6th Barcelona Initiative for Gravitation (BIG) Meeting

Institute of Space Sciences (ICE-CSIC)

04/07/2025

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ MODIFIED GRAVITY

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Fondazione Angelo Della Riccia Postdoctoral Fellowship 2025

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

- DARK ENERGY MODELS AND THE HUBBLE CONSTANT TENSION
- THE HUBBLE CONSTANT AS A DIAGNOSTIC TOOL
- REDSHIFT BINNED ANALYSIS OF THE SNe Ia PANTHEON SAMPLE
- DECREASING TREND OF THE HUBBLE CONSTANT WITH REDSHIFT
- THEORETICAL INTERPRETATION
  IN THE JORDAN FRAME OF $f(R)$ MODIFIED GRAVITY THEORIES:
  THE IMPACT OF NON-MINIMALLY COUPLED SCALAR FIELDS
  ON THE COSMIC EXPANSION

f(R) gravity in the Jordan Frame as a Paradigm for the Hubble Tension

MNRAS Letters, 522, L72-L77 (2023) arXiv:2211.16737

Authors: **TS**, G. Montani, F. Bombacigno

On the Evolution of the Hubble Constant with the SNe Ia Pantheon Sample and Baryon Acoustic Oscillations: A Feasibility Study for GRB-Cosmology in 2030

Galaxies, 10, 24 (2022) arXiv:2201.09848

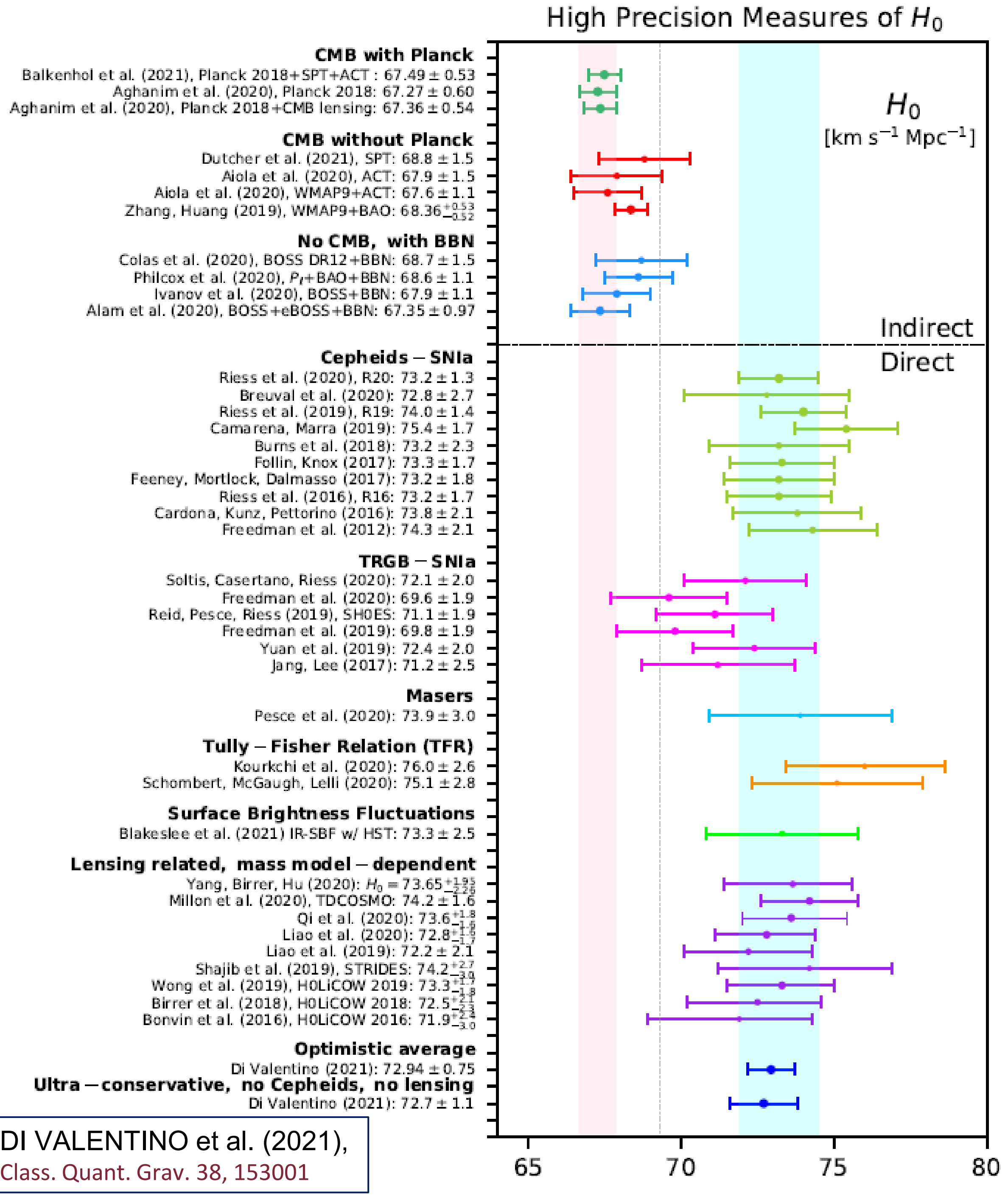
Authors: M. G. Dainotti, B. De Simone, **TS**, G. Montani, E. Rinaldi, G. Lambiase, M. Bogdan, S. Ugale

On the Hubble Constant Tension in the SNe Ia Pantheon Sample

ApJ 912, 150 (2021) arXiv:2103.02117

Authors: M. G. Dainotti, B. De Simone, **TS**, G. Montani, E. Rinaldi, G. Lambiase

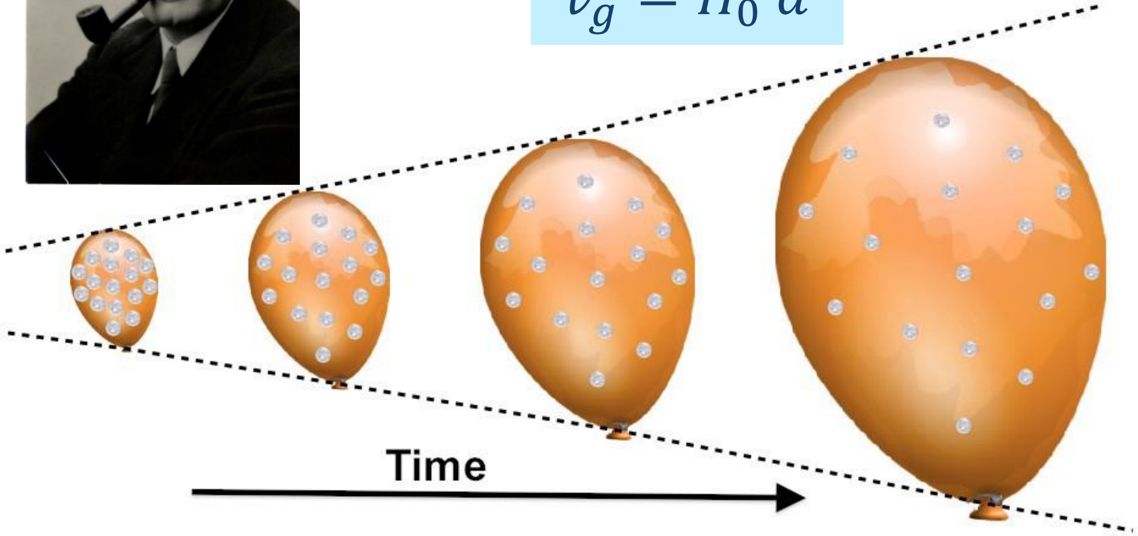
HUBBLE CONSTANT TENSION



DI VALENTINO et al. (2021),
Class. Quant. Grav. 38, 153001

Cosmic expansion and Hubble law

$$v_g = H_0\, d$$



Hubble constant definition

$$H_0 \equiv H(t = t_0) = H(z = 0)$$

Tiziano Schiavone

DARK ENERGY MODELS

Hubble function: $H(z) = H_0\, E(z)$

Ω_{m0}

cosmological density parameter of the matter component

For a flat geometry ($k = 0$), neglecting the radiation contribution (late times):

- ΛCDM model

$$E(z) = \sqrt{\Omega_{m0}\,(1+z)^3 + 1 - \Omega_{m0}}$$

- wCDM model

$w = w(z)$

$$E(z) = \sqrt{\Omega_{m0}\,(1+z)^3 + (1-\Omega_{m0}) \exp\left[3\int_0^z \frac{dz'}{1+z'}\,(1+w(z'))\right]}$$

- $w_0 w_a$CDM model

$$w(z) = w_0 + \frac{w_a\, z}{1+z}$$

CPL parameterization (Chevallier-Polarski-Linder)

CHEVALLIER & POLARSKI (2001), Int. J. Mod. Phys. D 10, 213

LINDER (2000), Phys. Rev. Lett. 90, 091301

THE HUBBLE CONSTANT AS A DIAGNOSTIC TOOL

MUST BE A CONSTANT

$$H_0 = \frac{H(z)}{E(z)}$$

Observational Hubble data (CC, BAOs)

The theoretical cosmological model is specified in $E(z)$

$$H_0 = \frac{(1+z)\int_0^z \frac{dz'}{E(z')}}{d_L(z)}$$

Luminosity distance Observational data (SNe Ia)

Observational data from probes at various redshifts z can be useful to test cosmological models.

Incompatible values of H_0 may indicate a **mismatch between observational data and the considered theoretical model**.

If we «compel» data analysis to conform to the prescribed ΛCDM model and still observe discrepancies, such as the H_0 tension, then H_0 effectively acquires z-dependence and varies across all redshifts

Krishnan et al. (2021) Phys. Rev. D 103, 103509

THE HUBBLE CONSTANT AS A DIAGNOSTIC TOOL

$$H = H_0\, E(z; \Omega_{m0}, \dots)$$

We can always express the Hubble function $H(z)$ of any model in terms of the reduced Hubble function $E^{\Lambda CDM}(z)$ of a flat ΛCDM model:

$(H_0, \Omega_{m0}, \dots)$
cosmological parameters of an arbitrary model

$$H(z) = \frac{H_0\, E(z;\ \Omega_{m0}, \dots)}{E^{\Lambda CDM}(z;\ \Omega_{m0}^{\Lambda CDM}, \dots)}\, E^{\Lambda CDM}(z;\ \Omega_{m0}^{\Lambda CDM}, \dots)$$
$$= H_0^{EFF}(z) \sqrt{\Omega_{m0}^{\Lambda CDM}\ (1+z)^3 + 1 - \Omega_{m0}^{\Lambda CDM}}$$

$(H_0^{\Lambda CDM}, \Omega_{m0}^{\Lambda CDM}, \dots)$
cosmological parameters of the flat ΛCDM model

Effective Hubble constant $H_0^{EFF}(z)$ as a **diagnostic tool.** It incorporates possible deviations from ΛCDM.

- **If $\boldsymbol{H_0^{EFF}(z)}$ evolves with $\boldsymbol{z}$**, it may indicate a **mismatch between observational data and the $\boldsymbol{\Lambda}$CDM model**;
- If $H_0^{EFF}(z)$ exhibits a flat profile, then there are no discrepancies.

PANTHEON SAMPLE BINNED ANALYSIS

Hubble constant tension within the SNe Ia redshift range?

1048 spectroscopically confirmed SNe Ia from different surveys
(PS1, SDSS, ESSENCE, SNLS, SCP, GOODS, CANDELS/CLASH)
Scolnic et al. (2018), ApJ 859, 101 Repository: https://github.com/dscolnic/Pantheon

$0.01 < z < 2.26$

- Equally populated subsamples of SNe Ia: 3, 4, 20, 40 redshift bins
- Statistical analysis for each redshift bin within the flat ΛCDM model, including statistical and systematic covariance matrices of SNe Ia χ^2 minimization, MCMC method
- Fixed $\Omega_{m0} = 0.298$ for the ΛCDM model [Scolnic et al. (2018), ApJ 859, 101]
- Uniform priors: $60 < H_0 < 80\ km\ s^{-1}\ Mpc^{-1}$
- Test to check the values of H_0 in different redshift bins

Test: non-linear fit

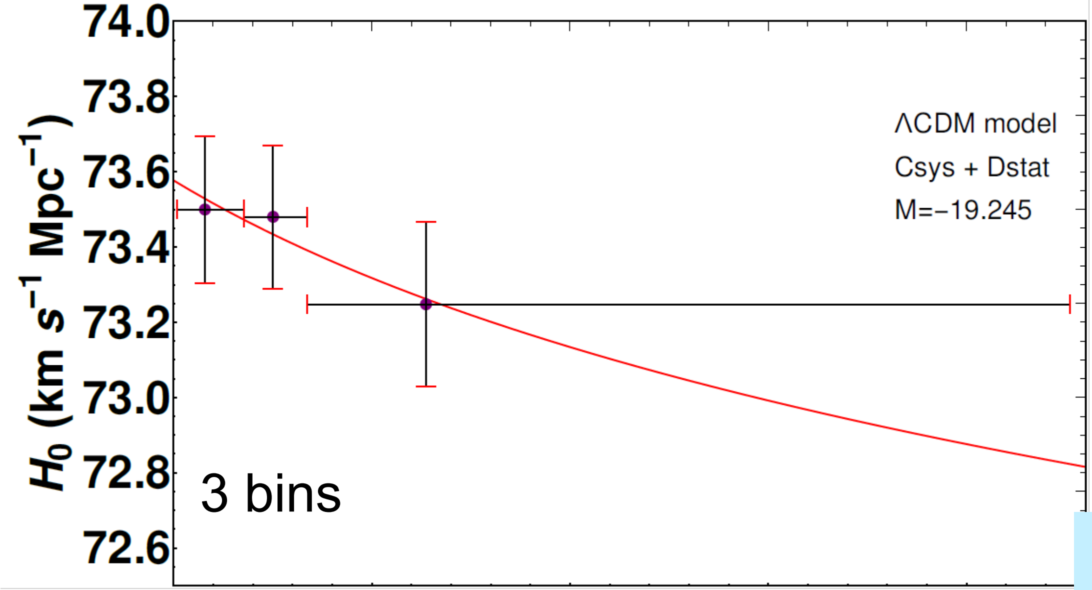
$$H_0^{fit}(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

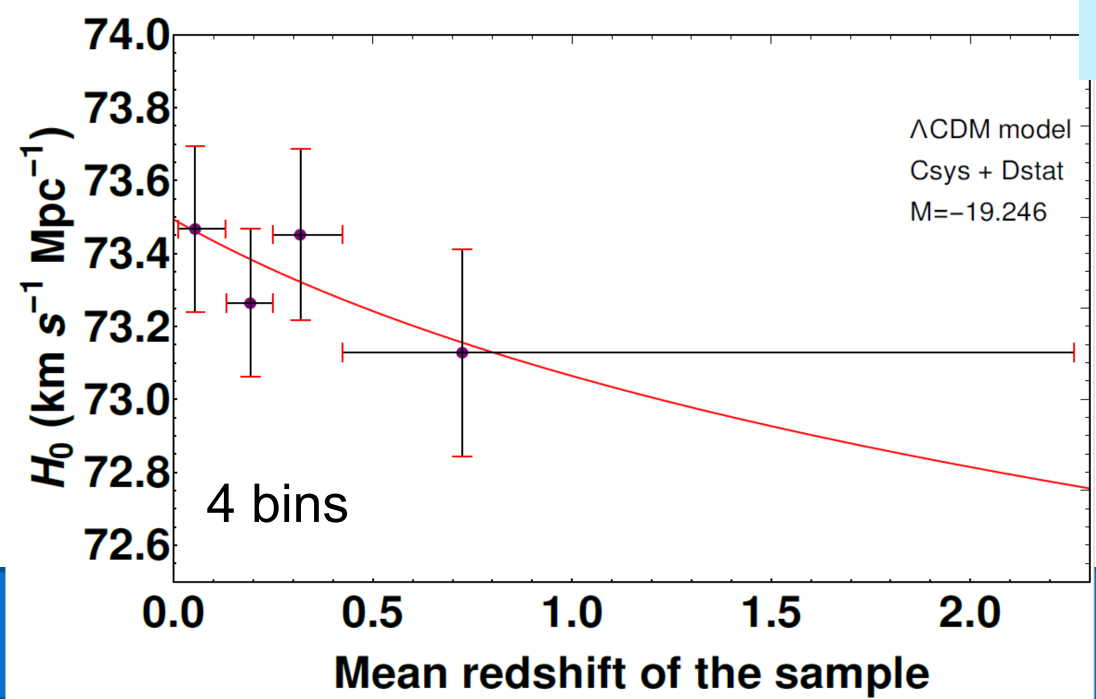
α: evolutionary parameter

$$\widetilde{H}_0 = H_0^{fit}(z=0)$$

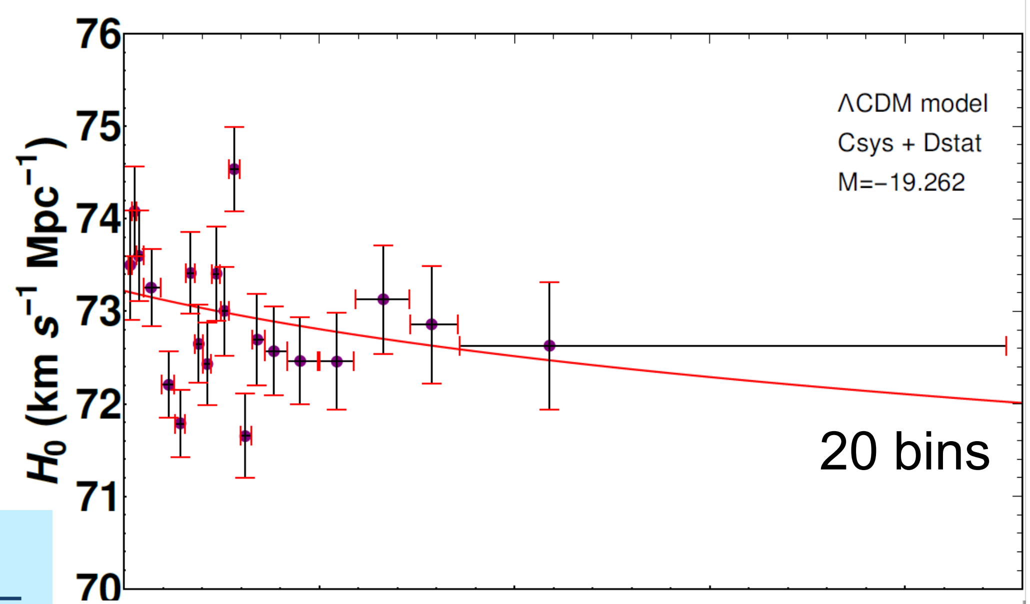
DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

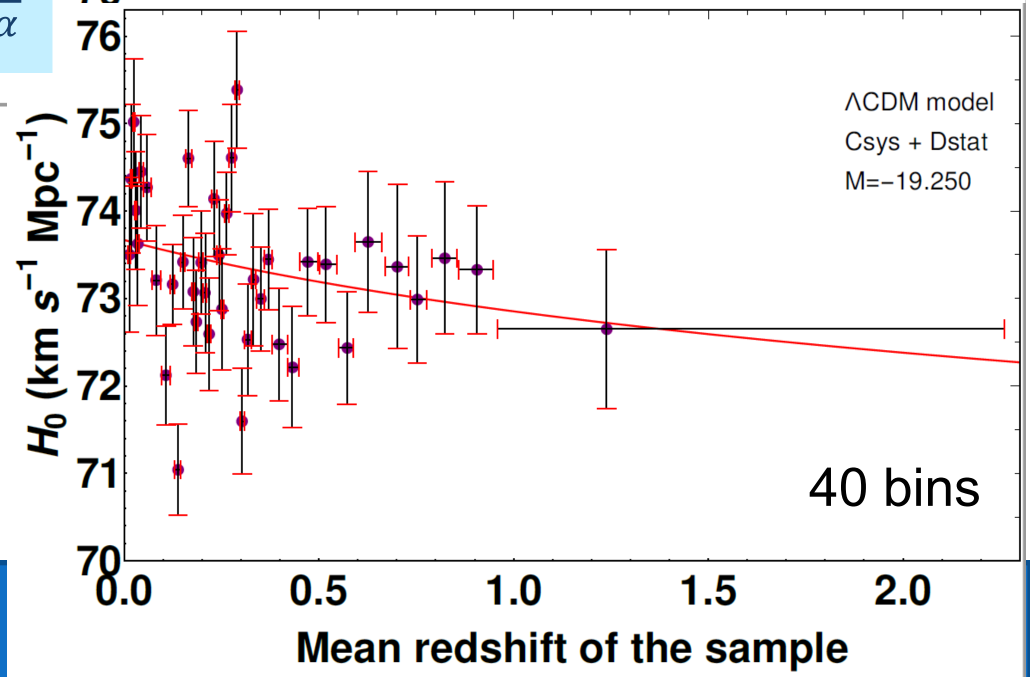
A NON-CONSTANT HUBBLE CONSTANT?





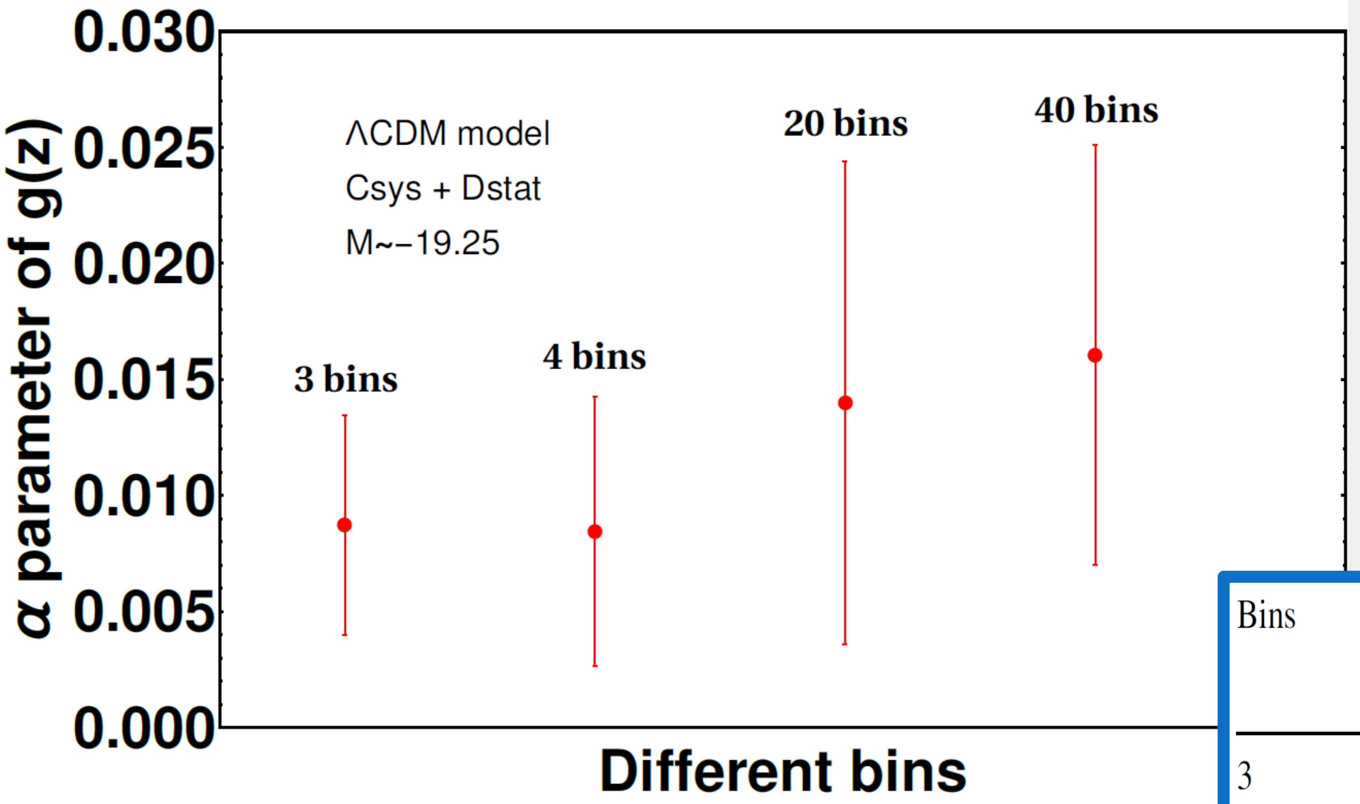
$$H_0^{fit}(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$





DAINOTTI,
DE SIMONE,
SCHIAVONE
et al. (2021),
ApJ 912, 150

A NON-CONSTANT HUBBLE CONSTANT?



$$H_0^{fit}(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

$\alpha = 0 \;\rightarrow$ no evolution

Error bars, 1 σ

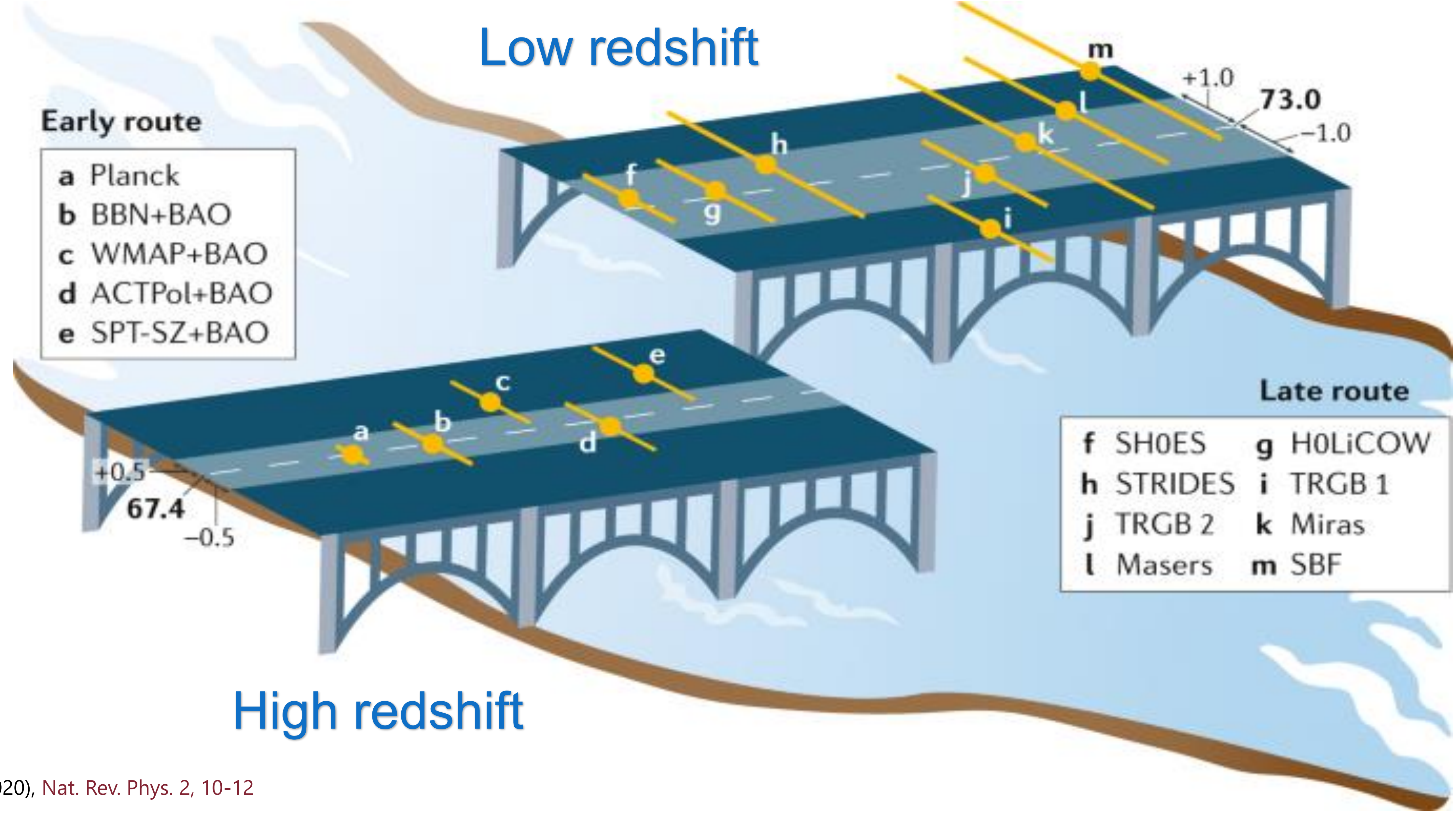
Slight and unexpected evolution of $H_0^{fit}(z)$

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

FIT RESULTS ΛCDM MODEL

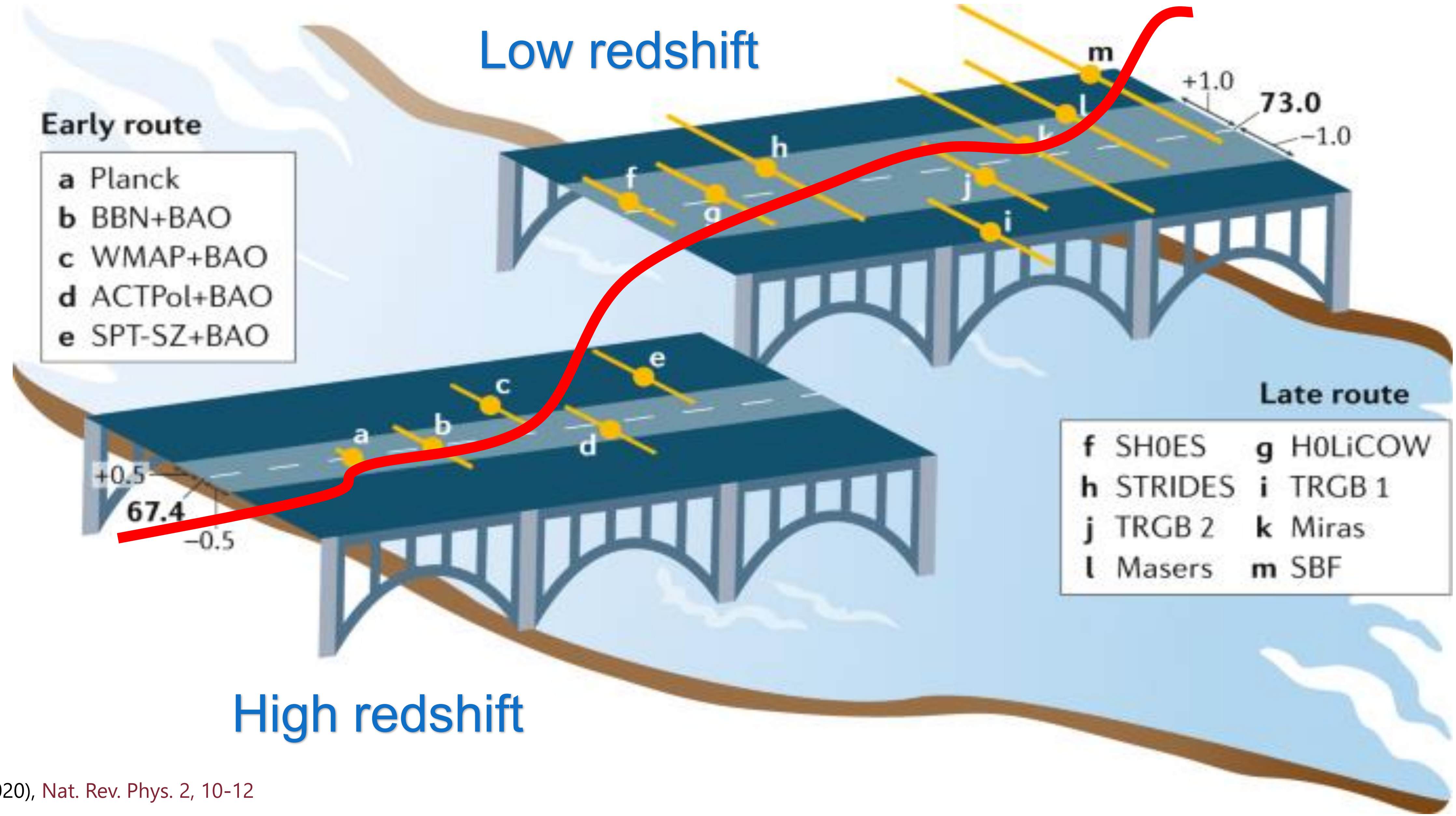
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	$\frac{\alpha}{\sigma_\alpha}$	M
3	73.577 ± 0.106	0.009 ± 0.004	2.0	-19.245 ± 0.006
4	73.493 ± 0.144	0.008 ± 0.006	1.5	-19.246 ± 0.008
20	73.222 ± 0.262	0.014 ± 0.010	1.3	-19.262 ± 0.014
40	73.669 ± 0.223	0.016 ± 0.009	1.8	-19.250 ± 0.021

Low redshift



High redshift

RIESS (2020), Nat. Rev. Phys. 2, 10-12



RIESS (2020), Nat. Rev. Phys. 2, 10-12

EXTRAPOLATED VALUES AT HIGH REDSHIFTS

ΛCDM model

Bins	$H_0(z = 11.09)$ (km s^{-1} Mpc^{-1})	$H_0(z = 1100)$ (km s^{-1} Mpc^{-1})
3	72.000 ± 0.805	69.219 ± 2.159
4	71.962 ± 1.049	69.271 ± 2.815
20	70.712 ± 1.851	66.386 ± 4.843
40	70.778 ± 1.609	65.830 ± 4.170

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

$$H_0^{fit}(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

Consistent in 1 σ with the Planck measurement from the CMB, at the redshift of the last scattering surface z=1100

$$H_0^{[CMB]} = (67.36 \pm 0.54)\ km\ s^{-1}\ Mpc^{-1}$$

PLANCK COLLABORATION
Planck 2018 result, VI: Cosmological parameters
A&A 641, A6 (2020).

OTHER INDEPENDENT EVIDENCE OF TRENDS IN COSMOLOGICAL PARAMETERS

Decreasing trends of H_0 (and/or increasing Ω_{m0}) with the redshift observed in different dataset combinations:

- Wong et al., MNRAS 498 (2019) 1420 [H0LiCOW data]
- Krishnan et al. (2020) Phys. Rev. D 102, 103525 [megamaser+CC+SNeIa+BAOs]
- Krishnan et al. (2021) Phys. Rev. D 103, 103509 [megamaser+CC+SNeIa+BAOs]
- Krishnan et al. arXiv:2201.13384 [megamaser+CC+SNeIa+BAOs]
- Horstmann et al. (2022), A&A 668 A34 [Pantheon SNeIa]
- Jia, Hu & Wang (2023) A&A 674 A45 [PantheonPlus+SH0ES SNeIa]
- Malekjani et al., *Eur.Phys.J.C* 84 (2024) 3, 317 [PantheonPlus SNeIa]
- Colgain et al. (2024) Physics of the Dark Universe, 44 101464 [CC+Pantheon SNeIa+QSOs]

POSSIBLE EXPLANATIONS OF $H_0^{EFF}(z)$

Astrophysical reasons

- Hidden redshift evolution of an astrophysical parameter of SNe Ia (stretch, metallicity, …)
- Astrophysical properties (host galaxies, selection effects)
- Biases not considered in the Pantheon Sample
- Systematics uncertainties of the sample

Local effects

- Inhomogeneities
- Cosmic voids

New Physics

- Late and/or early-time modification of gravity?

$f(R)$ MODIFIED GRAVITY in the JORDAN FRAME

- Extend GR to solve open problems in cosmology with extra degrees of freedom
- Geometrical modification of gravity theory
- Avoiding *ad hoc* components in the Universe, *e.g.* dark energy
- Generalized gravitational lagrangian $\mathcal{L}_g = f(R)$ $\quad$ R: Ricci scalar

- Dynamically equivalent action in the Jordan frame (JF), scalar-tensor theory
- The extra degree of freedom of $f(R)$ is replaced by a scalar field ϕ
- Non-minimal coupling between scalar field and metric

Scalar field

$$\phi = f'(R)$$

Potential

$$V(\phi) = R(\phi)\phi - f(R(\phi))$$

NOJIRI & ODINTSOV (2006), eConf C0602061, 06

SOTIRIOU & FARAONI (2010), Rev. Mod. Phys. 82, 451

$f(R)$ MODIFIED GRAVITY in the JORDAN FRAME

Dynamically equivalent action to $f(R)$ theories

Jordan Frame (JF)

$$S_g = \frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\;[\phi\,R - V(\phi)]$$

Scalar field

$$\phi\;=f'(R)$$

Potential

$$V(\phi) = R(\phi)\phi - f(R(\phi))$$

For a <u>flat FLRW metric</u>:

Generalized Friedmann eq.

$$H^2 = \frac{\chi\,\rho}{3\,\phi} - H\,\frac{\dot\phi}{\phi} + \frac{V(\phi)}{6\,\phi}$$

Scalar field eq.

$$3\,\ddot\phi - 2\,V(\phi) + \phi\,\frac{dV}{d\phi} + 9\,H\,\dot\phi = \chi\,\rho$$

Generalized cosmic acc. eq.

$$\frac{\ddot a}{a} = -\frac{\chi\,\rho}{6\,\phi} + \frac{V(\phi)}{6\,\phi} + \frac{1}{6}\,\frac{dV}{d\phi} + H\,\frac{\dot\phi}{\phi}$$

χ: Einstein constant

$\dot{(\ldots)} = \partial_t(\ldots)$

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

- Decreasing trend of $H_0^{fit}(z)$ from binned analysis of SNe Ia + BAOs, also considering two free parameters (H_0 and Ω_{m0}) in MCMC analysis [1,2]
- Failure of $w_0 w_a$CDM model and the $f(R)$ Hu-Sawicki proposal to explain $H_0^{fit}(z)$ [1,2]
- Need for a new $f(R)$ model able to both mimic a dark energy component and provide a mechanism for an effective Hubble constant $H_0^{\text{eff}}(z)$
- Non-minimally coupled scalar field plays a crucial role
- Goal: effective Hubble constant $H_0^{\text{eff}}(z)$ that evolves with z to match

$H_0^{[CMB]} = (67.36 \pm 0.54)$ km s^{-1} Mpc^{-1} and $H_0^{[loc]} = (73.04 \pm 1.04)$ km s^{-1} Mpc^{-1}

[1] DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

[2] DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2022), Galaxies 2022, 10, 24

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

From the generalized Friedmann eq. in JF:

$$\mathrm{H}(z) = \frac{H_0}{\sqrt{\phi - (1+z)\frac{d\phi}{dz}}} \sqrt{\Omega_{m0}(1+z)^3 + \frac{V(\phi)}{2\chi\rho_{c0}}}$$

Approximation: slight deviation from ΛCDM

$$V(\phi) \equiv 2\chi\rho_\Lambda + g(\phi)$$

$$g(\phi) \ll 2\chi\rho_\Lambda$$

We obtain a form similar to flat ΛCDM but with an effective Hubble constant (due to the extra scalar field)

ΛCDM: $$H(z) = H_0^{\Lambda CDM}\sqrt{\Omega_{m0}^{\Lambda CDM}(1+z)^3 + 1 - \Omega_{m0}^{\Lambda CDM}}$$

$f(R)$: $$H(z) \approx H_0^{\mathrm{eff}}(z)\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

$$H_0^{\mathrm{eff}}(z) = \frac{H_0}{\sqrt{\phi - (1+z)\frac{d\phi}{dz}}}$$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

Solving the cosmological dynamics in the JF, we can reconstruct analytically the scalar field potential, then the $f(R)$ functional form

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

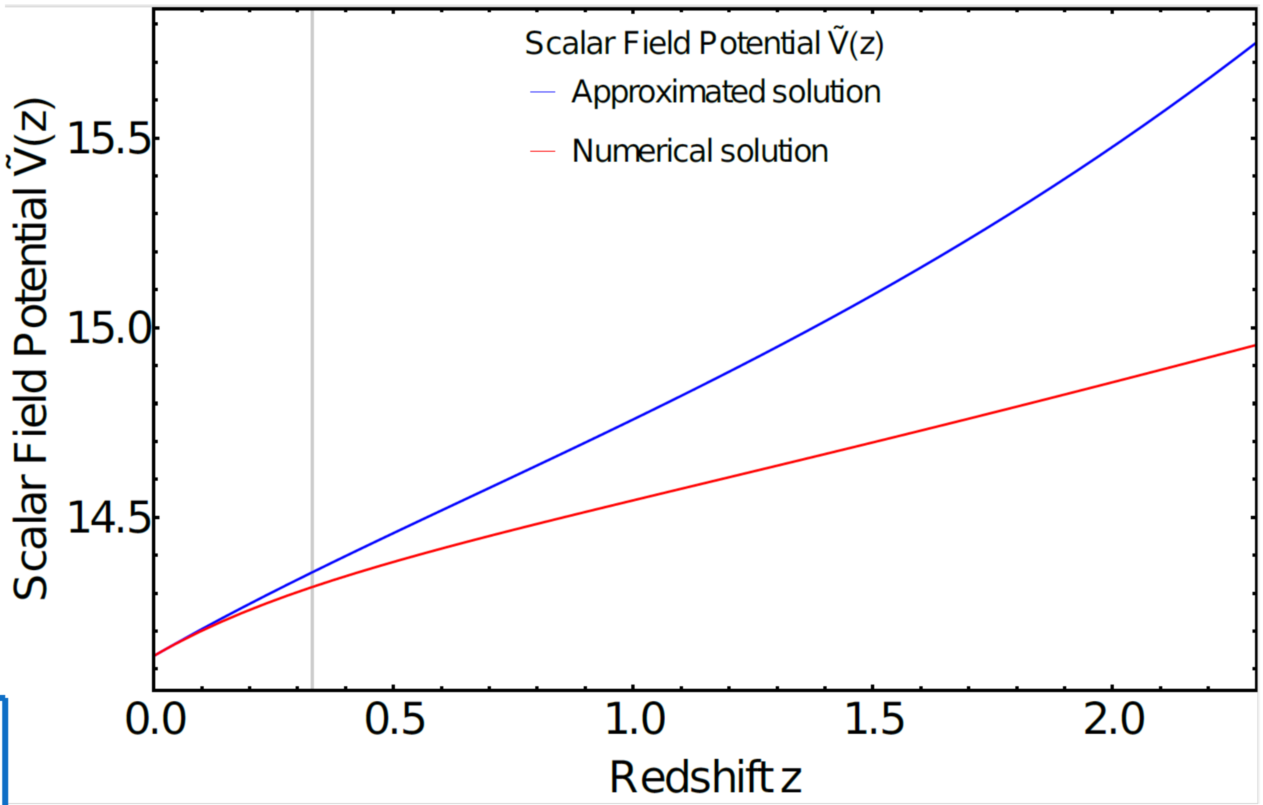
Approximated analytical solution:

$$\phi(z) = \phi_0\,(1+z)^{2\alpha}$$

$$H_0^{\mathrm{eff}}(z) = \frac{H_0}{\sqrt{\phi_0\,(1-2\,\alpha)}\,(1+z)^{\alpha}}$$

dimensionless

$$\tilde{V}(\phi) = \frac{V(\phi)}{m^2} \qquad m^2 \equiv \frac{\chi\,\rho_{m0}}{3}$$



Conditions to match $H_0^{[CMB]}$ and $H_0^{[loc]}$:

$$\phi_0 = \phi(0) = 1 - 10^{-7} \qquad \Omega_{m0} = 0.298$$

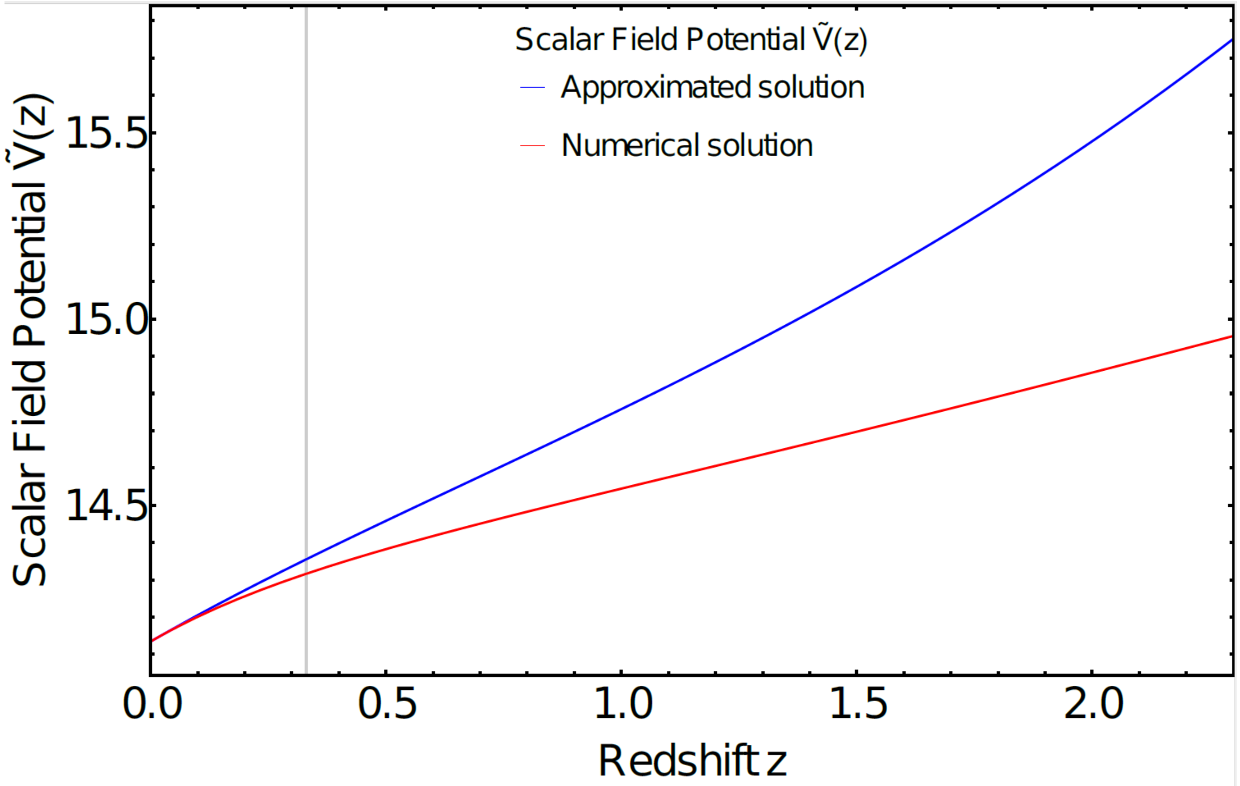
$$\alpha = 1.1 \times 10^{-2} \qquad \tilde{V}(\phi_0) = \tilde{V}(z=0) = 6\,\frac{1-\Omega_{m0}}{\Omega_{m0}}$$

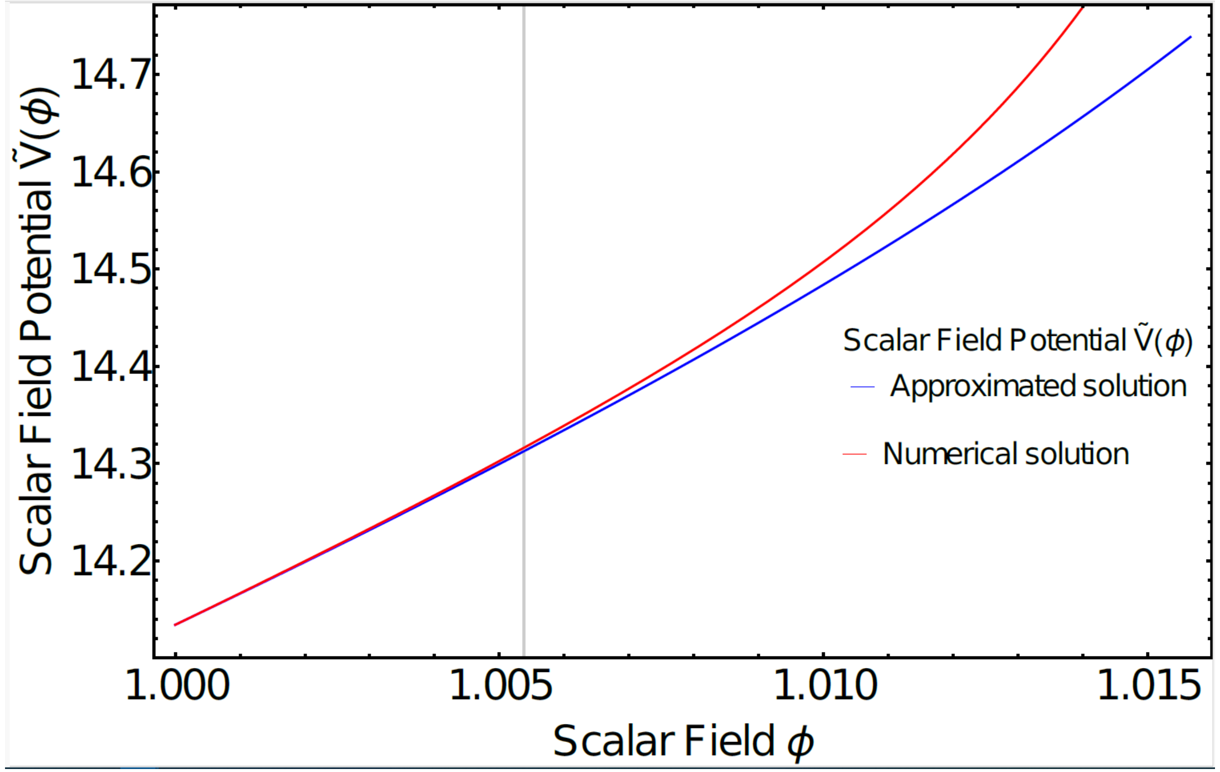
$$H_0 = 72.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

ΛCDM today

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY





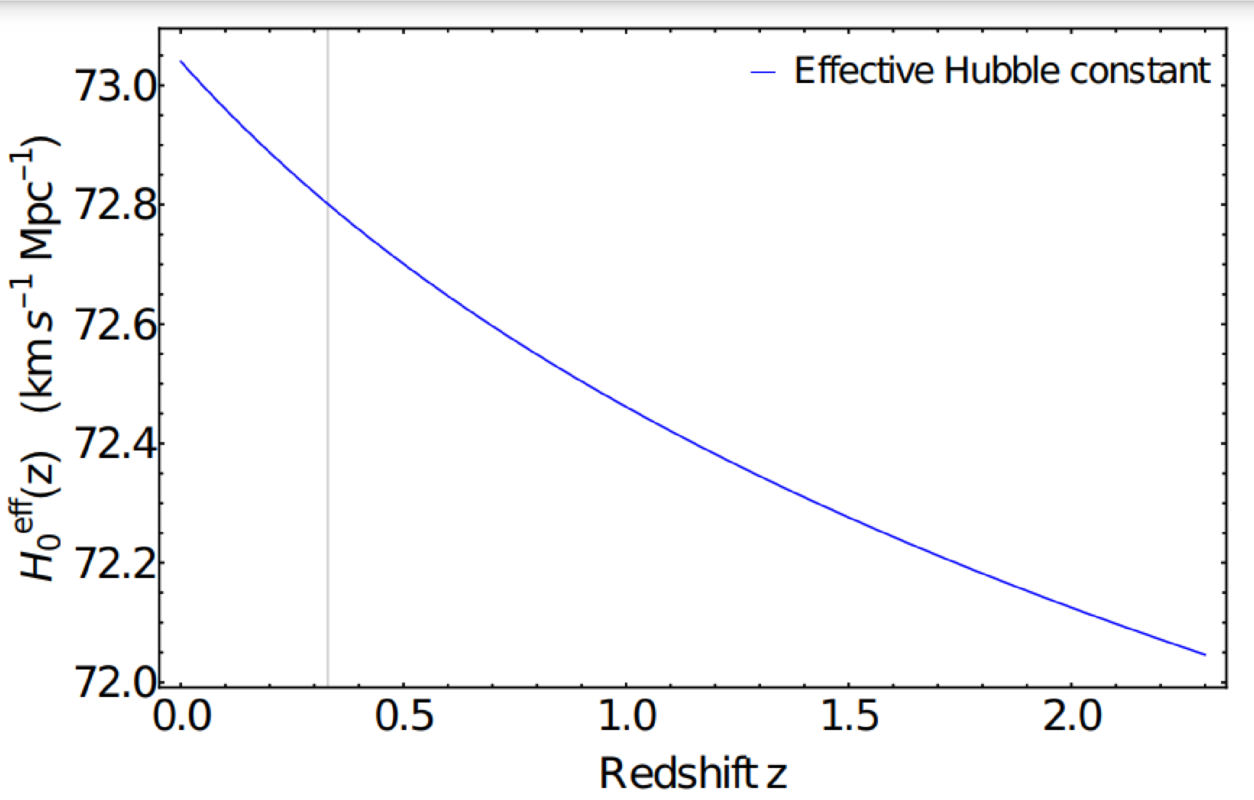
Percentage variation of $\tilde{V} \sim 1.6\%$

Nearly flat region occurs

for $0 < z < 0.3$

$$\tilde{V}(\phi) = \tilde{V}(\phi_0) + \frac{6\alpha}{1-\alpha}\left\{\frac{2+\alpha}{\alpha}\frac{1-\Omega_{m0}}{\Omega_{m0}}\ln\left(\frac{\phi}{\phi_0}\right) + \frac{1+2\alpha}{3}\left[\left(\frac{\phi}{\phi_0}\right)^{\frac{3}{2\alpha}} - 1\right]\right\}$$

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY



SCHIAVONE & MONTANI, arXiv:2408.01410

$$H_0^{\text{eff}}(z) = \frac{H_0}{\sqrt{\phi - (1+z)\frac{d\phi}{dz}}}$$

$H_0^{eff}(z)$ could be measured in principle at different redshifts.

The decreasing trend of $H_0^{eff}(z)$ approaches the Planck value for $z \approx 1100$.

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

The low-redshift $f(R)$ profile

For $z \ll 1$:

$$\phi(z) \approx \phi_0(1 + 2\,\alpha\, z) + O(z^3)$$

$$\tilde{V}(\phi) \approx \tilde{V}(\phi_0) + A_1\,(\phi - \phi_0) + A_2(\phi - \phi_0)^2 + O[(\phi - \phi_0)^3]$$

where the dimensionless constant A_i and B_i are algebraically related to the values of α, Ω_{m0} and ϕ_0

Relations in the Jordan frame

$$R = \frac{dV}{d\phi} \qquad V(\phi) = R(\phi)\phi - f(R(\phi))$$

$$f(R) \approx m^2\, B_0 + B_1\, R + B_2\, \frac{R^2}{m^2}$$

Approximated solution for $z \ll 1$: $f(R)$ – quadratic gravity

It can be shown that ΛCDM is recovered for $\alpha \to 0$ and $\phi_0 \to 1$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

SUMMARY AND CONCLUSIONS

- Binned analysis of the SNe Ia Pantheon Sample (low redshifts).
- Unexpected evolution and decreasing trend of $H_0(z)$ in different bins for various cosmological models.
- The non-minimally coupled scalar field in the Jordan frame of $f(R)$ modified gravity allows us to define an effective Hubble constant $\boldsymbol{H_0^{eff}(z)}$ that evolves with the redshift.
- The additional degree of freedom compared to GR is incorporated in $\boldsymbol{H_0^{eff}(z)}$.
- A running Hubble constant allows a **new interpretation of the tension: it could be no longer a discrepancy between local probes and Planck data, but an intrinsic evolutionary behavior of $\boldsymbol{H_0^{eff}(z)}$ as viewed in a modified gravity scenario.**

FUTURE PROJECTS

- If the plot Hubble constant VS z, obtained from different cosmological probes, is not flat in a considered theoretical model, then it may signal a mismatch between data and the model.
- An effective Hubble constant $\boldsymbol{H_0^{eff}(z)}$ can be used as a diagnostic tool to test cosmological models.
- The procedure can be generalized for a pletora of models that predict deviations from ΛCDM. The additional degrees of freedom can be restated in terms of $\boldsymbol{H_0^{eff}(z)}$.
- New binned analysis including different tracers (Pantheon+, BAOs, CC, quasars, GRBs, etc.) to better constrain α.
- Testing cosmological models with $\boldsymbol{H_0^{eff}(z)}$ via MCMC method, considering modified Hubble functions.
- Incoming paper: effective cosmological parameters in $f(R,T)$ gravity.

Backup slides

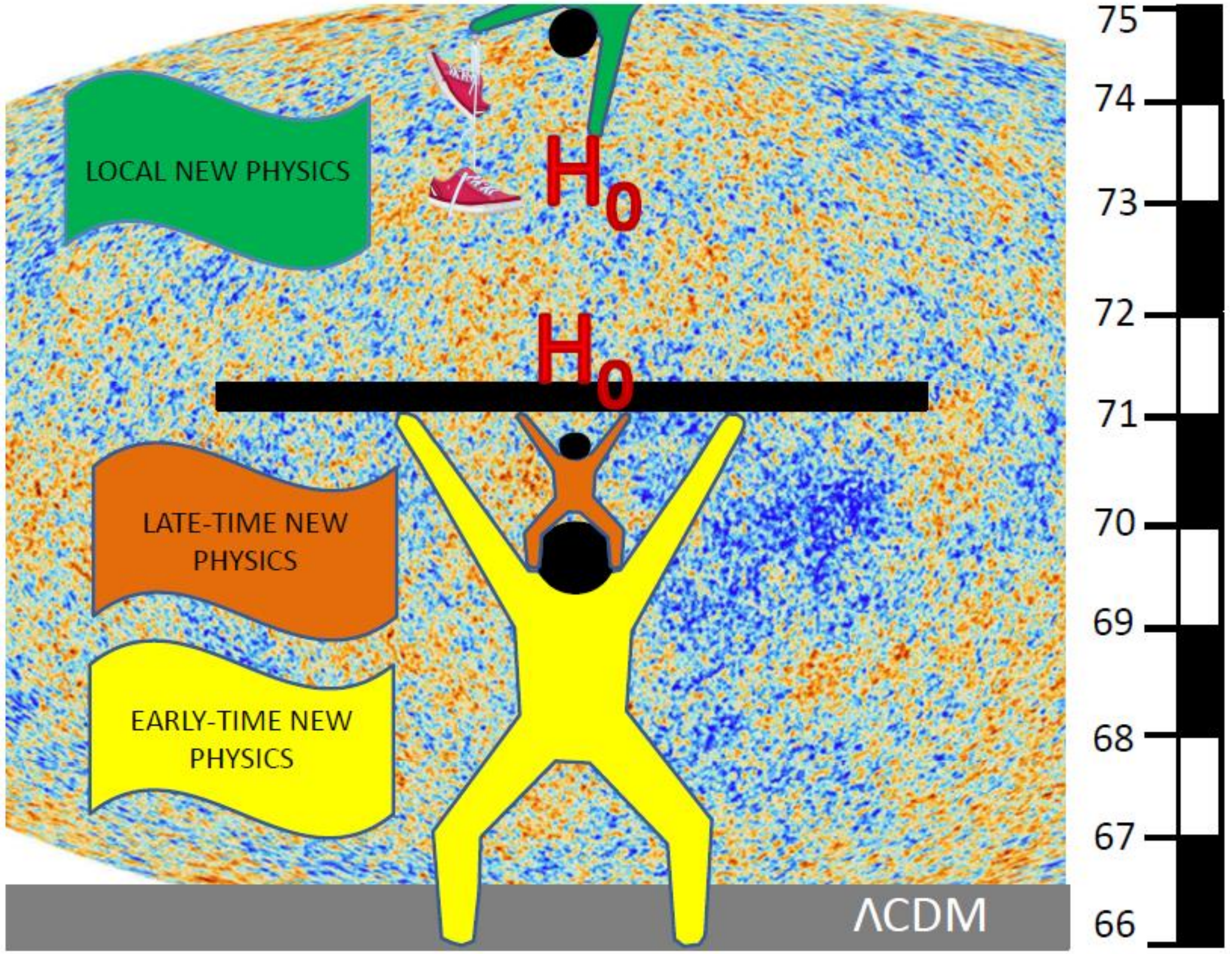
EARLY-TIME NEW PHYSICS + LATE-TIME CHANGES?

“Seven Hints that Early-Time New Physics Alone Is Not Sufficient to Solve the Hubble Tension”

SUNNY VAGNOZZI, *Universe* **2023**, *9*(9), 393

One bullet point in the list:

Descending trends observed in a wide range of low-z datasets



Credits: Cristina Ghirardini

SIMILAR PAPERS ABOUT MODIFICATIONS TO ADDRESS THE HUBBLE TENSION

- Odintsov et al., Nucl. Phys. B, 966 (2021) 115377 [f(R) gravity]
- Nojiri et al., Nucl. Phys. B, 980 (2022) 115850 [f(R) gravity]
- Montani, De Angelis, et al., MNRAS Letters 527(2024)1, L156 [f(R)+dynamical DE]
- Montani et al., Phys. Dark Univ. 44, 101486 (2024) [slow-rolling]
- Montani, Carlevaro, & De Angelis, Entropy 26, 662 (2024) [f(R)+matter creation]
- Schiavone & Montani, arXiv:2408.01410 [f(R) gravity]
- Montani, Carlevaro, Escamilla, & Di Valentino, arXiv:2404.15977 [DE-DM interaction]
- Chen et al., arXiv:2406.16503 [f(R) gravity]
- Luis Escamilla, et al. (including Di Valentino), arXiv:2408.04354 [modified gravity]

LUMINOSITY DISTANCE AND DISTANCE MODULUS

For a flat geometry $(k = 0)$

$$d_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}$$

$$\mu_{th} = 5 \log_{10} d_L + 25$$

These expressions depend on the considered cosmological model (see the function $E(z)$).

From SNe Ia observations:

Modified Tripp formula

TRIPP (1998), A&A 331, 815

BETOULE et al. (2014), A&A 568, A22

SCOLNIC et al. (2018), ApJ 859, 101

$$\mu_{obs} = m_B - M + \alpha\, x_1 - \beta\, c + \Delta M + \Delta B$$

B-band apparent magnitude

Absolute magnitude for $x_1 = 0 = c$

Stretch parameter

Color parameter

Host-galaxy mass correction

Bias correction

PANTHEON SAMPLE ANALYSIS

1048 spectroscopically confirmed SNe Ia from different surveys
(PS1, SDSS, ESSENCE, SNLS, SCP, GOODS, CANDELS/CLASH)

$$0.01 < z < 2.26$$

Statistical analysis:

$$\chi^2 = \Delta\mu^T C^{-1} \Delta\mu$$

where

$$\Delta\mu = \mu_{obs} - \mu_{th}$$

Theoretical distance modulus

$$\mu_{th} = 5 \log_{10} d_L + 25$$

$$C = D_{stat} + C_{sys}$$

Uncertainty matrix C (1048x1048)

Statistical matrix (diagonal matrix, 1048x1048)

Systematic covariance matrix (symmetric, 1048x1048)

Cobaya package in Python to minimize χ^2

Scolnic et al. (2018), ApJ 859, 101
Repository: https://github.com/dscolnic/Pantheon

PANTHEON SAMPLE ANALYSIS

$$C = D_{stat} + C_{sys}$$

D_{stat} Statistical matrix (diagonal matrix, 1048x1048)
Includes distance errors σ^2 for each SNe

C_{sys} Systematic covariance matrix (matrix, 1048x1048)
Includes N systematic (S_k) sources of errors

$$\underbrace{\sigma^2 = \sigma_N^2}_{\text{Photometric error}} + \underbrace{\sigma_{mass}^2}_{\text{Mass-step correction}} + \underbrace{\sigma_{\mu-z}^2}_{\text{Peculiar velocity and redshift}} + \underbrace{\sigma_{lens}^2}_{\text{Lensing}} + \underbrace{\sigma_{int}^2}_{\text{Intrinsic scatter}} + \underbrace{\sigma_{bias}^2}_{\text{Bias correction}}$$

$$C_{ij,sys} = \sum_{k=1}^{N} \frac{\partial \mu_i}{\partial S_k} \frac{\partial \mu_j}{\partial S_k} \sigma_{S_k}^2$$

S_k: systematics → $(m_B, x_1, c, m_B c, x_1 m_B, x_1 c)$

σ_{S_k}: systematic error

Tiziano Schiavone

PANTHEON SAMPLE BINNED ANALYSIS

Hubble constant tension within the SNe Ia redshift range?

$$0.01 < z < 2.26$$

- Equally populated subsamples of SNe Ia: 3, 4, 20, 40 redshift bins
- Building submatrices C and subvector $\Delta\mu$, considering the redshift order of SNe
- Statistical analysis for each redshift bin, χ^2 minimization, MCMC method
- Uniform priors: $60 < H_0 < 80 \; km \; s^{-1} \; Mpc^{-1}$
- Starting from the local value in the 1st bin: $H_0 = 73.5 \; km \; s^{-1} \; Mpc^{-1}$
- Fixed $\Omega_{m0} = 0.298$ for the ΛCDM model
- Fixed $\Omega_{m0} = 0.308, \;\; \mathrm{w}_0 = -1.009, \;\; \mathrm{w_a} = -0.129$ for the $w_0 w_a$CDM model
- Extracting H_0 value for each bin

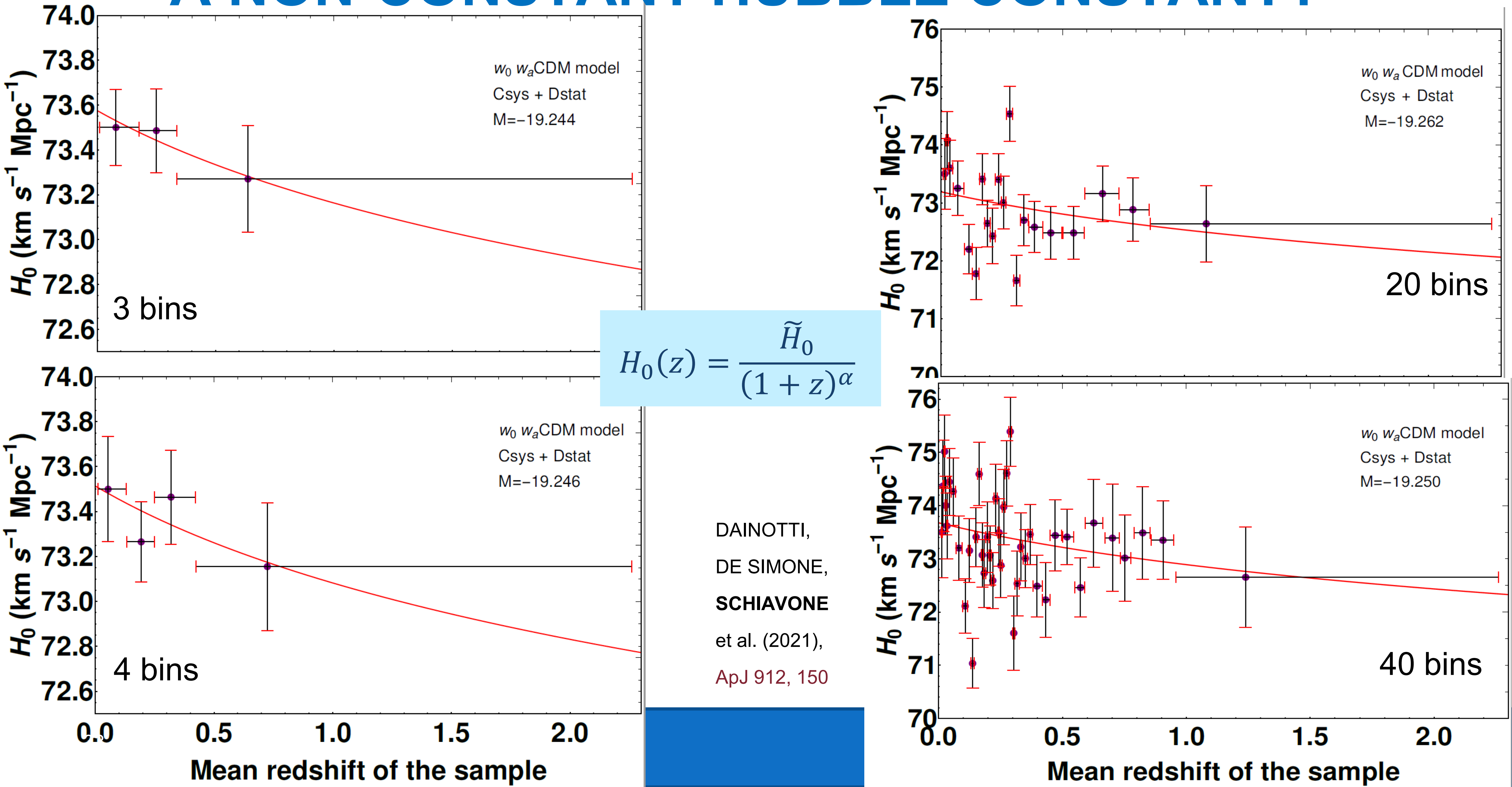
Test: non-linear fit

$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

α: evolutionary parameter

$\widetilde{H}_0 = H_0(z=0)$

A NON-CONSTANT HUBBLE CONSTANT?



$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

DAINOTTI,

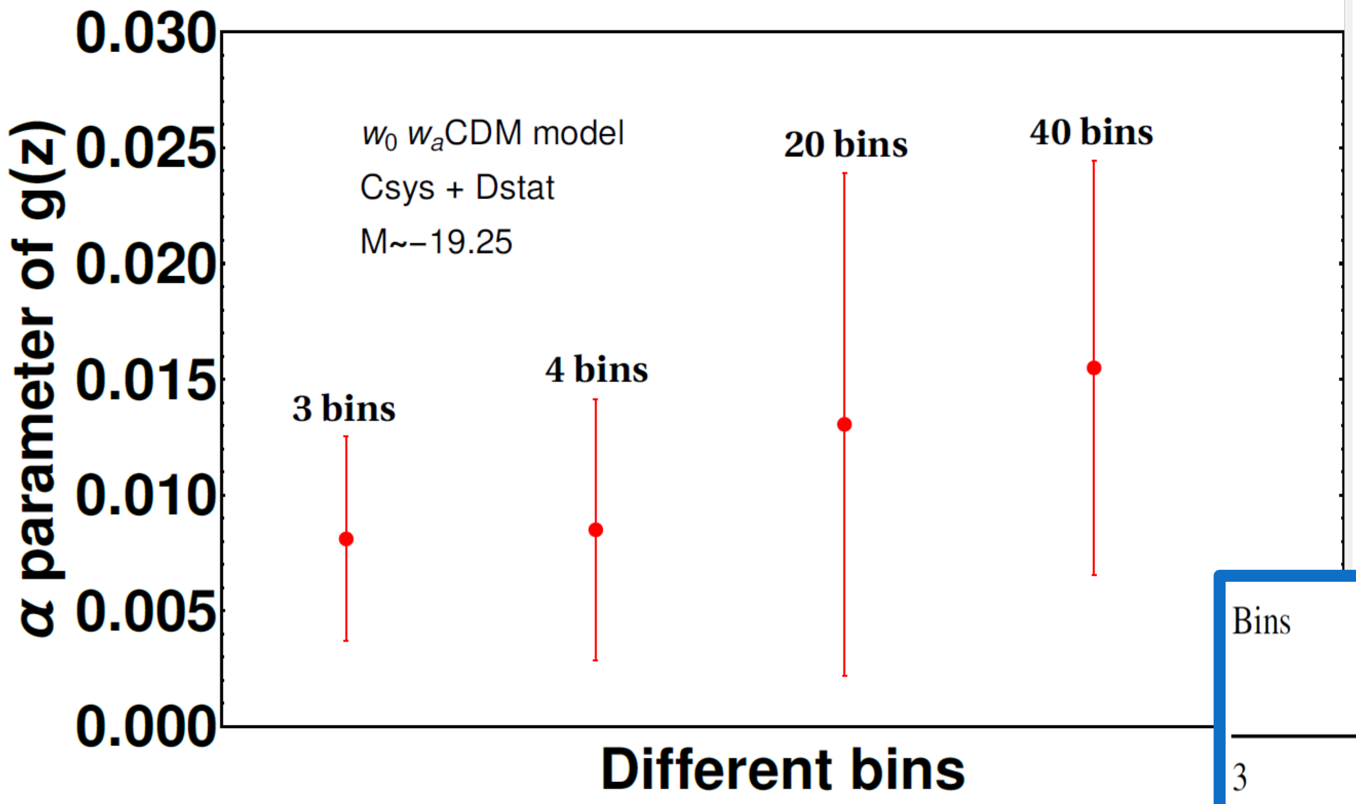
DE SIMONE,

SCHIAVONE

et al. (2021),

ApJ 912, 150

A NON-CONSTANT HUBBLE CONSTANT?



$$H_0(z) = \frac{\tilde{H}_0}{(1+z)^\alpha}$$

$\alpha = 0 \;\; \rightarrow$ no evolution

Error bars, 1 σ

Slight and unexpected evolution of $H_0(z)$

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

FIT RESULTS $w_0 w_a$CDM MODEL

Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	$\frac{\alpha}{\sigma_\alpha}$	M
3	73.576 ± 0.105	0.008 ± 0.004	1.9	-19.244 ± 0.005
4	73.513 ± 0.142	0.008 ± 0.006	1.2	-19.246 ± 0.004
20	73.192 ± 0.265	0.013 ± 0.011	1.9	-19.262 ± 0.018
40	73.678 ± 0.223	0.015 ± 0.009	1.7	-19.250 ± 0.022

EXTRAPOLATED VALUES AT HIGH REDSHIFTS

$w_0 w_a$CDM model

Bins	$H_0(z = 11.09)$ (km s^{-1} Mpc^{-1})	$H_0(z = 1100)$ (km s^{-1} Mpc^{-1})
3	72.104 ± 0.766	69.516 ± 2.060
4	71.975 ± 1.020	69.272 ± 2.737
20	70.852 ± 1.937	66.804 ± 5.093
40	70.887 ± 1.595	66.103 ± 4.148

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

$$H_0(z) = \frac{\tilde{H}_0}{(1+z)^{\alpha}}$$

Consistent in 1 σ with the Planck measurement from the CMB, at the redshift of the last scattering surface z=1100

$$H_0^{[CMB]} = (67.36 \pm 0.54)\ km\ s^{-1}\ Mpc^{-1}$$

PLANCK COLLABORATION
Planck 2018 result, VI: Cosmological parameters
A&A 641, A6 (2020).

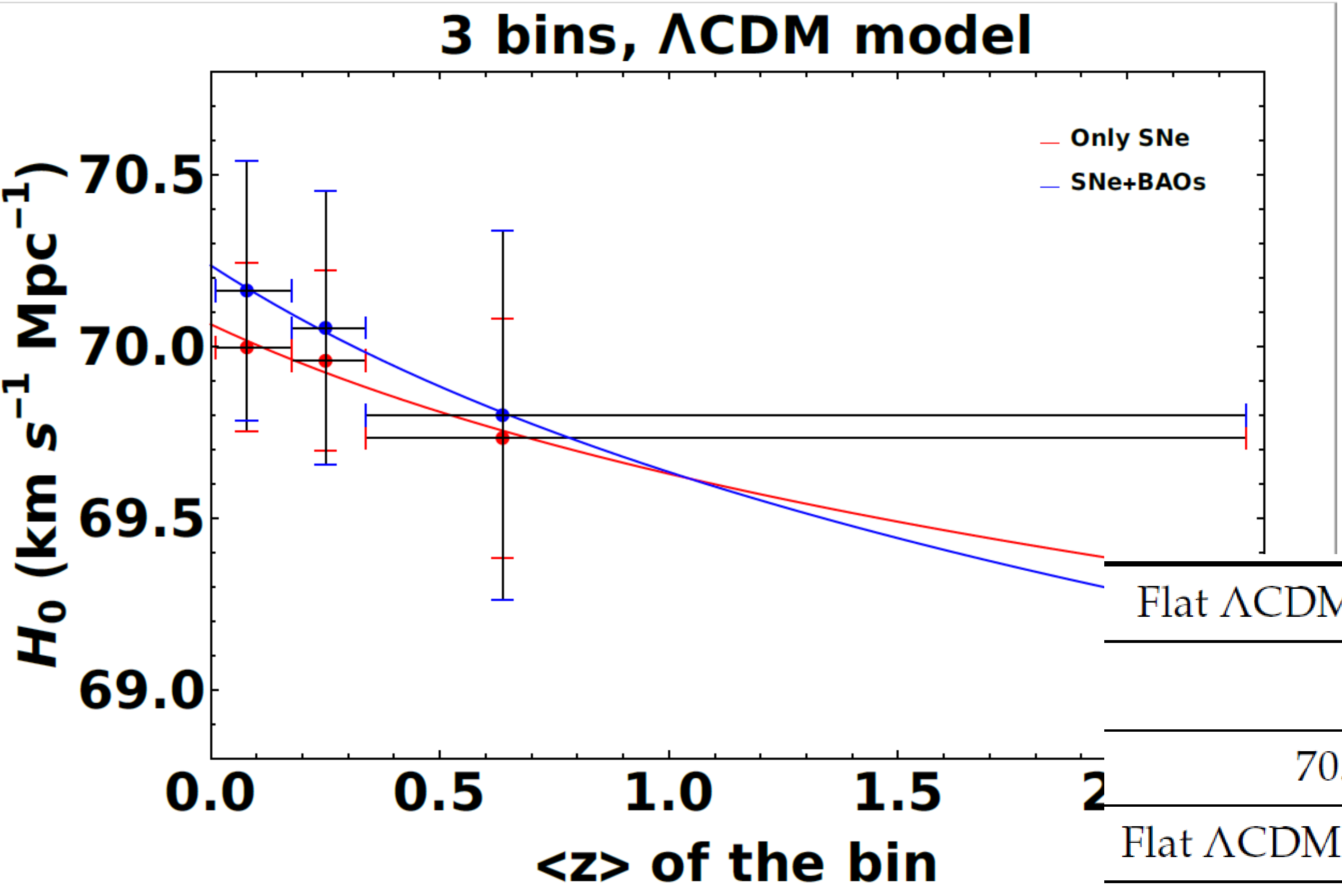
SNe + BAOs, BINNED ANALYSIS

- 3 redshift bins (≈ 350 SNe in each bin)
- Two free parameters in MCMC
  H_0 and Ω_{m0} for the ΛCDM model
  H_0 and w_a for the $w_0 w_a$CDM model
- $M = -19.35$ such that locally (in the first bin)
  $$H_0 = 70.0\ km\ s^{-1}\ Mpc^{-1}$$
  (conventional value of the PS release)
- New probes included, BAOs

- Gaussian priors:
  $\mu(H_0) = 70.393\ km\ s^{-1}\ Mpc^{-1}$
  $\sigma(H_0) = 2 * 1.079\ km\ s^{-1}\ Mpc^{-1}$
  $\mu(\Omega_{m0}) = 0.298 \qquad \sigma(\Omega_{m0}) = 2 * 0.022$
  [measurements from arXiv:1710.00845 in 2 σ]
  $\mu(w_a) = -0.129$
  $\sigma(w_a)$: $20\ \%$ deviation from its central value
- Fixed $w_0 = -0.905$ from arXiv:1710.00845

DAINOTTI, DE SIMONE, **SCHIAVONE**, et al. (2022), Galaxies 2022, 10, 24

SNe + BAOs, BINNED ANALYSIS



$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

$\alpha = 0 \;\rightarrow$ no evolution

Error bars, 1 σ

Flat ΛCDM model, without BAOs, varying H_0 and Ω_{0m}		
$\widetilde{H}_0$	α	α/σ_α
70.093 ± 0.102	0.009 ± 0.004	2.0
Flat ΛCDM model, including BAOs, varying H_0 and Ω_{0m}		
$\widetilde{H}_0$	α	α/σ_α
70.084 ± 0.148	0.008 ± 0.006	1.2

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2022), Galaxies 2022, 10, 24

$f(R)$ COSMOLOGY

- Extra degree of freedom in the parameterization, functional form of $f(R)$
- Mimic ΛCDM in the high-redshift regime, well-tested by the CMB
- Cosmic accelerated expansion with an effective cosmological constant
- Phenomenology of ΛCDM as a limiting case

Hu-Sawicki

$$f(R) = R - m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}$$

c_1, c_2 parameters; $n > 0$ $\qquad m^2 \equiv \frac{\chi\, \rho_{m0}}{3}$

Starobinski

$$f(R) = R - \mu\, R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right]$$

Tsujikawa

$$f(R) = R - \mu\, R_c \tanh\left(\frac{|R|}{R_c}\right)$$

$n, \mu, R_c > 0$

HU & SAWICKI (2007), Phys. Rev. D ,76, 064004

AMENDOLA & TSUJIKAWA (2010), Cambridge University Press

STAROBINSKi (2007), Jetp Lett. 86, 157

TSUJIKAWA (2008), Phys. Rev. D, 77, 023507

$f(R)$ HU-SAWICKI MODEL

Metric $f(R)$ formalism

$n = 1$

$$f(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

Jordan frame (equivalent scalar-tensor formalism)

$$V(\phi) = \frac{m^2}{c_2}\left[c_1 + 1 - \phi - 2\sqrt{c_1 (1-\phi)}\right]$$

c_1, c_2 parameters $\qquad m^2 \equiv \frac{\chi \rho_{m0}}{3} = H_0^2 \, \Omega_{m0}$

- Cosmological constant for $R \gg m^2$

  $f(R) \approx R - 2\Lambda_{eff}$ with $\Lambda_{eff} = \frac{c_1}{c_2} m^2$

- Constraining parameters, considering ΛCDM as a limiting case with $f(R) = R + F(R)$

  $\frac{c_1}{c_2} \approx 6 \frac{\Omega_{0\Lambda}}{\Omega_{0m}}$ and $F_R(z=0) = \left(\frac{dF}{dR}\right)_{z=0} = -\frac{c_1}{c_2^2}\left[3\left(1 + 4\frac{\Omega_{0\Lambda}}{\Omega_{0m}}\right)\right]^{-2}$ with $|F_R(z=0)| \lesssim 10^{-7}$

Liu, T., Zhang, X., & Zhao, W., Phys. Lett. B, 777, 286 (2018)

LUMINOSITY DISTANCE IN $f(R)$ GRAVITY

$$d_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}\left[(1+z')^3 + y_H(z')\right]}}$$

(arXiv: 0705.1158)

Dimensionless variables

$$y_H = \frac{H^2}{m^2} - (1+z)^3$$

$$y_R = \frac{R}{m^2} - 3\,(1+z)^3$$

$y_H(z)$ encodes information of a specific $f(R)$ model.

Modified field eqs. can be solved numerically

in terms of y_H, y_R and their derivatives

Initial conditions: z_i

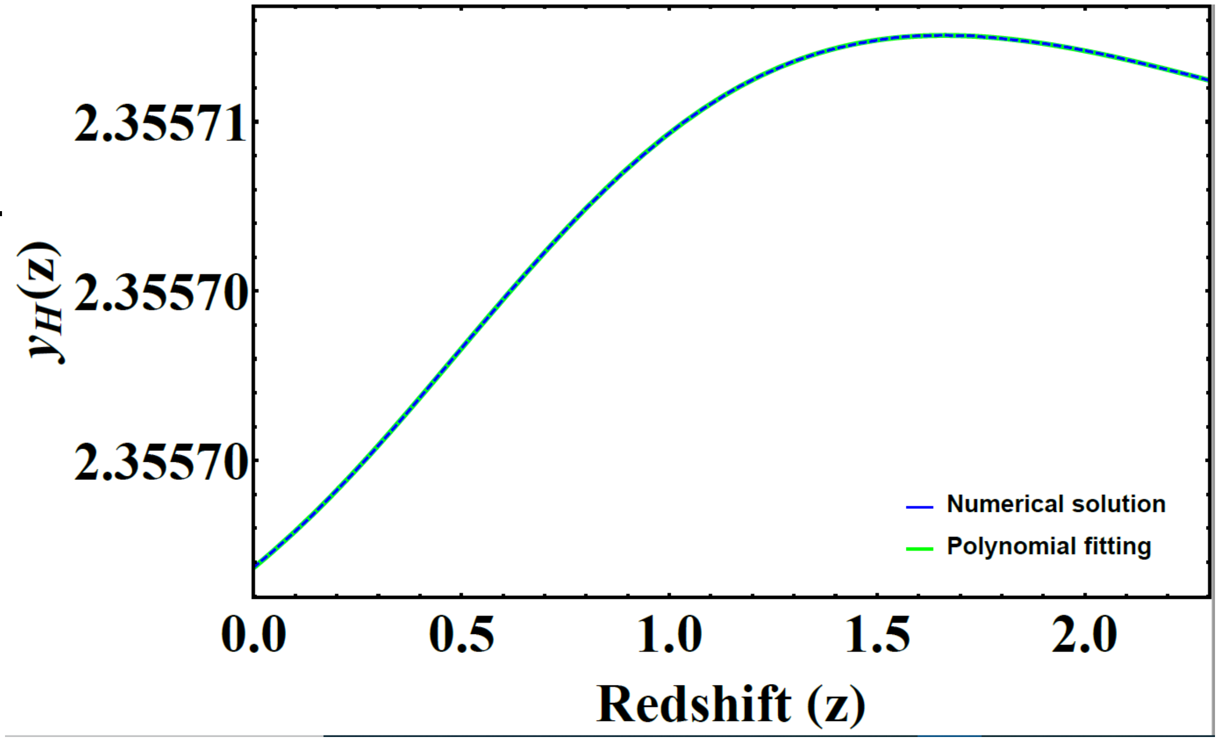
$$y_H(z_i) = \frac{\Omega_{\Lambda 0}}{\Omega_{m0}}$$

$$y_R(z_i) = 12\,\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}$$

BINNING APPROACH WITH f(R) HU-SAWICKI MODEL

$$f(R) \equiv R + F(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

$$y_H = \frac{H^2}{m^2} - (1+z)^3$$



$y_H(z)$ encodes information of a specific $f(R)$ model.

Modified field eqs. can be solved numerically

in terms of y_H, y_R and their derivatives

$$d_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}\left[(1+z')^3 + y_H(z')\right]}}$$

$\frac{c_1}{c_2} \approx 6 \frac{\Omega_{\Lambda 0}}{\Omega_{m0}}$ and $F_R(z=0) = \left(\frac{dF}{dR}\right)_{z=0} = -\frac{c_1}{c_2^2}\left[3\left(1 + 4\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}\right)\right]^{-2}$ with $|F_R(z=0)| \lesssim 10^{-7}$

BINNING APPROACH WITH f(R) HU-SAWICKI MODEL

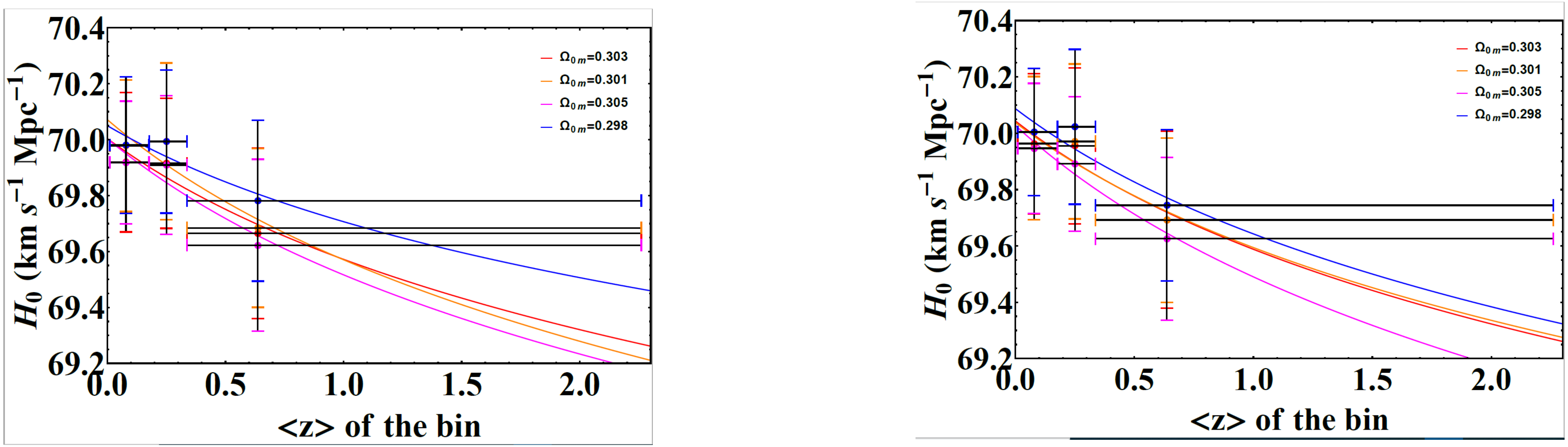


Figure 6. The Hubble constant versus redshift plots for the three bins of SNe Ia only, considering the Hu–Sawicki model. **Upper left panel.** The condition of $F_{R0} = -10^{-7}$ is applied to the case of SNe only, with the different values of $\Omega_{0m} = 0.301, 0.303, 0.305$. **Upper right panel.** The same of the upper left, but with the contribution of BAOs. **Lower left panel.** The SNe only case with the $F_{R0} = -10^{-4}$ condition, considering the different values of $\Omega_{0m} = 0.301, 0.303, 0.305$. **Lower right panel.** The same as the lower left, but with the contribution of BAOs. The orange color refers to $\Omega_{0m} = 0.301$, the red to $\Omega_{0m} = 0.303$, the magenta to $\Omega_{0m} = 0.305$, and the blue to $\Omega_{0m} = 0.298$.

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2022),

Galaxies 2022, 10, 24

RUNNING HUBBLE CONSTANT & $f(R)$ GRAVITY

[Naively]
Impact of a non-minimal coupled scalar field in the Jordan frame:
effective Einstein constant χ_{eff} or effective Newton constant G_{eff}

$$\chi \rightarrow \chi_{eff} = \frac{\chi}{\phi}$$

$$\chi = 8\pi G$$

If $\phi \sim (1+z)^{2\alpha}$ then $\chi_{eff} \sim (1+z)^{-2\alpha}$

$$H_0^{eff}(z) \sim \frac{1}{(1+z)^{\alpha}}$$

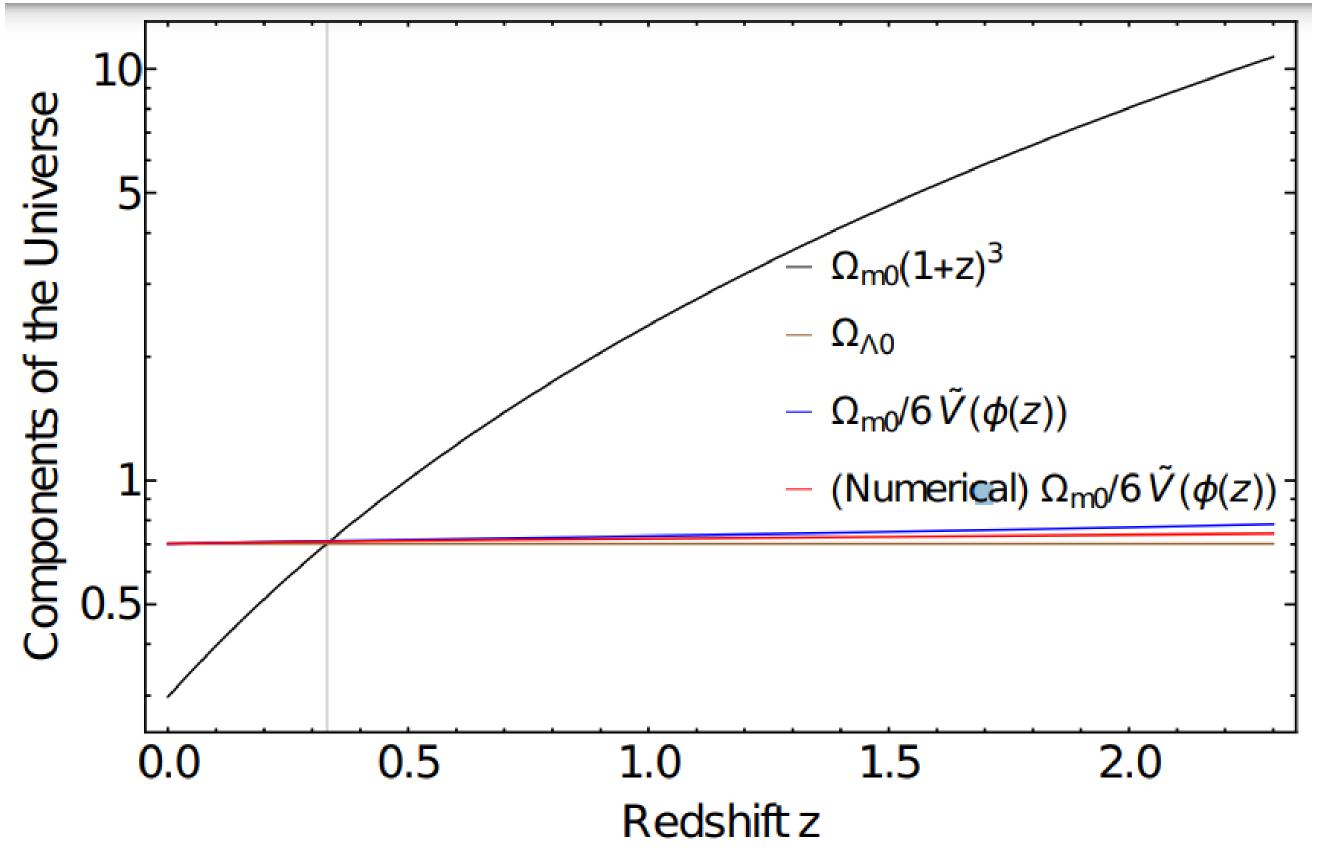
Critical density today

$$\rho_{c0} = \frac{3\,H_0^2}{\chi} \rightarrow \frac{3\,H_0^{eff\,2}(z)}{\chi_{eff}}$$

$$H(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

Phenomenological Hubble parameter

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY



SCHIAVONE & MONTANI, arXiv:2408.01410

Visualizing different contributions in the dimensionless-modified Friedmann eq. in the JF considering our numerical solutions

$V(\phi) \approx 2\chi\rho_\Lambda$ is a good approximation, it is a slowly varying cosmological constant. It was checked *a posteriori* the ansatz we made for the approximated analytical solution

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

The low-redshift $f(R)$ profile

For $z \ll 1$:

$$\phi(z) \approx \phi_0(1 + 2\,\alpha\,z) + O(z^3)$$

$$\tilde{V}(\phi) \approx \tilde{V}(\phi_0) + A_1\,(\phi - \phi_0) + A_2(\phi - \phi_0)^2 + O[(\phi - \phi_0)^3]$$

$$f(R) \approx m^2\,B_0 + B_1\,R + B_2\,\frac{R^2}{m^2}$$

Approximated solution for $z \ll 1$: $f(R)$ – quadratic gravity

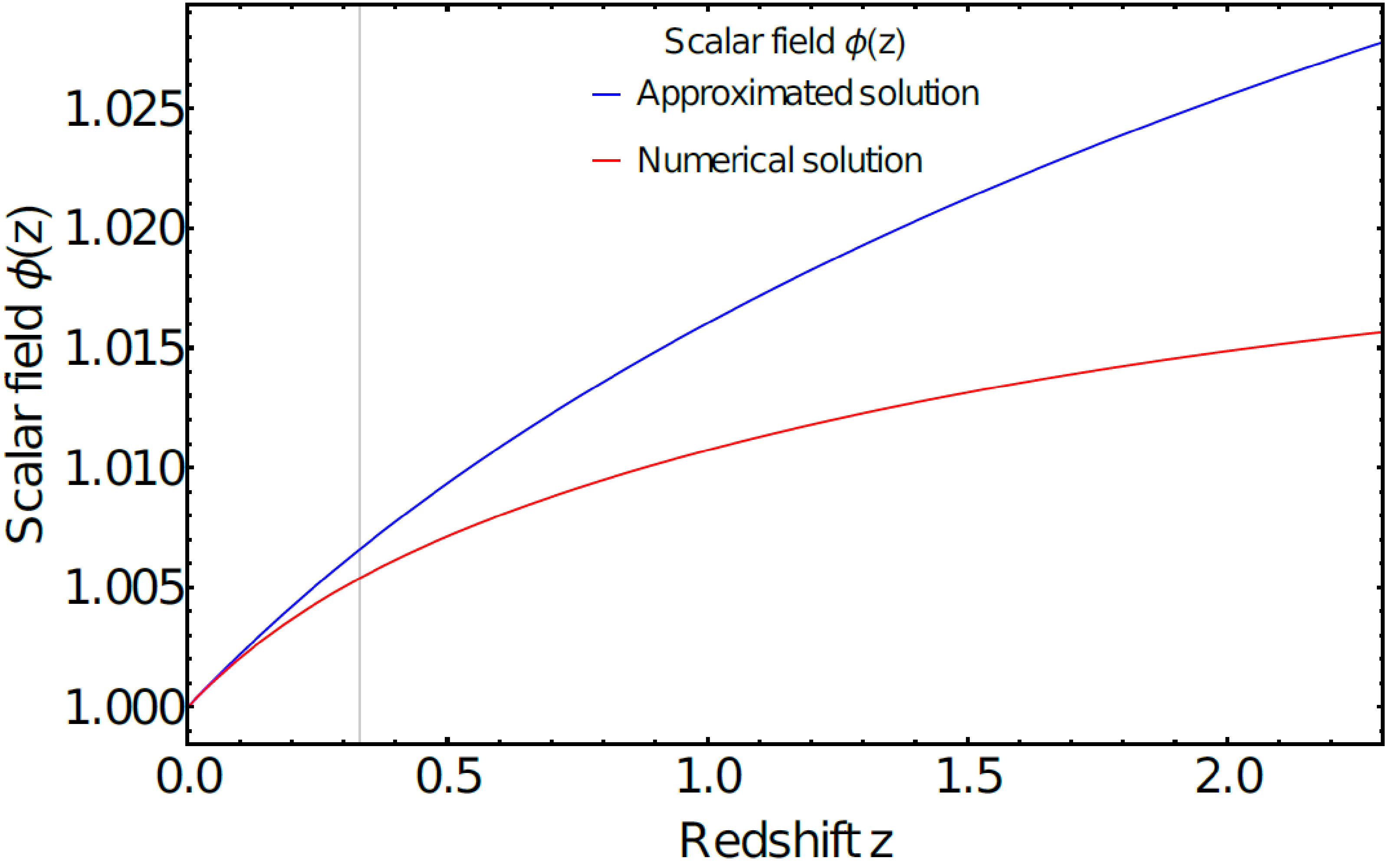
$$A_1 = \frac{6}{K\,(1-\alpha)}\left[\frac{1+2\alpha}{2} + (\alpha+2)\,\frac{1-\Omega_{m0}}{\Omega_{m0}}\right],$$

$$A_2 = \frac{3}{K^2\,(1-\alpha)}\left[\frac{1+2\alpha}{2}\left(\frac{3}{2\alpha} - 1\right) - (\alpha+2)\,\frac{1-\Omega_{m0}}{\Omega_{m0}}\right]$$

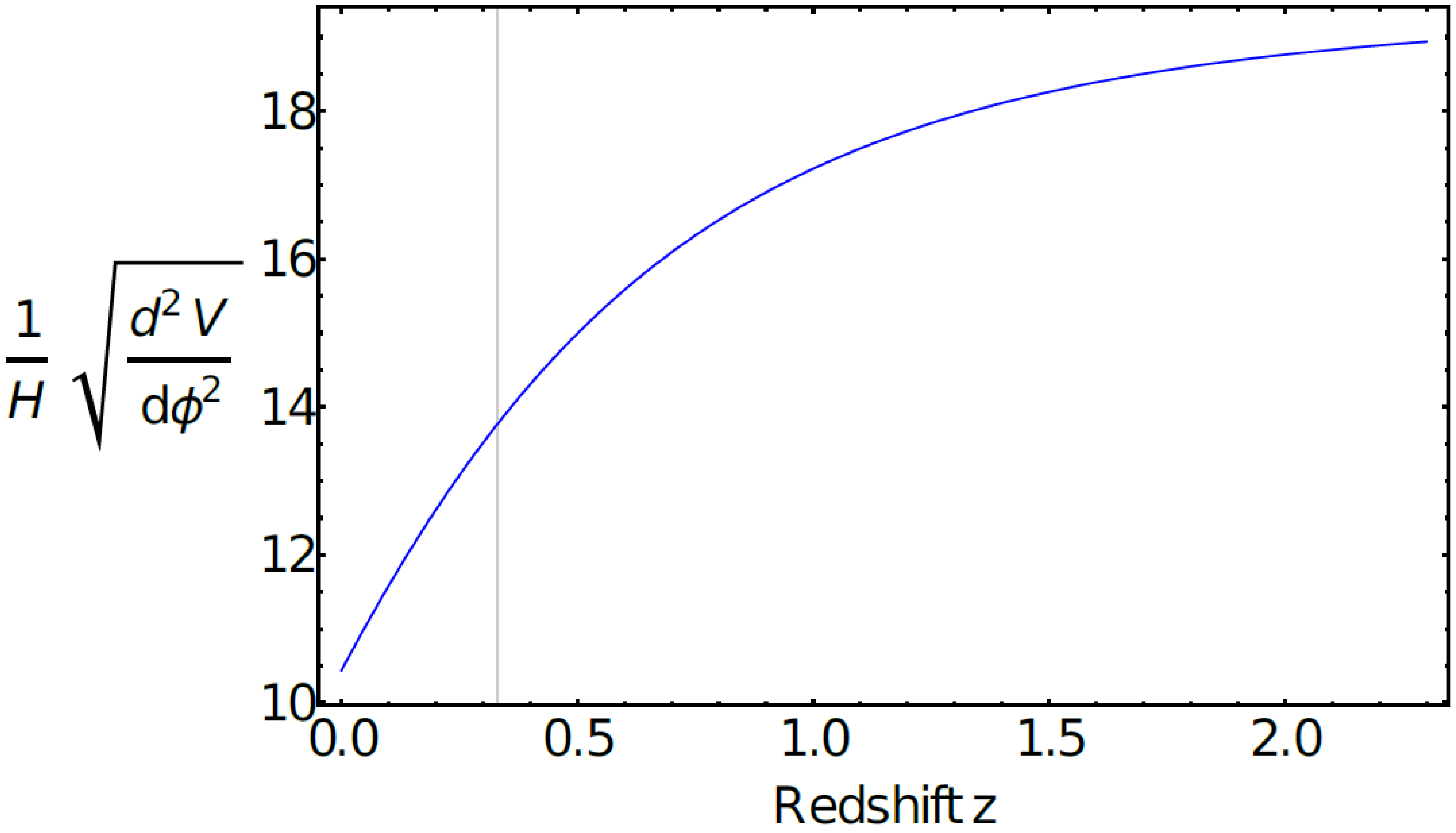
$$B_0 = \frac{A_1^2}{4A_2} - \tilde{V}\,(K)\,, \qquad B_1 = K - \frac{A_1}{2A_2}\,, \qquad B_2 = \frac{1}{4A_2}$$

It can be shown that ΛCDM is recovered for $\alpha \to 0$ and K=$\phi_0 \to 1$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77



SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77



SCHIAVONE, MONTANI, & BOMBACIGNO (2023),

MNRAS Letters, 522, L72-L77