

# Nonequilibrium dynamics in the inner crust of a neutron star

Daniel Pęcak

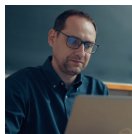


Institute of Physics Polish Academy of Sciences

22<sup>nd</sup> May 2026, CSQCD, Barcelona



Piotr  
Magierski



Gabriel  
Wlazłowski



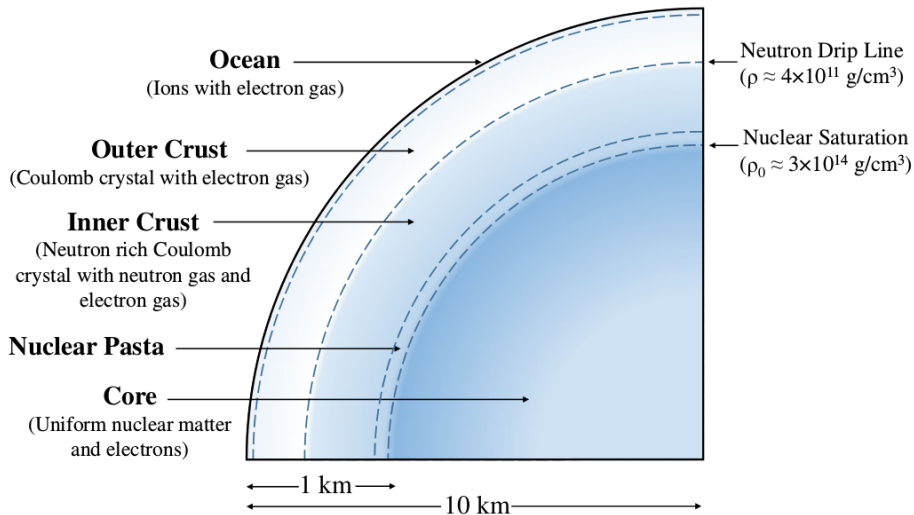
Agata  
Zdanowicz



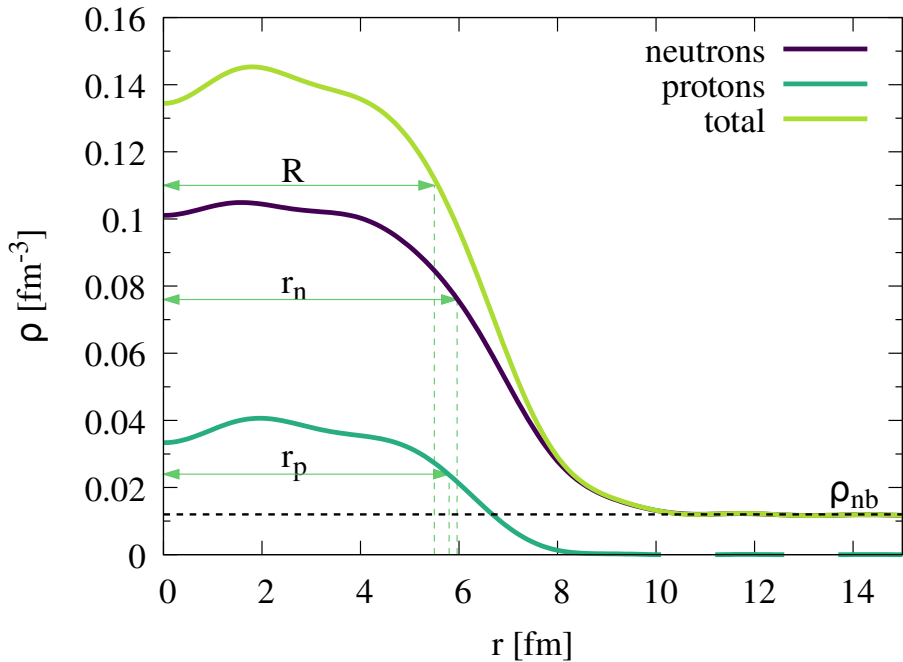
Nicolas Chamel

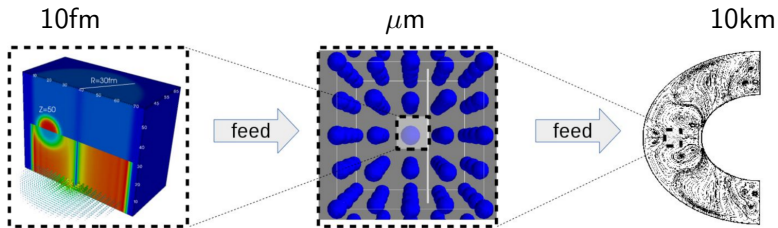
Physical Review X  
14, 041054 (2024)

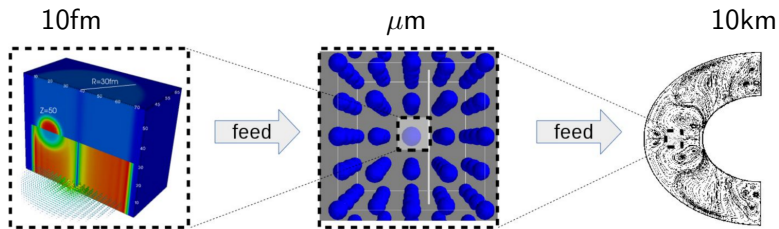




Caplan, M. E., and C. J. Horowitz, *Reviews of Modern Physics* 89, 041002 (2017)



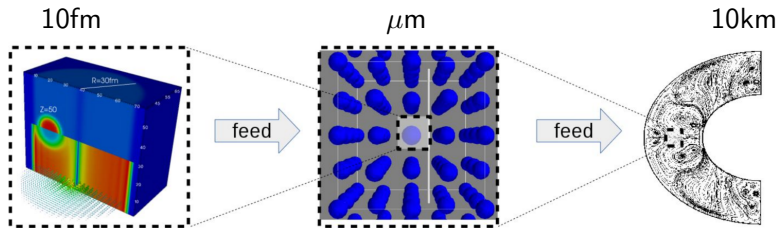




LUMI, Finland (#5 Top 500)



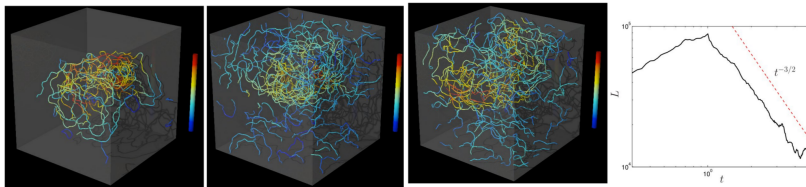
Piz Daint, Switzerland (#37 Top 500)



LUMI, Finland (#5 Top 500)



Piz Daint, Switzerland (#37 Top 500)



R. Hänninen, A. W. Baggaley, Proc. Nat. Acad. Sci. U.S.A. 111, 4667 (2014)

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\rho(r) = \sum_k |v_k(r)|^2$$

$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$

$$\nu(r) = \sum_k u_k(r)v_k^*(r)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\rho(r) = \sum_k |v_k(r)|^2$$

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$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla \frac{\delta\varepsilon}{\delta\tau} \nabla - \frac{i}{2} \left\{ \frac{\delta\varepsilon}{\delta\mathbf{j}}, \nabla \right\}$$

$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

## Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \mathbf{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \mathbf{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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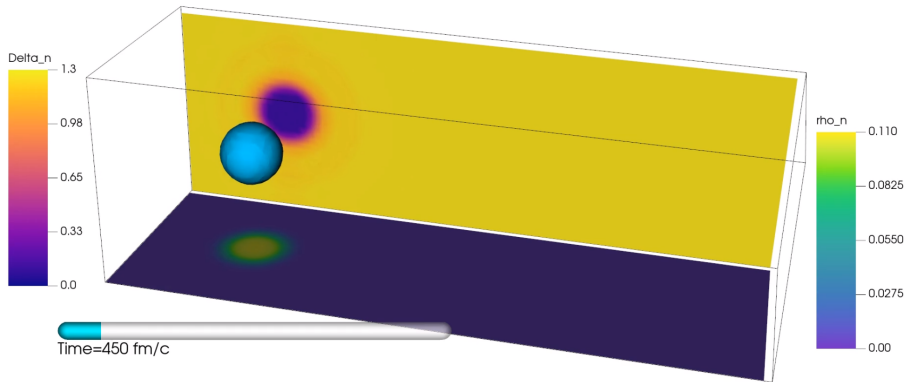
$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

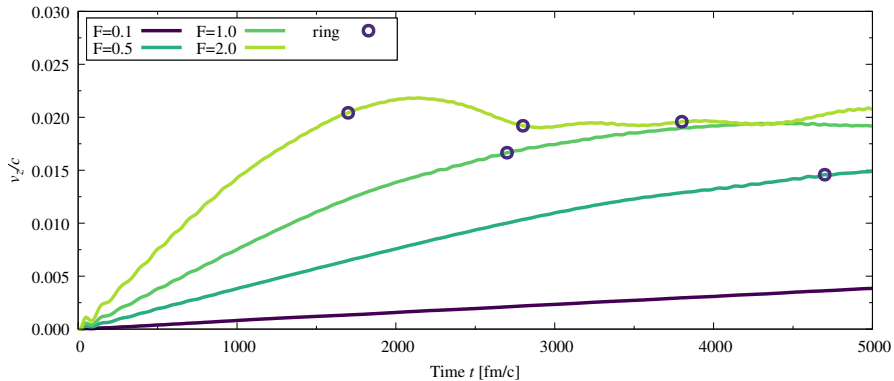
## Superfluid **Local Density Approximation**

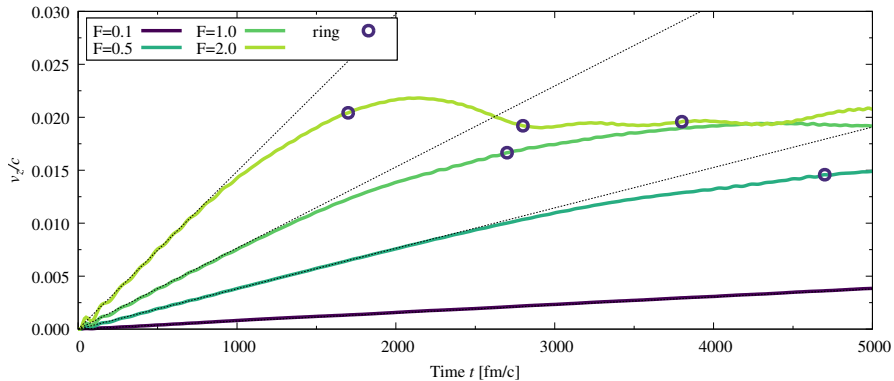
A. Bulgac, Physical Review A **76**, 040502 (2007)

### Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

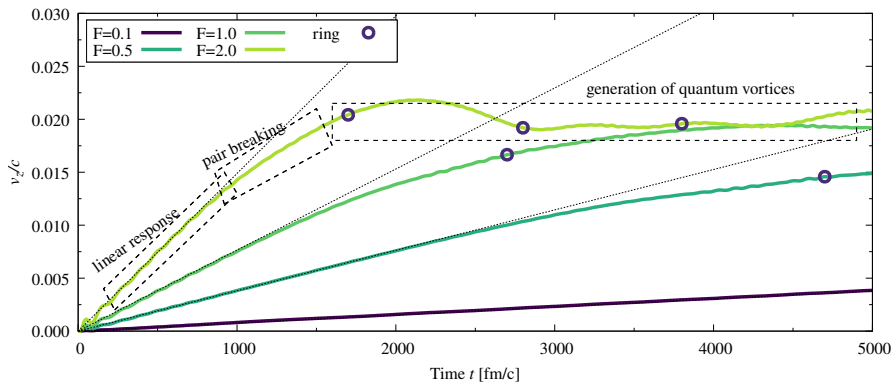






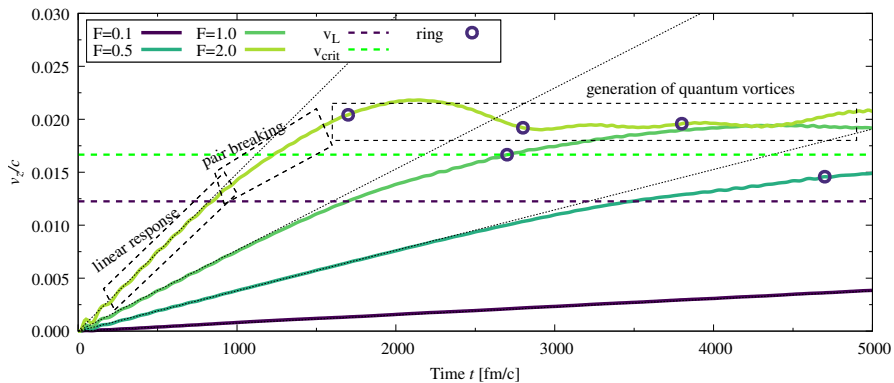
$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$

$$v_L = \frac{\Delta}{\hbar k_F}$$

$$v_{crit} = \frac{e \Delta}{2 \hbar k_F}$$





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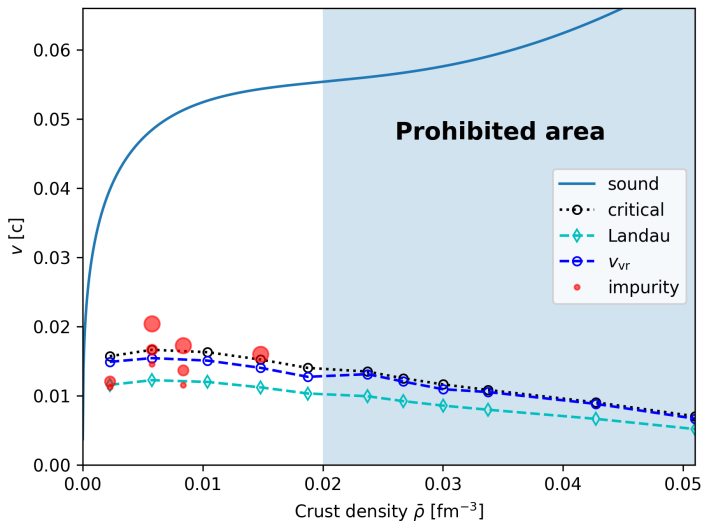


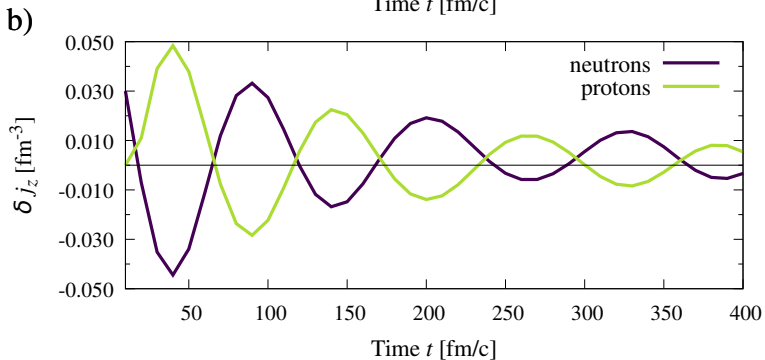
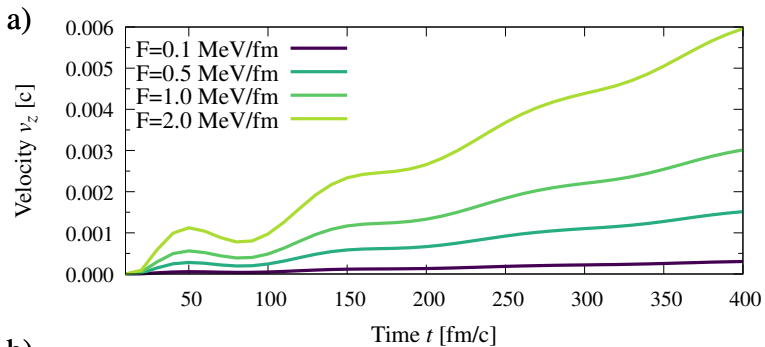
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Credit: BBC Earth

$$v_{vr} = \frac{1}{2\pi R} \frac{\hbar c}{M_n c^2} \left( \ln \frac{8R}{a_{core}} - \alpha \right)$$





## W-SLDA Toolkit

Self-consistent solver  
of mathematical problems  
which have structure  
formally equivalent to  
Bogoliubov-de Gennes equations.

static problems: st-wslida

$$\begin{pmatrix} h_a(\mathbf{r}) - \mu_a & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_b^*(\mathbf{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

time-dependent problems: td-wslida

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) - \mu_a & \Delta(\mathbf{r}, t) \\ \Delta^*(\mathbf{r}, t) & -h_b^*(\mathbf{r}, t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix}$$

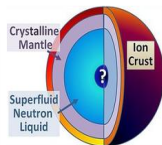
Extension to nuclear matter  
in neutron stars

### Extension to nuclear matter in neutron stars

Integration with VisIt:  
visualization, animation and  
analysis tool

Unified solvers for static and  
time-dependent problems

Dimensionalities of  
problems: 3D, 2D and 1D



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

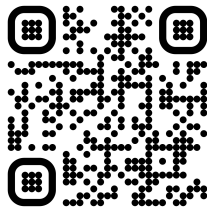
ALL FUNCTIONALITIES +

### Getting the code

 DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

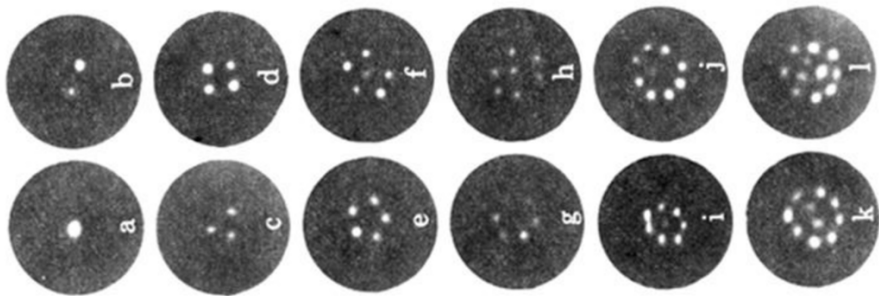
READ MORE +



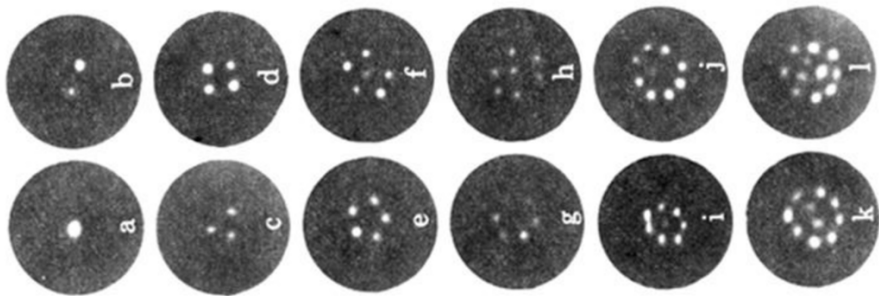
> CONTRIBUTORS

> How to cite W-SLDA

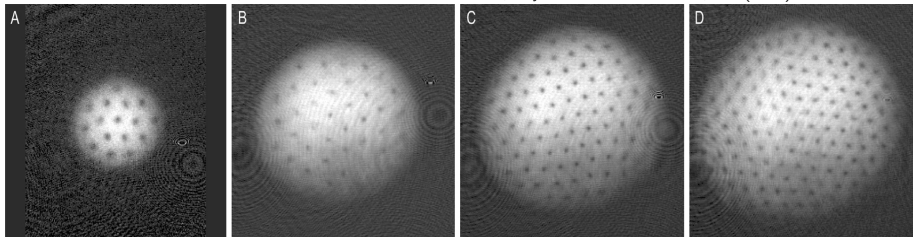
> Requirements & Documentation



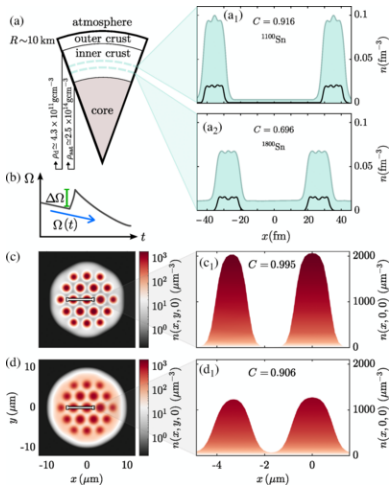
E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard, *Physical Review Letters* 43, 214 (1979)

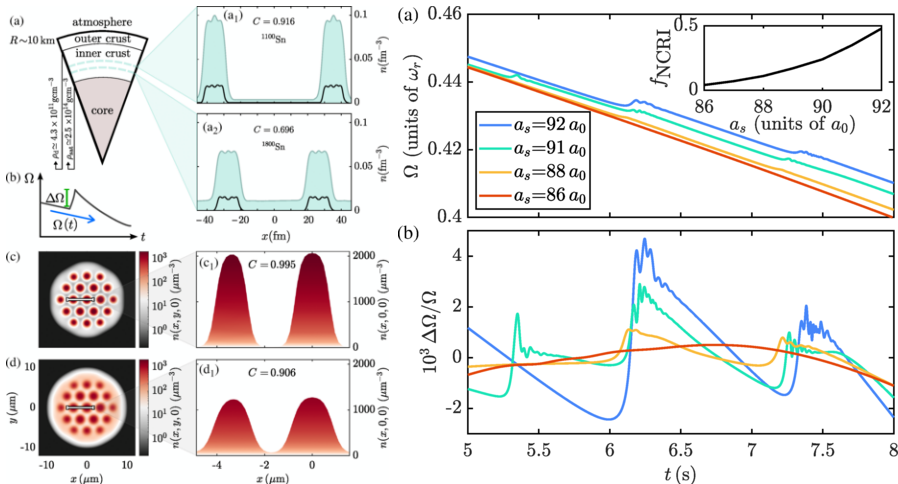


E. J. Yarmchuk, M. J. V. Gordon, and R. E. Packard, *Physical Review Letters* 43, 214 (1979)



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle, *Science* 292, 476 (2001)





Poli, Elena, et al. "Glitches in rotating supersolids." Physical Review Letters 131, 223401 (2023)

# CA24139 - Superfluid Condensates in Astrophysics and Laboratory Experiments (SCALES)

Downloads

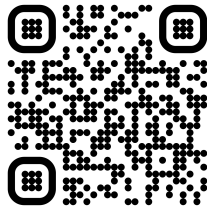
[Home](#) > [Browse Actions](#) > Superfluid Condensates in Astrophysics and Laboratory Experiments (SCALES)

Description

Management Committee

Main Contacts and Leadership

Working Groups and Membership



## Description

Superfluidity is a striking phenomenon observed in many quantum fluids, which can flow without viscosity when cooled to low temperatures. Recent experimental advances allow us to study and visualise the complex flows of these systems in the presence of vorticity, and track the dynamics of quantum vortices far from equilibrium, both for bosonic and fermionic superfluids. Experiments with helium-4 and helium-3 allow the analysis of quantum turbulence on different scales, while cold atomic gases allow exquisite studies of single vortex dynamics in a variety of regimes, spanning the entire crossover from molecular Bose-Einstein condensates (BEC) to Bardeen-Cooper-Schrieffer (BCS) superfluids. Moreover, a growing body of theoretical and observational evidence suggests that nucleons in neutron stars are paired and form a fermionic superfluid, the dynamics of which is thought to be at the origin of the observed radio pulsar glitches.

The next years will bring a wealth of data on neutron star dynamics, from new radio observatories such as the Square Kilometer Array (SKA), but especially from gravitational wave observations with next-generation detectors, like the planned European Einstein Telescope (ET). Direct laboratory analogues of superfluid neutron stars are now being studied with superfluids on rotating platforms, but this connection is neither systematically explored nor disseminated among the relevant scientific communities.

SCALES will bring together novel laboratory experiments, emerging massive parallel simulations, and neutron star experts to kickstart this new

### Action Details

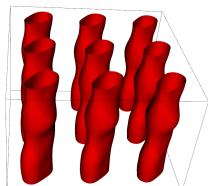
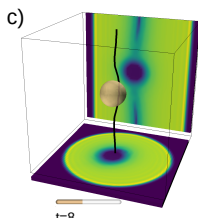
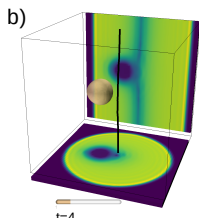
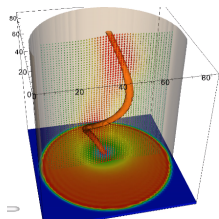
- MoU - 04/8/25
- CSO Approval date - 19/05/2025
- Start date - 26/09/2025
- End date - 25/09/2029

### How can I participate?

- Read the Action Description [MoU](#)
- Inform the Main Proposer/Chair of your interest ([email](#))
- [Apply](#) to join your Working Groups of interest

# Summary

- fully self-consistent 3D (TD)HFB calculations
- BSk31 Energy Density Functional
- effective parameters can be extracted
- effective mass
- dissipation channels
- creating vortex rings
- giant dipole resonance
- pinning force



Thank you!

¡Muchas gracias!

Moltes gràcies!