Rotational effects in quark stars: comparing different models

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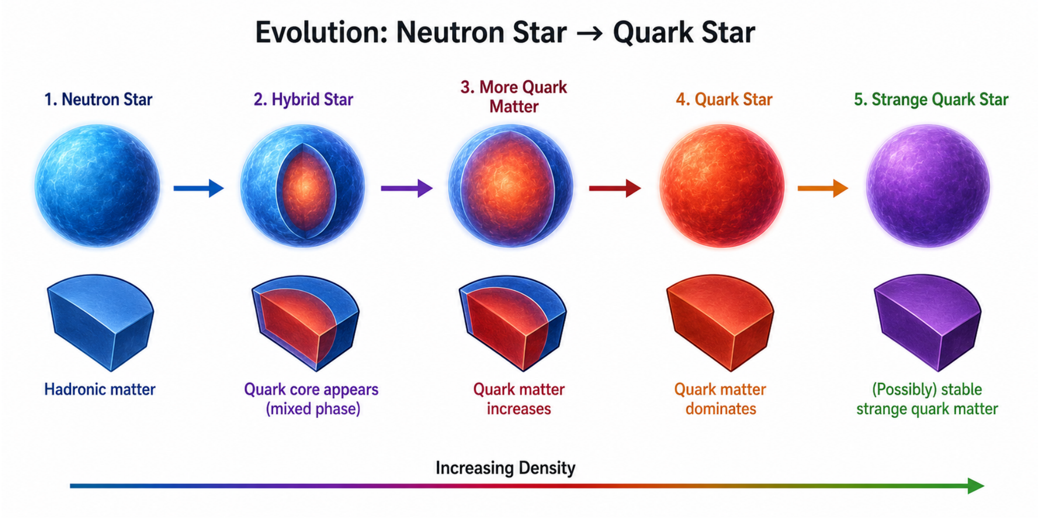
What is a quark star?



Motivation

Why Study Rotating Quark Stars?

- Test the strange quark matter hypothesis
- To distinguish between different quark matter EOSs using rotational observables
- How rotation can help distinguish quark matter from hadronic matter

Central Question

- Is the enforcement of the Bodmer–Witten conjecture sufficient to conclude that quark matter is self-bound?

Key Mathematical Expressions

1. Density-Dependent Quark Mass (DDQM) Model[1]

$$m_i(n_B) = m_{0i} + \frac{D}{n_B^{1/3}} + C n_B^{1/3}, \tag{1}$$

2. Vector MIT Bag Model[2]:

$$\mathcal{L} = \sum_i \left[\bar{\psi}_i (i\gamma^\mu \partial_\mu - m_{0i})\psi_i - B\right] \Theta(\bar{\psi}_i \psi_i) + \sum_i [g_{iV} \bar{\psi}_i \gamma^\mu V_\mu \psi_i - \frac{1}{2} m_V^2 V_\mu V^\mu] \Theta(\bar{\psi}_i \psi_i) + \frac{b_4}{4} (g^2 V_\mu V^\mu)^2 \tag{2}$$

The parameterizations of both models are taken from F. M. da Silva, A. Issifu, Luiz L. Lopes, *et al.*, Phys. Rev. D **109**, 043054 (2024), where the model parameters were constrained using recent astrophysical data.

[1] G.X. Peng, H. C. Chiang, J. J. Yang, *et al.*, Phys. Rev.C 61 (2000) 015201
[2] L. L. Lopes, C. Biesdorf, D. P. Menezes, Phys.Scripta 96 (2021) 6, 065303

EOS Constraints

Source	Type	Mass ($M_\odot$)	Spin ν (Hz)
PSR J0952-0607	RPP	2.35 ± 0.17	707
PSR J0740+6620	RPP (NICER)	2.072 ± 0.067	347
PSR J0030+0451	RPP (NICER)	1.34 ± 0.15	205
HESS J1731-347	CCO	0.77 ± 0.20	–

Table: Astrophysical sources used to constrain the DDQM and vector MIT bag models.

Additionally, we impose the Bodmer–Witten hypothesis, $(\varepsilon/n_B)_{\mathrm{SQM}} \leq 930\,\mathrm{MeV}$ and $(\varepsilon/n_B)_{\mathrm{2QM}} > 930\,\mathrm{MeV}$, together with the causality constraint at the stellar surface.

Key Mathematical Expressions

Stellar structure: Stellar configurations computed using the `RNS` code based on axisymmetric stationary spacetime[1]

$$I = \frac{J}{\Omega}, \quad T = \frac{J\Omega}{2}, \quad \text{with} \quad \Omega = 2\pi\nu. \tag{3}$$

Quadrupole Moment

$$Q = M_2 - \frac{4}{3}\left(\frac{1}{4} + b\right) M^3, \quad \bar{Q} = \frac{QM}{J^2}, \tag{4}$$

b is a dimensionless coefficient determined by the asymptotic behavior of the metric potentials, and M_2 denotes the coefficient of the $l = 2$ term in the expansion.

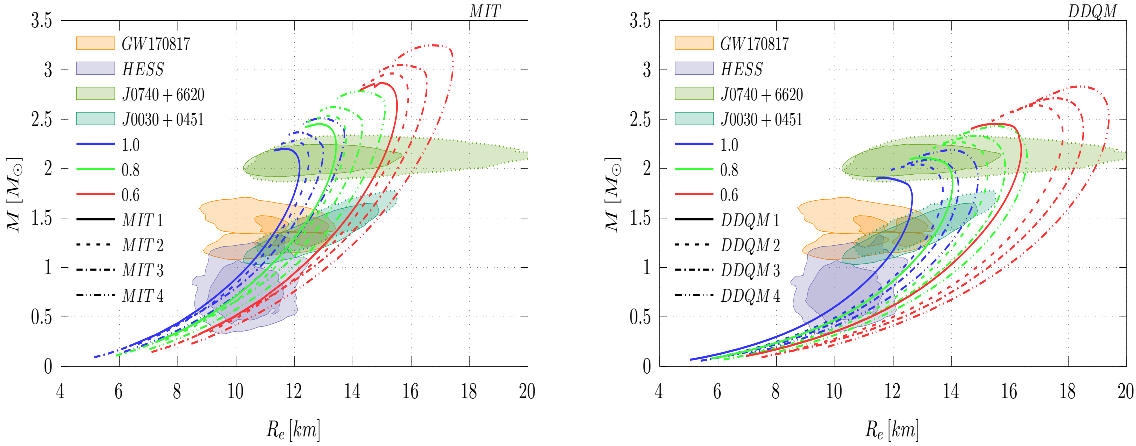
Mass Enhancement due to Rotation

$$\Delta M_{\max}(r_p/r_e) = M_{\max}(r_p/r_e) - M_{\max}(1.0). \tag{5}$$

Energy decomposition: Binding energy: $E_{\text{bind}} = M - M_0$, proper mass M_p, internal energy $U = M_p - M_0$, gravitational energy $|W| = |M - M_p - T|$.

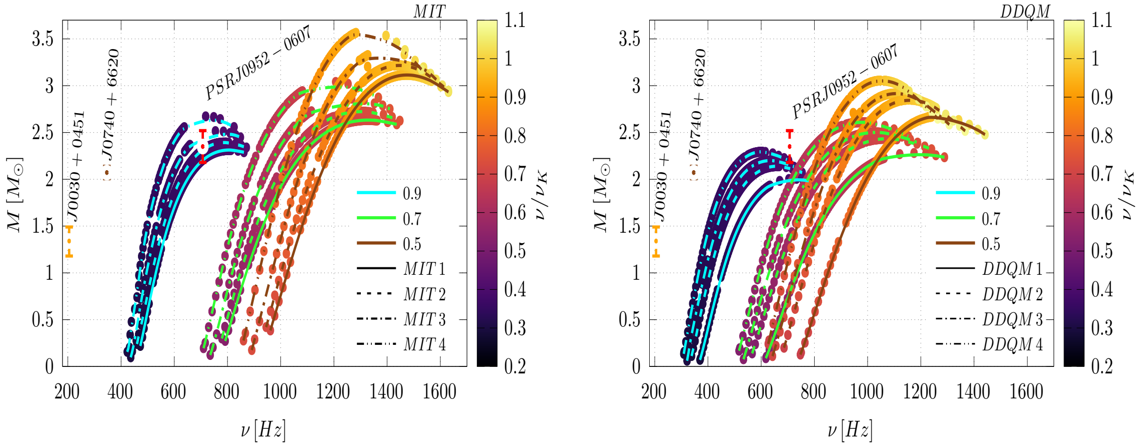
[1] N. Stergioulas, and J. L. Friedman, Astrophys.J. 444 (1995) 306.

Mass–Radius Relation



- Rotation increases both maximum mass and equatorial radius
- The stiffer vMIT bag model produces more massive and more compact stars

Mass versus rotation frequency



- The maximum stable spin frequency (Keplerian limit) is model-dependent
- MIT stars support: $\nu \gtrsim 1450$ Hz and DDQM support $\nu \lesssim 1300$ Hz

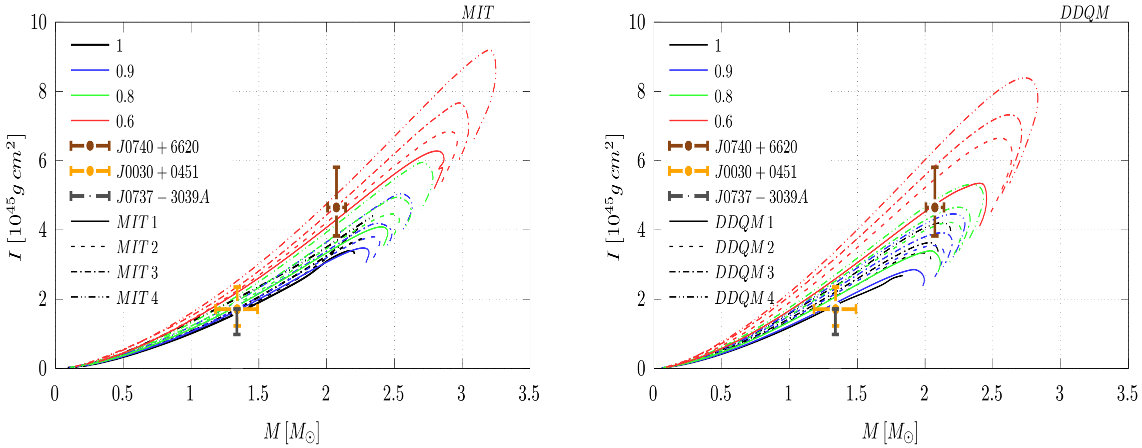
Maximum mass enhancement from rotation

Model	Static $M_{\max}(M_\odot)$	Kepler $M_{\max}(M_\odot)$	$\Delta M_{\max}(\%)$	ν_K (Hz)
DDQM1	1.91	2.47	29.3	1299
DDQM2	2.04	2.64	29.4	1164
DDQM3	2.10	2.71	29.1	1117
DDQM4	2.18	2.83	29.8	1060
MIT1	2.21	2.87	29.9	1452
MIT2	2.28	2.97	30.3	1427
MIT3	2.40	3.11	29.6	1409
MIT4	2.54	3.30	29.9	1342

Table: For $r_p/r_e = 0.6$, near the Kepler limit. Enhancement is model-independent ($\sim 30\%$).

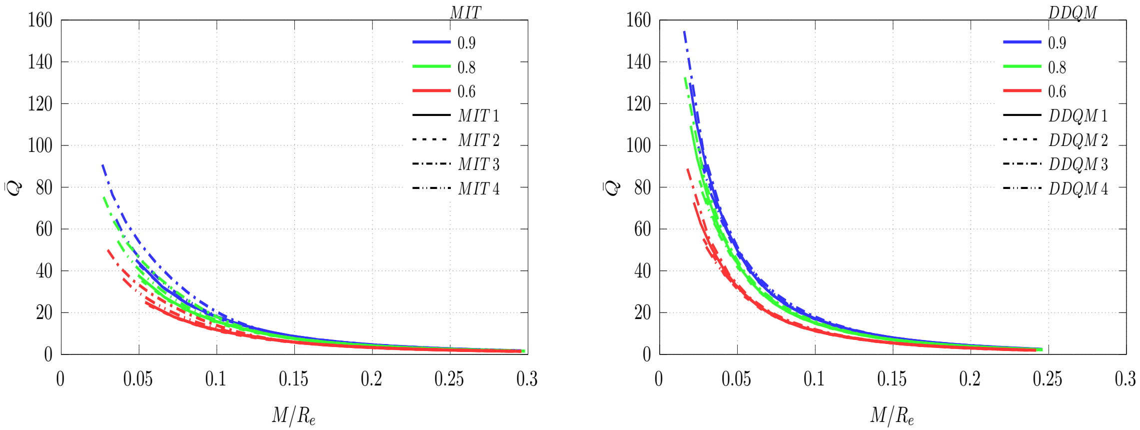
- DDQM1 and MIT1 correspond to larger ν_K, while DDQM4 and MIT4 correspond to smaller $\nu_K \propto \sqrt{M/R_e^3}$
- ΔM_{max} from hadronic models at ν_K is typically $\sim$ 15–20% (A. Konstantinou and S. Morsink, ApJ 934, 139 (2022)).

Moment of Inertia



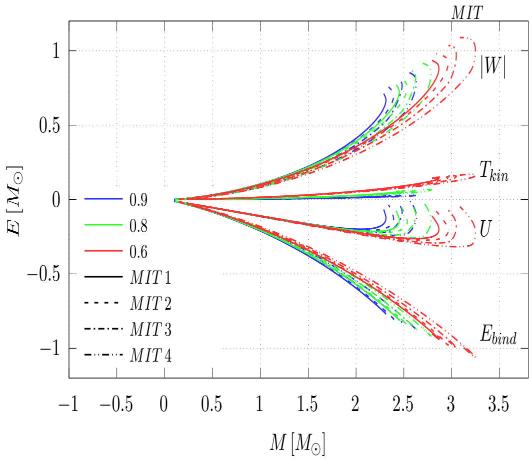
- At a fixed mass, the soft DDQM model produces a higher I than the siffer MIT bag model
- The stiff MIT model produces a higher I at $M_{\max}$

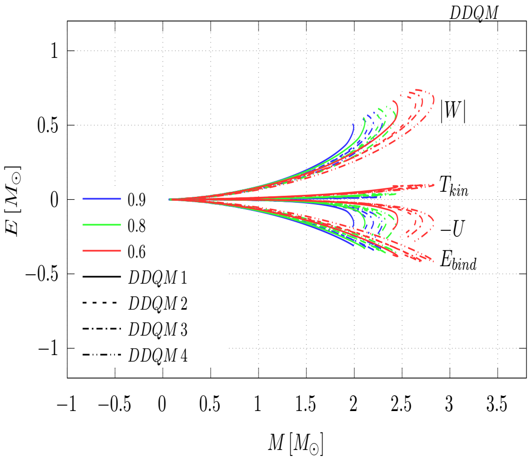
Quadrupole Moment



- At the same compactness, the DDQM model yields a larger $\bar{Q}$ than the MIT model

Energy Decomposition





- The sign of the U distinguishes self-bound from gravitationally bound stars
- Gravitational energy $|W|$ dominates the total energy budget in both models

Multimessenger Signatures

Observable	DDQM (soft)	vMIT (stiff)
$M_{\max}$	$\sim 2.8 M_\odot$	$\sim 3.3 M_\odot$
ν_K	< 1300 Hz	> 1450 Hz
I at $1.4 M_\odot$	High	Low
Q at $1.4 M_\odot$	High	Low
Z_p (max)	~ 0.55	> 0.8
U sign	> 0 (grav. bound)	< 0 (self-bound)
GW deformability	Larger	Smaller

Table: Combined measurements break EOS degeneracy.

Final Remarks

- **Energy decomposition probes microphysics**
  - vMIT: $U < 0 \rightarrow$ self-bound matter
  - DDQM: $U > 0 \rightarrow$ gravitationally bound
- **Observational discriminators**
  - DDQM predicts larger R_e, $\bar{Q}$, and I
  - vMIT supports higher ν_K and Z_p

Conclusion: The vector MIT bag model describes intrinsically self-bound quark matter, whereas the DDQM model produces configurations that are only gravitationally bound.

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Thank You!