Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA
EXCELENCIA MARÍA DE MAEZTU 2020-2023

First ICCUB Numerical Relativity Waveform Catalog of Eccentric Black Hole Binaries

Ana Climent

ICC - University of Barcelona

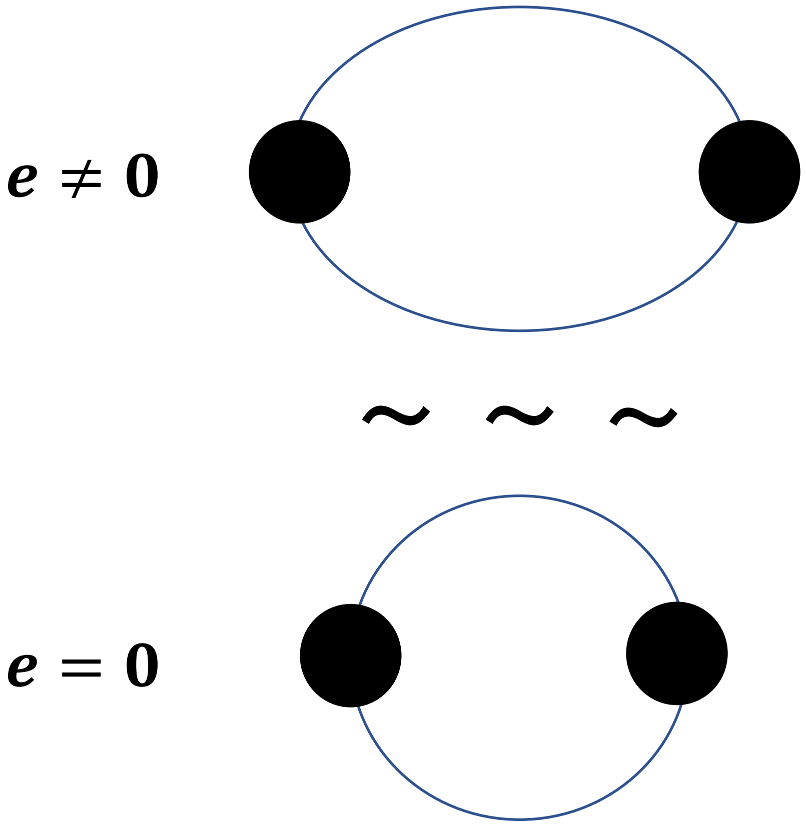
[Appear soon – Juan Trenado, Tomas Andrade, AC, and Maria-Antonia Ferrer]

Why?

Isolated BBH

Eccentricity

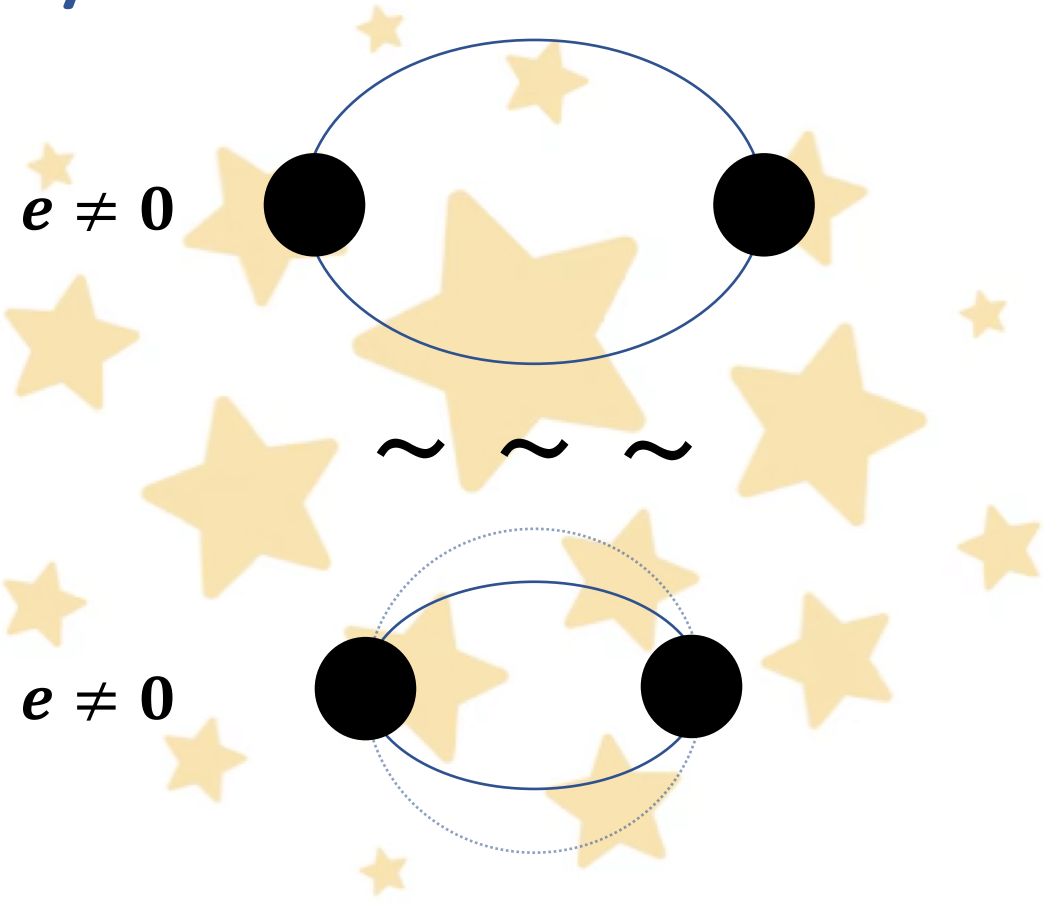
- Gravitational Waves **circularize** orbits



Dense environments

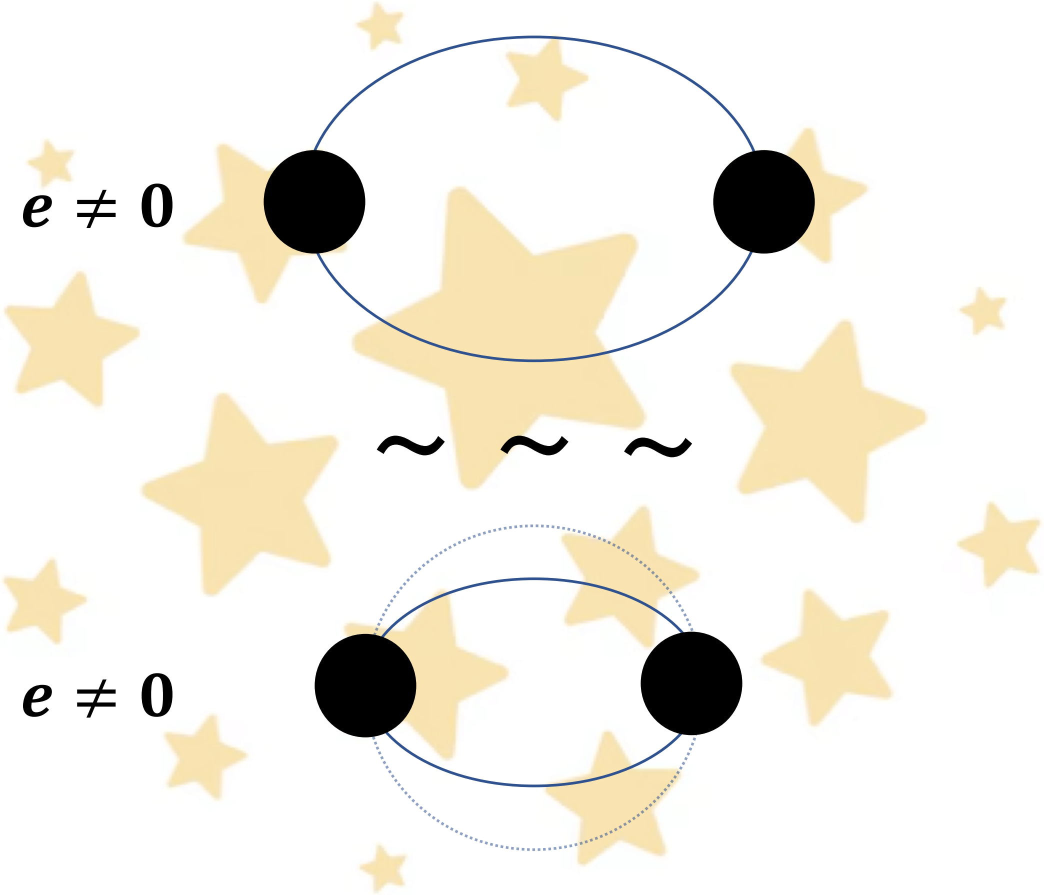
Eccentricity

- Gravitational Waves **circularize** orbits
- **Dense environments**
  - Tidal forces
  - Dynamical friction
  - Three body encounters



ICCUB Catalog

Dense environments



ICCUB Catalog




Eccentric binary black holes

- Improve detection strategies
- New astrophysical insights

ICCUB Catalog

Equal-mass

Highly eccentricity

128 simulations

Equal-spin

Spin aligned with angular momentum

How?

Numerical Relativity (NR)

- Numerical simulations of **General Relativity**

Numerical Relativity (NR)

- Numerical simulations of **General Relativity**
- NR Catalogues
  - SXS
  - **RIT**
  - CoRe
  - SACRA-MPI
  - Cardiff
  - MAYA

Numerical Relativity (NR)

- Numerical simulations of **General Relativity**

- NR Catalogues
  - SXS
  - **RIT**
  - CoRe
  - SACRA-MPI
  - Cardiff
  - MAYA

arXiv > gr-qc > arXiv:2202.00018

Search...

Help | Adva

General Relativity and Quantum Cosmology

[Submitted on 31 Jan 2022]

The Fourth RIT binary black hole simulations catalog: Extension to Eccentric Orbits

James Healy, Carlos O. Lousto (RIT)

Einstein Toolkit

- **Open source**

Einstein Toolkit

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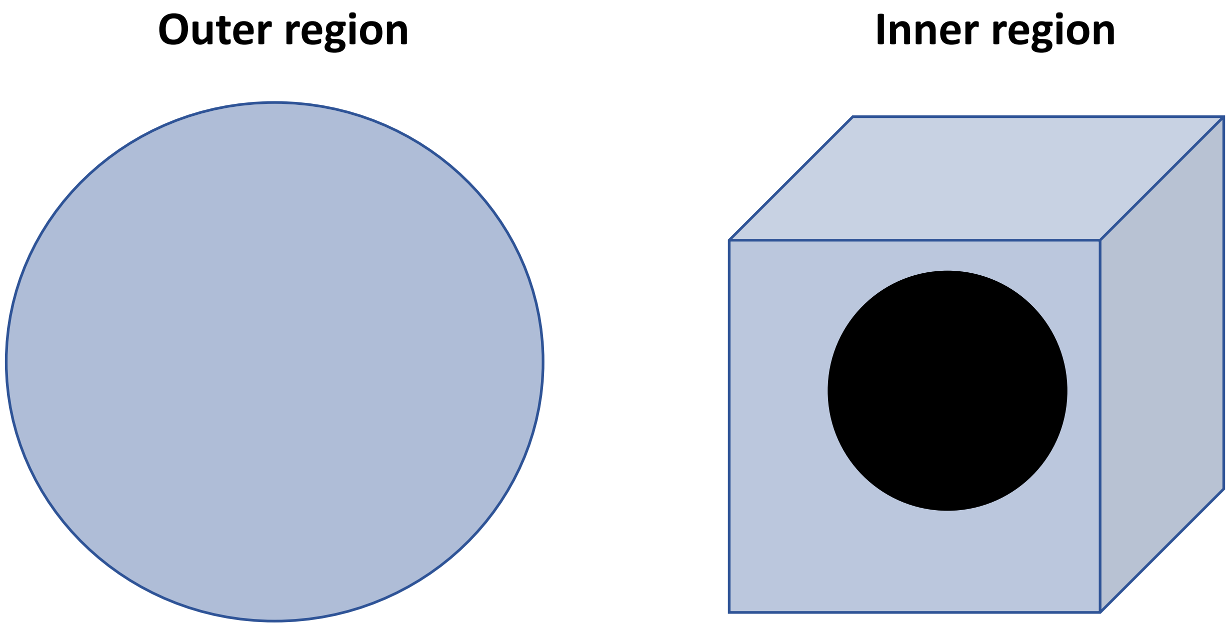
- **BSSN formulation**

Einstein Toolkit

- **Open source**
- **BSSN formulation**
- **Multi-patch grid structure**



Einstein Toolkit

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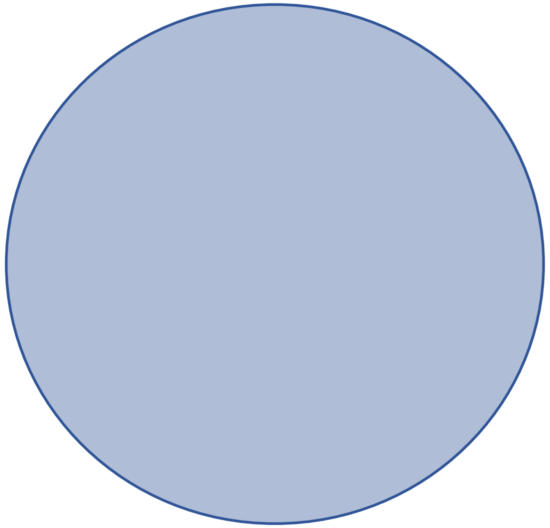
- **BSSN formulation**
- **Multi-patch grid structure**

Outer region

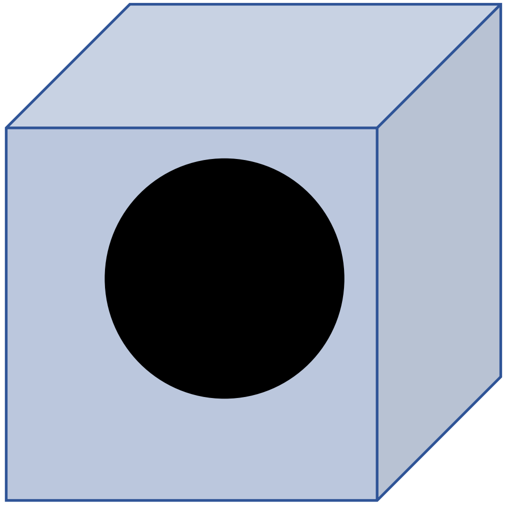


$h = 1.37M$

$n = 28$ points

$r = 1000 - 1500M$

Inner region



7 levels

$dx = 0.0214M$

$X_{max} = 1.37M$

Einstein Toolkit

- **Open source**
- **BSSN formulation**
- **Multi-patch grid structure**
- **Extraction of Weyl Scalar ψ_4 up to $l = 4$ at $R = 100, 115, 136, 167M$**

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- **Extraction of Weyl Scalar ψ_4 up to $l = 4$ at $R = 100, 115, 136, 167M$**

$$\boxed{\psi_4 = \ddot{h}_+ - i\ddot{h}_x}$$

Einstein Toolkit

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- **Multi-patch grid structure**
- **Extraction of Weyl Scalar ψ_4 up to $l = 4$ at $R = 100, 115, 136, 167M$**

$$\boxed{\psi_4 = \ddot{h}_+ - i\ddot{h}_x}$$

→ Extrapolation $\psi_4|_{r\to\infty}$

→ Strain computation h

Post-processing!

Post-Processing

Weyl Scalar ψ_4 at infinity at first order extrapolation

$$r\,\psi_4^{lm}\Big|_{r\to\infty} = \left(1-\frac{2M}{r}\right)\left(r\boldsymbol{\psi_4^{NR,lm}(t,r)} - \frac{(l-1)(l+2)}{2r}\int dt\left[r\boldsymbol{\psi_4^{NR,lm}(t,r)}\right]\right)$$

Post-Processing

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Strain modes

$$\psi_4^{lm}\Big|_{r\to\infty} = \partial_t^2 h$$

Post-Processing

Weyl Scalar ψ_4 at infinity at first order extrapolation

$$r\,\psi_4^{lm}\Big|_{r\to\infty} = \left(1-\frac{2M}{r}\right)\left(r\boldsymbol{\psi}_4^{NR,lm}(\boldsymbol{t},\boldsymbol{r}) - \frac{(l-1)(l+2)}{2r}\int \boldsymbol{dt}\left[r\boldsymbol{\psi}_4^{NR,lm}(\boldsymbol{t},\boldsymbol{r})\right]\right)$$

Strain modes

$$\psi_4^{lm}\Big|_{r\to\infty} = \boldsymbol{\partial_t^2} h$$

Triple time integral!

Fixed Frequency Integration (FFI)

Fourier Transform $\psi_4^{lm}(t) \rightarrow \psi_4^{lm}(f)$

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Fourier Transform $\psi_4^{lm}(t) \rightarrow \psi_4^{lm}(f)$

Perform the integral

$$h^{lm}(f) = \begin{cases} -\dfrac{\psi_4^{lm}(f)}{(2\pi f)^2} & \text{if } f \geq f_0 \\ -\dfrac{\psi_4^{lm}(f)}{(2\pi f_0)^2} & \text{if } f < f_0 \end{cases}$$

Fixed Frequency Integration (FFI)

Fourier Transform $\psi_4^{lm}(t) \rightarrow \psi_4^{lm}(f)$

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Inverse Fourier Transform $h^{lm}(f) \rightarrow h^{lm}(t)$

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Cutoff frequency

Inverse Fourier Transform $h^{lm}(f) \rightarrow h^{lm}(t)$

Fixed Frequency Integration (FFI)

$\boldsymbol{f_0}$ **that minimizes the energy and angular momentum balance**

Cutoff frequency

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Fixed Frequency Integration (FFI)

$\boldsymbol{f_0}$ **that minimizes the energy and angular momentum balance**

$$\delta_E \equiv E_{ADM} - E_f - E_{rad}$$

$$\delta_J \equiv J_{ADM} - J_f - J_{rad}$$

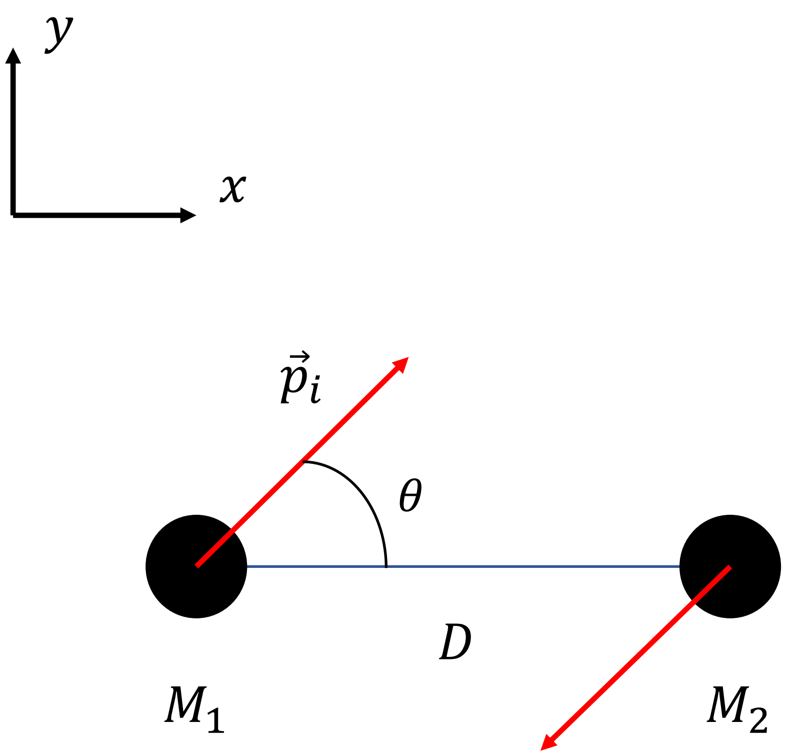
Cutoff frequency

$$h^{lm}(f) = \begin{cases} -\dfrac{\psi_4^{lm}(f)}{(2\pi f)^2} & \text{if } f \geq \boldsymbol{f_0} \\ -\dfrac{\psi_4^{lm}(f)}{(2\pi \boldsymbol{f_0})^2} & \text{if } f < \boldsymbol{f_0} \end{cases}$$

$$E_{rad} = \frac{1}{16\pi} \sum_{(l,m)} \int_{t_i}^{t_f} \dot{h}^{lm}\, \dot{h}^{lm} \qquad J_{rad} = \frac{1}{16\pi} \sum_{(l,m)} m \int_{t_i}^{t_f} \Im[h^{lm} \dot{h}^{*lm}]$$

ICCUB Catalog

Set-up



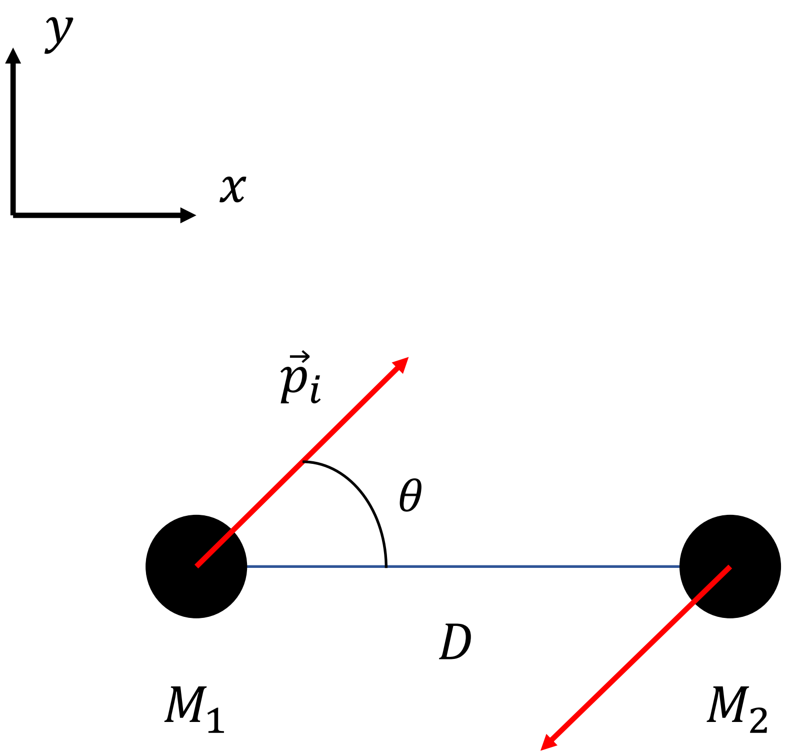
Initial separation

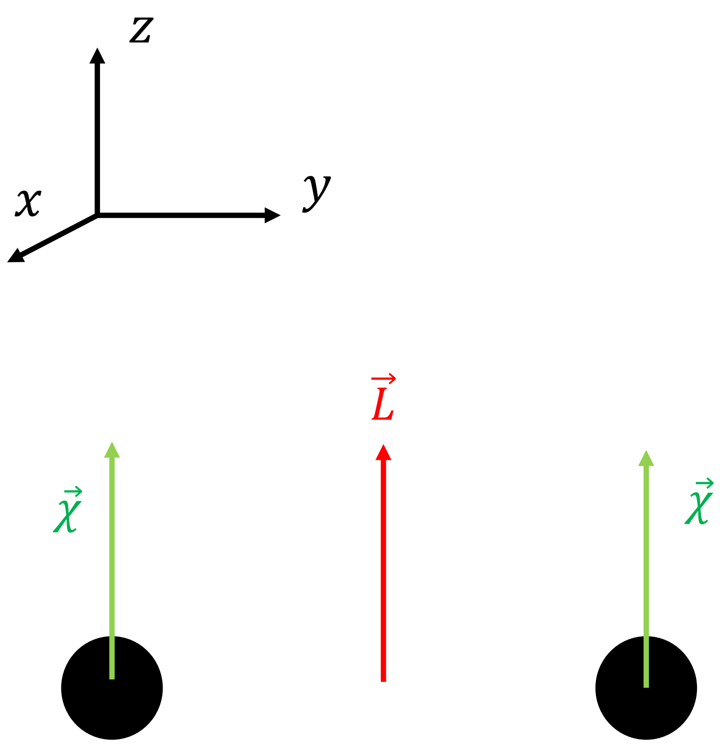
$$D = 40M$$

Initial Mass

$$M = 1, \qquad q = 1$$

Set-up





Initial separation

$D = 40M$

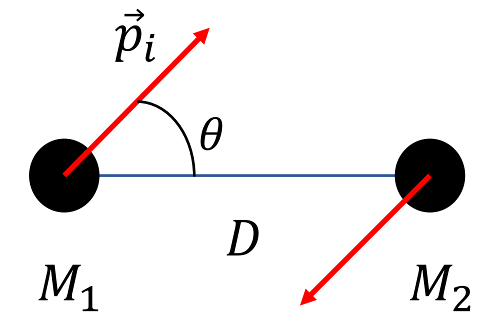
Initial Mass

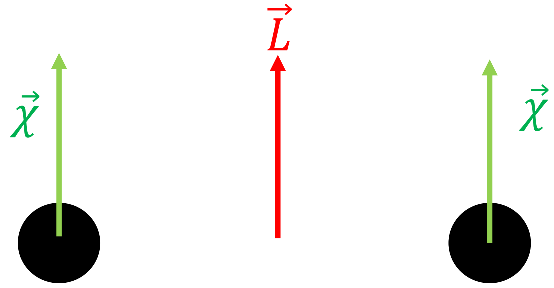
$M = 1, \quad q = 1$

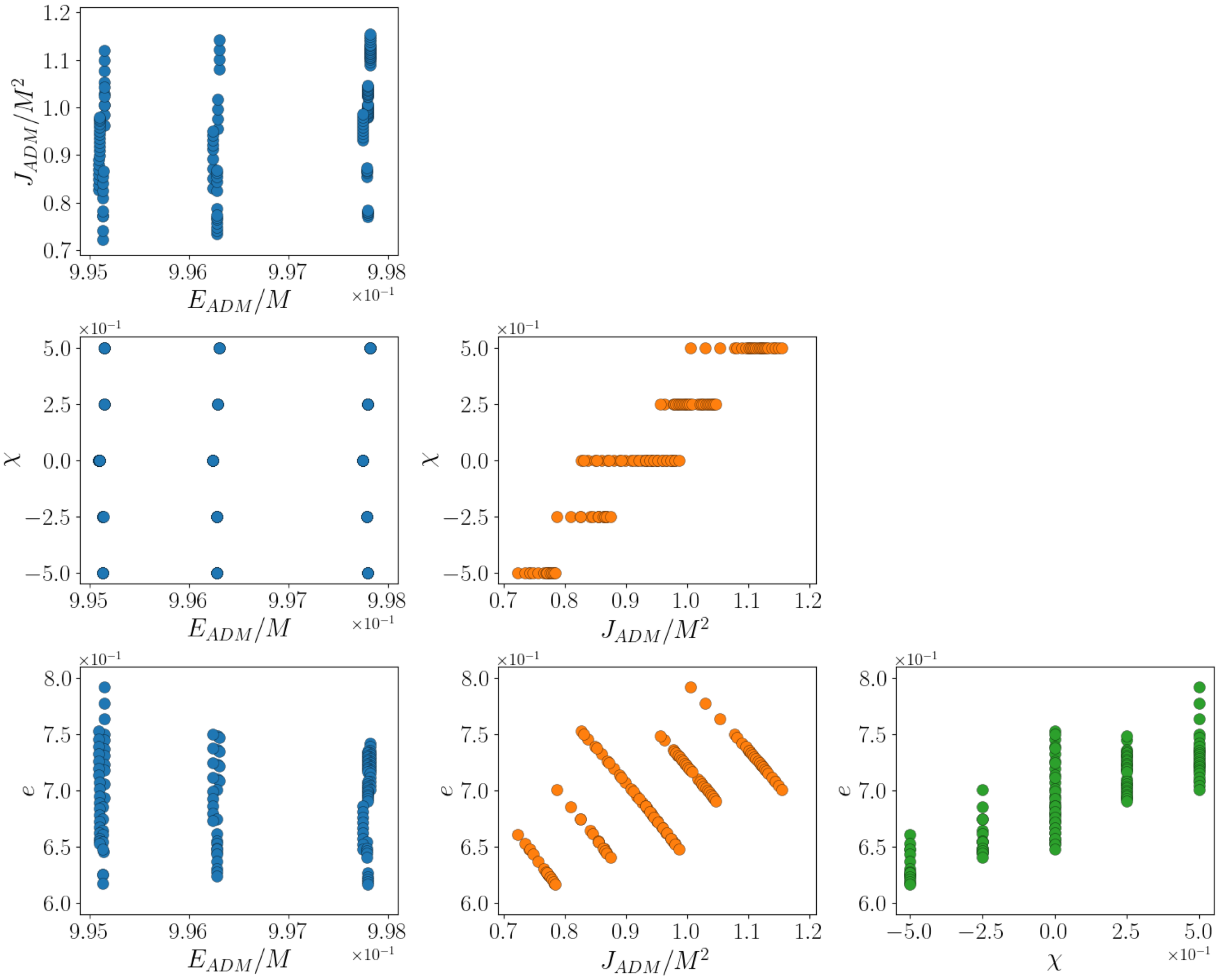
Total angular momentum

$\vec{J} = \vec{L} + \vec{\chi}M^2$

Initial values



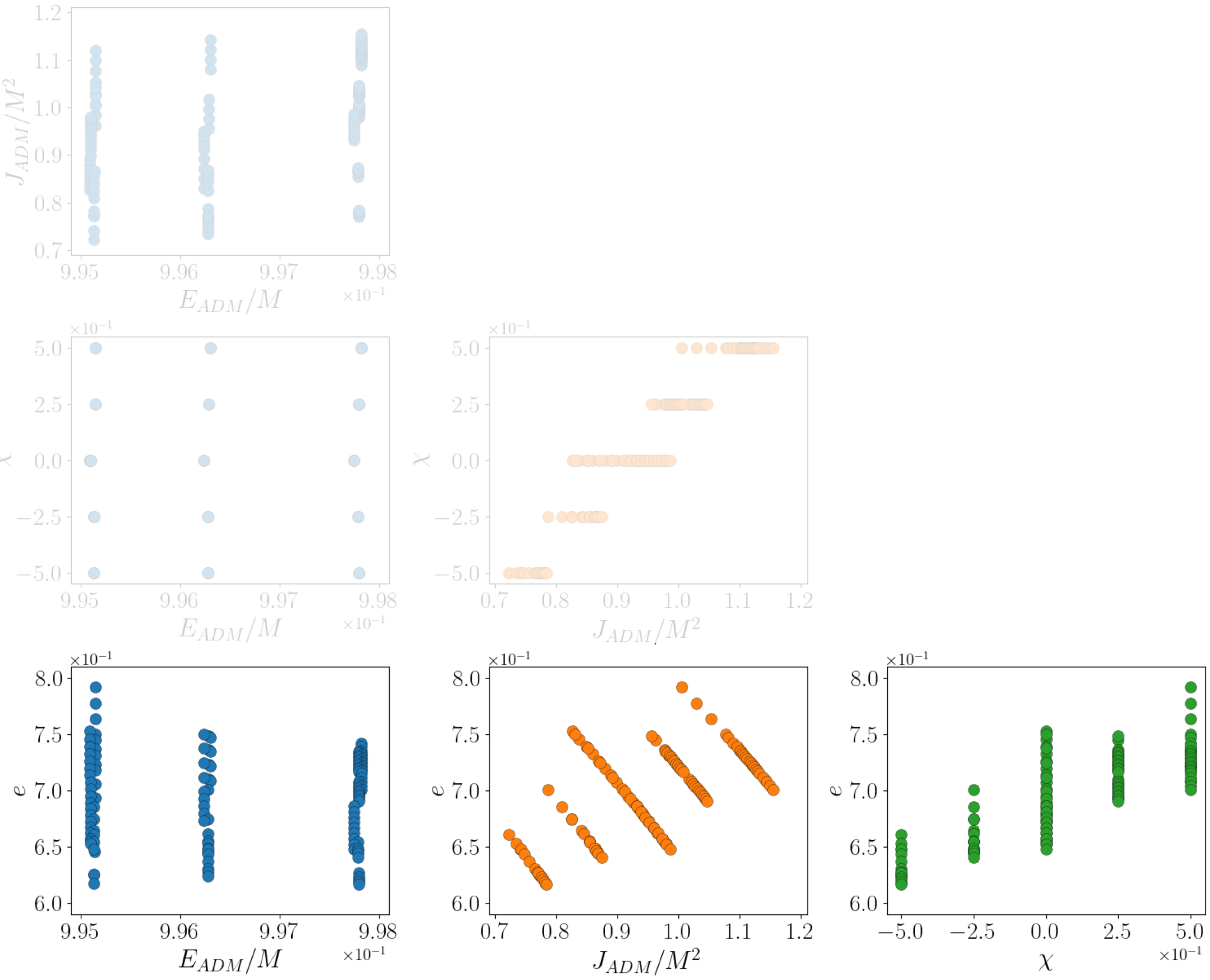




Initial values

Eccentricity

$e \in (0.62\,, 0.79)$



Eccentricity

For eccentric orbits $P_t = (1 - \varepsilon)\boldsymbol{P_{t,qc}}$

Eccentricity

For eccentric orbits $P_t = (1 - \varepsilon)\boldsymbol{P_{t,qc}}$

Eccentricity $e = \varepsilon(2 - \varepsilon)$

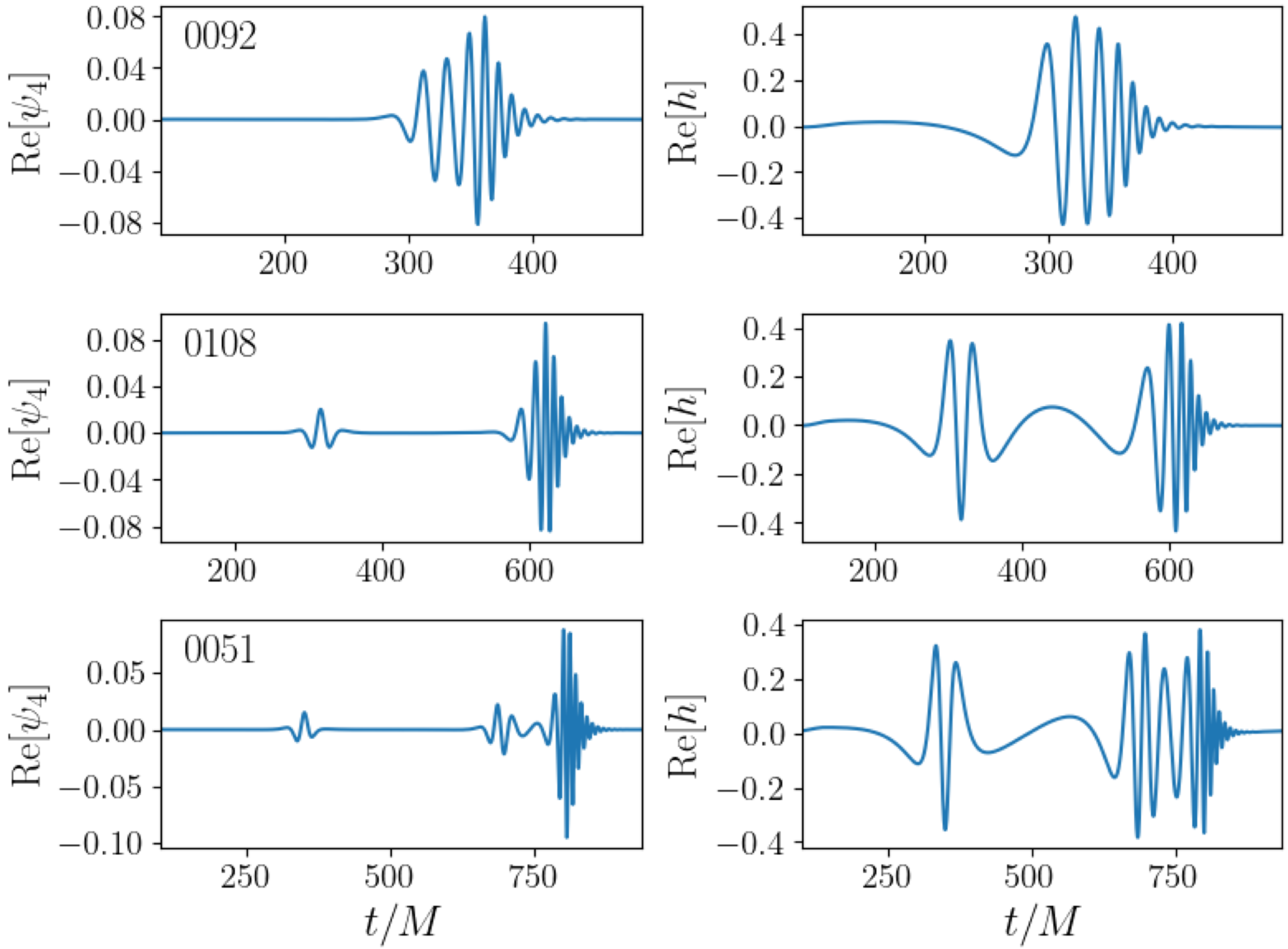
3rd order in PN approximation

$$
\begin{aligned}
\frac{P_t}{M} = \frac{q}{(1+q)^2}\sqrt{\frac{M}{r}}\Bigg[& 1 + 2\,\frac{M}{r} + \left(-\frac{3}{4}\,\frac{(3+4\,q)\,q\chi_{1z}}{(1+q)^2} - \frac{3}{4}\,\frac{(3\,q+4)\,\chi_{2z}}{(1+q)^2}\right)\left(\frac{M}{r}\right)^{3/2} \\
& + \Bigg(-\frac{3}{2}\,\frac{{\chi_{1x}}^2 q^2}{(1+q)^2} - 3\,\frac{\chi_{1x}\,\chi_{2x}\,q}{(1+q)^2} + \frac{3}{4}\,\frac{{\chi_{1y}}^2 q^2}{(1+q)^2} + \frac{3}{2}\,\frac{\chi_{1y}\,\chi_{2y}\,q}{(1+q)^2} + \frac{3}{4}\,\frac{{\chi_{1z}}^2 q^2}{(1+q)^2} + \frac{3}{2}\,\frac{\chi_{1z}\,\chi_{2z}\,q}{(1+q)^2} \\
& - \frac{3}{2}\,\frac{{\chi_{2x}}^2}{(1+q)^2} + \frac{3}{4}\,\frac{{\chi_{2y}}^2}{(1+q)^2} + \frac{3}{4}\,\frac{{\chi_{2z}}^2}{(1+q)^2} + \frac{1}{16}\,\frac{42\,q^2+41\,q+42}{(1+q)^2}\Bigg)\left(\frac{M}{r}\right)^2 \\
& + \left(-\frac{1}{16}\,\frac{q\left(72\,q^3+116\,q^2+60\,q+13\right)\chi_{1z}}{(1+q)^4} - \frac{1}{16}\,\frac{\left(13\,q^3+60\,q^2+116\,q+72\right)\chi_{2z}}{(1+q)^4}\right)\left(\frac{M}{r}\right)^{5/2} \\
& + \Bigg(-\frac{1}{16}\,\frac{q^2\left(80\,q^2-59\right)\chi_{1x}{}^2}{(1+q)^4} + \frac{1}{8}\,\frac{q\left(12\,q^2+35\,q+12\right)\chi_{2x}\,\chi_{1x}}{(1+q)^4} - \frac{1}{2}\,\frac{q^2\left(q^2+10\,q+8\right)\chi_{1y}{}^2}{(1+q)^4} \\
& - \frac{1}{4}\,\frac{q\left(27\,q^2+58\,q+27\right)\chi_{2y}\,\chi_{1y}}{(1+q)^4} + \frac{1}{32}\,\frac{q^2\left(128\,q^2+56\,q-27\right)\chi_{1z}{}^2}{(1+q)^4} + \frac{1}{16}\,\frac{q\left(60\,q^2+133\,q+60\right)\chi_{2z}\,\chi_{1z}}{(1+q)^4} \\
& + \frac{1}{16}\,\frac{\left(59\,q^2-80\right)\chi_{2x}{}^2}{(1+q)^4} - \frac{1}{2}\,\frac{\left(8\,q^2+10\,q+1\right)\chi_{2y}{}^2}{(1+q)^4} - \frac{1}{32}\,\frac{\left(27\,q^2-56\,q-128\right)\chi_{2z}{}^2}{(1+q)^4} \\
& + \frac{163\,\pi^2 q}{128\,(1+q)^2} + \frac{1}{32}\,\frac{120\,q^4-659\,q^3-1532\,q^2-659\,q+120}{(1+q)^4}\Bigg)\left(\frac{M}{r}\right)^3\Bigg],
\end{aligned}
$$

$p_i = 0.045$, $\chi = 0.25$,
$\theta = 28.4°$, $e = 0.72$

$p_i = 0.045$, $\chi = 0.25$,
$\theta = 29.8°$, $e = 0.69$

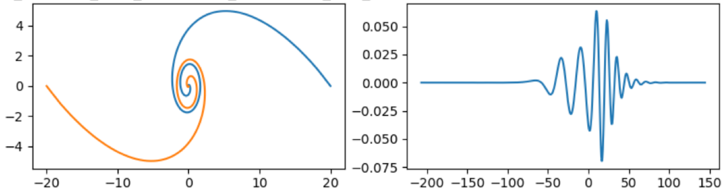
$p_i = 0.022$, $\chi = 0.50$,
$\theta = 38.0°$, $e = 0.71$



Video

https://drive.google.com/file/d/1KB7l1InpbaY5_ASywoOQETCpJb6QBgpF/view?usp=drive_link

BBH_P0p0346_th32_chi1m0p25_chi2m0p25_D40_mp



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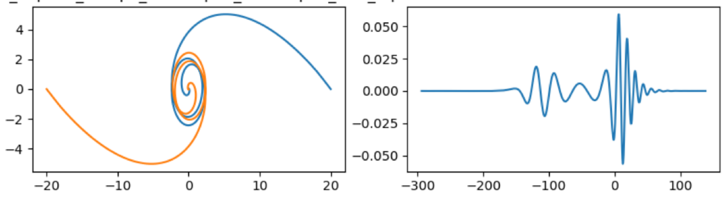

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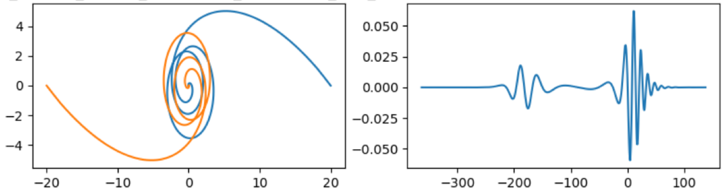

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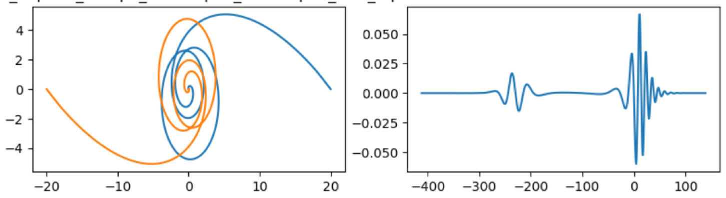

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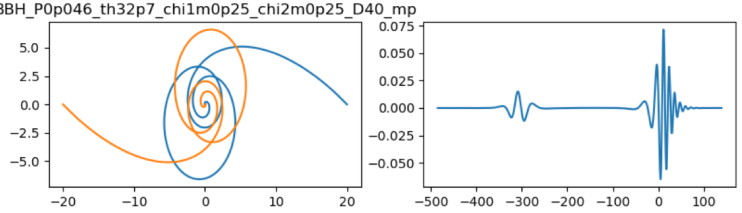

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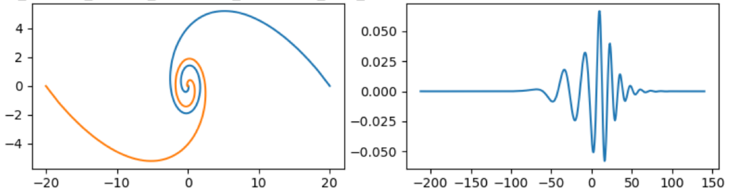

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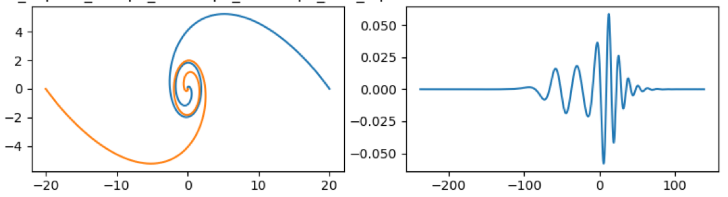

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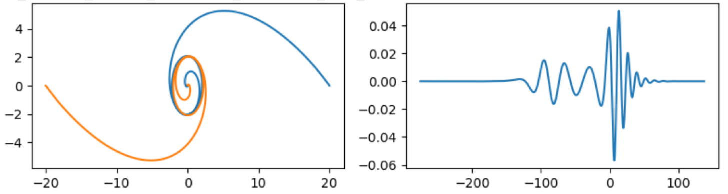

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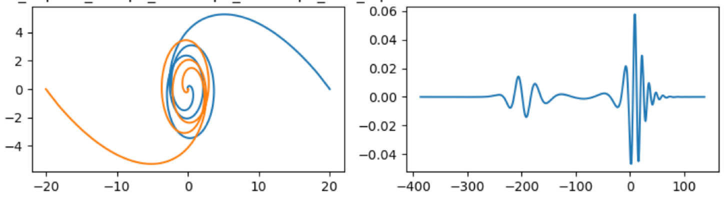

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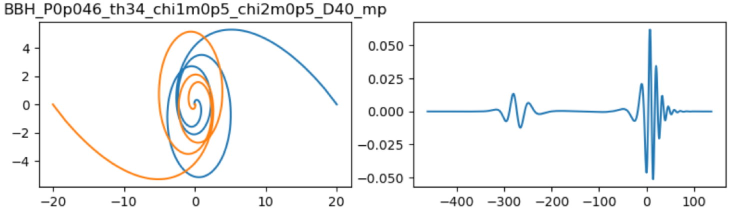

Where?

https://egrav.icc.ub.edu/

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LOAD LIST ABOUT LOGIN

Download Data

ID	D	M_i	J_i	q	M1_i	M2_i	s1x	s1y	s1z	chi1z	chi2z
1	40	0.99515	1.00456	1	0.5	0.5	0	0	0.125	0.5	0.5
2	40	0.99515	1.02953	1	0.5	0.5	0	0	0.125	0.5	0.5
3	40	0.99515	1.05355	1	0.5	0.5	0	0	0.125	0.5	0.5
4	40	0.99515	1.07659	1	0.5	0.5	0	0	0.125	0.5	0.5
5	40	0.99509	0.82659	1	0.5	0.5	0	0	0	0	0
6	40	0.99514	0.96274	1	0.5	0.5	0	0	0.0625	0.25	0.25

Columns

Where?

https://egrav.icc.ub.edu/

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- ICCUB-0001
- ICCUB-0051
- ICCUB-0092
- ICCUB-0108

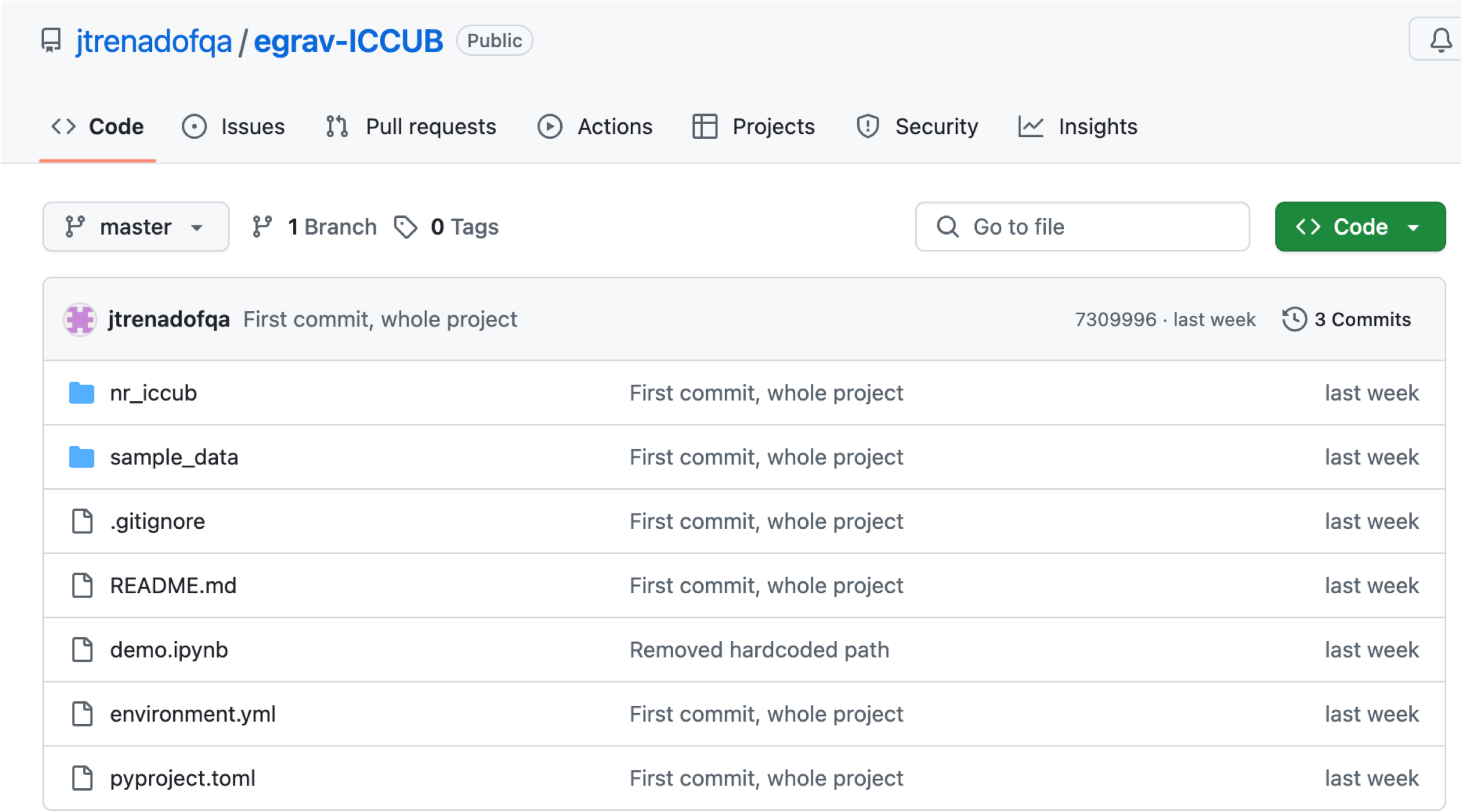
- 0001_metadata.json
- 0001_parameter.par
- 0001_wf.h5

ID	D	M_i	J_i	q	M1_i	M2_i	s1x	s1y	s1z	chi1z	chi2z
1	40	0.99515	1.00456	1	0.5	0.5	0	0	0.125	0.5	0.5
2	40	0.99515	1.02953	1	0.5	0.5	0	0	0.125	0.5	0.5
3	40	0.99515	1.05355	1	0.5	0.5	0	0	0.125	0.5	0.5
4	40	0.99515	1.07659	1	0.5	0.5	0	0	0.125	0.5	0.5
5	40	0.99509	0.82659	1	0.5	0.5	0	0	0	0	0
6	40	0.99514	0.96274	1	0.5	0.5	0	0	0.0625	0.25	0.25

Columns

Where?

https://github.com/jtrenadofqa/egrav-ICCUB



Further directions

- Unequal mass binaries
- Unequal spin
- Increase the number of higher modes
- **Machine learning techniques**

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Gràcies!

Ana Climent

ICC - University of Barcelona