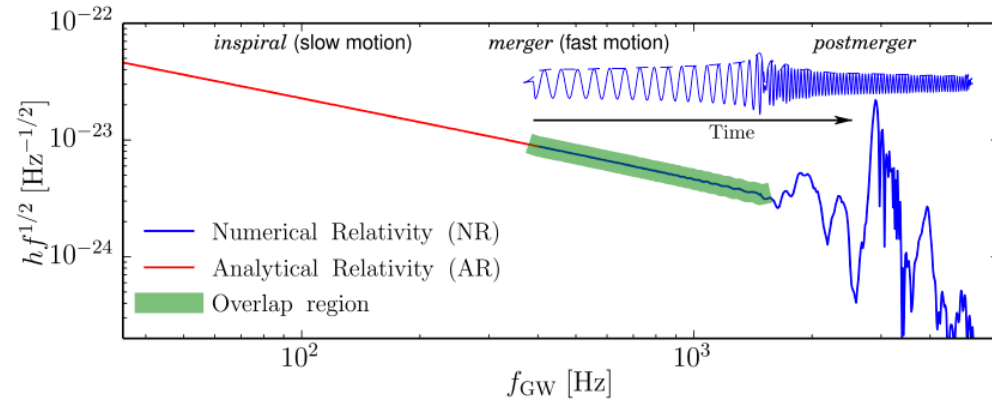
- Virial approximation & thermal effects in BNS



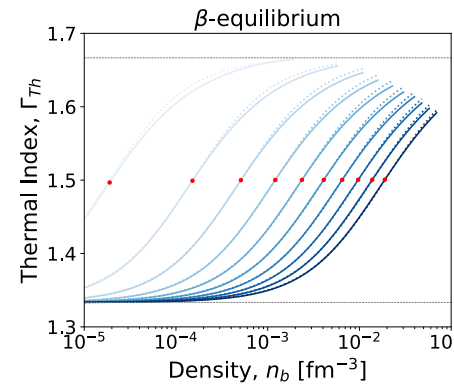
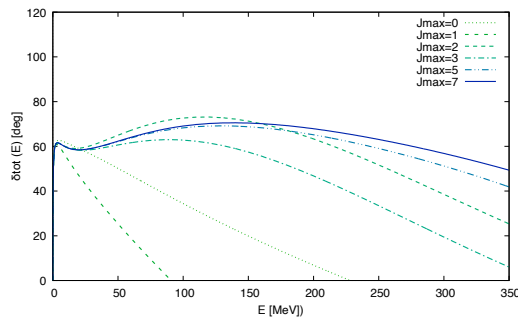
- Gaussian processes



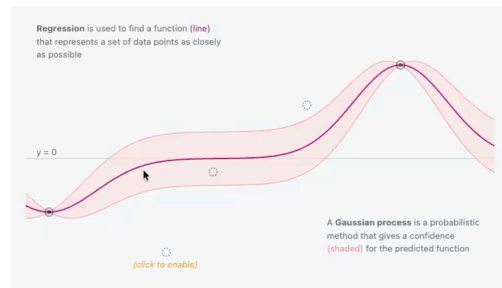
- Motivation



- Virial approximation & thermal effects in BNS



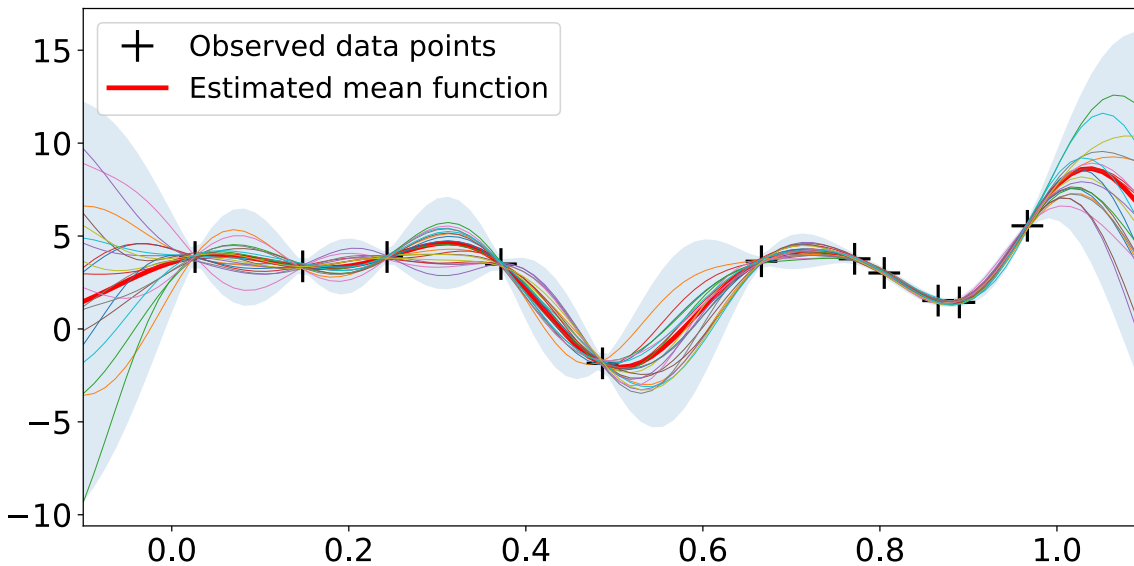
- Gaussian processes



If we denote a real-valued function $f(x)$ over an input space X , then saying that f is distributed as a *Gaussian Process* means that for any finite set of inputs $\{x_1, x_2, \dots, x_n\} \subset X$, the vector of function values

$$\mathbf{f} = (f(x_1), f(x_2), \dots, f(x_n))^{\top}$$

follows a multivariate Gaussian distribution.



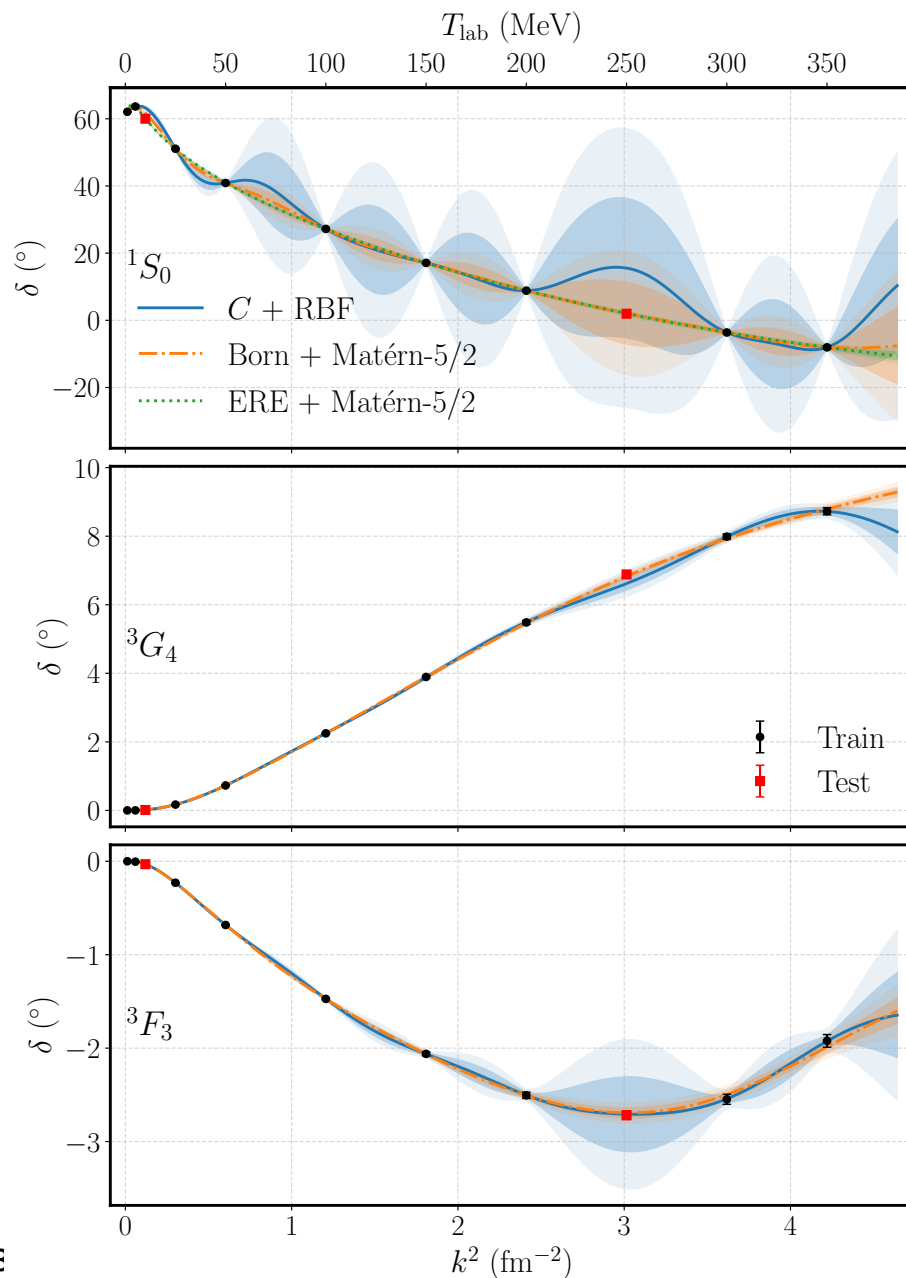
$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

mean

$$m(\mathbf{x}) = \mathbb{E}[f(\mathbf{x})]$$

kernel

$$k(\mathbf{x}, \mathbf{x}') = \mathbb{E}[(f(\mathbf{x}) - m(\mathbf{x})) (f(\mathbf{x}') - m(\mathbf{x}'))]$$



GP pros

- Guaranteed to reproduce data
- Bayesian interpolator
- Errors accounted for
- Differentiable model

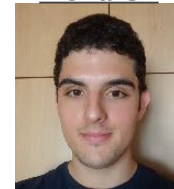
GP cons

- Extrapolation “misguided”

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

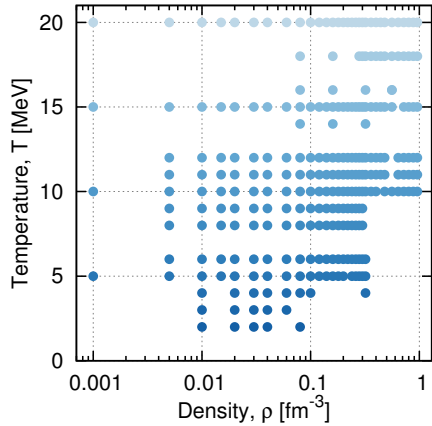
$$m(\mathbf{x}) = \mathbb{E}[f(\mathbf{x})] \approx C$$

Rozalén



Kochankovski





Mean model for EoS

$$m_{\text{par}}(\rho_B, T, Y_q) = (e_0(\rho_B) + f_{0,\text{th}}(\rho_B, T)) \\ + (e_{\text{sym}}(\rho_B) + f_{\text{sym,th}}(\rho_B, T)) \delta^2 \\ + m_{\text{el}}(\rho_B, T, Y_q)$$

Cold part

$$\chi = \frac{\rho_B - n_0}{3n_0}$$

$$e_0(\rho_B) = E_{\text{sat}} + \frac{1}{2}K_0 \chi^2 + \frac{1}{6}Q_0 \chi^3$$

$$e_{\text{sym}}(\rho_B) = J + L \chi + \frac{1}{2}K_{\text{sym}} \chi^2$$

Trainable parameters

$$E_{\text{sat}}, K_0, Q_0, J, L, K_{\text{sym}}, \alpha_0, \alpha_n, \beta_0, \beta_n$$

Thermal part

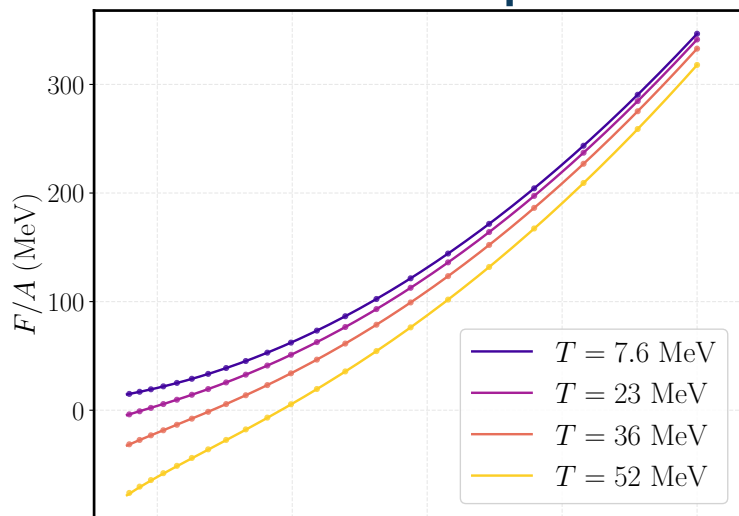
$$f_{0,\text{th}}(\rho_B, T) = -\frac{a_0(\rho_B) T^2}{1 + b_0(\rho_B) T}$$

$$f_{\text{sym,th}}(\rho_B, T) = -\frac{a_{\text{sym}}(\rho_B) T^2}{1 + b_S(\rho_B) T}$$

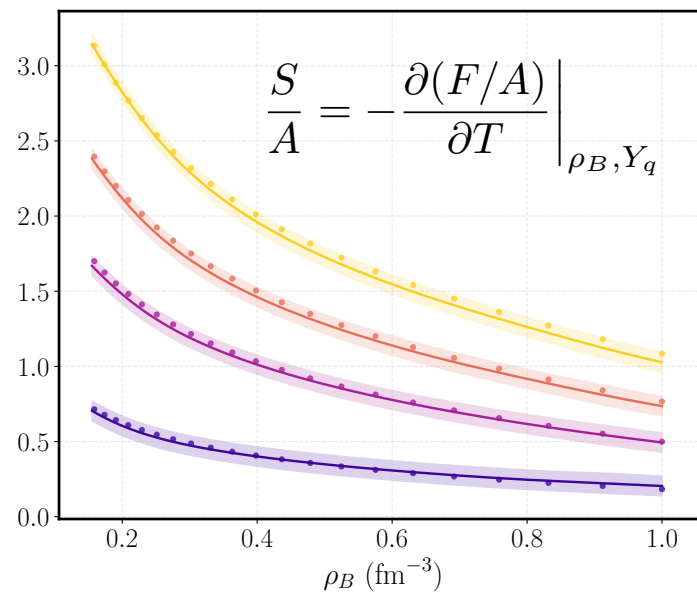
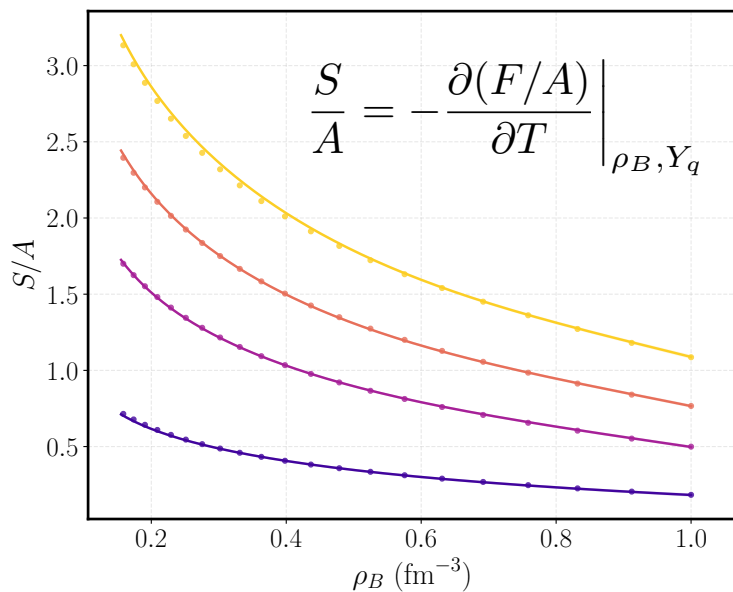
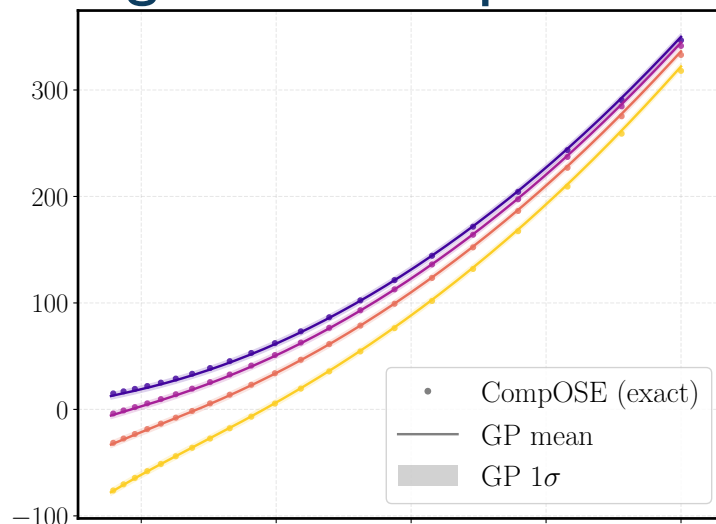
$$a_\tau(\rho) = \frac{\pi^2}{2} \frac{m_\tau^*(\rho)/m}{\varepsilon_{F,\tau}(\rho)}$$

$$\frac{m_\tau^*(\rho)}{m} = \frac{1}{1 + \alpha_\tau \rho / \rho_0}$$

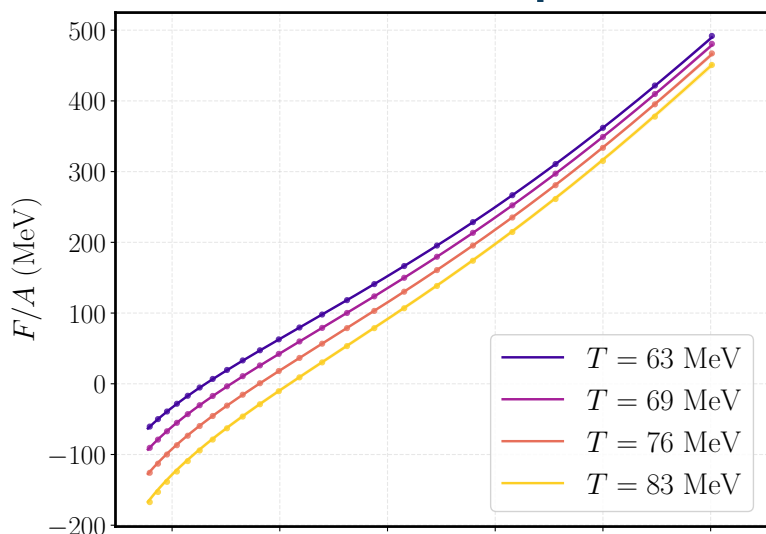
Informed interpolation



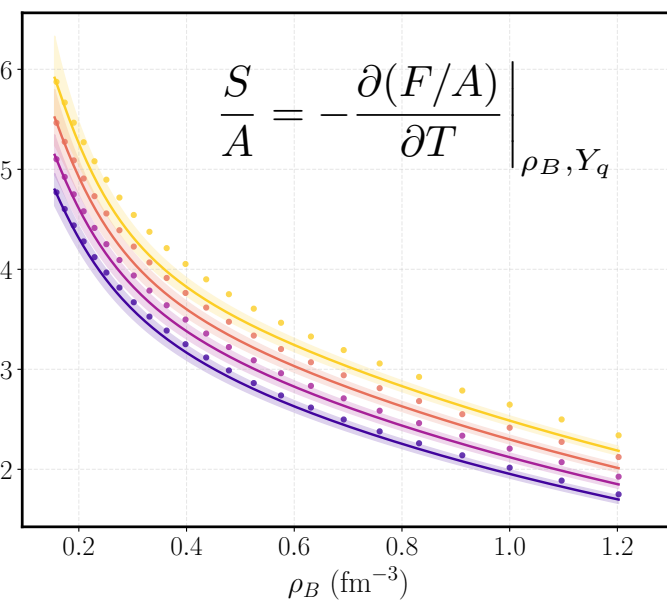
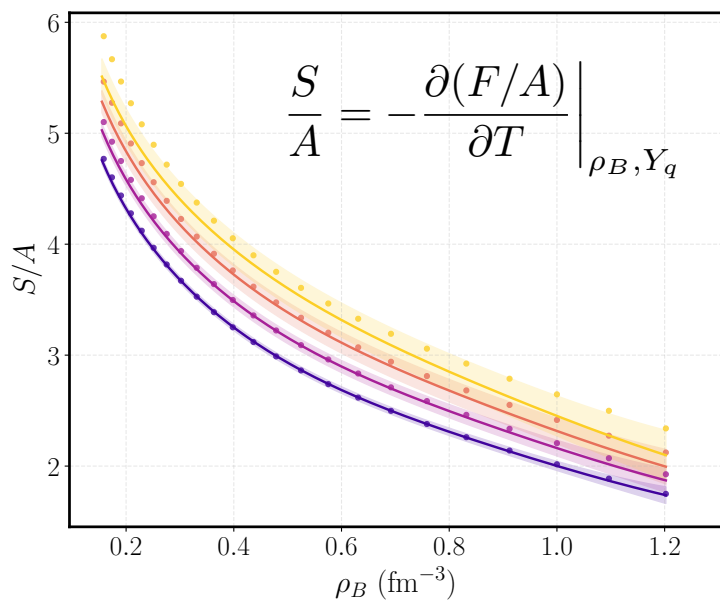
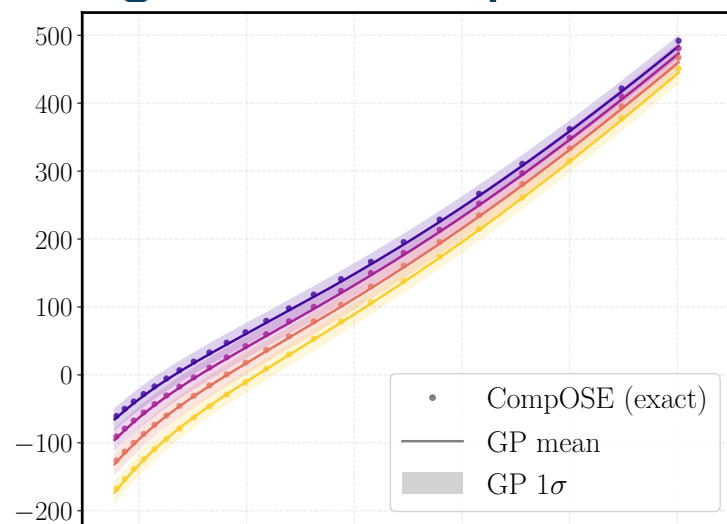
Agnostic interpolation



Informed extrapolation



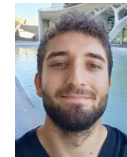
Agnostic extrapolation



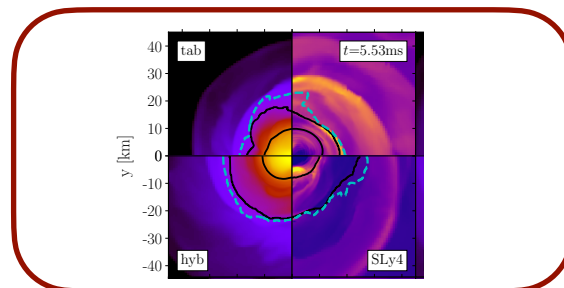
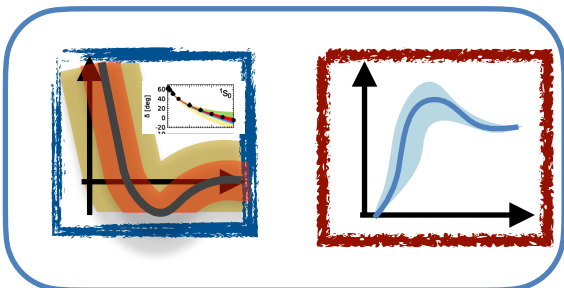
- Ab initio **finite temperature** effects can be simulated & quantified
- **ML** GP techniques can help **interpolate** & **extrapolate** (if physics guided)
- **Challenges**
 - **Uncertainty quantification**
 - **Practical simulability**
 - **Internal NS structure: modes? Phases?**

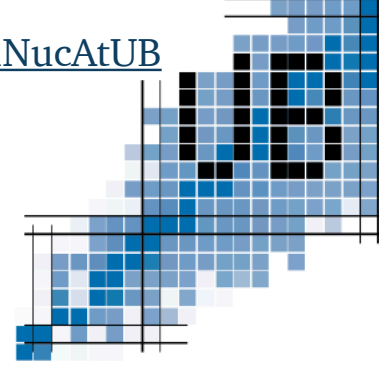


Márquez



G Riviaccio





Thank you!

arnau.rios@icc.ub.edu

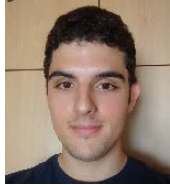
<https://sites.google.com/view/arnaorios/>



À Ramos



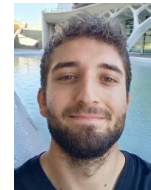
J Rozalén H Kochankovski



D Guerra



G Riviuccio



T Font



P Cerdá-Duran



M Ruiz



A Nadal



R Bondarescu



R Márquez

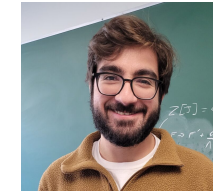


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M Drissi



G Palkanoglou



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