Generalized entropies in cosmology

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Black Hole Thermodynamics

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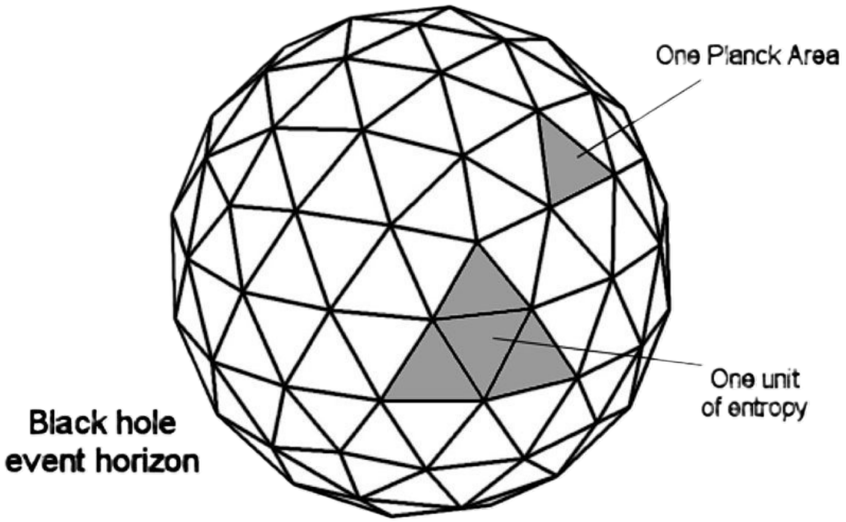

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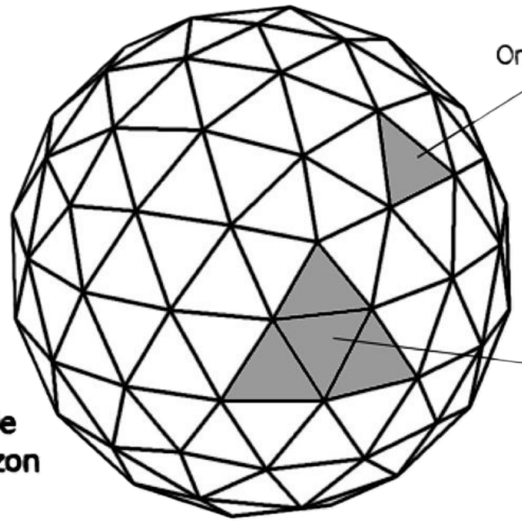

Problem: What happens if an encyclopedia falls inside a black hole?

Solution: $S \propto A$ where $A = 4\pi r_h$ is the area of the **EVENT HORIZON**

<u>1974</u> **Hawking** Hawking radiation : Black holes emits thermal bath at temperature $T \equiv \frac{\kappa}{2\pi} = \frac{1}{8\pi GM}$

Bekenstein-Hawking entropy $S = \frac{A}{4G} = \frac{\pi r_h^2}{G}$

Entropic cosmology

Cai and Kim, 2005, hep-th/0501055

FLRW metric possess an **APPARENT HORIZON**
(limit beyond which light is effectively unreachable due to the universe's expansion)

$$r_h = \frac{1}{H}$$

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Hawking temperature

$$T_H = \frac{|\kappa|}{2\pi} = \frac{H}{2\pi}\left|1 - \frac{\dot{H}}{2H^2}\right|$$

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1st Principle of Thermodynamics

$$T_H dS = -dE + W dV$$

$$E = \rho V \qquad W = \frac{1}{2}(\rho - p) \qquad V = \frac{4\,\pi}{3H^3}$$

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Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

$$\dot{H} = -4\pi G(\rho + p)$$

Generalized entropies

A generalized entropy must respect these proprieties:

a) **Positive**
b) **Monotonically increasing**
c) $S_g(0) = 0$
d) **Bekenstein-Hawking limit**

Examples:

$S_T = \frac{A_0}{4G}\left(\frac{A}{A_0}\right)^{\delta}$	Tsallis entropy	$S_q = \frac{1}{1-q}\left[e^{(1-q)\Lambda(\gamma_0)\,S} - 1\right]$	Loop quantum gravity entropy
$S_R = \frac{1}{\alpha}\ln\left(1+\alpha S\right)$	Renyi entropy	$S_K = \frac{1}{K}\sinh\left(K\,S\right)$	Kaniadakis entropy
$S_B = \left(\frac{A}{A_{Pl}}\right)^{1+\Delta/2}$	Barrow entropy	$S_{SM} = \frac{1}{R}\left[(1+\delta\,S)^{R/\delta} - 1\right]$	Sharma-Mittal entropy

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1st Principle of Thermodynamics

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Generalized Friedmann equations

1st : $$\int d\left(H^2\right)\left(\frac{\partial S_g}{\partial S}\right) = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

2nd : $$\dot{H}\left(\frac{\partial S_g}{\partial S}\right) = -4\pi G(\rho + p)$$

4-parameters generalized entropy

Nojiri, Odintsov and Paul, 2205.08876

$$S_4(\alpha_\pm,\beta,\gamma)=\frac{1}{\gamma}\left[\left(1+\frac{\alpha_+}{\beta}S\right)^{\beta}-\left(1+\frac{\alpha_-}{\beta}S\right)^{-\beta}\right]$$

S_4	$\alpha_-=0,\alpha_+=\gamma$	S_{SM}
	$\alpha_+\to\infty,\alpha_-=0$	S_T, S_B
	$\alpha_-=0,\alpha_+=\gamma,\beta\to 0$ with $\frac{\alpha_+}{\beta}$ finite	S_R
	$\beta\to\infty,\alpha_-=0,\alpha_+=\gamma$	S_q
	$\beta\to\infty,\alpha_+=\alpha_-$	S_K

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Friedmann equations for the 4-parameter entropy

$$\frac{\beta G H^4}{\pi\gamma}\left[\frac{1}{2+\beta}\left(\frac{\beta G H^2}{\pi\alpha_-}\right)^{\beta}{}_2F_1\left(1+\beta,2+\beta,3+\beta,-\frac{\beta G H^2}{\pi\alpha_-}\right)\right.$$
$$\left.+\frac{1}{2-\beta}\left(\frac{\beta G H^2}{\pi\alpha_+}\right)^{-\beta}{}_2F_1\left(1-\beta,2-\beta,3-\beta,-\frac{\beta G H^2}{\pi\alpha_+}\right)\right]=\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}$$

$$\dot{H}\left\{\frac{1}{\gamma}\left[\alpha_+\left(1+\frac{\alpha_+}{\beta}S\right)^{\beta-1}+\alpha_-\left(1+\frac{\alpha_-}{\beta}S\right)^{-\beta-1}\right]\right\}=-4\pi G\left(\rho+p\right)$$

Inflation with entropy

During the inflation the typical energy scale is of order $H \sim 10^{-4} M_{Pl}$ $\Rightarrow$

Inflationary limit

$$GH^2 \ll 1$$

The first Friedmann equation can be expanded at leading order as:

$$\frac{1}{2-\beta}\frac{\beta G H^4}{\pi\gamma}\left(\frac{\beta G H^2}{\pi\alpha_+}\right)^{-\beta}\left(1-\frac{(1-\beta)(2-\beta)}{3-\beta}\frac{\beta G H^2}{\pi\alpha_+}\right)=\frac{8\pi G}{3}\rho$$

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In absence of matter fields or in presence of a perfect fluid we have

$$H = const.$$

de-Sitter inflation

No exit mechanism, eternal inflation,

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Fixing the scalar field potential

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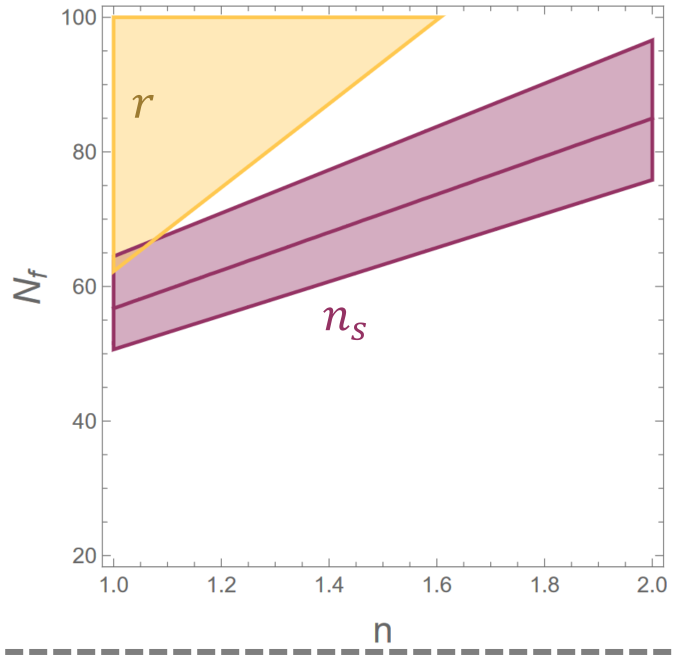
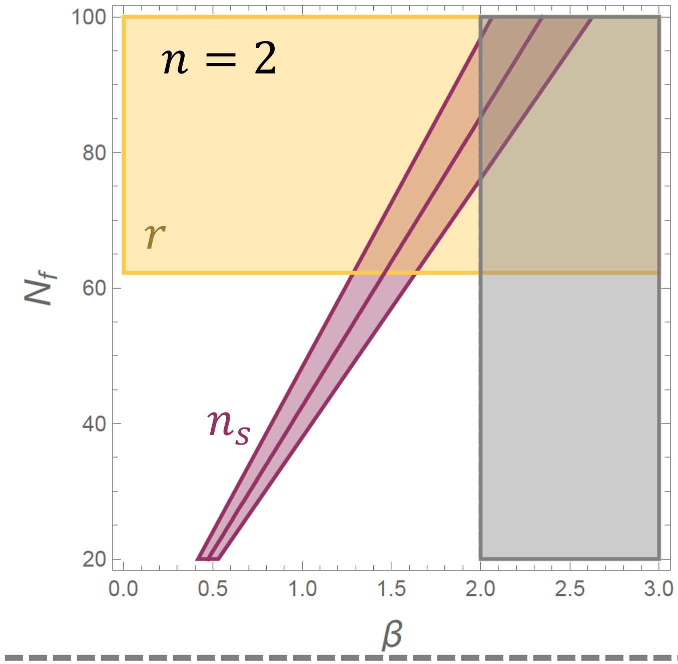
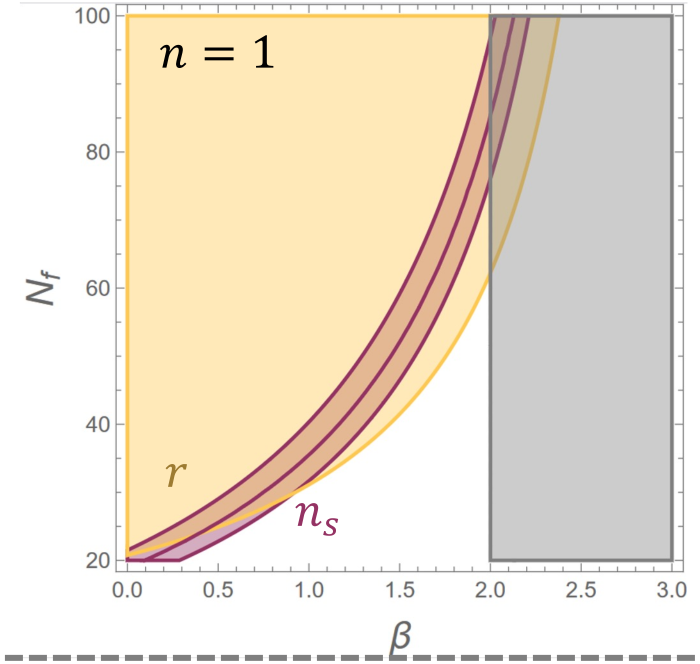


For the **generalized entropy S_4** :

$$n_s = 1 + \frac{2(4-5\beta)n - 8(2-\beta)}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_{\rm f}}$$

$$r = \frac{16n}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_{\rm f}}$$

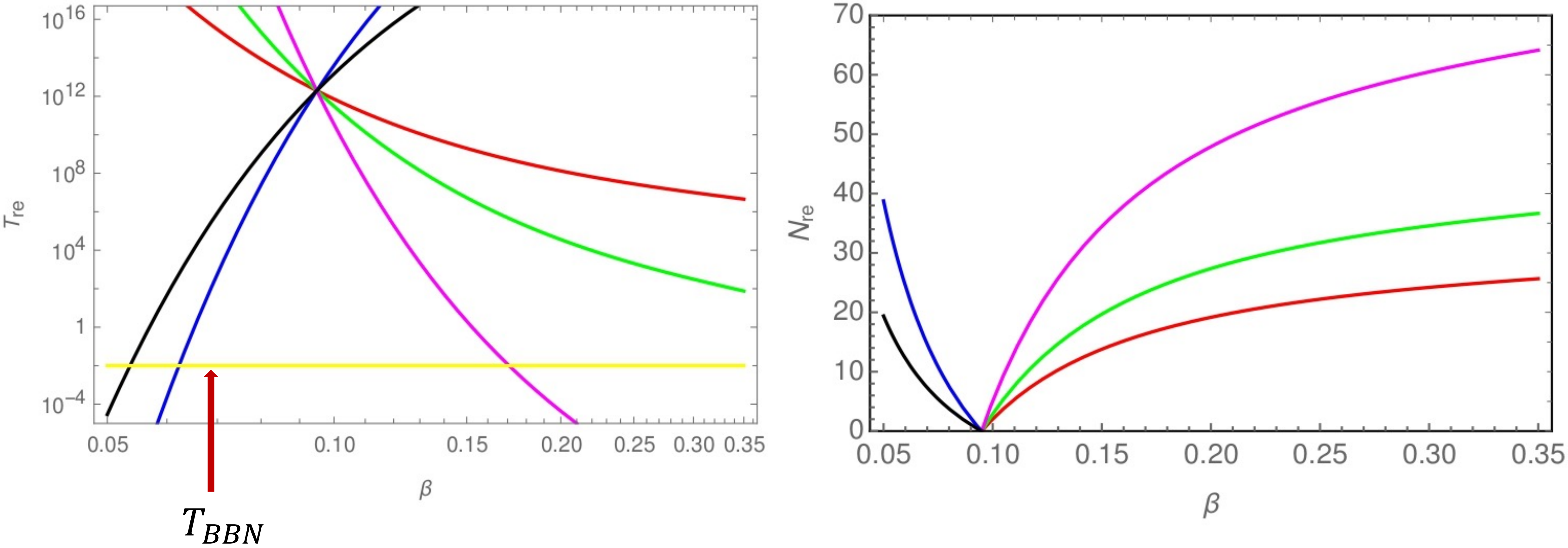




Reheating

D'Onofrio, Odintsov and Paul, 2306.15225

$$N_{\mathrm{re}} = \frac{4}{(1-3w_{\mathrm{eff}})}\left\{61.6 - \frac{1}{4\beta}\ln\left[\frac{\beta\exp\left[-(1+\sigma_0 N_{\mathrm{f}})\right]\left\{1+\sqrt{1+\exp\left[2(1+\sigma_0 N_{\mathrm{f}})\right]\left[\left(\frac{1+\sigma_0}{2\beta}\right)^2-1\right]}\right\}^2}{(16\pi^2\alpha_+/3\beta)^{\beta}\ \ \{1+\sigma_0+2\beta\}}\right] - N_{\mathrm{f}}\right\}$$



Primordial gravitational waves

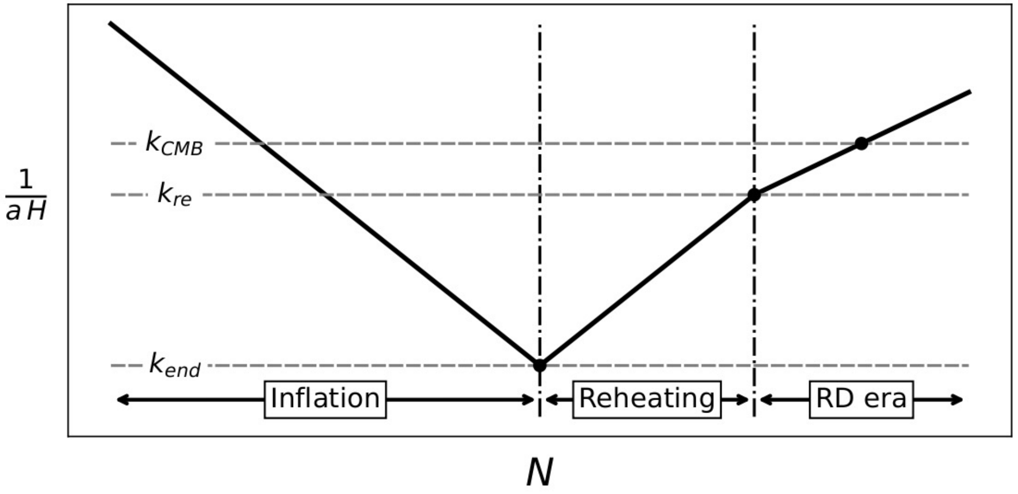
D'Onofrio, Odintsov and Paul, 2407.05855

Possible tool for the measurement of the entropic parameters.

$$ds^2 = -dt^2 + a^2(t)\left[(\delta_{ij} + h_{ij})\,dx^i dx^j\right]$$

$$\ddot{h}(k,t) + 3H\dot{h}(k,t) + \frac{k^2}{a^2}h(k,t) = 0$$

GWs spectrum today is flat for the modes that re-enter the horizon during the radiation era ($k < k_{re}$), while the spectrum carries a **tilted nature** over the modes re-entering the horizon during the reheating stage.



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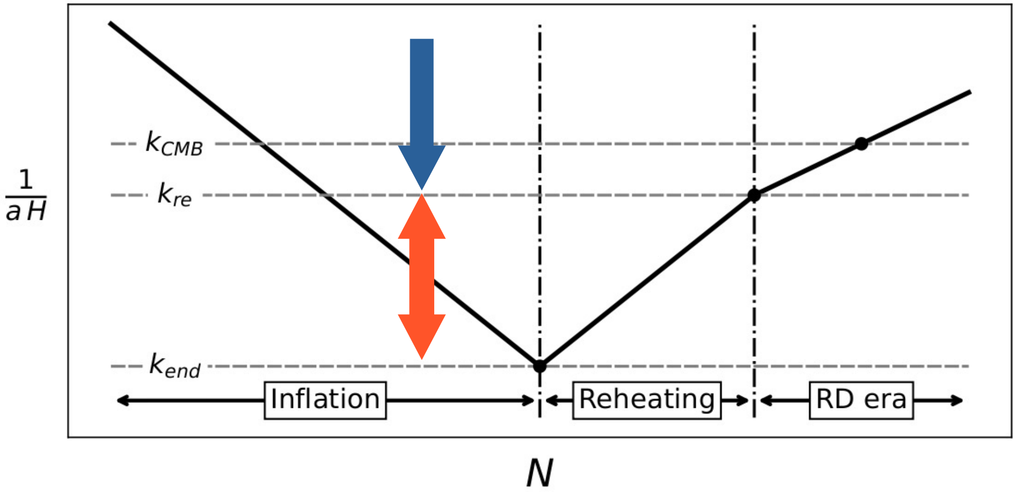
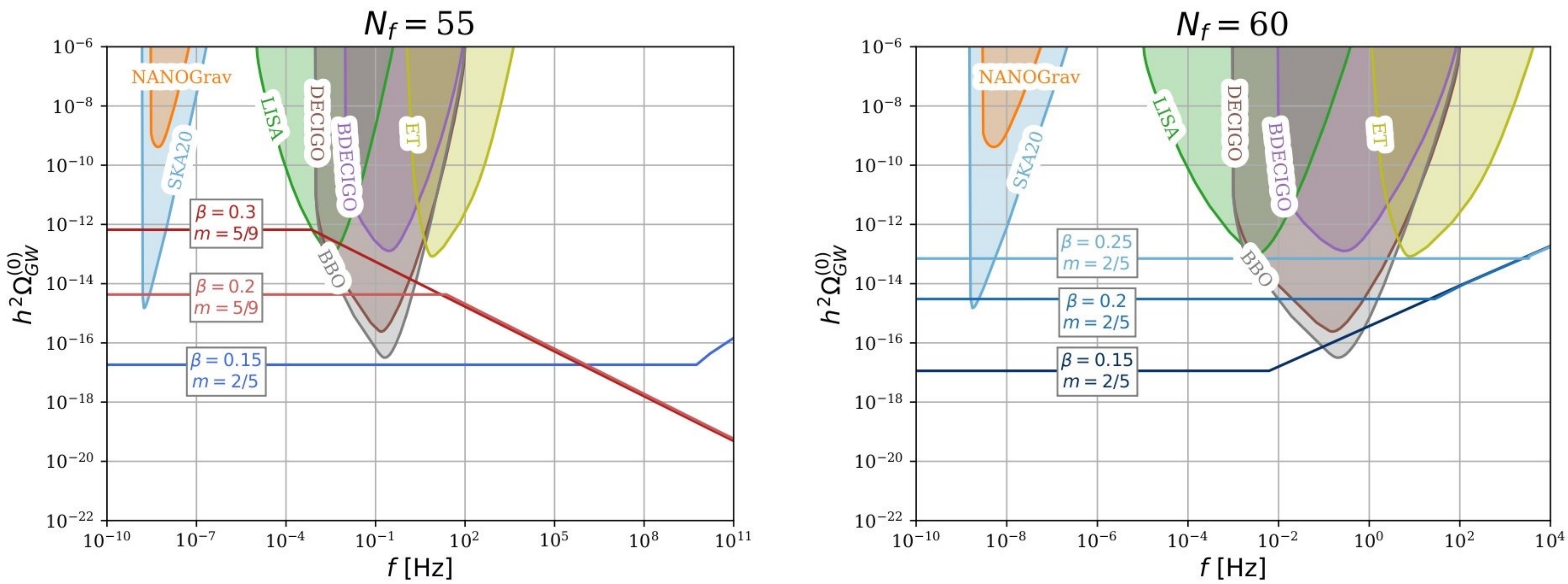


$k < k_{re}$ lies in the super-Hubble scale and remain frozen during the reheating stage

$k_{re} < k < k_{end}$ re-enters the horizon during the reheating stage and are not frozen

D'Onofrio, Odintsov and Paul, 2407.05855

$$\Omega_{\rm GW}^{(0)}(k)h^2 \simeq \left(\frac{1}{6\pi^2}\right)\Omega_{\rm R}h^2\left(\frac{H_{\rm i}}{M_{\rm Pl}}\right)^2(1+3w_{\rm eff})^{\frac{4}{1+3w_{\rm eff}}}\left(\frac{\Gamma(1-\nu)}{\sqrt{\pi}}\right)^2\left(\frac{k}{k_{\rm re}}\right)^{2\left(\frac{3w_{\rm eff}-1}{3w_{\rm eff}+1}\right)}$$



Late time evolution

D'Onofrio, Odintsov and Paul, 2504.03470

The generalized entropy induces an effective energy density in the modified Friedmann equations, which turns out to be favorable for the late time acceleration.

The dark energy **fractional density** and the **dark energy EoS** parameter have been found in closed **analytic forms**

$$\Omega_{\mathrm{D}}(z) = 1 + \left\{ \frac{\Omega_{\mathrm{k0}}}{\tilde{\Omega}(z)} - \left[\frac{1 + \frac{\Omega_{\mathrm{k0}}}{\tilde{\Omega}(z)} + \frac{\Lambda}{3H_0^2(1+z)^2\tilde{\Omega}(z)}}{\frac{\sigma}{2-\beta}\left[H_0^2(1+z)^2\tilde{\Omega}(z)\right]^{1-\beta}} \right]^{\frac{1}{2-\beta}} \right\}^{-1}$$

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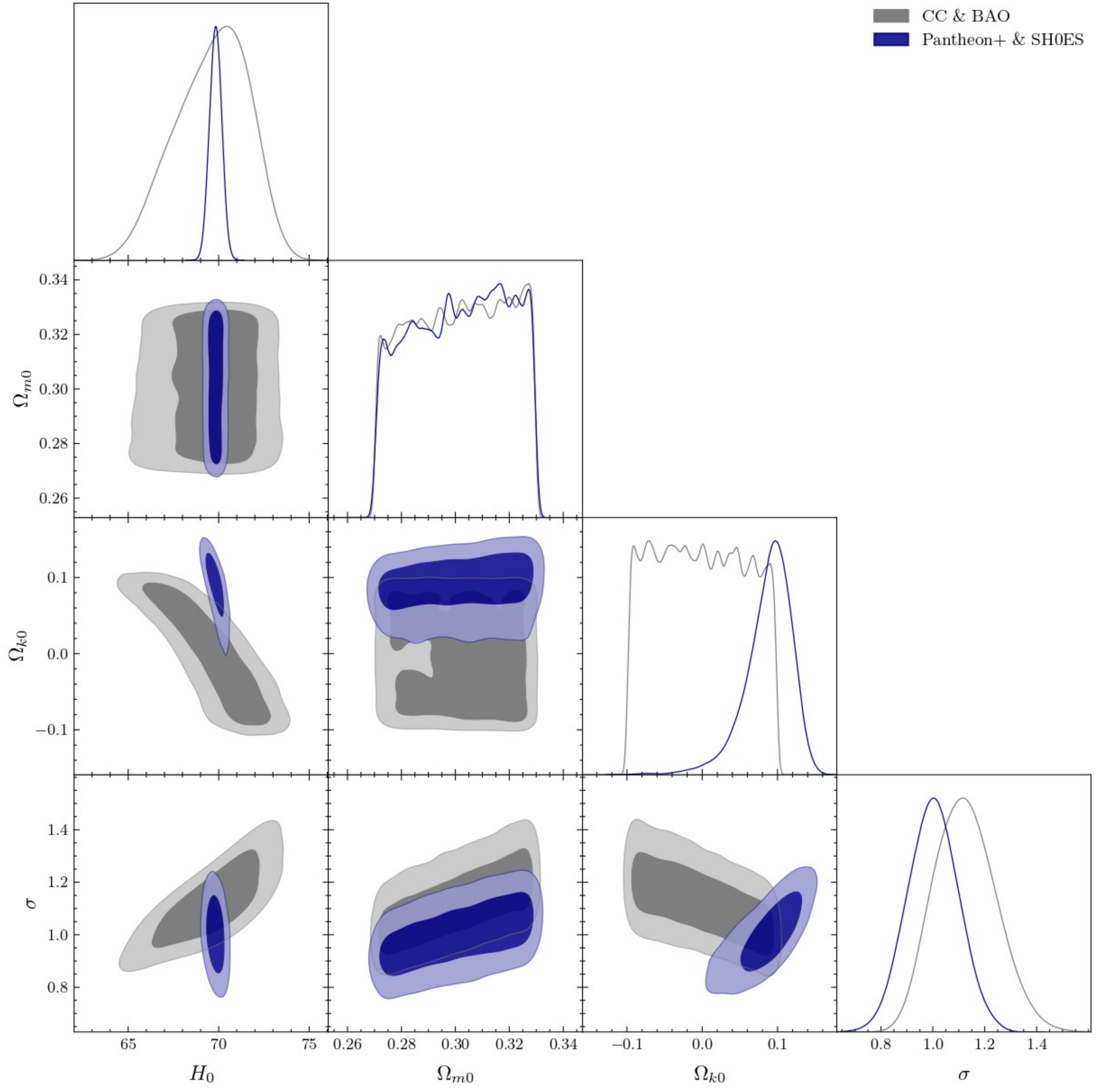
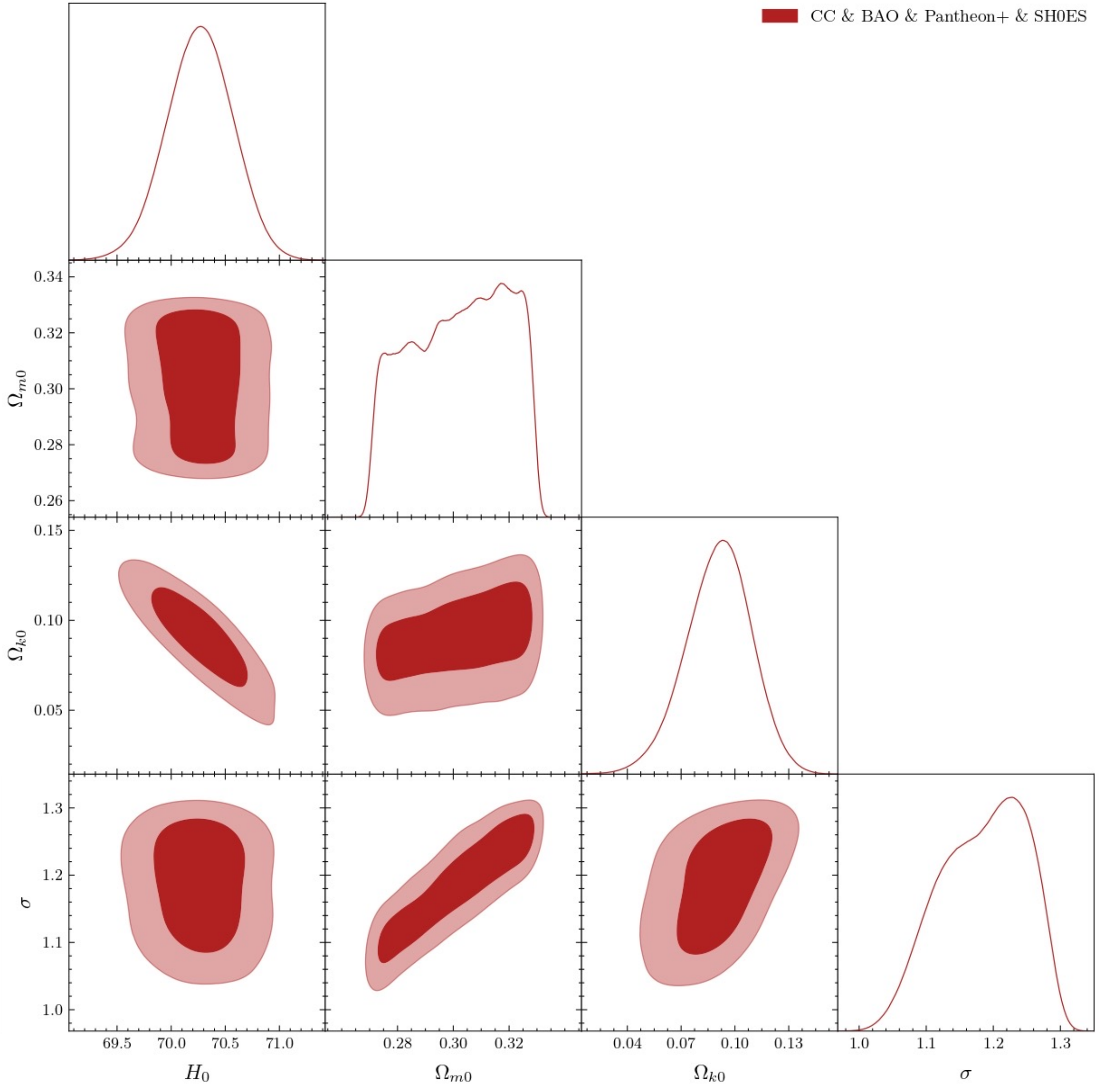
The MCMC analysis yields the present value of the Hubble parameter larger than that of in the ΛCDM scenario, and thus the model may serve as a **possible resolution of the Hubble tension issue.**

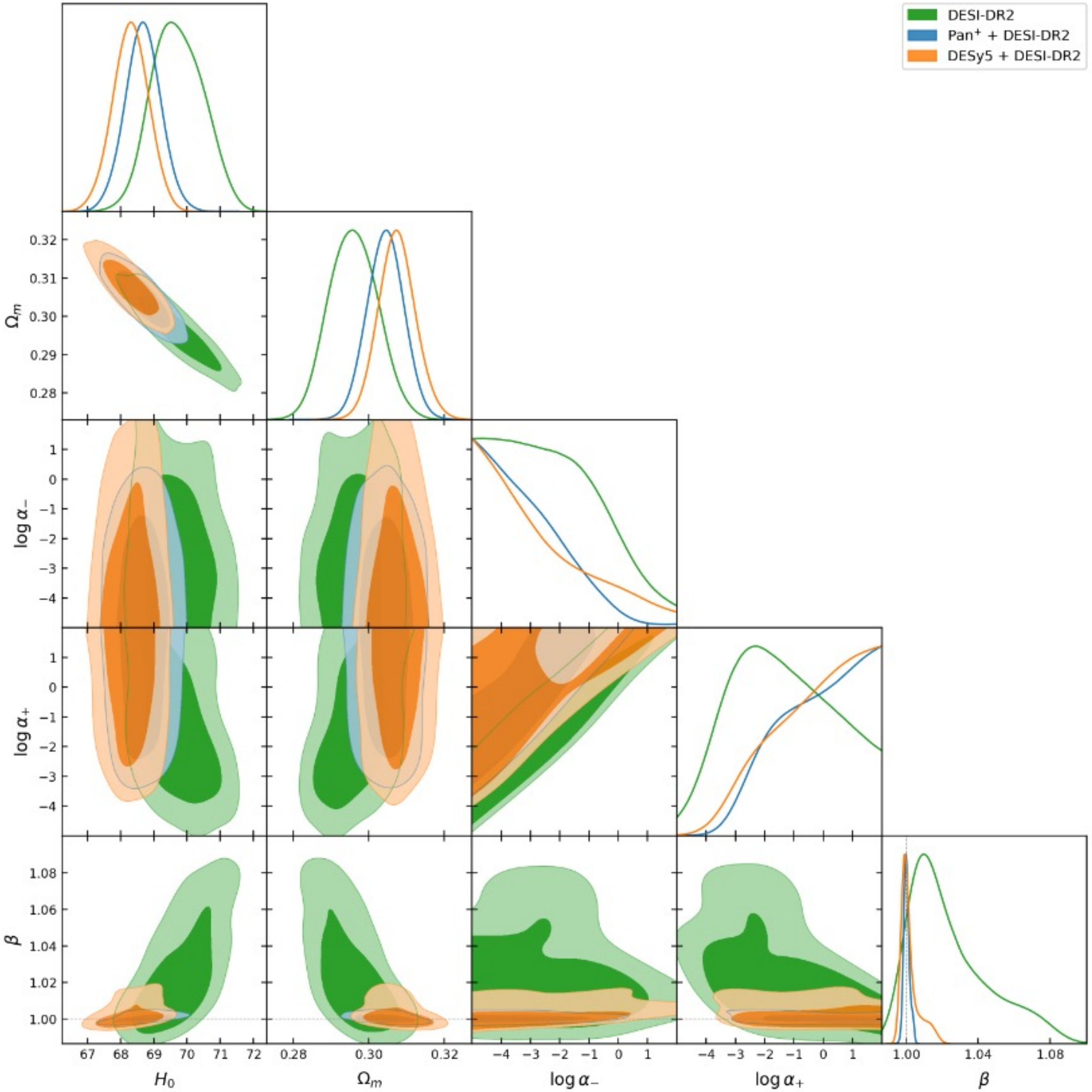
$$H(z) = \frac{H_0(z+1)}{\sqrt{1-\Omega_{\mathrm{D}}(z)}}\sqrt{(z+1)\Omega_{\mathrm{m0}} - \Omega_{\mathrm{k0}}}$$

D'Onofrio, Odintsov and Paul, *2504.03470*





Tyagi, Haridasu and Basak, 2504.11308



Conclusions

- Entropic cosmology is an intriguing framework for understanding the evolution of the universe through the lens of entropy and information theory

- It can give an explanation of why the universe is expanding

- Entropic models can help us to understand the role and behavior of dark energy in the expansion of the universe

- It has the potential for bridging gravity and quantum mechanics

Simone D’Onofrio
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Thanks for the attention!

“A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts.”

Albert Einstein


BACKUP

Tsallis entropy (1988)

$$S_T = \frac{A_0}{4G}\left(\frac{A}{A_0}\right)^{\delta}$$

Comes from the analysis of **non-extensive statistic**, in particular is applicable to systems with **long range interaction** in which the standard partition function diverges and the standard Boltzmann-Gibbs entropy is not applicable.

The parameter **δ quantifies the non-extensivity of the system**.

Renyi entropy (1960)

$$S_R = \frac{1}{\alpha}\ln\left(1+\alpha S\right)$$

Introduced in information theory, was introduced as a parameter to estimate the information of a system and, originally, had no relation with the statistics of physical systems. After has **been related to physical systems such as non-isothermal processes**, in which the parameter **α estimates the ratio of different temperatures.**
It is a relaxation of Shannon entropy and α indicates its deformation from such entropy. It is additive but non extensive.

Barrow entropy (2020)

$$S_B = \left(\frac{A}{A_{Pl}}\right)^{1+\Delta/2}$$

Inspired from Covid-19 pandemic, it takes into account the possibility of **fractal structure in the black hole horizon**, which may be generated in quantum gravity systems.

The parameter **Δ is the fractal order**, Δ = 0 is a non fractal horizon and Δ = 1 is the most fractal configuration

Shamra-Mittal entropy (2018) It is a combination of Renyi and Tsallis entropy. It estimates **the free energy difference between the off-equilibrium and equilibrium distributions.**

$$S_{SM} = \frac{1}{R}\left[(1+\delta\, S)^{R/\delta} - 1\right]$$

It is a generalization of Renyi entropy.

Kaniadakis entropy (2005)

Generalization of **Beckenstein-Hawking entropy for relativistic statistical systems.**

$$S_K = \frac{1}{K}\sinh\left(K\, S\right)$$

K is a parameter that represents the **deviation from a classical (non-relativistic) statistic.**

Loop quantum gravity entropy (2016)

This entropy comes from **non-extensive statistical mechanics in Loop Quantum Gravity.**

$$S_q = \frac{1}{1-q}\left[e^{(1-q)\Lambda(\gamma_0)\, S} - 1\right]$$

The parameter **q estimates how the probability of frequent events is enhanced with respect to infrequent ones.**

Why should we associate a thermodynamics to the cosmological horizon?

- Matter field show a flux trough the horizon. For not violating the 2° principle we should associate an entropy to the horizon

- Expanding and contracting solutions are equivalent. If we consider the universe as a thermodynamic system, the preference for the expanding solution naturally arises from the 2° law of thermodynamics.

Entropic cosmology generic horizon

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2) = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega^2$$

$$with \quad \tilde{r}(r,t) \equiv a(t)r \;\; and \quad h_{ab} \equiv (-1, a(t)^2)$$

$$h^{ab}\partial_a \tilde{r}\, \partial_b \tilde{r} = 0 \qquad\qquad r_h = \frac{1}{H}$$

Hawking temperature

$$T_H = \frac{|\kappa|}{2\pi} = \frac{1}{2\pi r_h}\left|1 - \frac{\dot{r_h}}{2Hr_h}\right|$$

$$\kappa = \frac{1}{2\sqrt{-h}}\partial_a\left(\sqrt{-h}\, h^{ab}\partial_b \tilde{r}\right)\Big|_{\tilde{r}=r_h}$$
$$= -\frac{1}{r_h}\left(1 - \frac{\dot{r_h}}{2Hr_h}\right)$$

Bekenstein-Hawking entropy $\quad S = \dfrac{\pi r_h^2}{G}$

1st principle

$$E = \rho V \qquad W = \frac{1}{2}(\rho - p)$$

$$TdS = -dE + WdV = -Vd\rho - \frac{1}{2}(\rho + p)dV$$

2nd Friedmann Equations

$$\frac{\dot{r_h}}{r_h} = -\frac{4\pi G}{3}\,\dot{\rho}$$

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During the inflation the typical energy scale is of order $H \sim 10^{-4} M_{Pl}$

Inflationary limit

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The first Friedmann equation can be expanded at leading order as:

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Entropic Inflation in Presence of Scalar Field

D'Onofrio, Odintsov and Paul, 2312.13587

Describing the matter fluid as a scalar field

$$\mathscr{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$
$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0$$

Slow-roll approximation

$$\ddot{\phi} \ll H\dot{\phi} \qquad \frac{\dot{\phi}^2}{2} \ll V$$

Friedmann equations during inflation

$$H^2 = \left[\frac{8\pi G}{3}\frac{\gamma}{\alpha}\left(\frac{\beta G}{\alpha\pi}\right)^{\beta-1}(2-\beta)\,V(\phi)\right]^{\frac{1}{2-\beta}}$$

$$\dot{H} = -\frac{2\pi G\gamma}{\alpha}\left(\frac{\beta G H^2}{\alpha\pi}\right)^{\beta-1}\dot{\phi}^2\ ,$$

Inflationary parameters

$$\epsilon(t) = -\frac{\dot{H}}{H^2}$$
$$\eta(t) = -\frac{\ddot{H}}{2H\dot{H}}$$

Scalar spectral index $\quad n_s = 1 - 6\epsilon + 2\eta\Big|_{t=t_c}$

Tensor–to-scalar ratio $\quad r = 16\epsilon\Big|_{t=t_c}$

Time of horizon crossing of the CMB modes

Planck 2018

$$n_s = 0.9649 \pm 0.0042 \quad \text{and} \quad r < 0.064$$

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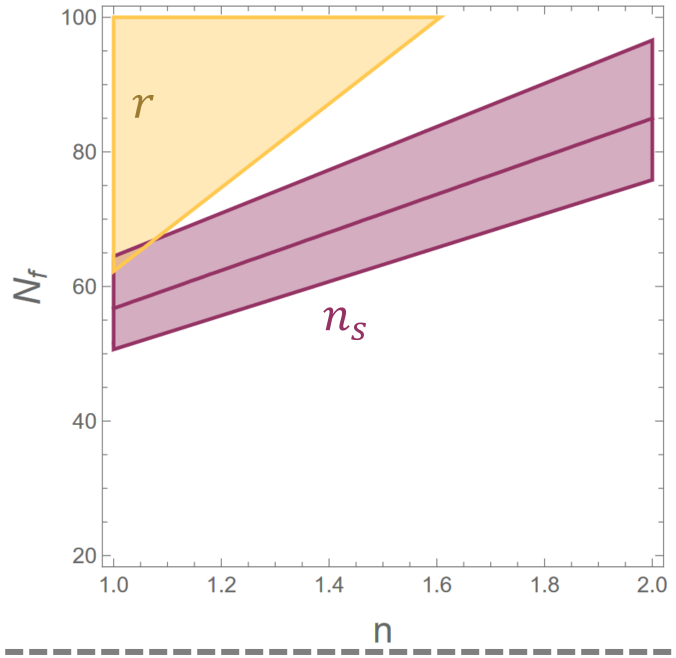
Fixing the scalar field potential

$$V(\phi) = V_0 \phi^n$$

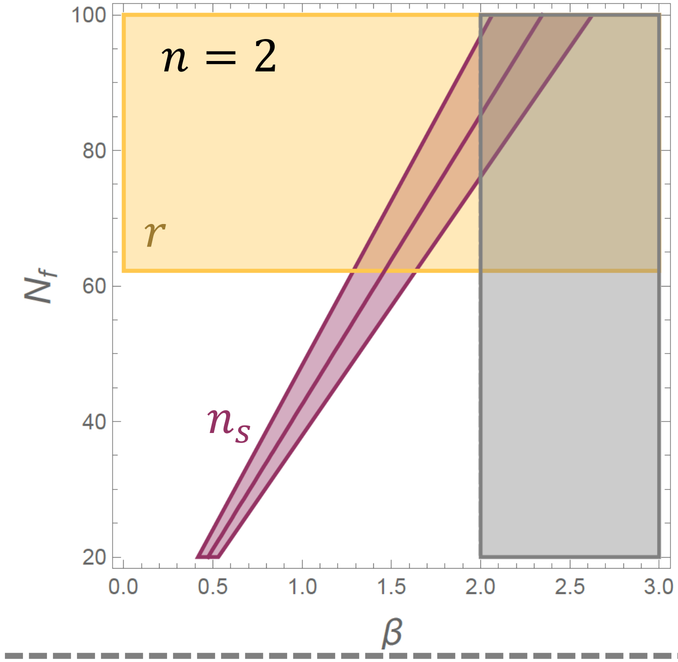
For the **Bekenstein-Hawging entropy** would be excluded

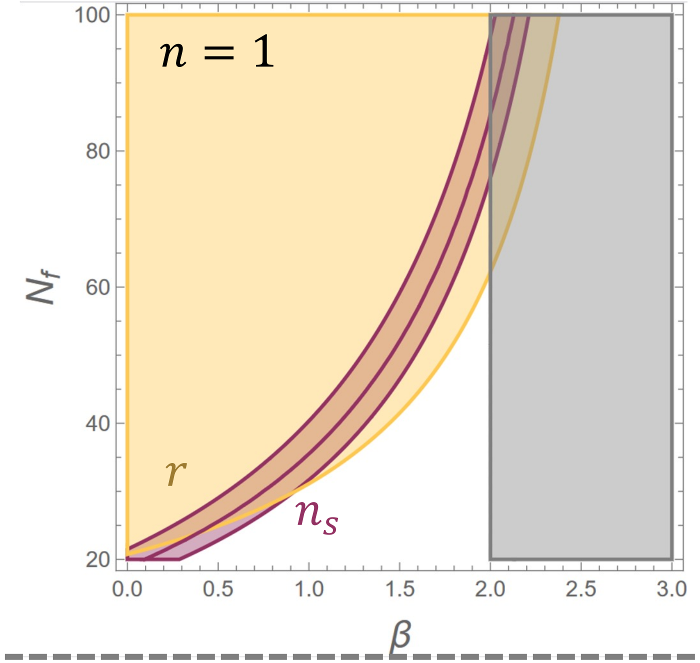


For the **generalized entropy S_4** :

$$n_s = 1 + \frac{2(4-5\beta)n - 8(2-\beta)}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_f}$$

$$r = \frac{16n}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_f}$$





Entropic Inflation varying the parameters

Nojiri, Odintsov and Paul, 2205.08876

$$S_4(\alpha_\pm,\beta,\gamma) = \frac{1}{\gamma}\left[\left(1+\frac{\alpha_+}{\beta}S\right)^{\beta} - \left(1+\frac{\alpha_-}{\beta}S\right)^{-\beta}\right]$$

$$\gamma(N) = \begin{cases} \gamma_0 \;\exp\left[-\int_N^{N_f}\sigma(N)\,dN\right] & ;\; N \leq N_f \\ \gamma_0 & ;\; N \geq N_f \end{cases}$$

$$-\left(\frac{2\pi}{G}\right)\left(\frac{\partial S_{\mathrm{g}}}{\partial S}\right)\frac{H'(N)}{H^3} + \left(\frac{\partial S_{\mathrm{g}}}{\partial \gamma}\right)\gamma'(N) = 0$$

$$H(N) = 4\pi M_{\mathrm{Pl}}\sqrt{\frac{\alpha_+}{\beta}}\left[\frac{2^{1/(2\beta)}\exp\left[-\frac{1}{2\beta}\int_0^N\sigma(N)dN\right]}{\left\{1+\sqrt{1+4\left(\alpha_+/\alpha_-\right)^{\beta}\exp\left[-2\int_0^N\sigma(N)dN\right]}\right\}^{1/(2\beta)}}\right]$$

$$n_s = 1 - \frac{2\sigma_0\sqrt{1+4\left(\alpha_+/\alpha_-\right)^{\beta}\exp\left[-2\left(1+\sigma_0 N_f\right)\right]}}{\left(1+\sigma_0\right)\sqrt{1+4\left(\alpha_+/\alpha_-\right)^{\beta}}} - \frac{8\sigma_0\left(\alpha_+/\alpha_-\right)^{\beta}}{1+4\left(\alpha_+/\alpha_-\right)^{\beta}}$$

$$r = \frac{16\sigma_0\sqrt{1+4\left(\alpha_+/\alpha_-\right)^{\beta}\exp\left[-2\left(1+\sigma_0 N_f\right)\right]}}{\left(1+\sigma_0\right)\sqrt{1+4\left(\alpha_+/\alpha_-\right)^{\beta}}}$$

$$N_{\mathrm{re}} = \frac{4}{\left(1-3w_{\mathrm{eff}}\right)}\left\{61.6 - \frac{1}{4\beta}\ln\left[\frac{\beta\exp\left[-\left(1+\sigma_0 N_{\mathrm{f}}\right)\right]\left\{1+\sqrt{1+\exp\left[2\left(1+\sigma_0 N_{\mathrm{f}}\right)\right]\left[\left(\frac{1+\sigma_0}{2\beta}\right)^2-1\right]}\right\}^2}{\left(16\pi^2\alpha_+/3\beta\right)^{\beta}\;\left\{1+\sigma_0+2\beta\right\}}\right] - N_{\mathrm{f}}\right\}$$

Constraining the parameters in early universe

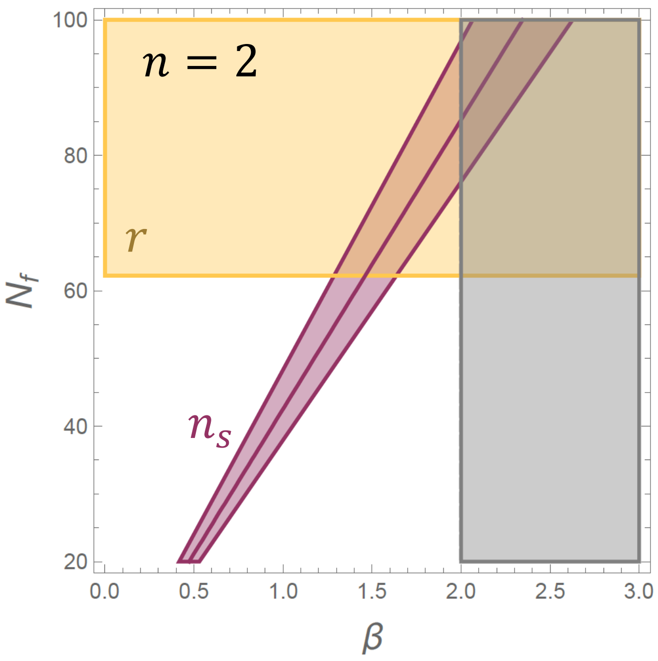
Inflation with a scalar field

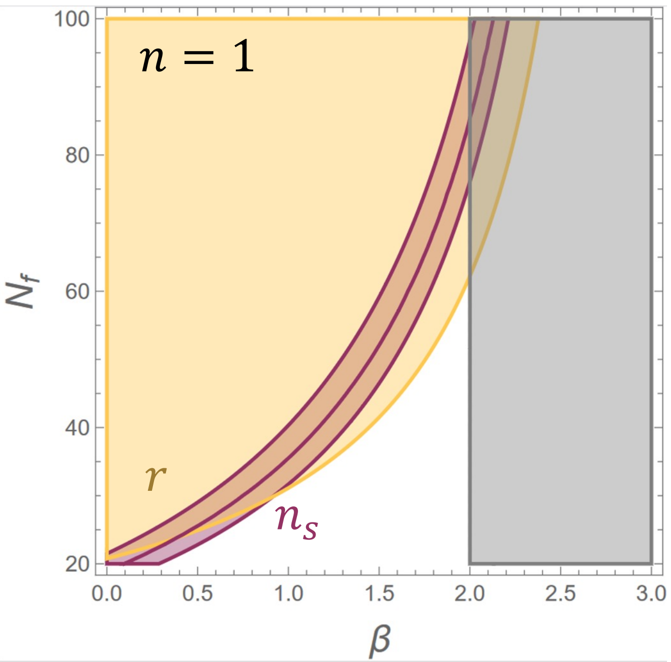
D'Onofrio, Odintsov and Paul, 2312.13587

$$V(\phi) = V_0 \phi^n$$

$$n_s = 1 + \frac{2(4-5\beta)n - 8(2-\beta)}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_\mathrm{f}}$$

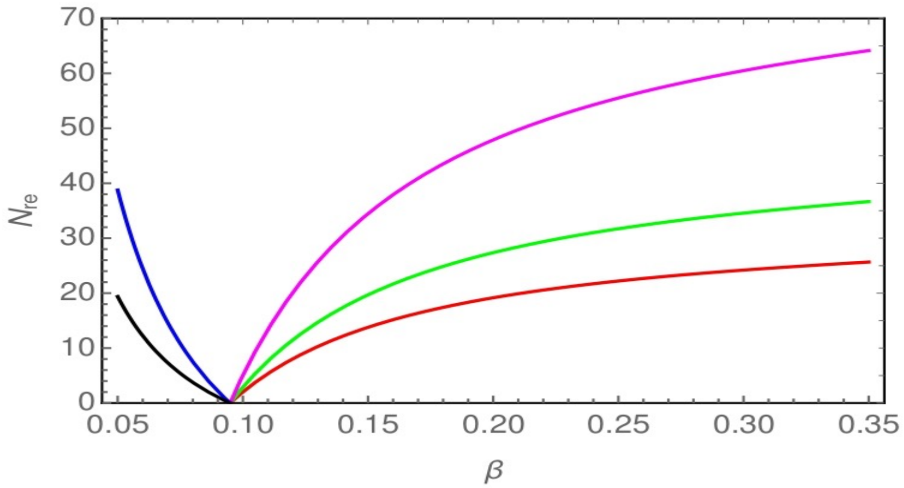
$$r = \frac{16n}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_\mathrm{f}}$$

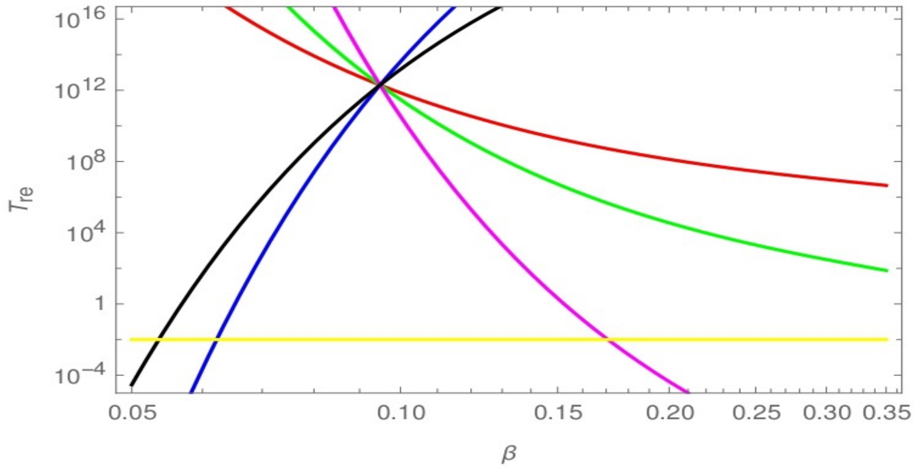




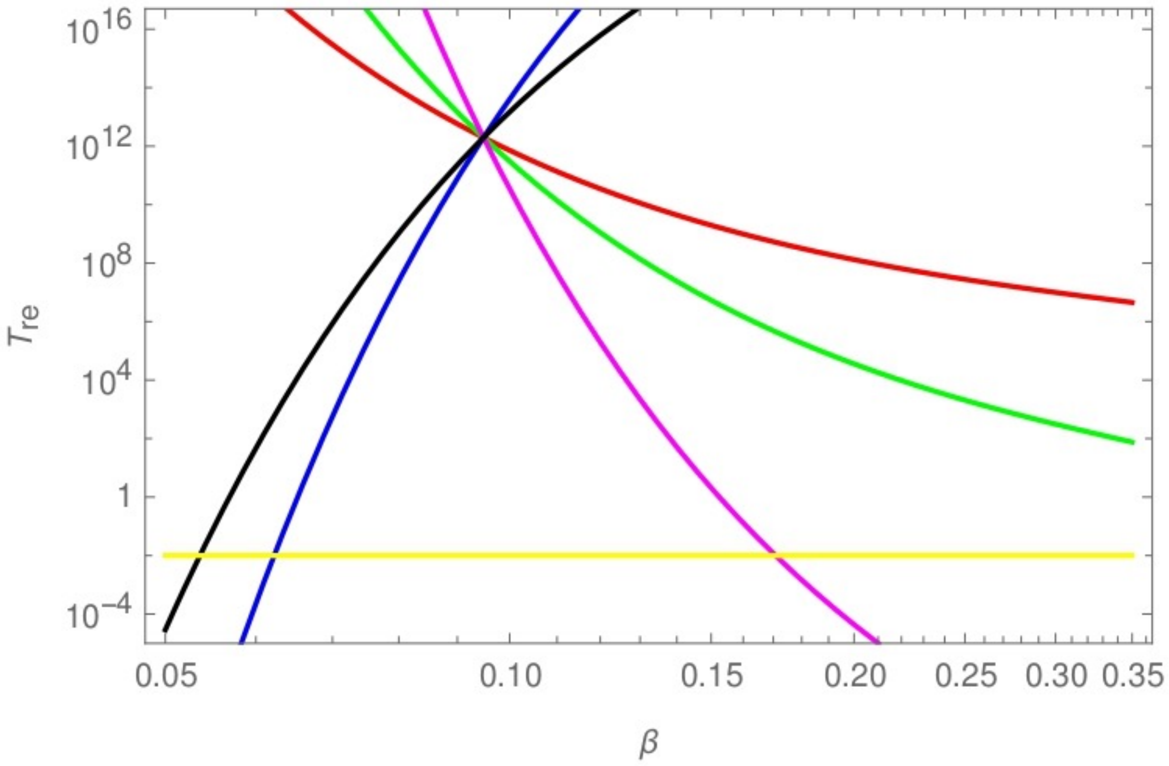
Reheating era

D'Onofrio, Odintsov and Paul, 2306.15225

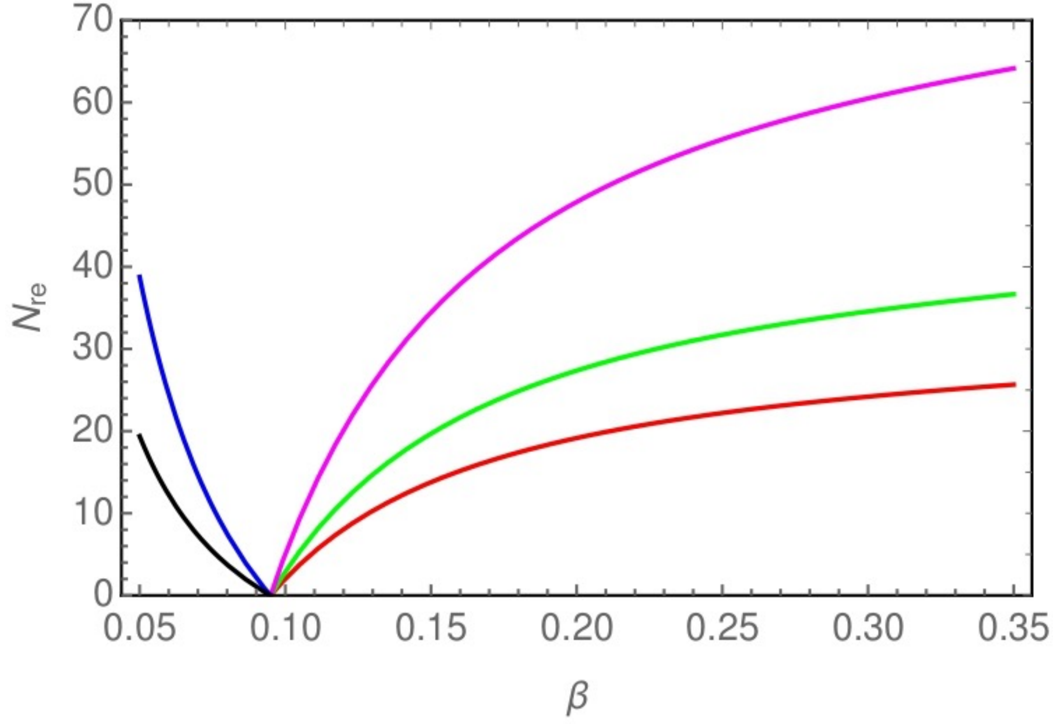




Reheating



10^{16}
10^{12}
10^{8}
10^{4}
1
10^{-4}
T_{re}
0.05
0.10
0.15
0.20
0.25
0.30
0.35
β



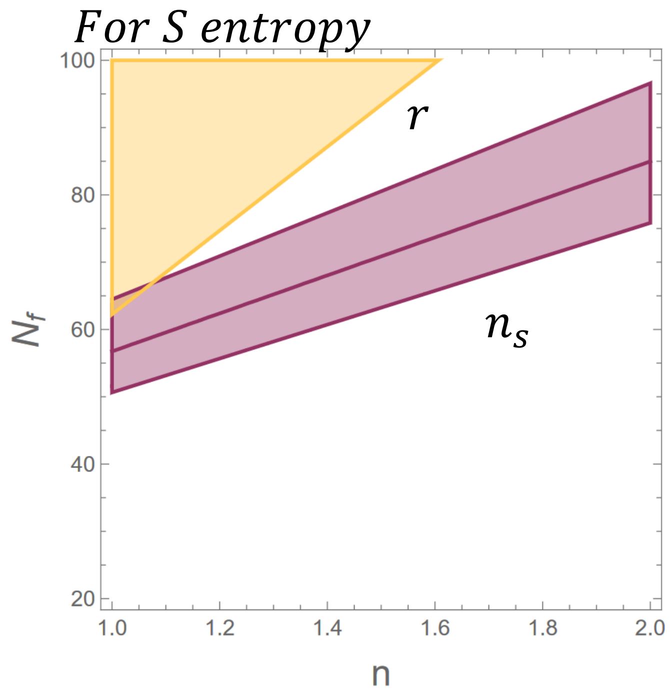
70
60
50
40
30
20
10
0
N_{re}
0.05
0.10
0.15
0.20
0.25
0.30
0.35
β

Fixing the scalar field potential

$$V(\phi) = V_0 \phi^n$$

For the Bekenstein-Hawging entropy would be excluded

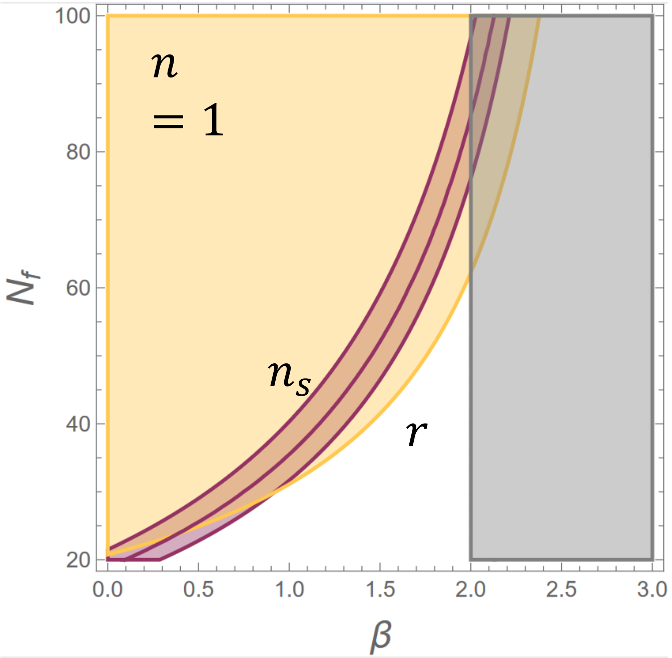
$$dN \equiv H dt$$

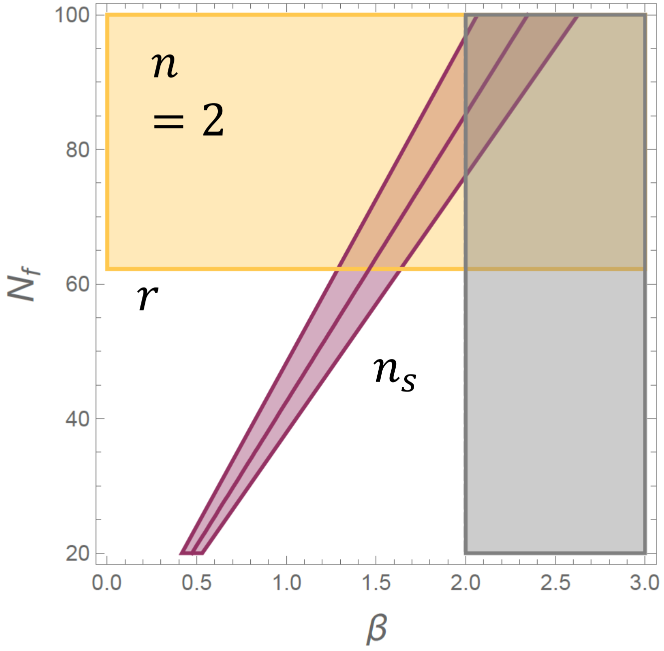


For the generalized entropy :

$$n_s = 1 + \frac{2(4-5\beta)n - 8(2-\beta)}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_{\mathrm{f}}}$$

$$r = \frac{16n}{n + 4(2-\beta)\left(2 - \frac{1-\beta}{2-\beta}n\right) N_{\mathrm{f}}}$$





$$\tilde{\Omega}(z) \equiv (z+1)\Omega_{m0} - \Omega_{k0}$$

$$H(z) = \frac{H_0(z+1)}{\sqrt{1-\Omega_{\rm D}(z)}}\sqrt{(z+1)\Omega_{\rm m0} - \Omega_{\rm k0}}$$

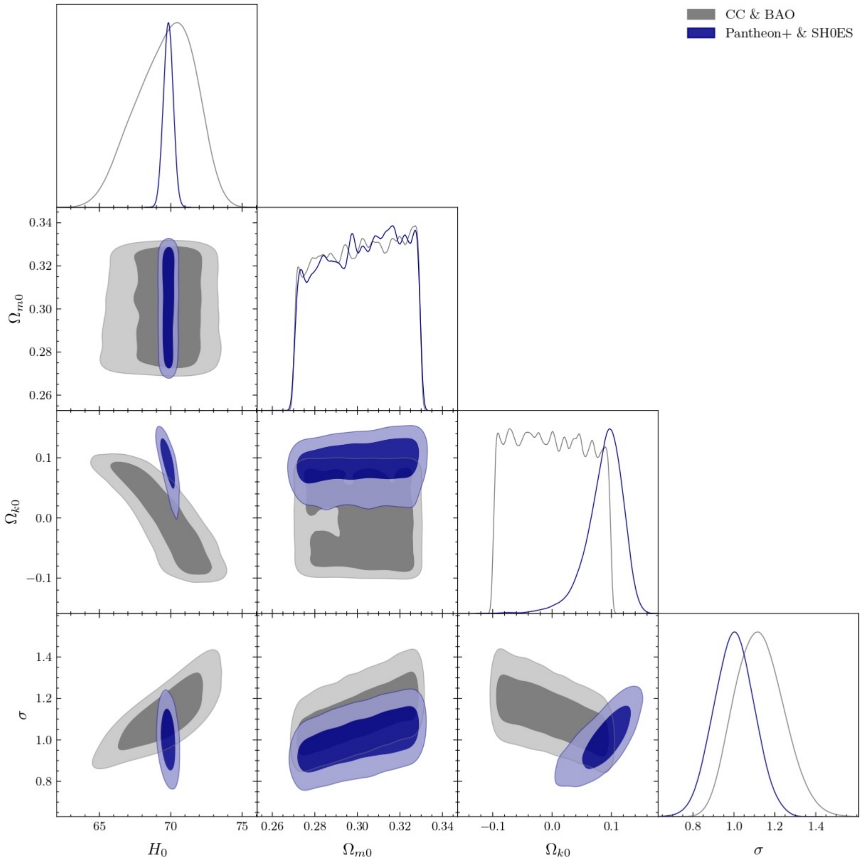
$$w_{\rm D} = -1 + \frac{\mathscr{N}}{\mathscr{D}}$$

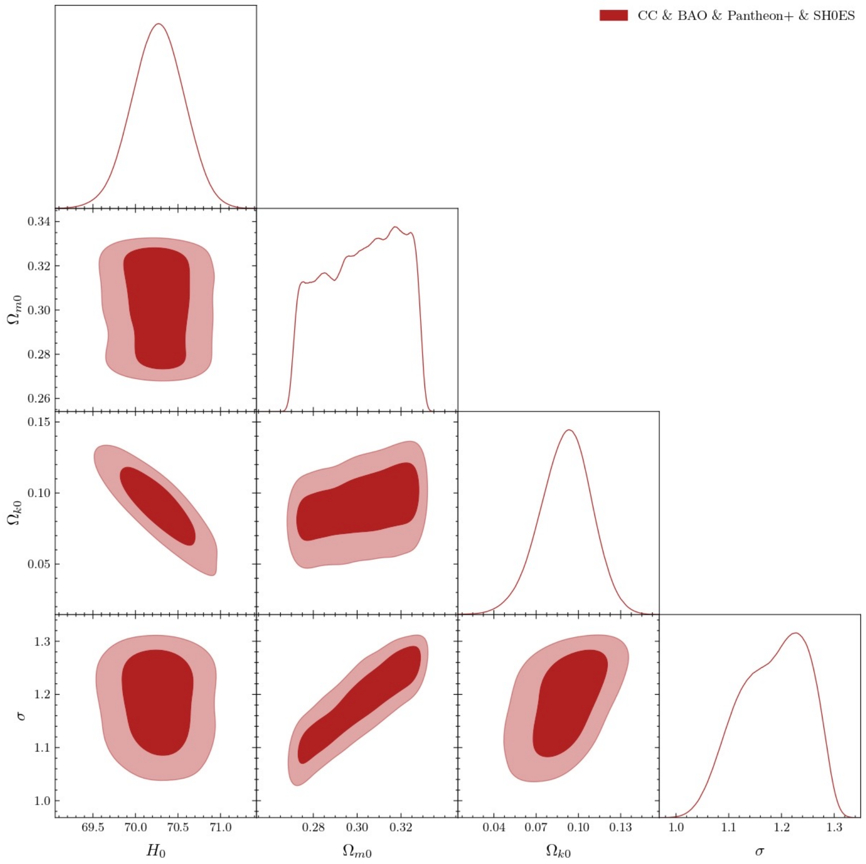
$$\mathscr{N} \equiv \frac{H_0^2(z+1)^2}{(1-\Omega_{\rm D})^2}\left[(z+1)\Omega_{\rm m0} - \Omega_{\rm k0}\right]\left\{3(1+\Omega_{\rm D}) + (1+z)\frac{d\Omega_{\rm D}}{dz} + \frac{(3-2\Omega_{\rm D})(1-\Omega_{\rm D})}{(z+1)\Omega_{\rm m0}-\Omega_{\rm k0}}\Omega_{\rm k0}\right\}$$
$$\times\left\{1-\sigma\left[\frac{H_0^2(z+1)^2}{1-\Omega_{\rm D}}\left[(z+1)\Omega_{\rm m0}-\Omega_{\rm k0}\Omega_{\rm D}\right]\right]^{1-\beta}\right\}$$

$$\mathscr{D} \equiv \Lambda + \frac{3H_0^2(z+1)^2}{1-\Omega_{\rm D}}\left[(z+1)\Omega_{\rm m0}+\Omega_{\rm k0}\Omega_{\rm D}\right]\left\{1-\frac{\sigma}{2-\beta}\left[\frac{H_0^2(z+1)^2}{1-\Omega_{\rm D}}\left[(z+1)\Omega_{\rm m0}-\Omega_{\rm k0}\Omega_{\rm D}\right]\right]^{1-\beta}\right\}$$

$$\Omega_{\rm D}(z) = 1 + \left\{\frac{\Omega_{\rm k0}}{\tilde{\Omega}(z)} - \left[\frac{1+\frac{\Omega_{\rm k0}}{\tilde{\Omega}(z)}+\frac{\Lambda}{3H_0^2(1+z)^2\tilde{\Omega}(z)}}{\frac{\sigma}{2-\beta}\left[H_0^2(1+z)^2\tilde{\Omega}(z)\right]^{1-\beta}}\right]^{\frac{1}{2-\beta}}\right\}^{-1}$$

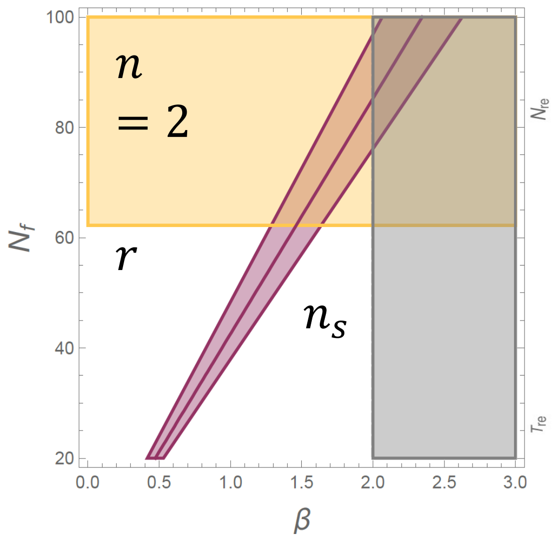
Dataset	H_0	Ω_{m0}	Ω_{k0}	σ
CC & BAO	$69.804^{1.878}_{-2.351}$	$0.302^{0.019}_{-0.021}$	$-0.002^{0.068}_{-0.066}$	$1.121^{0.125}_{-0.116}$
Pantheon+ & SH0ES	$69.832^{0.334}_{-0.339}$	$0.302^{0.019}_{-0.021}$	$0.092^{0.025}_{-0.032}$	$1.002^{0.097}_{-0.100}$
CC & BAO & Pantheon+ & SH0ES	$70.266^{0.304}_{-0.303}$	$0.303^{0.018}_{-0.021}$	$0.092^{0.017}_{-0.019}$	$1.193^{0.062}_{-0.077}$





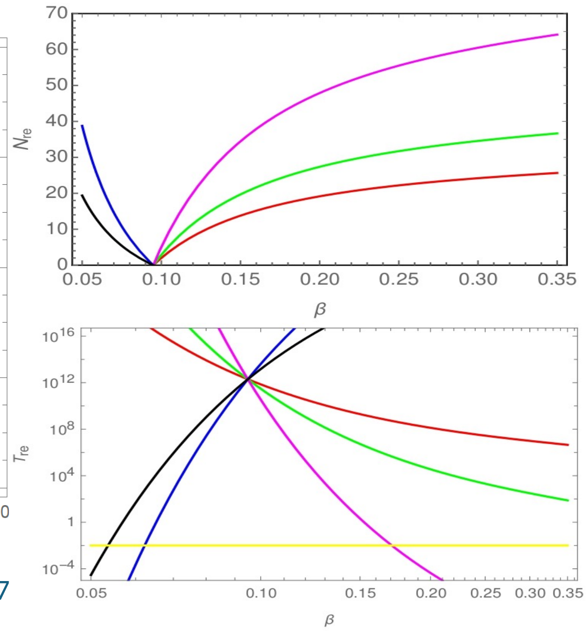
Constraining the parameters

Inflation



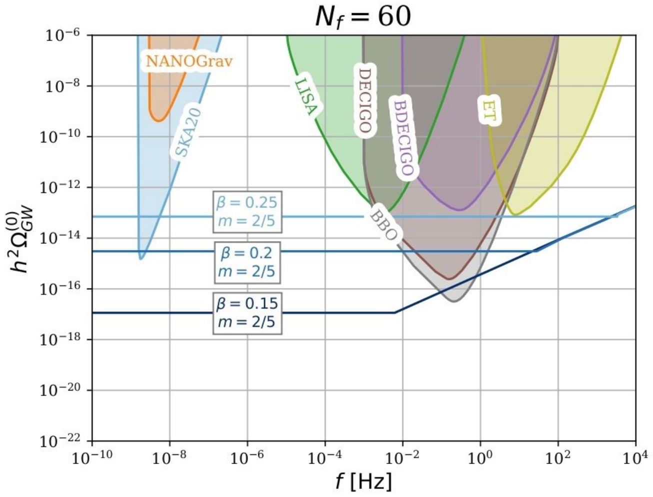
D'Onofrio, Odintsov and Paul, 2312.13587

Reheating



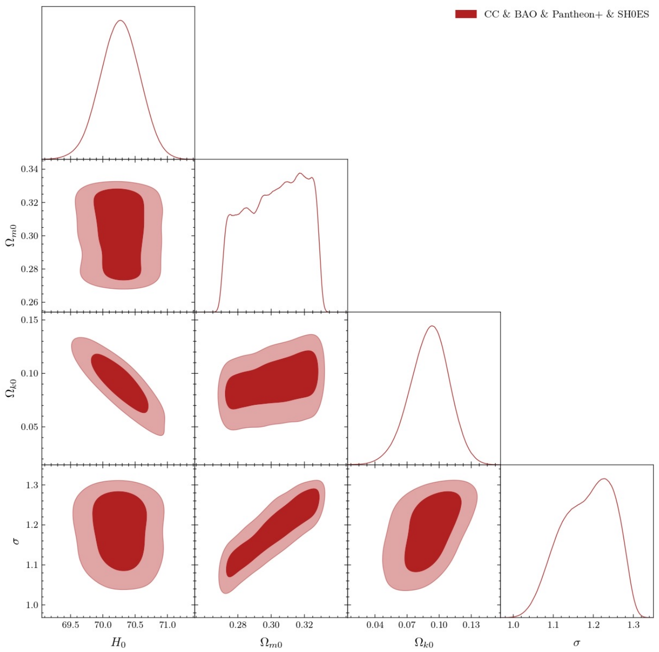
D'Onofrio, Odintsov and Paul, 2306.15225

Primordial GW



D'Onofrio, Odintsov and Paul, 2407.05855

Late universe



D'Onofrio, Odintsov and Paul, *in review*

Comoving Horizon $v_c = 1$
Apparent Horizon $v_h = -\frac{\dot{H}}{H^2}$