

Dense Matter Equation of State from Astrophysical Observations

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arXiv: 2307.12364, 2502.04211, 2510.00997

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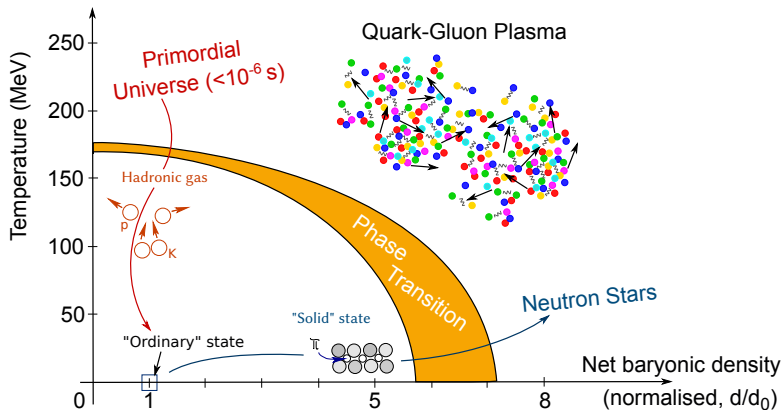
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Outline

- A very brief introduction to neutron stars (NSs)
- Description of nuclear matter
- Models specific to our work and the constraints used
- Results
- Summary

QCD Phase Diagram



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Neutron stars

- Dead stars, produced from the gravitational collapse of a massive star after a supernova
- Mass $\sim 1 - 3M_{\odot}$
- $R \sim 10 - 15$ km
- Density $\sim 10^{15}$ g cm $^{-3}$
- Period ~ 1 ms $- 10$ s
- Magnetic field $B_{\text{surface}} \sim 10^8 - 10^{15}$ G
- ~ 3000 NS discovered so far

Structure of a Neutron Star

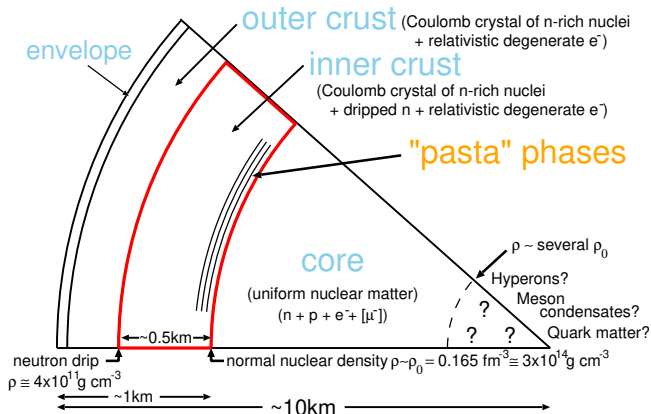


Figure: Schematic picture of a NS Interior

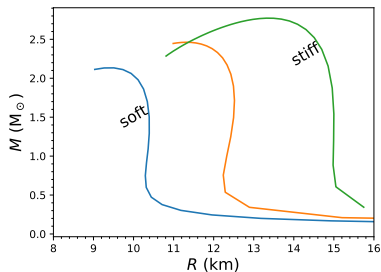
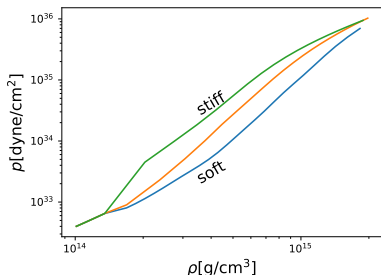
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Equation of State and Mass-Radius

$$P = P(\rho)$$



$$M = M(R)$$



One to one correspondence between Equation of State (EOS) and mass-radius

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Tidally Deformed Stars

- If a static spherically symmetric star of mass M and radius R is placed in a time-independent external tidal field \mathcal{E}_{ij} , a quadrupole moment Q_{ij} is induced onto the star and to linear order

$$Q_{ij} = -\lambda \mathcal{E}_{ij},$$

- Tidal deformation parameter λ related to the $l = 2$ dimensionless Love number k_2

$$\lambda = \frac{2}{3} k_2 R^5.$$

- Observational parameter in LIGO-Virgo, ET: $\Lambda = \lambda/M^5$

NS Observations that an EOS must satisfy

- Precise mass-measurement of massive NSs:
 - (1.908 ± 0.016) M_{\odot} Arzoumanian et al, ApJS 235, 37 (2018).
 - (2.01 ± 0.04) M_{\odot} Antoniadis et al, Science 340, 448 (2013).
 - (2.08 ± 0.07) M_{\odot} E. Fonseca et al, ApJL 915 L12 (2021).
- BNS merger event GW170817 provides bounds on tidal deformability (Λ), and pressure at $2\rho_0$; Abbott et al, PRL 121, 161101 (2018):
 - $\Lambda_{1.4} = 190_{-120}^{+390} \Rightarrow \Lambda_{1.4} \leq 580, P(2\rho_0) = 3.5_{-1.7}^{+2.7} \times 10^{34} \text{ dyn/cm}^2$
- NICER collaboration provided:
 - 1) Simultaneous mass-radius measurements of PSR J0030+0451
 - $M = 1.34_{-0.16}^{+0.15} M_{\odot}, R = 12.71_{-1.19}^{+1.14} \text{ km}$ Riley et al, ApJL, 887, L21 (2019).
 - $M = 1.44_{-0.14}^{+0.15} M_{\odot}, R = 13.02_{-1.06}^{+1.24} \text{ km}$ Miller et al, ApJL, 887, L24 (2019).
 - 2) Radius measurements of J0740+6620
 - $R = 12.39_{-0.98}^{+1.30} \text{ km}$ Riley et al, ApJL, 918, L27 (2021).
 - $R = 13.7_{-1.5}^{+2.6} \text{ km}$ Miller et al, ApJL, 918, L28 (2021).
 - 3) Radius measurement of PSR J0437-4715
 - $R = 11.36_{-0.63}^{+0.95} \text{ km}$ Choudhury et al, ApJL, 971, L20 (2024).

Description of Nuclear Matter:

pressure as a function of energy

energy per particle of nuclear matter

$$P(\rho, \delta) = \rho^2 \frac{d}{d\rho} (E(\rho, \delta))$$

symmetric nuclear matter

symmetry energy

$$E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}\delta^2$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\chi^2 + \frac{Q_0}{6}\chi^3$$

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L_{\text{sym}}\chi + \frac{K_{\text{sym}}}{2}\chi^2 + \frac{Q_{\text{sym}}}{6}\chi^3$$

$$\delta = \text{“Isospin asymmetry”} = (\rho_n - \rho_p)/\rho, \quad \chi = (\rho - \rho_0)/3\rho_0$$

J. Margueron, R. Hoffmann Casali, and F. Gulminelli, Phys. Rev. C 97, 025805 (2018)

Approaches to EOS of Dense matter:

- Many-body description of matter
 - ▶ Ab-initio microscopic methods
 - ▶ Brueckner-Hartree-Fock
 - ▶ Self-consistent Green's function
 - ▶ Approaches based on the variational method
 - ▶ Chiral effective field theory
 - ▶ Quantum Monte Carlo methods
 - ▶ Perturbative QCD
 - ▶ Phenomenological approaches
 - ▶ Non-relativistic Skyrme-Interaction
 - ▶ Relativistic Mean Field (RMF) models
- Agnostic approaches
 - ▶ Piecewise polytropes
 - ▶ Spectral method
 - ▶ Nonparametric Gaussian processes

Chatziioannou et al. [arXiv:2407.11153](https://arxiv.org/abs/2407.11153); Burgio et al. *Prog. Part. Nucl. Phys.* 120 (2021) 103879; Oertel et al. *RMP* 89, 015007 (2017); Sun et al. *Phys. Rev. C* 109, 055801 (2024)

RMF model:

- Interaction between baryons is described via exchange of mesons.
- The most general form of the interaction Lagrangian density:

$$\begin{aligned}\mathcal{L}_{\text{DD}} &= \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \Gamma_\sigma(\rho)\sigma\bar{\psi}\psi - \Gamma_\omega(\rho)\bar{\psi}\gamma^\mu\omega_\mu\psi \\ &- \frac{\Gamma_\rho(\rho)}{2}\bar{\psi}\gamma^\mu\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau}\psi + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu,\end{aligned}$$

σ , ω_μ , and $\boldsymbol{\rho}_\mu$ are meson fields.

- For the density dependent (DD) models, the coupling parameters Γ_σ , Γ_ω , and Γ_ρ are density dependent and do not have nonlinear terms.

Our main objective: Exploring the parameter space to understand the uncertainties.

Choice of density dependence:

- TW:

$$\Gamma_i(\rho) = \Gamma_i(n_{sat})f_i(x), \quad \text{with}$$
$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2},$$

for $i = \sigma, \omega$, and

$$\Gamma_\rho(\rho) = \Gamma_\rho(n_{sat})e^{-a(x-1)}, \quad \text{with } x = n/n_{sat},$$

with conditions: $f_i(1) = 1$, $f_i''(0) = 0$ and $f_\sigma''(1) = f_\omega''(1)$.

S. Typel and H. H. Wolter, Nucl. Phys. A656, 331 (1999)

- GDFM:

$$\Gamma_i(\rho) = a_i + (b_i + d_i x^3)e^{-c_i x},$$

for $i = \sigma, \omega, \rho$, and $x = n/n_0$.

P. Gogelein, E. N. E. van Dalen, C. Fuchs, and H. Muther, Phys. Rev. C 77, 025802 (2008)

Saturation properties of nuclear matter:

Parameter sets are obtained by exploring the uncertainties of the saturation properties of nuclear matter:

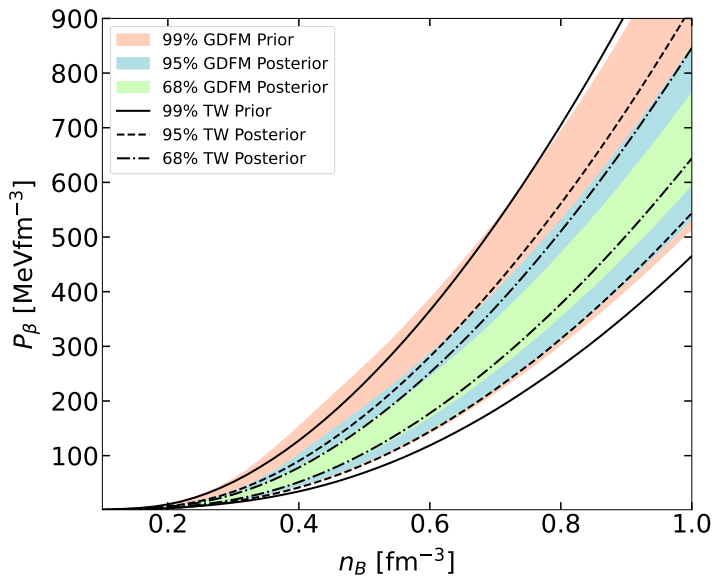
- Saturation density: $\rho_{sat} = (0.14, 0.17) \text{ fm}^{-3}$
- Binding energy per nucleon: $E_{sat} = (-17, -14) \text{ MeV}$.
- Incompressibility: $K_{sat} = (150, 350) \text{ MeV}$.
- Symmetry energy: $E_{sym} = (20, 45) \text{ MeV}$.
- Symmetry energy slope : $L_{sym} = (20, 180) \text{ MeV}$.

Additionally, we use the constraints coming from chiral EFT calculations from [Huth et al, Phys Rev C 103, 025803 \(2021\)](#)

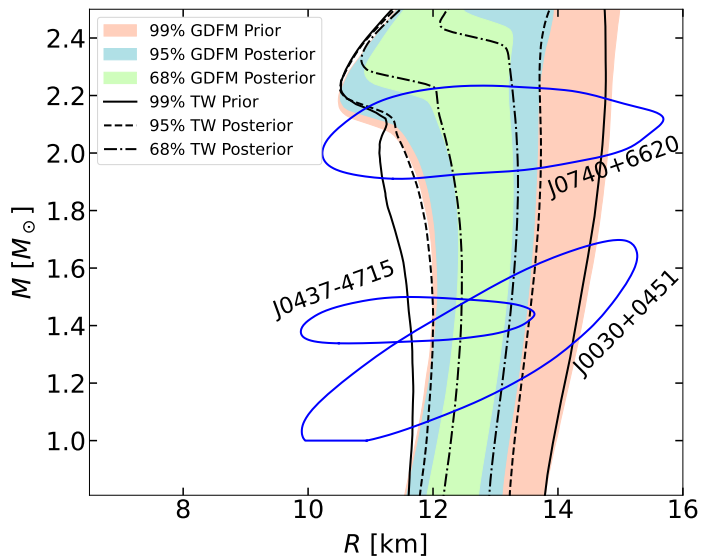
Results: Unified EOS

- High density EOS is constructed for a set of model parameters corresponding to a unique set of nuclear matter parameters
- Low density EOS is calculated within the compressible liquid drop model (CLDM) model for the aforementioned set of nuclear matter parameters.
- β -equilibrium is applied over the whole range.
- The crust and the core are matched with the continuity of pressure and chemical potential.

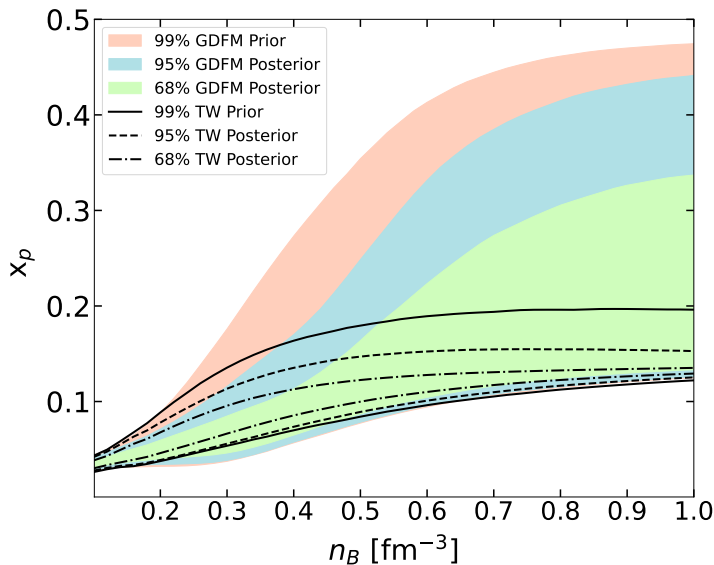
Results: EOS at β -equilibrium



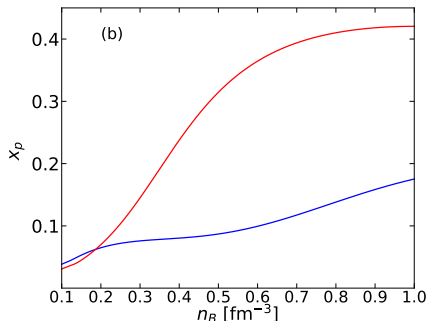
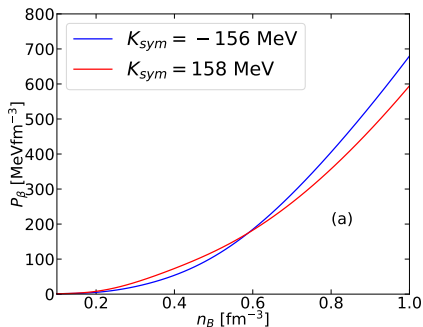
Results: Mass - Radius



Results: Proton fraction

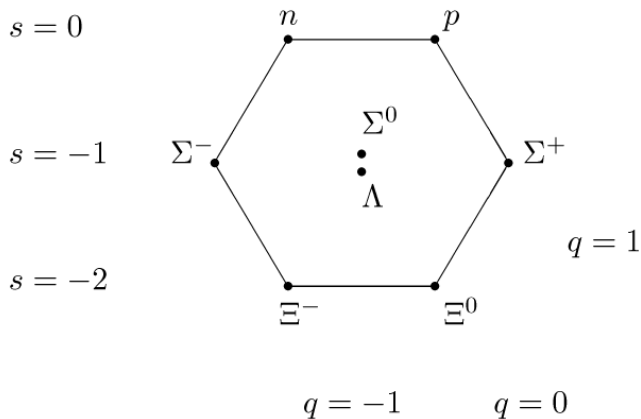


Results:



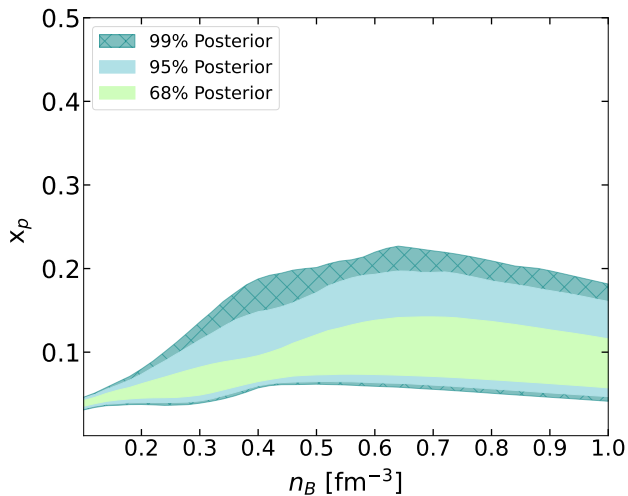
Pressure (panel a) and proton fraction (panel b) as a function of baryon density n_B for two example EOS models with negative and positive K_{sym} , respectively.

Hyperons

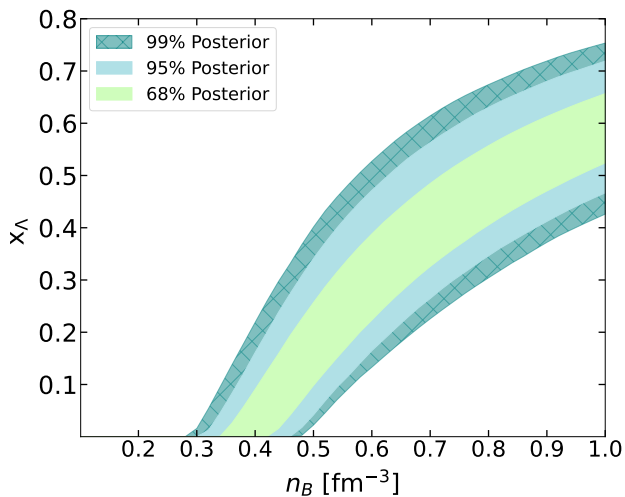


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Results: Including Λ hyperons



Results: Including Λ hyperons



Results: Including Λ hyperons

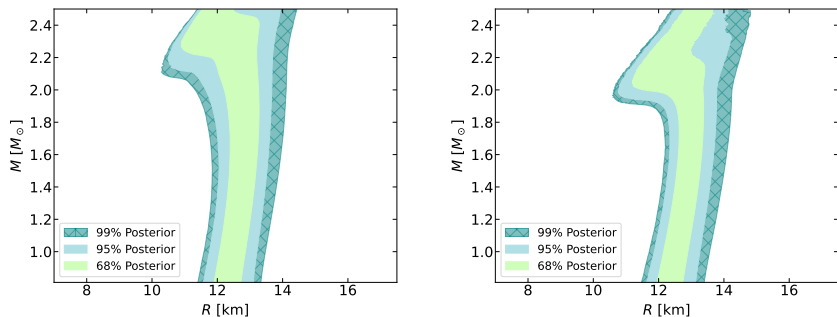


Figure: Mass-radius diagram for Nucleon-only (left) and Hyperon (right) matter, respectively.

Summary:

- Any study of dense matter EOS is heavily model dependent. Therefore, a metamodelling approach to dense matter is very helpful to refine our knowledge.
- Within the GDFM type density-dependent RMF model, a wide range of EOSs can be generated with diverse nuclear matter properties that will be able to satisfy present observational constraints.
- One key finding is the large variation of proton fraction within this model. Future observations of Cooling of Neutron Stars will be very useful to distinguish among models.
- Ongoing project: Search for hyperon signatures in future observations.

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Thank You